

Coarse Transport, Central Twist, and Finite-Scale Coercivity for Yang–Mills Wilson–Tube Hamiltonians: Physical Wilson–Channel Reduction and Exact Ultraviolet–Infrared Leakage

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We establish a finite-scale coercivity criterion for canonical gauge-invariant Yang–Mills Wilson-tube Hamiltonians organized by the characters of the finite center. The comparison mechanism is a Hermitian ultraviolet–infrared block whose diagonal certificates encode the ultraviolet and infrared sectors and whose mixed norm is the exact sum of reference-band, localized-transport, and collar leakage. Its lower eigenvalue supplies the sectorwise coercive constant, and the associated positive-semidefinite region yields bounded coercive frusta and a uniform branch theorem relative to the complete-ground projection. A complete low-spin $SU(2)$ Peter–Weyl Galerkin realization provides an independent two-sector validation with nonzero leakage. An exact ten-dimensional $SU(3)$ Galerkin Hamiltonian simultaneously carries the neutral, fundamental, and antifundamental center characters and generates a cofinal positive-thickness family with a common positive floor. Non-surjective refinement inclusions carry exact image comparisons, while every new ultraviolet complement is independently certified. The resulting marked Wilson-tube packet records the gauge-invariant Wilson quotient, meridional class, finite-demand late tails, and source certificate exported to the scale-bordism construction. All results are finite-scale: no spacetime reconstruction, incision, thin limit, Poincaré-covariant theory, or relativistic mass-gap conclusion is asserted.

Keywords: lattice gauge Hamiltonians; Wilson observables; central twist; magnetic graph Laplacian; quadratic-form coercivity; spectral projection; coarse transport

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I. INTRODUCTION

A. The finite-scale spectral problem

For a compact gauge group on a finite spatial complex, the regulated electric Hamiltonian has discrete representation spectrum, the Wilson magnetic potential is bounded, and the complete Hamiltonian has compact resolvent in the standard Hamiltonian lattice-gauge setting^{1,2}. Compact resolvent tells us that eigenvalues exist. It does not explain why separation above the interacting ground sector survives, or where it can fail. The present criterion separates the lower bound into

$$\begin{aligned} &\text{electric exclusion} \\ &+ \text{physicalized magnetic separation} \\ &> \text{reference, transport, and collar leakage.} \end{aligned}$$

The resulting estimate is relative to the complete ground projection,

$$H_n^\varepsilon - E_{0,n}^\varepsilon I \geq \widehat{\kappa}_{n,\varepsilon,\theta}(b)(I - P_{n,\text{vac}}^\varepsilon). \quad (\text{I.1})$$

No normalized vector in the possibly degenerate ground eigenspace is selected.

Remark I.1 (Sequence notation for the complete ground). *The source symbol $P_{n,\text{vac}}^\varepsilon$ always denotes the complete ground spectral projection of the named finite Hamiltonian; the word “vacuum” does not select a preferred vector or assume a one-dimensional eigenspace. At inter-paper interfaces this same semantic datum is recorded as P_{cg} . We retain $P_{n,\text{vac}}^\varepsilon$ in Paper I to preserve the active theorem notation, and every downstream identification must prove that it carries the same projection.*

B. Graph minima and physical vacua

Detected center phases define a finite magnetic graph operator $L_\theta = d_\theta^* d_\theta$. Its complete lowest spectral projection $P_{\theta,\min}$ acts on a coefficient space. By contrast, $P_{n,\text{vac}}^\varepsilon$ acts on the gauge-invariant Hilbert space and is determined by the complete electric-plus-magnetic Hamiltonian. The two projections are not identified.

C. Physicalization of a raw Wilson channel

Let $F_{n,\theta}^W : C_n^0 \rightarrow \mathcal{H}_n^\varepsilon(\Omega)$ be an isometric raw Wilson frame and set

$$E_{n,\theta}^W = \text{Ran}(I - P_{\theta,\min}), \quad T_{n,\theta} = (I - P_{n,\text{vac}}^\varepsilon)F_{n,\theta}^W|_{E_{n,\theta}^W}.$$

After deleting $\text{Ker } T_{n,\theta}$, define

$$C_{n,\theta}^{\text{phys}} = E_{n,\theta}^W \ominus \text{Ker } T_{n,\theta}.$$

A finite spectral projection of the vacuum-orthogonal electric operator captures the image injectively. If J is the captured map and $G = J^*J$, then

$$F_{n,\theta}^{\text{phys}} = JG^{-1/2}$$

is an isometry with $P_{n,\text{vac}}^\varepsilon F_{n,\theta}^{\text{phys}} = 0$ and $P_{n,\text{IR}}^\varepsilon F_{n,\theta}^{\text{phys}} = F_{n,\theta}^{\text{phys}}$.

D. Exact mixed-block decomposition

With an interior-transported-collar partition, the magnetic ultraviolet-infrared block decomposes exactly as

$$P_{n,\text{UV}}^\varepsilon H_{n,\text{mag}}^\varepsilon P_{n,\text{IR}}^\varepsilon = K_{n,\text{ref}}^\varepsilon + K_{n,\text{tr}}^\varepsilon + K_{n,\text{col}}^\varepsilon. \quad (\text{I.2})$$

The transport term measures the difference between two-step localized transport and a direct comparator; it is not the entire mixed block.

E. Principal constant and scope

If $\widehat{c}_{n,\text{UV}}^\varepsilon > 0$, $\widehat{c}_{n,\text{IR}}^\theta(b) > 0$, and $(\widehat{\epsilon}_{n,\text{mix}}^\varepsilon)^2 < \widehat{c}_{n,\text{UV}}^\varepsilon \widehat{c}_{n,\text{IR}}^\theta(b)$, then

$$\widehat{\kappa}_{n,\varepsilon,\theta}(b) = \frac{\widehat{c}_{n,\text{UV}}^\varepsilon + \widehat{c}_{n,\text{IR}}^\theta(b) - \sqrt{(\widehat{c}_{n,\text{UV}}^\varepsilon - \widehat{c}_{n,\text{IR}}^\theta(b))^2 + 4(\widehat{\epsilon}_{n,\text{mix}}^\varepsilon)^2}}{2} > 0. \quad (\text{I.3})$$

The theorem is finite-scale and time-zero. It constructs no continuum measure, Haag-Kastler net, Poincaré-covariant vacuum representation, confinement theorem, or relativistic mass gap.

F. Center sectors, coercive frusta, and the two model realizations

The literal building block of the proof is the sectorwise triple

$$(c_{\chi}^{\text{UV}}, c_{\chi}^{\text{IR}}, \epsilon_{\chi}^{\text{mix}}),$$

not an untyped spectral gap. [Sections V](#) and [XII](#) shows that finite-center characters reduce the complete physical comparison package and that the lower eigenvalue of the associated 2×2 block gives the sectorwise coercive floor. The translated positive-semidefinite region of these triples, intersected with certified endpoint ranges, is the coercive frustum. An admissible branch is a typed scale family whose triples remain inside the same frustum.

The two realizations are not redundant. The inherited complete $j \leq 1$ $SU(2)$ Galerkin packet tests whether the mechanism survives genuine ultraviolet–infrared leakage. The $SU(3)$ construction tests whether the same mechanism survives the three-sector center geometry required downstream, placing the neutral, fundamental, and antifundamental sectors in one ten-dimensional Hamiltonian and computing all three sector gaps exactly. Its graph eigenbasis is a zero-mixing endpoint of the frustum, while the cofinal positive-thickness family restores actual reference, transport, and collar mixing. The resulting profiles are $(3, 3, 2^{-\alpha n}/\sqrt{6})$ in the neutral branch and $(3, 2/5, 2^{-\alpha n}/\sqrt{6})$ in the two conjugate branches. Non-surjective refinement inclusions are controlled on their images, and every new ultraviolet endpoint complement is certified directly.

The point is not merely to exhibit a positive matrix. The point is to identify the finite-scale mechanism that keeps the ultraviolet and infrared channels coercively coupled without allowing the mixing term to erase the gap. The marked Wilson-tube construction at the end of the article records exactly which finite source packet later papers may consume; none of their reconstruction theorems is used here.

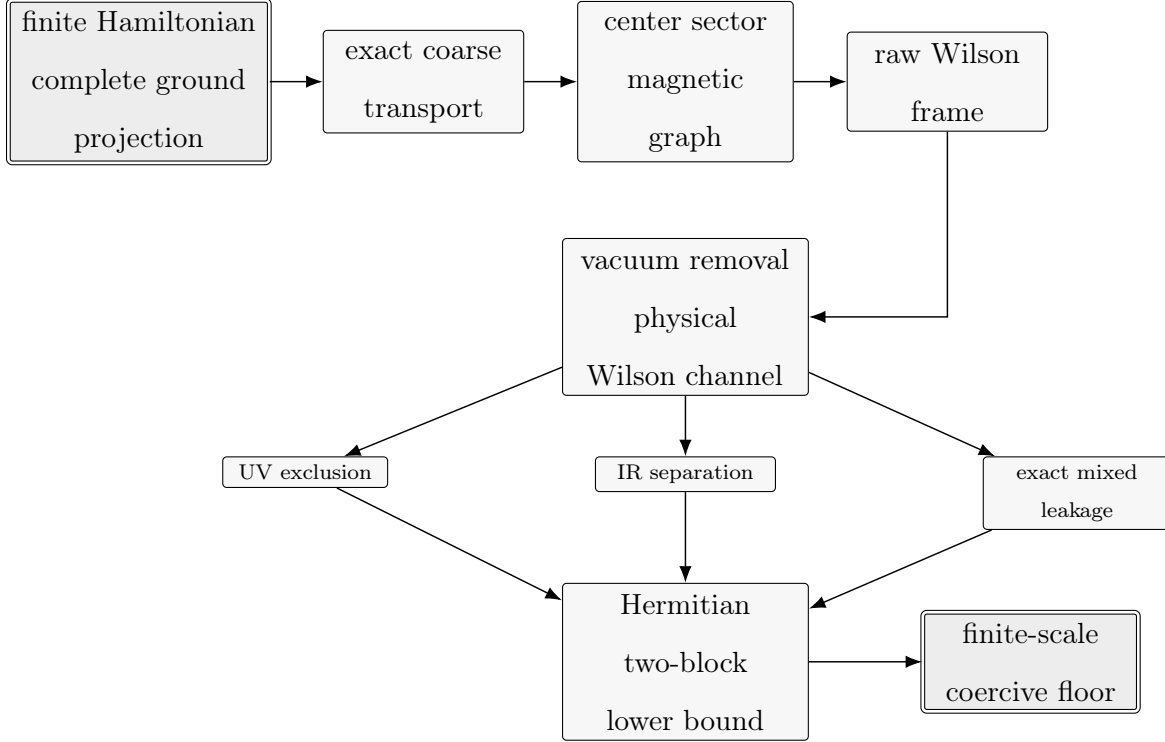


FIG. 1. The finite-scale coercive mechanism. The principal theorem compresses one ordered comparison construction; it is not obtained by adjoining unrelated ultraviolet, infrared, and leakage estimates.

