

Supplementary Material for “Spectral Transfer, Interaction, and No-Leakage for Thin Yang–Mills Observable Representations”

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This supplementary material records the non-load-bearing transport-groupoid and operational mass-realization outlook, detailed branch and obligation matrices, the notation and normalization ledger, supersession rules, the final scope and circularity firewall, and the machine-readable interaction certificate. Every theorem and proof required for the spectral-transfer, no-leakage, mass-threshold, and Haag-type non-equivalence conclusions remains in the main article.

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I. TRANSPORT-GROUPOID INTERPRETATION AND CONJECTURAL OUTLOOK

A. Logical firewall

This chapter is entirely non-load-bearing. None of its conjectures is used in the construction of the thin observable net, the Poincaré implementation, the proof of the forward-cone spectrum condition, the Glue–Mosco–Riesz theorem, the Physical Hyperbolic Identification and Ground-Triviality theorem, the exact zero-mode theorem, or the branchwise mass-gap theorem. The rigorous conclusion is already contained in the unified final theorem of the main article.

B. The natural global object is a transport groupoid

Let $\mathbf{Bkg}_{\text{YM}}$ denote the admissible background groupoid whose objects contain spacetime, incision, gauge bundle, branch, chart, collar, and scale data,

$$\mathfrak{b} = (M, \Sigma, \mathcal{P}, \xi, \mathfrak{c}, \dots),$$

and whose arrows include admissible spacetime transformations, gauge transformations, Cauchy transports, chart refinements, corridor transitions, scale-bordism maps, and coefficient-line or correspondence seams. The natural global Yang–Mills object is an assignment

$$\mathfrak{b} \longmapsto (\mathcal{A}_{\mathfrak{b}}, \omega_{\mathfrak{b}}, \mathcal{H}_{\mathfrak{b}}, \pi_{\mathfrak{b}}, U_{\mathfrak{b}})$$

with coherent transport along the arrows.

Conjecture I.1 (Transport-groupoid ontology). The natural nonperturbative state of Yang–Mills theory is a coherent state section over an admissible transport groupoid rather than a single globally preferred vector in one fixed Hilbert space. The ordinary vacuum representation is the global observable shadow of this higher transport object.

C. Gluons as Hilbert-gerbe sections

In scalar Sector III the represented system consists of Hilbert spaces $\mathcal{H}_{\alpha, \mathfrak{c}}$, coefficient lines $L_{\alpha\beta}^{\mathfrak{c}}$, and seam maps

$$V_{\alpha\beta}^{\mathfrak{c}} : L_{\alpha\beta}^{\mathfrak{c}} \otimes \mathcal{H}_{\beta, \mathfrak{c}} \longrightarrow \mathcal{H}_{\alpha, \mathfrak{c}}$$

with scalar-twisted triple-overlap law

$$V_{\alpha\beta}^c(1 \otimes V_{\beta\gamma}^c) = \Omega_{\alpha\beta\gamma}^c V_{\alpha\gamma}^c.$$

Conjecture I.2 (Gerbe-sectional gluons). A nonperturbative colored gluonic excitation is naturally a local or sectional object of the represented Yang–Mills Hilbert gerbe. Globally descended physical vectors arise only after local color, dressing, and transport data combine into a descent-compatible neutral composite.

Conjecture I.3 (No pointlike infrared color). There is no compactly localized, gauge-invariant, pointlike infrared one-gluon state in the global observable Hilbert space. Every nontrivial infrared color excitation requires noncompact dressing, cone or corridor localization, boundary attachment, or gerbe-sectional realization.