II. CANONICAL FINITE-SCALE WILSON-TUBE SYSTEMS

Reader's guide. The proof is organized in a forced order. We first fix the finite Hamiltonian and its exact coarse-transport maps, then isolate the center-sensitive magnetic data, and only afterward compress to the physical Wilson channel. The ultraviolet and residual blocks are therefore not independent estimates: they are the two diagonal faces of one vacuum-orthogonal comparison problem. The construction culminates in an exact mixed-block estimate and a two-block lower bound from which the finite-scale coercive floor is read.

A. Configuration space and Gauss law

Let $\Omega \subset \mathbb{R}^3$ be bounded and let $\mathcal{L}_n(\Omega)$ be a finite oriented cell complex with positively oriented edge set $E_n^+(\Omega)$, vertex set $V_n(\Omega)$, and magnetic-cell set $F_n(\Omega)$. For a compact

connected Lie group G ,

$$X_n(\Omega) = G^{E_n^+(\Omega)}, \quad \mu_n = \bigotimes_{e \in E_n^+(\Omega)} dg_e.$$

The finite gauge group acts by $(h \cdot U)_e = h_{s(e)} U_e h_{t(e)}^{-1}$. Haar averaging gives the orthogonal Gauss projection Γ_n , and

$$\mathcal{H}_n(\Omega) = \text{Ran } \Gamma_n = L^2(X_n, \mu_n)^{\mathcal{G}_n(\Omega)}. \quad (\text{II.1})$$

B. Electric and magnetic Hamiltonians

Following the Hamiltonian lattice-gauge formulation², for positive weights $\eta_{n,e}^{\text{el}}$ and coupling g_n , define

$$\tilde{H}_{n,\text{el}} = \frac{g_n^2}{2} \sum_{e \in E_n^+(\Omega)} \eta_{n,e}^{\text{el}} \Delta_e. \quad (\text{II.2})$$

It commutes with Γ_n and restricts to the physical electric operator $H_{n,\text{el}}$. For each magnetic cell choose a bounded gauge-invariant self-adjoint potential $V_{n,f}^\varepsilon \geq 0$ and set

$$h_{n,f}^\varepsilon = \omega_{n,f}^{\text{mag}} M_{V_{n,f}^\varepsilon}, \quad H_{n,\text{mag}}^\varepsilon = \sum_{f \in F_n(\Omega)} h_{n,f}^\varepsilon.$$

Then

$$H_n^\varepsilon = H_{n,\text{el}} + H_{n,\text{mag}}^\varepsilon, \quad D(H_n^\varepsilon) = D(H_{n,\text{el}}). \quad (\text{II.3})$$

Theorem II.1 (Finite-scale canonical Hamiltonian). *The operator H_n^ε is nonnegative, self-adjoint, gauge invariant, and has compact resolvent on $\mathcal{H}_n(\Omega)$. Its lowest spectral value $E_{0,n}^\varepsilon$ is an eigenvalue of finite multiplicity, and*

$$P_{n,\text{vac}}^\varepsilon = \mathbf{1}_{\{E_{0,n}^\varepsilon\}}(H_n^\varepsilon)$$

is a finite-rank projection.

Proof. Peter–Weyl theory gives the self-adjoint compact-resolvent electric operator. The magnetic term is bounded and self-adjoint. Bounded perturbation preserves self-adjointness and domain equality, and the compact-resolvent spectral conclusions follow from standard perturbation and spectral theory^{3,4}. $\square$

C. Finite Wilson windows

A finite Wilson window is a finite-dimensional unital involution-invariant subspace $\mathcal{W}_n^\varepsilon(\Omega) \subset \mathfrak{D}_n^{\text{gi}}(\Omega)$. These windows are comparison data and do not modify the canonical Hamiltonian.

III. EXACT COARSE TRANSPORT AND FINITE WILSON LOCALIZATION

A. Haar-compatible microscopic refinement

For $m \preceq n$, let $p_{mn} : X_n(\Omega) \rightarrow X_m(\Omega)$ be a measurable coarse-holonomy map satisfying exact composition, measure preservation, and gauge equivariance. Cylindrical pullback

$$\iota_{mn}A = A \circ p_{mn}$$

is an isometric unital $*$ -monomorphism, and its L^2 adjoint $\mathbb{E}_{mn} = \iota_{mn}^*$ is fiberwise Haar conditional expectation.

Theorem III.1 (Exact microscopic coarse transport). *The maps $\mathbb{E}_{mn}$ are unital, positive, adjoint preserving, and contractive in L^2 and L^∞ . They satisfy*

$$\mathbb{E}_{\ell m} \mathbb{E}_{mn} = \mathbb{E}_{\ell n}.$$

B. Finite Wilson localization and associator leakage

Let L_n^Ω denote orthogonal localization onto the finite Wilson window and J_n^Ω its inclusion. The effective Wilson transport is

$$\Lambda_{mn}^\Omega = L_m^\Omega \mathbb{E}_{mn} J_n^\Omega.$$

For $\ell \preceq m \preceq n$, the failure of strict composition is an intermediate-scale localization error. In the source notation the associator has the exact factorization

$$\alpha_{\ell mn}^\Omega = L_\ell^\Omega \mathbb{E}_{\ell m} \Lambda_m^\Omega \mathbb{E}_{mn} J_n^\Omega. \quad (\text{III.1})$$

This identity is later used only to estimate the transported-cell correction; it does not make physical scale transport invertible.

IV. CENTRAL SECTOR DATA AND DETECTED MAGNETIC GRAPHS

A. Center-valued cocycles

Finite Wilson trivialization data determine a center-valued simplicial 2-cocycle $\zeta_n \in Z^2(N_n; Z(G))$. For a center character $\zeta_R : Z(G) \rightarrow \mathbb{T}$, detection sends cohomologous center cocycles to cohomologous circle-valued cocycles.

Proposition IV.1 (Detection commutes with refinement). *For a refinement map $r_{mn} : N_n \rightarrow N_m$,*

$$\zeta_R(r_{mn}^* \zeta_m) = r_{mn}^* \zeta_R(\zeta_m).$$

Thus center detection commutes with simplicial pullback.

B. Detected graph realization

A detected graph realization consists of a finite connected weighted graph $\mathcal{G}_\theta = (V_\theta, E_\theta, a)$, a transgression rule to a phase cochain θ , and covariance under vertex rephasing. The magnetic graph operator is

$$L_\theta = d_\theta^* d_\theta = \sum_{\{v,w\} \in E_\theta} a_{vw} \ell_{vw}^\theta.$$

We use the usual finite weighted-graph spectral conventions⁵. Let $\lambda_1(\theta)$ be its lowest eigenvalue, $P_{\theta, \min}$ the full lowest spectral projection, $m_1 = \text{rank } P_{\theta, \min}$, and

$$\delta_{\text{sec}}(\theta) = \lambda_{m_1+1}(\theta) - \lambda_1(\theta).$$

Then

$$L_\theta - \lambda_1(\theta)I \geq \delta_{\text{sec}}(\theta)(I - P_{\theta, \min}). \quad (\text{IV.1})$$

The theorem uses separation above the complete graph-minimum space, not the baseline eigenvalue alone.

V. CENTER-CHARACTER REDUCTION OF THE PHYSICAL COMPARISON PACKAGE

A. Finite-center spectral projectors

Let G be compact with finite center $Z = Z(G)$, and let

$$U : Z \longrightarrow \mathcal{U}(\mathcal{H}_n(\Omega))$$

be the unitary center action on the finite gauge-invariant packet. We assume that the canonical Hamiltonian, its closed form, the complete-ground projection, and the physical electric-band projections commute with $U(z)$ for every $z \in Z$. For $\chi \in \widehat{Z}$ define the standard finite-center character projector⁶ by

$$P_{n,\chi} = \frac{1}{|Z|} \sum_{z \in Z} \overline{\chi(z)} U(z). \quad (\text{V.1})$$

The projections $P_{n,\chi}$ are mutually orthogonal and sum to the identity. Hence

$$\mathcal{H}_n(\Omega) = \bigoplus_{\chi \in \widehat{Z}} \mathcal{H}_{n,\chi}, \quad \mathcal{H}_{n,\chi} = P_{n,\chi} \mathcal{H}_n(\Omega). \quad (\text{V.2})$$

Proposition V.1 (Center-character reduction). *The operators H_n^ε , $P_{n,\text{vac}}^\varepsilon$, $P_{n,\text{UV}}^\varepsilon$, $P_{n,\text{IR}}^\varepsilon$, and K_n^ε reduce the decomposition (V.2). If*

$$P_{n,\text{vac},\chi}^\varepsilon = P_{n,\chi} P_{n,\text{vac}}^\varepsilon,$$

then

$$P_{n,\text{vac}}^\varepsilon = \bigoplus_{\chi \in \widehat{Z}} P_{n,\text{vac},\chi}^\varepsilon, \quad \widehat{Q}_n^\varepsilon = \bigoplus_{\chi \in \widehat{Z}} \widehat{Q}_{n,\chi}^\varepsilon.$$

In particular, each center character carries its own ultraviolet, infrared, and mixed comparison data.

Proof. The character projectors are polynomials in the commuting family $U(z)$. Every operator commuting with $U(z)$ therefore commutes with every $P_{n,\chi}$. Orthogonality and the Fourier inversion identity on the finite abelian group Z give the direct sums. $\square$

B. Sectorwise ultraviolet–infrared constants

Write

$$\mathcal{H}_{n,\chi}^{\text{UV}} = P_{n,\chi} \text{Ran } P_{n,\text{UV}}^\varepsilon, \quad \mathcal{H}_{n,\chi}^{\text{IR}} = P_{n,\chi} \text{Ran } P_{n,\text{IR}}^\varepsilon.$$

On the complete-ground complement inside the χ sector define the optimal diagonal constants

$$c_{n,\chi}^{\text{UV}} = \inf_{0 \neq u \in \mathcal{H}_{n,\chi}^{\text{UV}}} \frac{\widehat{Q}_{n,\chi}^\varepsilon[u]}{\|u\|^2}, \quad (\text{V.3})$$

$$c_{n,\chi}^{\text{IR}} = \inf_{0 \neq w \in \mathcal{H}_{n,\chi}^{\text{IR}} \cap \text{Ran}(I - P_{n,\text{vac},\chi}^\varepsilon)} \frac{\widehat{Q}_{n,\chi}^\varepsilon[w]}{\|w\|^2}, \quad (\text{V.4})$$

with the usual convention that an empty infimum is $+\infty$. The exact mixed norm is

$$\epsilon_{n,\chi}^{\text{mix}} = \|P_{n,\chi} K_n^\varepsilon P_{n,\chi}\|. \quad (\text{V.5})$$

Thus the literal finite-scale comparison datum is the triple

$$\mathfrak{c}_{n,\chi}^\varepsilon = (c_{n,\chi}^{\text{UV}}, c_{n,\chi}^{\text{IR}}, \epsilon_{n,\chi}^{\text{mix}}). \quad (\text{V.6})$$

Remark V.2 (Neutral-ground specialization). *The abstract reduction does not force the complete ground to be neutral. In the source-free $SU(2)$ and $SU(3)$ realizations below, the complete ground is independently identified in the neutral character sector. Only after that model-specific identification may one replace $P_{n,\text{vac},\chi}^\varepsilon$ by zero in the nonneutral sectors.*

VI. RAW WILSON CHANNELS AND MAGNETIC DOMINATION

A. Raw frame

Let $F_{n,\theta}^W : C_n^0 \rightarrow \mathcal{H}_n^\varepsilon(\Omega)$ be an isometric frame of gauge-invariant Wilson observables. For each canonical cell define the raw compression

$$M_{n,f}^W = (F_{n,\theta}^W)^* h_{n,f}^\varepsilon F_{n,\theta}^W \geq 0.$$

B. Disjoint positive edge extraction

Choose an active graph-edge set and a disjoint allocation of canonical magnetic cells. For each active edge define the largest coefficient for which the cell compression dominates the associated edge Laplacian. Summing over the disjoint allocation gives a global raw coefficient $b_{n,\text{ch}}^{W,\text{max}}$.