D. Scattering as dynamical stackification

The stack need not be a passive background container. It may be generated by local excitations and their admissible compositions. Local scattering maps

$$\mathcal{S}_\alpha : \mathfrak{H}_{\text{in},\alpha} \longrightarrow \mathfrak{H}_{\text{out},\alpha}$$

should satisfy

$$V_{\alpha\beta}^{\text{out}}(1 \otimes \mathcal{S}_\beta) = \mathcal{S}_\alpha V_{\alpha\beta}^{\text{in}}.$$

Conjecture I.4 (Dynamical stackification by scattering). The physical Yang–Mills stack is the coherence completion of local gluonic excitation, transport, and scattering data. Local excitations define objects; scattering and transport define morphisms; comparisons among multi-channel factorizations define two-morphisms; and residual associativity defects define the cocycle data required for stackification.

Conjecture I.5 (Descent-compatible scattering). The exact asymptotic scattering theory of a nonperturbative Yang–Mills branch is a transformation between asymptotic Hilbert gerbes. In Sector I it strictifies to an ordinary global scattering operator whenever the usual asymptotic hypotheses hold. In Sector III it remains a coefficient-line-compatible family whose compositions retain the scalar associator.

Conjecture I.6 (Infrared cancellation as descent). An infrared-safe observable is an asymptotic functional that descends through the relevant soft-transport groupoid. Virtual corrections, unresolved radiation, and dressing terms are not separately descended objects; their inclusive combination is finite because it is the global shadow of one coherent sectional process.

E. Hadronization, confinement, and effective strings

Conjecture I.7 (Hadronization as descent completion). Hadronization is the process by which locally defined colored sections become globally admissible neutral objects. A hadron or glueball is a minimal stable global section obtained by completing the dressing, transport, and coherence data required for physical descent.

Conjecture I.8 (Space-filling confinement). The microscopic support of confining Yang–Mills transport is a percolating or effectively space-filling network of local obstruction data. The observed flux tube is a mesoscopic compression of this network rather than a fundamental one-dimensional object.

gerbe transport network $\longrightarrow$ space-filling confining structure
 $\longrightarrow$ coarse flux tube $\longrightarrow$ effective worldsheet.

Conjecture I.9 (Effective string as a secondary shadow). A confining string is the long-distance worldsheet shadow of a descent-compatible gluonic transport network. Its worldsheet degrees of freedom are collective coordinates of a coarse flux tube, which is itself an emergent compression of a more distributed microscopic transport obstruction.

Strings are not fundamental; coherence is fundamental.

F. Critical spectral-stack emergence in three spatial dimensions

A companion spectral-action analysis derives, under its declared regularity and emergence hypotheses,

$$(d, s, \lambda) = (d, 1, d - 2),$$

for first-order Dirac matter coupled to its own second-order static gauge mediator. The critical diagonal $s = \lambda$ is attained only at $d = 3$. This exponent-level result is not used in the mass-gap proof.

Conjecture I.10 (Critical spectral-stack emergence). A stable relativistic stack of self-generated matter and gauge excitations is naturally realized at the critical spectral balance $s = \lambda$. For first-order Dirac matter whose gauge mediator is generated by a second-order spectral-action sector, this balance selects three spatial dimensions.

G. Stacky duality and the landscape as an atlas

Conjecture I.11 (Stacky sectional duality). The familiar S -, T -, and U -dualities are equivalences between higher stacks of local physical presentations rather than pointwise identifications of fundamental objects. They act on local sections, coefficient lines, defects, scattering morphisms, and boundary data, and their compositions close only up to coherent natural transformations.

Conjecture I.12 (Spectrum-preserving coherence permutation). A duality may preserve the global observable spectrum while permuting represented coefficient lines, boundary characters, associator classes, and defect-fusion data. Equality of mass spectra does not imply equality of represented transport structures.

Conjecture I.13 (Landscape-as-atlas). A substantial portion of the apparent theory landscape is an atlas of local presentations, sectors, phases, compactifications, or descent realizations of one higher transport stack. Dualities provide transition equivalences; anomalies and obstruction classes provide higher overlap data.

H. A topological theory controlling gluing bordisms

Conjecture I.14 (Gluing-bordism topological field theory). There exists an extended topological field theory $\mathfrak{T}_{\text{glue}}$ whose bordism assignments encode admissible coefficient lines, center sectors, associators, boundary characters, defect junctions, and transport compositions. The physical Yang–Mills theory is a module, boundary condition, or dynamical fibration over $\mathfrak{T}_{\text{glue}}$.

Possible low-energy models may resemble a center-valued BF theory, a finite Dijkgraaf–Witten theory, a higher-form symmetry TFT, or a correspondence-valued extended theory. No such identification is proved here.

Conjecture I.15 (Boundary inflow for incision data). The retained collar-current and boundary-character data are governed by an induced lower-dimensional topological theory on or near Σ , with a bulk-boundary inflow relation

$$\delta S_{\text{bulk}} + \delta S_{\Sigma} = 0.$$

I. Strict-versus-twisted observables

Conjecture I.16 (Twisted-sector interferometry). There exist linked extended-operator configurations for which the normalized ratio between twisted and strict expectation values detects the scalar associator class,

$$\frac{\langle W_1 W_2 W_3 \rangle_{\text{III}}}{\langle W_1 W_2 W_3 \rangle_{\text{I}}} = \exp(2\pi i \langle [\Omega], [C] \rangle),$$

for a suitable linked compositional cycle $[C]$.

Conjecture I.17 (Higher Berry two-holonomy). Adiabatic transport of an incision, defect, background, or scale corridor around a nontrivial two-parameter cycle produces a measurable two-holonomy whose local Čech representative is the associator class retained by the scalar Sector-III Hilbert gerbe.

J. The mass gap as a coherence threshold

The rigorous theorem gives

$$\text{supp } E \subseteq \{0\} \cup \{p \in \overline{V}_+ : p^2 \geq \kappa_*\}, \quad E(\{0\}) = E_{M^2}(\{0\}) = P_{\Omega},$$

and

$$E_{M^2}((0, \kappa_*)) = 0.$$

Conjecture I.18 (Mass gap as a coherence threshold). The Yang–Mills mass gap is the minimum invariant spectral cost required to deform the complete-ground transport section into a nontrivial globally admissible physical section. The vacuum transport object is isolated from every nontrivial descended excitation by a positive coherence threshold.

The mass gap is the price of becoming globally physical.

Conjecture I.19 (Gap-controlled stack rigidity). The inverse mass gap determines the characteristic length scale beyond which the physical transport prestack behaves rigidly for gauge-invariant observables. Below this scale local dressing and defect data retain substantial sectional dependence; above it the observable content organizes into stable descended neutral composites.

Conjecture I.20 (Gap–confinement coherence relation). The mass gap measures the minimum local spectral cost of nontrivial transport deformation, while the confining string tension measures the accumulated spatial cost of maintaining a nontrivial color-transport obstruction over macroscopic separation. They are distinct observables of one coercive transport geometry.

Conjecture I.21 (Twist as sector locking above a common gap). Strict and scalar-twisted theories may share the same certified mass threshold while differing in multiplicities, extended-operator selection rules, scattering composition, finite-volume splitting, and higher interference phases. The twist organizes the spectrum above the gap rather than necessarily generating the numerical gap itself.

K. Operational mass realization and the meaning of $E = mc^2$

The invariant mass is the Poincaré Casimir,

$$M^2 = P_\mu P^\mu.$$

The familiar relation $E = mc^2$ is a rest-frame statement following from

$$E^2 = |\mathbf{p}|^2 c^2 + m^2 c^4$$

when $\mathbf{p} = 0$. The vacuum has $p = 0$ but is not a material excitation. A nonzero null excitation has energy and momentum but no rest frame. The theorem eliminates every nonzero future-null physical sector, so the first nonvacuum globally physical orbit is timelike.

Conjecture I.22 (Operational Mass-Realization Conjecture).

The Yang–Mills mass gap is the threshold at which a nontrivial quantum transport section first admits a stable, frame-independent classical measurement as a massive object.