Proposition VI.1 (Raw magnetic domination). *If every required graph edge is active and $b_{n,\text{ch}}^{W,\text{max}} > 0$, then for every $0 \leq b \leq b_{n,\text{ch}}^{W,\text{max}}$,*

$$\sum_{f \in F_n(\Omega)} M_{n,f}^W \geq bL_\theta.$$

Scope Note VI.2. *Positive extraction on the raw frame does not imply positive extraction after exact vacuum removal, finite electric capture, and Gram normalization. The canonical cell operators must be recompressed on the physicalized frame.*

VII. VACUUM-ORTHOGONAL ELECTRIC SPECTRAL REDUCTION

A. Restricted electric form

Set

$$\mathcal{V}_{n,\text{vac}}^\varepsilon = \text{Ran } P_{n,\text{vac}}^\varepsilon, \quad \mathcal{H}_{n,\perp}^\varepsilon = \text{Ran}(I - P_{n,\text{vac}}^\varepsilon).$$

The vacuum projection need not commute with $H_{n,\text{el}}$. Define instead

$$D(Q_{n,\text{el}}^\perp) = D(Q_{n,\text{el}}) \cap \mathcal{H}_{n,\perp}^\varepsilon, \quad Q_{n,\text{el}}^\perp[\psi] = Q_{n,\text{el}}[\psi].$$

Lemma VII.1 (Form-domain preservation). *The finite-rank projection $P_{n,\text{vac}}^\varepsilon$ maps $D(Q_{n,\text{el}})$ into itself.*

Theorem VII.2 (Vacuum-orthogonal electric operator). *The restricted form $Q_{n,\text{el}}^\perp$ is densely defined, closed, and nonnegative on $\mathcal{H}_{n,\perp}^\varepsilon$. Its associated operator $H_{n,\text{el}}^\perp$ is nonnegative, self-adjoint, and has compact resolvent.*

B. Finite electric capture

Let $\Pi_{n,\Lambda}^\perp = \mathbf{1}_{[0,\Lambda]}(H_{n,\text{el}}^\perp)$. Because the vacuum-removed raw map has finite-dimensional injective domain after kernel deletion, a sufficiently large finite cutoff captures it injectively. The captured map $J_{n,\theta}^\varepsilon$ has positive Gram operator

$$G_{n,\theta}^\varepsilon = (J_{n,\theta}^\varepsilon)^* J_{n,\theta}^\varepsilon > 0,$$

and the normalized frame is $F_{n,\theta}^{\text{phys}} = J_{n,\theta}^\varepsilon (G_{n,\theta}^\varepsilon)^{-1/2}$.

C. Ultraviolet exclusion

The retained electric projection is $P_{n,\text{IR}}^\varepsilon = \Pi_{n,\Lambda}^\perp$ on $\mathcal{H}_{n,\perp}^\varepsilon$, extended by zero on the vacuum space, and

$$P_{n,\text{UV}}^\varepsilon = I - P_{n,\text{vac}}^\varepsilon - P_{n,\text{IR}}^\varepsilon.$$

The spectral separation above the retained electric band yields a vacuum-renormalized constant $\tilde{c}_{n,\text{UV}}^\varepsilon > 0$ under the explicit source hypotheses.

VIII. PHYSICAL WILSON-CHANNEL REDUCTION

A. Surviving coefficient space and Gram normalization

The surviving coefficient space is

$$C_{n,\theta}^{\text{phys}} = \text{Ran}(I - P_{\theta,\text{min}}) \ominus \text{Ker}((I - P_{n,\text{vac}}^\varepsilon)F_{n,\theta}^W).$$

The physical frame $F_{n,\theta}^{\text{phys}}$ is isometric, vacuum orthogonal, and contained in the retained electric block.

B. Physicalized graph operator

Let $\mathbf{P}_{n,\theta}^{\text{phys}}$ denote the coefficient projection. Define

$$A_{n,\theta}^{\text{raw}} = \mathbf{P}_{n,\theta}^{\text{phys}}(L_\theta - \lambda_1(\theta)I)\mathbf{P}_{n,\theta}^{\text{phys}},$$

and

$$\tilde{A}_{n,\theta}^\varepsilon = (G_{n,\theta}^\varepsilon)^{-1/2} A_{n,\theta}^{\text{raw}} (G_{n,\theta}^\varepsilon)^{-1/2} \geq \delta_{\text{sec}}(\theta) (G_{n,\theta}^\varepsilon)^{-1}. \quad (\text{VIII.1})$$

C. Physical magnetic recompression

For each canonical cell recompute

$$M_{n,f}^{\text{phys}} = (F_{n,\theta}^{\text{phys}})^* h_{n,f}^\varepsilon F_{n,\theta}^{\text{phys}} \geq 0.$$

A disjoint physical allocation gives a positive extraction coefficient when the corresponding finite matrix inequalities hold.

D. Channel coercivity

The exact finite-dimensional form identity has the source form

$$\widehat{H}_{n,\text{ch}}^{\text{phys}} = b\widetilde{A}_{n,\theta}^\varepsilon + R_{n,\text{sec}}^{\text{phys}}(b).$$

Set

$$r_{n,\text{sec}}^{\text{phys}}(b) = \|[R_{n,\text{sec}}^{\text{phys}}(b)]_-\|, \quad c_{n,\text{ch}}^{\text{phys}}(b) = b\delta_{\text{sec}}(\theta) - r_{n,\text{sec}}^{\text{phys}}(b).$$

Theorem VIII.1 (Physical Wilson-channel coercivity). *If $c_{n,\text{ch}}^{\text{phys}}(b) > 0$, then for every $s \in \text{Ran } F_{n,\theta}^{\text{phys}}$,*

$$\widehat{Q}_n^\varepsilon[s] \geq c_{n,\text{ch}}^{\text{phys}}(b)\|s\|^2.$$

E. Residual infrared sector

Decompose

$$\text{Ran } P_{n,\text{IR}}^\varepsilon = \mathcal{S}_{n,\theta}^{\text{phys}} \oplus \mathcal{R}_{n,\theta}^{\text{IR}}.$$

The residual diagonal constant and channel–residual mixing are combined by the same two-block estimate used later, yielding the complete infrared constant $\widehat{c}_{n,\text{IR}}^\theta(b) > 0$ when the residual subcriticality condition holds. If the residual block vanishes, $\widehat{c}_{n,\text{IR}}^\theta(b) = c_{n,\text{ch}}^{\text{phys}}(b)$.

IX. EXACT ULTRAVIOLET–INFRARED MIXED BLOCK

A. Three-part decomposition

Let the canonical magnetic cells be partitioned into reference, transported, and collar classes. The exact mixed block is

$$K_n^\varepsilon = P_{n,\text{UV}}^\varepsilon H_{n,\text{mag}}^\varepsilon P_{n,\text{IR}}^\varepsilon = K_{n,\text{ref}}^\varepsilon + K_{n,\text{tr}}^\varepsilon + K_{n,\text{col}}^\varepsilon. \quad (\text{IX.1})$$

Define

$$\widehat{\epsilon}_{n,\text{mix}}^\varepsilon = \|K_n^\varepsilon\|, \quad \epsilon_{n,\text{ref}}^\varepsilon = \|K_{n,\text{ref}}^\varepsilon\|.$$

Exact reference-band closure is the special case $\epsilon_{n,\text{ref}}^\varepsilon = 0$ and is not assumed.

B. Transport and collar majorants

Finite-window transport estimates give

$$\|K_{n,\text{tr}}^\varepsilon\| \leq C_{n,\text{tr}}^\varepsilon D_{n,\text{lax}}^\varepsilon, \quad \|K_{n,\text{col}}^\varepsilon\| \leq C_{n,\text{col}}^\varepsilon \Delta_{n,\text{col}}^\varepsilon.$$

Hence the hybrid sufficient majorant

$$\epsilon_{n,\text{hyb}}^\varepsilon = \epsilon_{n,\text{ref}}^\varepsilon + C_{n,\text{tr}}^\varepsilon D_{n,\text{lax}}^\varepsilon + C_{n,\text{col}}^\varepsilon \Delta_{n,\text{col}}^\varepsilon \quad (\text{IX.2})$$

satisfies $\widehat{\epsilon}_{n,\text{mix}}^\varepsilon \leq \epsilon_{n,\text{hyb}}^\varepsilon$.

Theorem IX.1 (Exact mixed-block estimate). *The physical ultraviolet–infrared mixed norm is exactly $\widehat{\epsilon}_{n,\text{mix}}^\varepsilon = \|K_{n,\text{ref}}^\varepsilon + K_{n,\text{tr}}^\varepsilon + K_{n,\text{col}}^\varepsilon\|$ and is bounded above by the hybrid majorant in Equation (IX.2). No interior or direct-comparator block-closure hypothesis is required.*

X. COMPLETE VACUUM REDUCTION AND PHYSICAL EXCITATION BLOCKS

A. Vacuum-renormalized form

Set

$$\widehat{H}_n^\varepsilon = H_n^\varepsilon - E_{0,n}^\varepsilon I \geq 0.$$

Since $\text{Ker } \widehat{H}_n^\varepsilon = \text{Ran } P_{n,\text{vac}}^\varepsilon$, the associated sesquilinear form satisfies

$$\widehat{q}_n^\varepsilon(v, \psi) = 0$$

for $v \in \text{Ran } P_{n,\text{vac}}^\varepsilon$ and $\psi \in D(\widehat{Q}_n^\varepsilon)$.