The claim is operational, not definitional: M^2 is already a quantum spectral observable. The first nonvacuum descended sector is the first sector admitting a rest frame, proper-time parametrization, invariant rest energy, and stable apparatus correlation as a massive object.

local quantum section $\longrightarrow$ descent-compatible excitation
 $\longrightarrow$ isolated timelike Poincaré orbit
 $\longrightarrow$ stable apparatus correlation
 $\longrightarrow$ classical rest-mass value.

The mass gap is where energy first becomes rest energy.

Mass begins as a Casimir; at the gap, it becomes an object.

L. Criticality stabilized by a dynamical gap

Conjecture I.23 (Criticality stabilized by a dynamical gap). The $3 + 1$ -dimensional Yang–Mills mass gap is the nonperturbative rigidity mechanism stabilizing the critical spectral balance $s = \lambda = 1$. Criticality selects the distinguished dimension; the dynamically generated gap prevents the critical theory from degenerating into a continuum of arbitrarily soft globally descended excitations.

spectral-action emergence $\longrightarrow d = 3$ critical scaling
 $\longrightarrow$ Yang–Mills coercivity
 $\longrightarrow$ positive invariant-mass gap
 $\longrightarrow$ no soft spectral leakage
 $\longrightarrow$ rigid causal and scattering transport
 $\longrightarrow$ stable critical spectral stack.

M. Conjectural synthesis

Local gluonic excitations are sections. Their transports and scatterings generate morphisms. Their competing compositions generate higher coherence data. Stackification produces the

represented physical object. Globally neutral states and observables descend to the ordinary observable shadow. Confinement is the energetic obstruction to global colored descent. Flux tubes are coarse compressions of that obstruction. Effective strings are long-distance worldsheet descriptions. Dualities are equivalences among sectional presentations. The landscape is the atlas of the resulting stack. A topological gluing theory controls admissible bordisms and defect compositions.

The infrared physical content of Yang–Mills theory is encoded not by fundamental pointlike colored particles or fundamental strings, but by descent-compatible sections, scattering morphisms, transport holonomies, and globally neutral composites. Confinement, effective strings, duality, and sector structure are distinct observable shadows of one critical spectral transport stack.

There is no unique fundamental object; there is a globally consistent hierarchy of coherence.

Appendix A: Definitive Branch and Obligation Matrices

1. Branch matrix

Branch	Ordinary observable object	Represented object
Sector I	One global observable vacuum GNS system	One global strict represented
Sector II	Global ordinary system after physical state-section verification	Strict in each corridor, relative
Scalar Sector III	One global observable vacuum GNS system	Chart–corridor coefficient-linear
Corr.	Ordinary or Markov shadow where available	Normal correspondences with

2. SP1–SP12 definitive closure and branch certification

ID	Obligation	Definitive closure
SP1	Spectral restart	Closed for certified scalar branches; declared conditional data retained.
SP2	Covariant vacuum state	Global ordinary invariant state; represented object typed by sector.
SP3	Strong implementation	Global, corridorwise, or chart–corridor as required.
SP4	Generators	Joint PVM, strongly commuting momenta, invariant mass, and common
SP5	Forward cone	Past and spacelike Lorentz-orbit fibers vanish.
SP6	Physical bridge	Glue–Mosco–Riesz plus $P_0^2 \geq K^{\text{thin}}$; orbit rigidity gives the invariant bound
SP7	Exact zero sector	$E(\{0\}) = E_{M^2}(\{0\}) = P_\Omega$; nonzero null support vanishes.
SP8	Covariance and compatibility	All-Borel seam covariance; nonlinear surgery compression; Gauss–Ward g
SP9	Gluing stability	Strict, corridor, coefficient-line, product, quotient, direct sum, and direct
SP10	Threshold transfer	Certified normalized mass-square lower bound κ_* .
SP11	Vacuum multiplicity	Stabilized complete-ground rank and sector-refined zero object.
SP12	Final theorem	Outcome A for fully certified mobile-incision Sector-I and scalar Sector-II