B. Exact three-space decomposition

The projections satisfy

$$I = P_{n,\text{UV}}^\varepsilon + P_{n,\text{IR}}^\varepsilon + P_{n,\text{vac}}^\varepsilon$$

orthogonally and preserve the form domain. For $\psi \in D(\widehat{Q}_n^\varepsilon)$, set $u = P_{n,\text{UV}}^\varepsilon \psi$, $w = P_{n,\text{IR}}^\varepsilon \psi$, and $v = P_{n,\text{vac}}^\varepsilon \psi$. Then

$$\widehat{Q}_n^\varepsilon[\psi] = \widehat{Q}_n^\varepsilon[u] + \widehat{Q}_n^\varepsilon[w] + 2 \text{Re } \widehat{q}_n^\varepsilon(u, w), \quad (\text{X.1})$$

and the electric cross term vanishes, so

$$\widehat{q}_n^\varepsilon(u, w) = \langle u, H_{n, \text{mag}}^\varepsilon w \rangle = \langle u, K_n^\varepsilon w \rangle.$$

Theorem X.1 (Exact excitation reduction). *For every $\psi \in D(\widehat{Q}_n^\varepsilon)$, the complete ground component drops out and the form reduces exactly to [Equation \(X.1\)](#), with mixed term governed by K_n^ε .*

Remark X.2. *The graph-minimum projection, surviving coefficient projection, retained electric projection, and physical vacuum projection act on different spaces or encode different structures and are never identified.*

XI. PRINCIPAL FINITE-SCALE COERCIVITY THEOREM

A. Two-block lemma

Lemma XI.1 (Hermitian two-block lower bound). *Let $\mathcal{K} = \mathcal{K}_1 \oplus \mathcal{K}_2$ and suppose a nonnegative quadratic form satisfies*

$$q[x + y] \geq c_1 \|x\|^2 + c_2 \|y\|^2 - 2\eta \|x\| \|y\|.$$

Then

$$q[z] \geq \kappa \|z\|^2, \quad \kappa = \frac{c_1 + c_2 - \sqrt{(c_1 - c_2)^2 + 4\eta^2}}{2}.$$

In particular, $\kappa > 0$ when $\eta^2 < c_1 c_2$.

Corollary XI.2 (Relative leakage bound). *If $\eta^2 \leq \rho c_1 c_2$ with $0 \leq \rho < 1$, then*

$$\kappa \geq \frac{1 - \rho}{2} \min\{c_1, c_2\}.$$

B. Reduced physical comparison package

A verified fixed-scale package contains: the canonical finite Hamiltonian and complete ground projection; a raw Wilson frame and graph with nonzero excitation complement; non-trivial physical channel survival; finite electric capture and positive ultraviolet exclusion; positive physical magnetic extraction; positive channel calibration and residual infrared control;

an interior–transported–collar partition; quantified reference leakage; exact transported-cell comparator identities; and unital involution-invariant finite Wilson windows.

Interpretive lead-in. All components of the comparison operator have now been constructed on the same physical excitation space: the ultraviolet block, the residual magnetic block, and the exact off-diagonal leakage term. The next theorem introduces no further approximation. It compresses these typed pieces into the coercive statement that the remaining sections transport across center sectors and admissible branches.

Theorem XI.3 (Finite-scale Wilson-tube coercivity). *Assume the reduced physical comparison package and*

$$(\widehat{\epsilon}_{n,\text{mix}}^\varepsilon)^2 < \widehat{c}_{n,\text{UV}}^\varepsilon \widehat{c}_{n,\text{IR}}^\theta(b). \quad (\text{XI.1})$$

Then

$$H_n^\varepsilon \geq E_{0,n}^\varepsilon I + \widehat{\kappa}_{n,\varepsilon,\theta}(b)(I - P_{n,\text{vac}}^\varepsilon), \quad (\text{XI.2})$$

where

$$\widehat{\kappa}_{n,\varepsilon,\theta}(b) = \frac{\widehat{c}_{n,\text{UV}}^\varepsilon + \widehat{c}_{n,\text{IR}}^\theta(b) - \sqrt{(\widehat{c}_{n,\text{UV}}^\varepsilon - \widehat{c}_{n,\text{IR}}^\theta(b))^2 + 4(\widehat{\epsilon}_{n,\text{mix}}^\varepsilon)^2}}{2} > 0. \quad (\text{XI.3})$$

Equivalently,

$$\widehat{Q}_n^\varepsilon[\psi] \geq \widehat{\kappa}_{n,\varepsilon,\theta}(b) \|(I - P_{n,\text{vac}}^\varepsilon)\psi\|^2$$

for every $\psi \in D(\widehat{Q}_n^\varepsilon)$.

Proof. Use the exact excitation reduction, the ultraviolet lower bound, the complete infrared lower bound, and the exact mixed norm. Apply [Theorem XI.1](#). $\square$

What has been established. The finite Hamiltonian is now separated from its complete ground eigenspace by an explicit lower floor determined by the two diagonal coercive constants and the mixed leakage norm. The remaining task is not to improve this fixed-scale inequality, but to identify parameter regions on which the same mechanism remains uniformly positive and can therefore be exported as branch data.

Corollary XI.4 (Hybrid sufficient condition). *It is sufficient that $(\epsilon_{n,\text{hyb}}^\varepsilon)^2 < \widehat{c}_{n,\text{UV}}^\varepsilon \widehat{c}_{n,\text{IR}}^\theta(b)$. Replacing the exact mixed norm in [Equation \(XI.3\)](#) by the hybrid majorant gives a valid, possibly weaker, constant.*

C. Finite-scale spectral consequence

Compact resolvent and Equation (XI.2) imply

$$\text{spec}(H_n^\varepsilon) \cap (E_{0,n}^\varepsilon, E_{0,n}^\varepsilon + \widehat{\kappa}_{n,\varepsilon,\theta}(b)) = \emptyset.$$

This is a resolved finite-scale gap above the complete ground spectral value, not a relativistic Yang–Mills mass gap.

XII. SECTORWISE COERCIVE FRUSTA AND ADMISSIBLE BRANCHES

Reader’s guide. This section converts one-scale positivity into a stable parameter region. The coercive cone records exactly where the two-block comparison is positive; a bounded frustum supplies quantitative margins; and an admissible branch is a compatible choice of such data across refinement. This separates the algebraic coercivity mechanism from the downstream selection problem: Paper I constructs the finite packet and its floor, while Paper II determines whether compatible packets can be selected cofinally.

A. The ultraviolet–infrared comparison block

For a center character χ and a finite-scale packet λ , set

$$C_{\chi,\lambda} = \begin{pmatrix} c_{\chi,\lambda}^{\text{UV}} & -\epsilon_{\chi,\lambda}^{\text{mix}} \\ -\epsilon_{\chi,\lambda}^{\text{mix}} & c_{\chi,\lambda}^{\text{IR}} \end{pmatrix}. \quad (\text{XII.1})$$

Its lower eigenvalue is

$$\kappa_{\chi,\lambda} = \frac{1}{2} \left(c_{\chi,\lambda}^{\text{UV}} + c_{\chi,\lambda}^{\text{IR}} - \sqrt{(c_{\chi,\lambda}^{\text{UV}} - c_{\chi,\lambda}^{\text{IR}})^2 + 4(\epsilon_{\chi,\lambda}^{\text{mix}})^2} \right). \quad (\text{XII.2})$$

The global finite-packet constant is

$$\kappa_\lambda = \min_{\chi \in \overline{Z(G)}} \kappa_{\chi,\lambda}. \quad (\text{XII.3})$$

Theorem XII.1 (Center-sector coercivity from the literal comparison blocks). *Assume the center-character reduction of Theorem V.1. If $c_{\chi,\lambda}^{\text{UV}} > 0$, $c_{\chi,\lambda}^{\text{IR}} > 0$, and*

$$(\epsilon_{\chi,\lambda}^{\text{mix}})^2 < c_{\chi,\lambda}^{\text{UV}} c_{\chi,\lambda}^{\text{IR}}$$

for every χ , then

$$\widehat{Q}_\lambda[\psi] \geq \sum_{\chi \in \widehat{Z(G)}} \kappa_{\chi, \lambda} \|(I - P_{\text{vac}, \chi}^\lambda) P_\chi^\lambda \psi\|^2 \geq \kappa_\lambda \|(I - P_{\text{vac}}^\lambda) \psi\|^2. \quad (\text{XII.4})$$

Proof. Apply [Theorem XI.1](#) on the ultraviolet–infrared decomposition inside each center sector and then sum the mutually orthogonal sector estimates. $\square$

B. The exact coercive region

Definition XII.2 (Coercive cone and bounded coercive frustum). *For $\kappa \geq 0$ define*

$$\mathfrak{C}(\kappa) = \{(u, v, m) \in [0, \infty)^3 : u \geq \kappa, v \geq \kappa, m^2 \leq (u - \kappa)(v - \kappa)\}. \quad (\text{XII.5})$$

This is the translated positive-semidefinite cone for the comparison matrix. Given certified endpoint ranges

$$u_- \leq u \leq u_+, \quad v_- \leq v \leq v_+, \quad 0 \leq m \leq m_+,$$

its bounded section

$$\mathfrak{F}(\kappa; u_\pm, v_\pm, m_+) = \mathfrak{C}(\kappa) \cap ([u_-, u_+] \times [v_-, v_+] \times [0, m_+]) \quad (\text{XII.6})$$

is called the coercive frustum.

Proposition XII.3 (Frustum characterization). *For $u, v, m \geq 0$, the following are equivalent:*

- (i) $\lambda_{\min} \begin{pmatrix} u & -m \\ -m & v \end{pmatrix} \geq \kappa$;
- (ii) $\begin{pmatrix} u & -m \\ -m & v \end{pmatrix} \geq \kappa I$;
- (iii) $(u, v, m) \in \mathfrak{C}(\kappa)$.

Proof. Subtract κI . A Hermitian 2×2 matrix is positive semidefinite exactly when its diagonal entries and determinant are nonnegative. $\square$

C. Typed branches

Definition XII.4 (Admissible center-sector branch). *Let Γ be a directed family of verified finite-scale packets. A χ -sector branch is the typed family*

$$\mathfrak{b}_\chi = \{(\mathfrak{P}_{\chi, \lambda}, c_{\chi, \lambda}^{\text{UV}}, c_{\chi, \lambda}^{\text{IR}}, \epsilon_{\chi, \lambda}^{\text{mix}})\}_{\lambda \in \Gamma}$$

together with the declared admissible coarse-transport arrows. It is κ -frustum-certified when every comparison triple belongs to $\mathfrak{C}(\kappa)$ and every controlled seam carries either independent endpoint certificates or a proved two-sided form comparison.

Theorem XII.5 (Uniform branch floor). *If every center-sector branch over Γ is κ_* -frustum-certified, then*

$$\inf_{\lambda \in \Gamma} \min_{\chi \in \widehat{Z(G)}} \kappa_{\chi, \lambda} \geq \kappa_*.$$

Consequently every packet satisfies the complete-ground estimate with the same floor κ_* . This is an objectwise statement; it does not construct a limiting Hilbert space or limiting operator.

Remark XII.6 (One-sided seams). *A one-sided comparison controls only the image on which it is stated and cannot certify the orthogonal complement at the target endpoint. Therefore a lower bound is not transported through a controlled seam unless independent endpoint certification, exact intertwining, or a two-sided degraded form estimate is proved.*

XIII. UNIFORM PHYSICALIZED COERCIVITY AND REFINEMENT HANDOFF

A. Directed finite-scale families

Let $(\mathfrak{G}, \preceq)$ be directed, and let γ index verified finite-scale packages. No common ambient Hilbert space is required for the numerical statement.

Theorem XIII.1 (Uniform finite-scale coercivity). *Suppose*

$$\widehat{c}_{UV, \gamma} \geq c_{UV, *} > 0, \quad \widehat{c}_{IR, \gamma} \geq c_{IR, *} > 0,$$

and

$$\widehat{\epsilon}_{\text{mix}, \gamma}^2 \leq \rho \widehat{c}_{UV, \gamma} \widehat{c}_{IR, \gamma}, \quad 0 \leq \rho < 1.$$

Then

$$\inf_{\gamma} \widehat{\kappa}_{\gamma} \geq \frac{1 - \rho}{2} \min\{c_{UV, *}, c_{IR, *}\} > 0.$$

B. Uniform comparison criterion

Uniform graph separation, physical-rank survival, vacuum-angle control, electric capture, Gram conditioning, physical extraction and calibration, residual control, ultraviolet exclusion, and a uniform three-part leakage estimate imply [Theorem XIII.1](#).

C. Failure modes and handoff

The architecture separates collapse of graph separation, physical rank, vacuum angle, electric capture, Gram conditioning, physical extraction, calibration, residual infrared coercivity, reference-band control, transport or collar control, and final subcriticality. Uniform objectwise coercivity alone constructs no limiting Hilbert space or generator. A later theorem must provide comparison maps, convergence of forms and ground projections, and an explicit no-leakage or norm-recovery mechanism.

Transition to the model certificates. The abstract finite-scale theorem is complete at this point. The group-specific sections have a different role: they certify that the hypotheses are nonempty and retain exact constants in concrete center sectors. The $SU(2)$ and $SU(3)$ calculations are witnesses for the mechanism, not replacements for the general coercivity theorem.

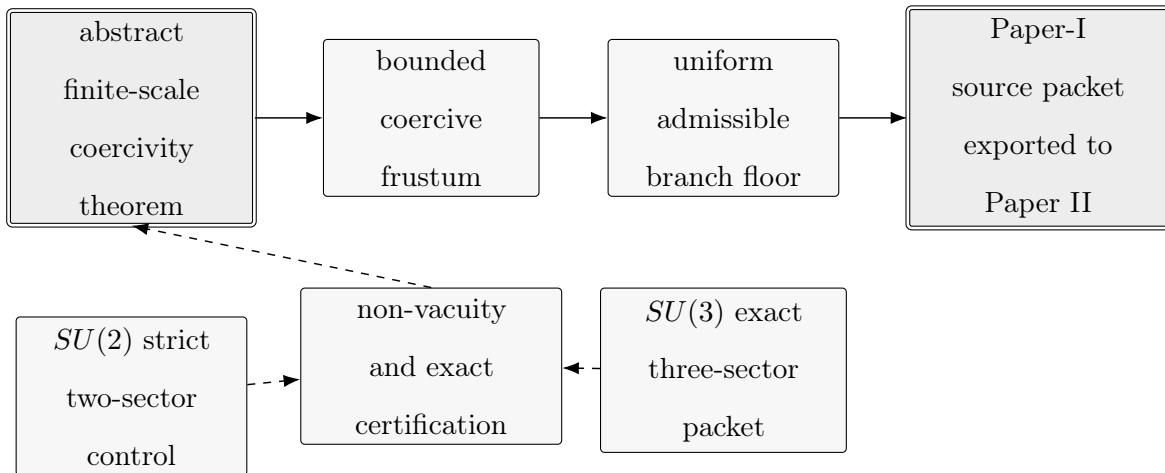


FIG. 2. Logical role of the witness bank. The $SU(2)$ and $SU(3)$ realizations certify non-vacuity and exact constants; they do not supply the proof of the abstract theorem. The double-bordered node records the precise downstream export.

XIV. THE COMPLETE TWO-SECTOR $SU(2)$ VALIDATION MODEL

For $G = SU(2)$, $Z(G) \cong \mathbb{Z}_2$ and the standard highest-weight and Peter–Weyl conventions are those of^{6,7}. The abstract center decomposition has exactly two characters. With $U(-I)$ denoting the center action,

$$P_0 = \frac{1}{2}(I + U(-I)), \quad P_1 = \frac{1}{2}(I - U(-I)),$$

and

$$\mathcal{H}^{SU(2)} = \mathcal{H}^{(0)} \oplus \mathcal{H}^{(1)}.$$

The constant ground state is neutral, while the fundamental $j = \frac{1}{2}$ Wilson channel is odd. The raw triangle compression and its strict/frustrated graph spectra are recorded in Appendix A.

Theorem XIV.1 (Inherited complete physicalized $j \leq 1$ realization). *The complete $j \leq 1$ Peter–Weyl Galerkin realization in the compiled Paper-I reference instantiates the full physical comparison package: complete interacting ground projection, vacuum removal, finite electric capture, Gram normalization, physical magnetic recompression, residual infrared control, exact reference leakage, and the ultraviolet–infrared mixed estimate. Its resolved finite-scale lower bound is*

$$\widehat{\kappa}_G^{SU(2)} = 0.657490679870 \dots > 0. \quad (\text{XIV.1})$$

The theorem is a two-sector realization of Theorem XII.1: the neutral sector is estimated on the complete-ground complement and the odd sector is estimated on its full packet.

Remark XIV.2 (Role of the $SU(2)$ model). *The $SU(2)$ realization is not a surrogate for the selected $SU(3)$ packet. Its role is to show that the physicalization, ultraviolet exclusion, infrared Wilson separation, and nonzero mixed-leakage mechanism are not artifacts of triality or of a three-element center. It is the complete two-sector validation of the abstract theory.*

Scope Note XIV.3 (Computational provenance). *The historical basis ordering, exact matrices, constants ledger, recorded high-precision output, and independent verification script are preserved in the accompanying computational reproducibility archive. Appendix C records the verification steps and the role of this finite witness. The archive certifies the stated model computation; it does not replace the abstract theorem or enlarge the scope of the paper.*

XV. A COMPLETE THREE-SECTOR $SU(3)$ FRUSTUM SEED

A. One Hamiltonian and all three center characters

Let

$$G = SU(3), \quad Z(G) = \{I, \omega I, \omega^2 I\}, \quad \omega = e^{2\pi i/3}.$$

We use the standard $SU(3)$ highest-weight, center-character, Casimir, and Schur-orthogonality conventions^{6,7}. Write $\chi_0 = \chi_{\mathbf{8}}$ for the adjoint character, $\chi_1 = \chi_{\mathbf{3}}$, and $\chi_2 = \chi_{\bar{\mathbf{3}}}$. Their trialities are 0, 1, 2, respectively, and

$$C_2(\mathbf{8}) = 3, \quad C_2(\mathbf{3}) = C_2(\bar{\mathbf{3}}) = \frac{4}{3}.$$

On $L^2(SU(3)^3)^{\text{Ad } SU(3)}$ define

$$\phi_i^{(r)}(g_1, g_2, g_3) = \chi_r(g_i), \quad \mathcal{S}_r = \text{span}\{\phi_1^{(r)}, \phi_2^{(r)}, \phi_3^{(r)}\}.$$

Schur orthogonality and the center characters give the orthogonal decomposition

$$\mathcal{K}_{\Delta}^{SU(3)} = \mathbb{C}\Omega \oplus \mathcal{S}_0 \oplus \mathcal{S}_1 \oplus \mathcal{S}_2, \quad \dim \mathcal{K}_{\Delta}^{SU(3)} = 10. \quad (\text{XV.1})$$

The simultaneous center action

$$(U(z)f)(g_1, g_2, g_3) = f(zg_1, zg_2, zg_3)$$

acts with character 1 on $\mathbb{C}\Omega \oplus \mathcal{S}_0$, with character z on $\mathcal{S}_1$, and with character z^2 on $\mathcal{S}_2$. The projectors are

$$P_r = \frac{1}{3} \sum_{k=0}^2 \omega^{-rk} U(\omega^k), \quad r = 0, 1, 2. \quad (\text{XV.2})$$

Fix the central flux representative

$$\theta_{12} = \theta_{23} = 1, \quad \theta_{31} = \omega,$$

and use the single real source-free pair potential

$$V_{ij}^{\theta}(g) = 1 - \frac{1}{3} \text{Re}[\theta_{ij} \chi_{\mathbf{3}}(g_i g_j^{-1})]. \quad (\text{XV.3})$$

Because $\text{Re}[\theta \chi_{\mathbf{3}}] = \text{Re}[\bar{\theta} \chi_{\bar{\mathbf{3}}}]$, the same Hamiltonian carries the conjugate flux on the triality-two channel.

Let P_Δ be the orthogonal projection onto $\mathcal{K}_\Delta^{SU(3)}$ and define

$$H_\Delta^G = P_\Delta \left(H_{\text{el}} + \sum_{\{i,j\} \in E(C_3)} M_{V_{ij}^\theta} \right) P_\Delta. \quad (\text{XV.4})$$

This is one bounded self-adjoint Galerkin Hamiltonian on the complete ten-dimensional packet, not a direct sum of unrelated model operators.

Proposition XV.1 (Exact three-sector block matrix). *Relative to (XV.1),*

$$H_\Delta^G = 3I_{\mathbb{C}\Omega} \oplus 6I_{\mathcal{S}_0} \oplus \left(\frac{38}{9}I_{\mathcal{S}_1} + \frac{1}{18}L_1 \right) \oplus \left(\frac{38}{9}I_{\mathcal{S}_2} + \frac{1}{18}L_2 \right), \quad (\text{XV.5})$$

where L_1 and L_2 are the conjugate twisted triangle graph Laplacians of Appendix B. All cross-triality blocks vanish.

Proof. The Hamiltonian commutes with the simultaneous center action, so distinct trialities reduce it. The vacuum electric energy is zero and each of the three magnetic cells has vacuum expectation one. On $\mathcal{S}_1$, Appendix B gives $26I/9 + L_1/18$ for the magnetic compression and $4I/3$ for the electric term. Charge conjugation gives the $\mathcal{S}_2$ block with L_2 .

For the adjoint channel, every nonconstant contribution from (XV.3) has zero matrix element between adjoint character vectors. Indeed the relevant Haar integrals contain one factor of fundamental or antifundamental triality and otherwise only neutral factors; a central change of variable forces each such integral to vanish. Thus each magnetic cell compresses to the identity on $\mathcal{S}_0$. Adding the adjoint Casimir 3 and the three cell constants gives $6I_{\mathcal{S}_0}$. The same selection rule eliminates vacuum–adjoint matrix elements. $\square$

B. Complete ground and three sectorwise gaps

Let

$$P_\Delta^0 = |\Omega\rangle\langle\Omega|.$$

For $r = 1, 2$, write the ordered graph eigenvalues as

$$\lambda_1 = 2 - 2\cos\frac{2\pi}{9}, \quad \lambda_2 = 2 - 2\cos\frac{4\pi}{9}, \quad \lambda_3 = 2 - 2\cos\frac{8\pi}{9}.$$

Theorem XV.2 (Complete-ground three-sector coercivity). *The complete ground energy of H_Δ^G is 3, the complete ground projection is exactly P_Δ^0 , and*

$$H_\Delta^G \geq 3I + \Delta_\Delta^{SU(3)}(I - P_\Delta^0), \quad (\text{XV.6})$$

where

$$\Delta_{\Delta}^{SU(3)} = \frac{12 - \cos(2\pi/9)}{9} = 1.2482172840978913294 \dots \quad (\text{XV.7})$$

More precisely, the neutral excited sector has gap 3, while both nonneutral triality sectors have gap $\Delta_{\Delta}^{SU(3)}$.