Appendix B: Definitive Notation, Normalization, and Supersession

1. Notation and constants

Symbol	Definitive meaning
$M_{\text{inc}} = \mathbb{R}^{3,1} \setminus \Sigma$	Incised spacetime; never a quotient by Σ .
$\mathcal{A}_{G,\xi,0}^W$	Ordinary thin observable net.
$(\mathcal{H}^{\text{obs}}, \pi^{\text{obs}}, U^{\text{obs}})$	Global ordinary observable vacuum representation.
$E(B)$	Joint translation PVM on momentum space.
P_μ	Strongly commuting translation generators.
$M^2 = P_\mu P^\mu$	Invariant-mass-square operator after the forward-cone theorem.
$\bar{q}_0, K^{\text{thin}}$	Paper-IV stiffness form and associated operator; not physical mass by notation.
P_Ω	Complete-ground projection and exact zero-mass projection.
κ_*	Inherited normalized threshold and certified mass-square lower bound.
$m_{\text{cert}} = \sqrt{\kappa_*}$	Certified normalized mass lower bound.
m_{spec}	Exact lowest surviving nonzero mass shell; only $(m_{\text{spec}})^2 \geq \kappa_*$ is universal.
$L_{\alpha\beta}^c, V_{\alpha\beta}^c$	Sector-III coefficient lines and seam unitaries.
$\Omega_{\alpha\beta\gamma}^c$	Scalar associator retained by the represented theory.
$\mathcal{S}_*$	Generally nonlinear entropy-surgery map on normal states.

2. Dimensional restoration

If the form was normalized using a reference length $\ell_{G,\xi}$, then

$$(m_{G,\xi}^{\text{cert}})^2 = \ell_{G,\xi}^{-2} \hat{\kappa}_*.$$

The certified lower bound is distinguished from the exact lowest surviving shell.

3. Supersession rules

The invariant-state residual of Phase V.0 is superseded by the invariant complete-ground state theorem. The preliminary PHIGT residual is superseded by the physical hyperbolic identification theorem. The primitive candidate inequality $M^2 \geq cK^{\text{thin}}$ is rejected as a general starting theorem;

the authoritative bridge is

$$P_0^2 \geq K^{\text{thin}} \geq \kappa_*(I - P_\Omega),$$

followed by the orbit argument. Norm-resolvent convergence is not used to infer arbitrary sharp physical projections. Sector-III represented data are never collapsed to one global Hilbert space. Surgery is treated as a nonlinear state map, not as an operator commuting with every spectral projection.

Appendix C: Final Scope and Circularity Firewall

1. Mass gap versus energy gap

The final theorem concerns the joint translation spectrum and the invariant Casimir M^2 , not merely a positive lower bound for P_0 in one frame. The forward-cone theorem is independent of stiffness. Lorentz-orbit rigidity upgrades the energy-square estimate to the invariant-mass statement.

2. Vacuum multiplicity

The identity $E(\{0\}) = P_\Omega$ does not imply rank one. Zero-momentum Hilbert multiplicity, multiplicity of Lorentz-fixed vacuum vectors, and the dimension of the normal ground-state face are distinct.

3. Clay-scope statement

The manuscript proves a branch-certified Haag–Kastler existence-and-gap theorem inside the constructed mobile-incision, incision-relative framework. It does not declare an unqualified resolution of the Clay Yang–Mills existence and mass-gap problem⁷. Such a claim requires an external end-to-end verification of the complete series against the exact accepted axiomatic and nontriviality requirements.

4. Conjecture firewall

The transport ontology, sectional gluons, scattering stack, space-filling confinement, emergent strings, stacky dualities, landscape-as-atlas, gluing TFT, and operational mass-realization statements are conjectures only. They introduce no premise into the proof of the unified final theorem in the main article.

Appendix D: Machine-Readable Interaction Certificate

The accompanying files `interaction_wick_defect_certificate.py` and `interaction_wick_defect_certificate` provide the exact symbolic and rational-interval verification used in the interaction chapter of the main article.

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