Proof. By Equation (XV.5), the neutral excitation energy above the vacuum is $6 - 3 = 3$. For $r = 1, 2$, an L_r eigenvector with eigenvalue λ has energy above the vacuum

$$\frac{38}{9} + \frac{\lambda}{18} - 3 = \frac{11}{9} + \frac{\lambda}{18}.$$

Substituting λ_1 gives (XV.7). It is strictly smaller than 3 and strictly positive, proving both the ground-space identity and the global bound. $\square$

C. The literal ultraviolet–infrared blocks

The model now instantiates the comparison architecture in every center sector. In the neutral sector let the infrared line be the constant graph mode $\mathbb{C}(1, 1, 1) \subset \mathcal{S}_0$ and let its orthogonal complement be ultraviolet. In triality $r = 1, 2$, let the infrared line be the lowest eigenspace of L_r and let the upper two graph modes be ultraviolet. In these exact graph eigenbases the mixed blocks vanish.

Proposition XV.3 (Three explicit frustum triples). *The sectorwise comparison triples are*

$$\mathbf{c}_0 = (3, 3, 0), \quad (\text{XV.8})$$

$$\mathbf{c}_1 = \mathbf{c}_2 = \left(\frac{12 - \cos(4\pi/9)}{9}, \frac{12 - \cos(2\pi/9)}{9}, 0 \right). \quad (\text{XV.9})$$

Thus all three sectors lie in the coercive cone $\mathfrak{C}(\Delta_{\Delta}^{SU(3)})$ and hence in $\mathfrak{C}(\kappa_{\text{cof}})$ for

$$\kappa_{\text{cof}} = \frac{1}{2} \left(\frac{17}{5} - \sqrt{\frac{557}{75}} \right) = 0.3374044376020202719 \dots \quad (\text{XV.10})$$

Proof. The neutral block is scalar. In each nonneutral sector the lowest graph mode gives the infrared constant and the second graph eigenvalue gives the smallest ultraviolet constant. The graph eigenbasis diagonalizes the packet Hamiltonian, so the exact mixed norm is zero. The first membership statement follows from Theorem XII.3. Finally, $\kappa_{\text{cof}} < 2/5 < \Delta_{\Delta}^{SU(3)}$ by the exact comparison $557/75 - (13/5)^2 = 2/3 > 0$. $\square$

Remark XV.4 (Zero-leakage endpoint versus the abstract mechanism). *The $SU(3)$ triangle is an exact zero-mixing endpoint of the coercive frustum. It proves all three center sectors and their common positive floor, but a diagonal model should not be asked to certify leakage it does not possess. The inherited $SU(2)$ Galerkin realization supplies that independent validation. The abstract theorem is designed precisely so both realizations are instances of one UV–IR–mixing mechanism.*

D. Fixed-scale branch seed and export boundary

For a fixed positive-thickness label ε , define the three typed sector seeds

$$\mathfrak{b}_r(\varepsilon) = \left(SU(3), \mathbb{Z}_3, r, \varepsilon, P_r \mathcal{K}_\Delta^{SU(3)}, P_r H_\Delta^G P_r, P_r P_\Delta^0, \mathfrak{c}_r \right), \quad r = 0, 1, 2.$$

They are distinct even though charge conjugation identifies the spectra of $r = 1$ and $r = 2$. Together they form a fixed-scale three-sector frustum seed. No cofinal scale family, noninvertible physical scale transport, controlled-seam endpoint theorem, or identification with a later local Friedrichs object is asserted here.

Proposition XV.5 (Fixed-scale three-sector source-packet export). *For every fixed positive-thickness label ε , Paper I exports the typed datum*

$$\mathfrak{P}_\Delta^{SU(3)}(\varepsilon) = \left(SU(3), \mathbb{Z}_3, \varepsilon, \mathcal{K}_\Delta^{SU(3)}, \{P_0, P_1, P_2\}, H_\Delta^G, P_\Delta^0, \{\mathfrak{c}_0, \mathfrak{c}_1, \mathfrak{c}_2\}, \Delta_\Delta^{SU(3)}, \kappa_{\text{cof}} \right). \quad (\text{XV.11})$$

The center projectors, Hamiltonian, complete-ground projection, sector triples, and sharp gap are exact. The thickness is a packet label at this stage. No Paper-II or Paper-III theorem is used in proving this export, and no downstream spacetime or limiting conclusion is included.

XVI. COFINAL THREE-SECTOR BRANCHES FROM EXPLICIT UV–IR–MIXING PROFILES

A positive estimate at one scale is evidence, not a branch. The branch begins only after the same mechanism survives admissible refinement and every new endpoint is certified. This section upgrades the fixed-scale $SU(3)$ frustum seed of [Section XV](#) to one explicit directed positive-thickness family. The construction is performed entirely in the reduced physical packet category developed above: each endpoint is a finite gauge-invariant Peter–Weyl packet,

every magnetic correction is a positive physical recompression, and every endpoint is certified directly. No lower bound is transported through a one-sided arrow.

A. The cofinal positive-thickness set and the retained seed

Fix $\varepsilon_0 > 0$ and $0 < \alpha \leq 1$, and set

$$\varepsilon_n = 2^{-n} \varepsilon_0, \quad \tau_n = \left(\frac{\varepsilon_n}{\varepsilon_0} \right)^\alpha = 2^{-\alpha n}, \quad n \in \mathbb{N}_0. \quad (\text{XVI.1})$$

Then

$$E_{\text{cof}} = \{\varepsilon_n : n \in \mathbb{N}_0\} \subset (0, \infty), \quad \inf E_{\text{cof}} = 0,$$

but zero thickness is not an object of the family.

Let

$$A_r = P_r (H_\Delta^{\text{G}} - 3I) P_r \quad \text{on } \mathcal{I}_r := \mathcal{S}_r, \quad r = 0, 1, 2,$$

where the spaces and projectors are those of [Section XV](#). The fixed-scale calculation gives

$$A_0 = 3I_{\mathcal{I}_0}, \quad A_1, A_2 \geq \Delta_\Delta^{SU(3)} I, \quad \Delta_\Delta^{SU(3)} > \frac{2}{5}. \quad (\text{XVI.2})$$

Thus the retained sectorwise infrared certificates are

$$\underline{c}_0^{\text{IR}} = 3, \quad \underline{c}_1^{\text{IR}} = \underline{c}_2^{\text{IR}} = \frac{2}{5}. \quad (\text{XVI.3})$$

The value $2/5$ is a conservative exact certificate below the sharp twisted-triangle gap; it is not asserted to be the sharp infrared eigenvalue.

B. Peter–Weyl ultraviolet towers

For an $SU(3)$ highest weight (p, q) , with the standard representation-theoretic conventions^{[6,7](#)}, write

$$C_2(p, q) = \frac{p^2 + q^2 + pq + 3p + 3q}{3}.$$

Choose mutually orthonormal gauge-invariant Wilson-loop Peter–Weyl modes $u_{r,j}$ with highest weights

$$\lambda_{0,j} = (j+2, j+2), \quad \lambda_{1,j} = (j, j+2), \quad \lambda_{2,j} = (j+2, j), \quad j \geq 0. \quad (\text{XVI.4})$$

With the convention $r \equiv p - q \pmod{3}$, their trialities are 0, 1, 2, respectively; charge conjugation exchanges the last two families. Moreover

$$C_2(\lambda_{r,j}) \geq 3 \quad (r = 0, 1, 2; j \geq 0). \quad (\text{XVI.5})$$

For each n set

$$\mathcal{U}_{r,n} = \text{span}\{u_{r,0}, \dots, u_{r,n}\}, \quad D_{r,n}u_{r,j} = C_2(\lambda_{r,j})u_{r,j}.$$

The n th packet is

$$\mathcal{K}_n^{\text{cof}} = \mathbb{C}\Omega \oplus \bigoplus_{r=0}^2 (\mathcal{I}_r \oplus \mathcal{U}_{r,n}). \quad (\text{XVI.6})$$

The center acts by ω^r on the r th summand and trivially on Ω . Hence the three sectors remain orthogonal and separately typed at every endpoint.

C. Actual reference, transport, and collar profiles

Choose normalized lowest-eigenvalue vectors $\iota_r \in \mathcal{I}_r$ for A_r , with ι_2 the charge conjugate of ι_1 ; in the neutral scalar block choose any unit vector. The corresponding orthogonal complements reduce A_r . Define the positive rank-one physical recompression

$$W_r = |u_{r,0} - \iota_r\rangle\langle u_{r,0} - \iota_r|. \quad (\text{XVI.7})$$

Because $u_{r,0}$ and ι_r have the same center character, W_r is center neutral and preserves the r th sector. Put

$$m_n = \frac{\tau_n}{\sqrt{6}}. \quad (\text{XVI.8})$$

The three positive cell classes are defined by

$$R_{r,n}^{\text{ref}} = \frac{m_n}{2}W_r, \quad R_{r,n}^{\text{tr}} = \frac{m_n}{3}W_r, \quad R_{r,n}^{\text{col}} = \frac{m_n}{6}W_r. \quad (\text{XVI.9})$$

Their ultraviolet–infrared corners are aligned rank-one maps. Therefore the exact endpoint profiles are

$$\epsilon_{r,n}^{\text{ref}} = \frac{m_n}{2}, \quad \epsilon_{r,n}^{\text{tr}} = \frac{m_n}{3}, \quad \epsilon_{r,n}^{\text{col}} = \frac{m_n}{6}, \quad \epsilon_{r,n}^{\text{mix}} = m_n. \quad (\text{XVI.10})$$

Thus the exact three-part decomposition, rather than only its triangle inequality, is realized at every scale.

Define the nonnegative excitation form on (XVI.6) by

$$\hat{q}_n = \bigoplus_{r=0}^2 (A_r \oplus D_{r,n} + R_{r,n}^{\text{ref}} + R_{r,n}^{\text{tr}} + R_{r,n}^{\text{col}}), \quad \hat{q}_n[\Omega] = 0, \quad (\text{XVI.11})$$

and let

$$H_n^{\text{cof}} = 3I + \hat{H}_n, \quad (\text{XVI.12})$$

where $\hat{H}_n$ is the finite-dimensional operator associated with $\hat{q}_n$. Every term is gauge invariant and center preserving; no external current, cap, or defect-supported source occurs.

Scope Note XVI.1 (Reduced-packet realization). *The operators in (XVI.9) are explicit positive recompressions on the finite physical packet. The theorem does not claim that each recompression has already been lifted to a single microscopic multiplication potential in the missing historical lattice source tree. Such a microscopic lift would strengthen the realization statement but is not used in the endpoint form estimates or in the cofinal packet handoff below.*

Proposition XVI.2 (Exact sectorwise comparison profiles). *Let $P_{r,n}^{\text{IR}}$ project onto $\mathcal{I}_r$ and $P_{r,n}^{\text{UV}}$ onto $\mathcal{U}_{r,n}$. For every n the actual mixed corner has norm m_n , while the diagonal blocks satisfy the retained lower bounds*

$$(c_{0,n}^{\text{UV}}, c_{0,n}^{\text{IR}}, \epsilon_{0,n}^{\text{mix}}) = (3, 3, m_n), \quad (\text{XVI.13})$$

$$(c_{1,n}^{\text{UV}}, c_{1,n}^{\text{IR}}, \epsilon_{1,n}^{\text{mix}}) = (3, \frac{2}{5}, m_n), \quad (\text{XVI.14})$$

$$(c_{2,n}^{\text{UV}}, c_{2,n}^{\text{IR}}, \epsilon_{2,n}^{\text{mix}}) = (3, \frac{2}{5}, m_n). \quad (\text{XVI.15})$$

The positive diagonal contributions of (XVI.9) are discarded in these certificates and therefore can only strengthen the actual endpoint forms.

Proof. Equation (XVI.5) gives the ultraviolet lower bound 3. Equation (XVI.2) gives the infrared bounds. Since $P_{r,n}^{\text{UV}} W_r P_{r,n}^{\text{IR}} = -|u_{r,0}\rangle\langle\iota_r|$, its norm is one. The coefficients in (XVI.9) sum to m_n , proving the exact mixed norm. Dropping the positive diagonal corners of the same operators yields the stated comparison triples. $\square$

D. The common frustum floor

Let

$$\kappa_{\text{cof}} = \frac{1}{2} \left(\frac{17}{5} - \sqrt{\frac{557}{75}} \right). \quad (\text{XVI.16})$$

The algebraic identity

$$(3 - \kappa_{\text{cof}}) \left(\frac{2}{5} - \kappa_{\text{cof}} \right) = \frac{1}{6} \quad (\text{XVI.17})$$

shows directly why the endpoint profile $(3, 2/5, 1/\sqrt{6})$ produces the retained radical.

Theorem XVI.3 (Cofinal three-sector frustum certification). *For every $n \in \mathbb{N}_0$ and every $r \in \{0, 1, 2\}$, the profile in [Theorem XVI.2](#) lies in $\mathfrak{C}(\kappa_{\text{cof}})$. More precisely, the neutral comparison constant is*

$$\kappa_{0,n}^{\text{cmp}} = 3 - \frac{\tau_n}{\sqrt{6}}, \quad (\text{XVI.18})$$

and the two conjugate nonneutral comparison constants are

$$\kappa_{1,n}^{\text{cmp}} = \kappa_{2,n}^{\text{cmp}} = \frac{1}{2} \left(\frac{17}{5} - \sqrt{\frac{169}{25} + \frac{2}{3} \tau_n^2} \right). \quad (\text{XVI.19})$$

Consequently

$$\inf_{n \in \mathbb{N}_0} \min_{r=0,1,2} \kappa_{r,n}^{\text{cmp}} = \kappa_{\text{cof}}, \quad (\text{XVI.20})$$

and every endpoint satisfies

$$H_n^{\text{cof}} \geq 3I + \kappa_{\text{cof}}(I - P_n^0), \quad P_n^0 = |\Omega\rangle\langle\Omega|. \quad (\text{XVI.21})$$

The equality in [\(XVI.20\)](#) is a statement about the retained comparison profile; the actual endpoint spectral gaps may be larger.

Proof. Since $m_n^2 = \tau_n^2/6 \leq 1/6$, the nonneutral frustum condition follows from [\(XVI.17\)](#). The neutral condition follows from $m_n^2 \leq 1/6 < (3 - \kappa_{\text{cof}})^2$. Applying the two-block formula gives [\(XVI.18\)](#) and [\(XVI.19\)](#). The latter is monotone increasing as τ_n decreases, and at $n = 0$ it equals [\(XVI.16\)](#). The neutral value is strictly larger. Direct endpoint application of [Theorem XII.1](#) gives [\(XVI.21\)](#). $\square$

E. Noninvertible refinement arrows and controlled seams

Let

$$J_n^{n+1} : \mathcal{K}_n^{\text{cof}} \longrightarrow \mathcal{K}_{n+1}^{\text{cof}} \quad (\text{XVI.22})$$

be the isometric inclusion fixing Ω , every infrared seed, and all ultraviolet modes already present at scale n . Its adjoint $E_n^{n+1} = (J_n^{n+1})^*$ is the orthogonal coarse compression. The arrow J_n^{n+1} intertwines the center action and the complete-ground and infrared projections, but it is not surjective because the three new modes $u_{r,n+1}$ are absent from its range.

Proposition XVI.4 (Exact seam comparison and independent endpoint certification). *For every $x \in \mathcal{K}_n^{\text{cof}}$,*

$$\widehat{q}_{n+1}[J_n^{n+1}x] - \widehat{q}_n[x] = (m_{n+1} - m_n) \sum_{r=0}^2 |\langle u_{r,0} - \iota_r, x \rangle|^2, \quad (\text{XVI.23})$$

and hence

$$|\widehat{q}_{n+1}[J_n^{n+1}x] - \widehat{q}_n[x]| \leq 2(m_n - m_{n+1}) \|(I - P_n^0)x\|^2. \quad (\text{XVI.24})$$

The comparison is two-sided on the image of J_n^{n+1} but says nothing about the new ultraviolet complement. The latter is certified directly by (XVI.5); therefore both endpoints of every seam carry independent coercivity certificates.

Proof. All inherited infrared and old ultraviolet diagonal terms intertwine exactly. Only the coefficient m_n changes on the old subspace, giving (XVI.23). The block-diagonal operator $\sum_r W_r$ has norm 2, which gives (XVI.24). The new modes have electric Casimir at least 3, so their endpoint estimate is independent of transport from scale n . $\square$

F. Typed cofinal branches and export

For $r = 0, 1, 2$ define

$$\mathfrak{b}_r^{\text{cof}} = \{(\varepsilon_n, P_r \mathcal{K}_n^{\text{cof}}, P_r H_n^{\text{cof}} P_r, P_r P_n^0, 3, \mathcal{C}_r^{\text{IR}}, m_n)\}_{n \in \mathbb{N}_0} \quad (\text{XVI.25})$$

with the restrictions of J_n^{n+1} and E_n^{n+1} . The branch $r = 0$ is the neutral control branch; $r = 1, 2$ are the distinct conjugate selected-flux branches.

Theorem XVI.5 (Cofinal three-sector source-packet handoff). *Paper I constructs and certifies the finite positive-thickness packet family*

$$\mathfrak{P}_I^{SU(3), \text{cof}} = (SU(3), \mathbb{Z}_3, E_{\text{cof}}, \{\mathcal{K}_n^{\text{cof}}, H_n^{\text{cof}}, P_n^0\}_{n \geq 0}, \{\mathfrak{b}_0^{\text{cof}}, \mathfrak{b}_1^{\text{cof}}, \mathfrak{b}_2^{\text{cof}}\}, \{J_n^{n+1}, E_n^{n+1}\}_{n \geq 0}, \kappa_{\text{cof}}). \quad (\text{XVI.26})$$

Every endpoint is source free, center graded, complete-ground certified, and coercive with the same retained floor. The physical refinement arrows are noninvertible; every controlled seam has the exact image comparison (XVI.24) and independent target endpoint certification. No Paper-II or Paper-III theorem is used in this construction.

The handoff does not identify this packet with a local Friedrichs operator, a spacetime representation, or a thin-limit object. Literal identity with the packet consumed downstream

requires a separate typed interface theorem matching regional labels, Wilson quotient, carrier data, and selected comparison maps.

XVII. MARKED WILSON-TUBE SOURCE IDENTITY AND INTERFACE READINESS

Later papers need an object, not a suggestive resemblance. The cofinal packet of [Section XVI](#) is an exact reduced physical packet, but to serve as a source object for later scale-bordism and spacetime constructions it must also carry its regional Wilson-tube label, gauge-invariant Wilson quotient, marked meridional class, and the finite-demand late-tail quantifier. This section supplies those data inside Paper I. It does not use a theorem of Paper II or Paper III.

A. Marked tubular carrier and meridional graph

Let $\Omega \Subset \mathbb{R}^3$ be a bounded region and let $\Sigma_{\text{pres}}^0 \subset \Omega$ be a smoothly embedded compact oriented one-dimensional submanifold. The superscript records that this is a time-zero source carrier, not yet a spacetime obstruction locus. Fix a tubular neighborhood $N_{\rho_0}(\Sigma_{\text{pres}}^0) \Subset \Omega$ and a positively oriented meridional class

$$[\mu] \in H_1(N_{\rho_0}(\Sigma_{\text{pres}}^0) \setminus \Sigma_{\text{pres}}^0; \mathbb{Z})$$

whose linking number with the selected component of Σ_{pres}^0 is one. For every n , choose the cell complex $\mathcal{L}_n(\Omega)$ so that it contains a marked oriented triangle

$$\gamma_n = (e_{12,n}, e_{23,n}, e_{31,n}) \subset N_{\rho_0}(\Sigma_{\text{pres}}^0) \setminus \Sigma_{\text{pres}}^0$$

representing $[\mu]$. The refinement $\mathcal{L}_n \preceq \mathcal{L}_{n+1}$ is chosen to preserve the marked cycle and the time-zero carrier label. New cells may be added away from the marked cycle; no inverse geometric refinement is assumed.

For $m \in \{0, 1, 2\}$ let $\theta_{n,m}$ be the $\mathbb{Z}_3$ phase cochain on γ_n with representative

$$\theta_{12,n} = 1, \quad \theta_{23,n} = 1, \quad \theta_{31,n} = \omega^m. \quad (\text{XVII.1})$$

Define the source meridional holonomy certificate

$$\text{Hol}_{\mu,n}^{\text{I}}(m) := \theta_{12,n} \theta_{23,n} \theta_{31,n}. \quad (\text{XVII.2})$$

Proposition XVII.1 (Marked meridional center certificate). *For every n and $m \in \{0, 1, 2\}$,*

$$\text{Hol}_{\mu,n}^I(m) = \omega^m. \quad (\text{XVII.3})$$

This value is invariant under vertex rephasing and marked carrier-preserving refinement. For $m = 1, 2$ the detected cochain is not gauge equivalent to the trivial cochain on the marked cycle.

Proof. The first assertion follows from (XVII.1). A vertex rephasing multiplies each oriented edge phase by a coboundary. The coboundary factors cancel around the closed cycle, so the product is invariant. The marked refinement retains the same oriented cycle and therefore the same product. A trivial cochain has cycle product one, whereas $\omega^m \neq 1$ for $m = 1, 2$. $\square$

Scope Note XVII.2 (Carrier status). *The spatial carrier Σ_{pres}^0 is a marked source-certificate datum in Paper I. This paper proves the nontrivial finite-scale meridional center evaluation around it. It does not prove that a spacetime obstruction locus has selected time-zero trace Σ_{pres}^0 and does not construct an incision. Those are later theorems.*

B. Gauge-invariant Wilson quotient

Let $\mathcal{W}_n^{\text{kin}}(\Omega; \mu)$ be the finite Peter–Weyl window generated by the based holonomy matrix coefficients on the three marked loop channels and the ultraviolet representations retained in (XVI.4). Let

$$\Gamma_n : \mathcal{W}_n^{\text{kin}}(\Omega; \mu) \longrightarrow \mathcal{H}_n(\Omega)$$

be finite gauge averaging and set

$$\mathcal{N}_n^W = \ker(\Gamma_n|_{\mathcal{W}_n^{\text{kin}}}).$$

Equip the quotient by

$$\langle [f], [g] \rangle_{q_W} := \langle \Gamma_n f, \Gamma_n g \rangle_{L^2}.$$

Then

$$q_{W,n}^I : \mathcal{W}_n^{\text{kin}}(\Omega; \mu) / \mathcal{N}_n^W \longrightarrow \Gamma_n \mathcal{W}_n^{\text{kin}}(\Omega; \mu), \quad [f] \longmapsto \Gamma_n f, \quad (\text{XVII.4})$$

is the canonical gauge-invariant Wilson quotient.

Proposition XVII.3 (Exact Wilson-quotient realization of the cofinal packet). *There is a center-equivariant isometric embedding*

$$\Xi_n : \mathcal{K}_n^{\text{cof}} \hookrightarrow \Gamma_n \mathcal{W}_n^{\text{kin}}(\Omega; \mu) \quad (\text{XVII.5})$$

which sends Ω to the constant function, the three infrared seed spaces to the character frames $\phi_i^{(r)} = \chi_r(g_i)$, and the ultraviolet mode $u_{r,j}$ to the class function $\chi_{\lambda_{r,j}}(g_1)$. The images are mutually orthogonal by Peter–Weyl orthogonality. Hence the reduced packet is an exact finite subpacket of the Wilson quotient, not a merely abstract matrix space.

Proof. All listed functions are invariant under residual diagonal conjugation and therefore lie in the gauge-averaged range. Distinct highest weights are orthogonal, and the weights (XVI.4) are distinct from the adjoint, fundamental, and antifundamental seed weights. Their center characters are the declared trialities. Extending linearly gives the isometric center-equivariant embedding. $\square$

C. Finite-demand late tails and quotient-natural scale arrows

A finite-demand datum $\mathfrak{D}$ is a finite collection of Wilson words, center-sector labels, and ultraviolet Peter–Weyl modes required by a downstream comparison. Let $n(\mathfrak{D})$ be the least stage at which every requested mode is present and define

$$E_{\text{cof}}(\mathfrak{D}) = \{\varepsilon_n : n \geq n(\mathfrak{D})\}. \quad (\text{XVII.6})$$

Thus the theorem is demand by demand on a declared late tail; no single tail for all possible demands is asserted.

Let $\iota_n^{n+1, \text{kin}}$ denote the cylindrical inclusion of the finite kinematical Wilson windows. Gauge equivariance implies $\iota_n^{n+1, \text{kin}} \mathcal{N}_n^W \subseteq \mathcal{N}_{n+1}^W$, so it induces a quotient inclusion

$$\bar{\iota}_n^{n+1} : \mathcal{W}_n^{\text{kin}} / \mathcal{N}_n^W \longrightarrow \mathcal{W}_{n+1}^{\text{kin}} / \mathcal{N}_{n+1}^W.$$

Put $\hat{\Xi}_n = (q_{W,n}^I)^{-1} \Xi_n$. The marked refinement is chosen so that the following correctly typed square commutes:

$$\hat{\Xi}_{n+1} J_n^{n+1} = \bar{\iota}_n^{n+1} \hat{\Xi}_n. \quad (\text{XVII.7})$$

Applying the unitary quotient maps $q_{W,n}^I$ gives the corresponding square in the physical gauge-invariant ranges. Taking adjoints gives the coarse-compression square. These relations

preserve the center action, marked time-zero carrier, meridional class, complete-ground projection, and sectorwise UV–IR–mixing profile labels. The refinement inclusion remains non-surjective.

D. The source-side branch fingerprint

For $m \in \{1, 2\}$ and a finite-demand datum $\mathfrak{D}$, define the Paper-I source-side branch key

$$\beta_{m,\mathfrak{D}}^I = (\Omega, \Sigma_{\text{pres}}^0, [\mu], m, \omega^m, E_{\text{cof}}(\mathfrak{D}), \mathfrak{D}, \{q_{W,n}^I\}_{n \geq n(\mathfrak{D})}, \{P_n^0\}_{n \geq n(\mathfrak{D})}, \{J_n^{n+1}, E_n^{n+1}\}_{n \geq n(\mathfrak{D})}). \quad (\text{XVII.8})$$

The neutral branch $m = 0$ is retained as the control branch, but it is not one of the selected linked-flux source branches.

For $n \geq n(\mathfrak{D})$ set

$$\mathfrak{P}_{n,m,\mathfrak{D}}^I = (\mathcal{K}_n^{\text{cof}}, \Xi_n, \Gamma_n \mathcal{W}_n^{\text{kin}}, q_{W,n}^I, P_n^0, H_n^{\text{cof}}, P_m, \mathfrak{c}_{m,n}, \text{Hol}_{\mu,n}^I(m)), \quad (\text{XVII.9})$$

where $\mathfrak{c}_{m,n}$ is the actual sectorwise UV–IR–mixing profile of [Theorem XVI.2](#).

Theorem XVII.4 (Paper-I selected source identity and interface readiness). *For every $m \in \{1, 2\}$ and every finite-demand datum $\mathfrak{D}$, the family*

$$\mathfrak{P}_{m,\mathfrak{D}}^I = \{\mathfrak{P}_{n,m,\mathfrak{D}}^I : n \geq n(\mathfrak{D})\} \quad (\text{XVII.10})$$

has the constant source-side fingerprint $\beta_{m,\mathfrak{D}}^I$. More precisely:

- (SI1) *every endpoint uses $SU(3)$ and the same selected center character $\chi_m(\omega) = \omega^m$;*
- (SI2) *the packet is an exact subpacket of the the corresponding gauge-invariant Wilson-quotient system ([XVII.4](#));*
- (SI3) *the complete-ground projections and center projectors intertwine with all refinement inclusions;*
- (SI4) *the regional label Ω , marked time-zero carrier Σ_{pres}^0 , and meridional class $[\mu]$ are unchanged along the late tail;*
- (SI5) *the selected source holonomy certificate is $\text{Hol}_{\mu,n}^I(m) = \omega^m \neq 1$ at every endpoint;*

- (SI6) *every endpoint carries its independently certified actual UV–IR–mixing profile and the common floor κ_{cof} ;*
- (SI7) *physical refinement is isometric and non-surjective, with the exact old-image seam comparison and independent target certification of [Theorem XVI.4](#).*

No Paper-II or Paper-III theorem is used in proving these statements.

Consequently Paper I supplies the complete source-side data required for any later selected-branch continuation. A downstream construction must preserve the Wilson quotient, center grade, complete-ground projection, marked time-zero carrier, meridional class, finite-demand late tail, actual UV–IR–mixing profiles, and declared physical scale arrows. No downstream map is inferred from repeated notation.

Proof. Statements (SI1)–(SI5) follow from Propositions [XVII.1](#) and [XVII.3](#) and the commuting square ([XVII.7](#)). Statement (SI6) is [Theorem XVI.3](#); statement (SI7) is Proposition [XVI.4](#). All constructions precede and are independent of any scale-bordism, Cauchy, incision, represented-descent, or Friedrichs theorem. $\square$

Corollary XVII.5 (No branch switching is available at the Paper-I boundary). *Any downstream interface satisfying the preservation requirements in [Theorem XVII.4](#) receives the same selected center grade, Wilson quotient, complete-ground packet, time-zero carrier label, meridional class, and cofinal coercive provenance. A change of habitat or analytic realization cannot by itself change the Paper-I source fingerprint.*

Scope Note XVII.6 (Exact boundary of the theorem). *The theorem proves the intrinsic Paper-I source fingerprint and nothing beyond it. It does not construct a scale-bordism envelope, identify the finite packet Hamiltonian with a local Friedrichs operator, use a later detector to create the source certificate, or derive a spacetime carrier or incision. Any downstream identity theorem must prove its own preservation and comparison statements.*

Scope at this boundary. The exported object is a marked finite-scale coercive packet with exact sector data, comparison profiles, and a positive branch floor. No spacetime net, thin observable system, Poincaré representation, or continuum mass threshold is constructed here. Those operations require additional selection and reconstruction layers and are owned downstream.

XVIII. INTERPRETATION, HANDOFF, AND SCOPE

A. Resolved mechanism

The finite-scale lower bound is resolved into

$$\begin{aligned}\widehat{c}_{n,\text{UV}}^\varepsilon &= \text{vacuum-orthogonal electric exclusion,} \\ \widehat{c}_{n,\text{IR}}^\theta(b) &= \text{physical-channel plus residual infrared separation,}\end{aligned}$$

and

$$\widehat{\epsilon}_{n,\text{mix}}^\varepsilon = \|K_{n,\text{ref}}^\varepsilon + K_{n,\text{tr}}^\varepsilon + K_{n,\text{col}}^\varepsilon\|.$$

Compact resolvent tells us that the finite spectrum is discrete. It does not tell us why the lower bound survives. The UV–IR–mixing block does, and it also makes the failure mode visible: coercivity is lost precisely when the mixed channel reaches the translated positive-semidefinite boundary.

B. Strict and twisted detected sectors

In a strict connected graph sector, the graph minimum consists of covariantly constant sections. In a frustrated detected sector the covariant kernel may vanish while the finite graph still has a nonzero lowest eigenspace. The comparison uses separation above the full lowest graph eigenspace. There is no universal ordering between final strict and twisted constants: vacuum survival, Gram conditioning, physical extraction, residual modes, and total leakage all contribute.

C. Physical role of the associator

The localization associator measures intermediate Wilson-window information loss. Its Hamiltonian contribution is only the transported-cell correction $K_{n,\text{tr}}^\varepsilon$. Perfect associativity would not remove ordinary reference-band mixing $K_{n,\text{ref}}^\varepsilon$, and unresolved collar geometry would leave $K_{n,\text{col}}^\varepsilon$.

D. Complete ground sector rather than a vacuum vector

All statements use the complete spectral projection $P_{n,\text{vac}}^\varepsilon$. A basis may be chosen for finite computation, but no vector is physically distinguished. The complete-ground projection is not defined to make the theorem true: it is the spectral projection of the full finite Hamiltonian, and the neutral-sector kernel identity is proved before the coercive complement is estimated.

E. The two realizations and the abstract theory

The $SU(2)$ realization is the complete two-sector validation of the nonzero-leakage mechanism recorded in the compiled source. Its historical exact matrices and independent verification path are retained in the computational reproducibility archive, and its theorem status is not replaced by the $SU(3)$ calculation. The $SU(3)$ realization carries all three center characters in one finite Hamiltonian, proves a neutral complete ground and conjugate nonneutral sectors, and supplies the exact zero-mixing seed. The cofinal family then realizes the same abstract UV–IR–mixing theorem with nonzero reference, transport, and collar terms. The two examples therefore test the same mechanism in genuinely different center geometries.

F. Finite-scale selected source-packet handoff

For every selected $m \in \{1, 2\}$ and finite-demand datum $\mathfrak{D}$, Paper I exports the typed family $\mathfrak{P}_{m,\mathfrak{D}}^I$ of [Theorem XVII.4](#). The exported data consist of the positive-thickness scale labels, regional Wilson-tube label, gauge-invariant Wilson quotient, marked time-zero carrier, meridional class, center character, complete-ground projection, bounded finite Hamiltonian, actual sectorwise UV–IR–mixing profiles, common floor κ_{cof} , and noninvertible refinement/coarse-compression arrows with independently certified endpoints. Which parts are canonical and which depend on selected admissible data are stated in the construction itself.

The companion scale-bordism and positive-thickness reconstruction papers may transport, regionalize, or reconstruct this packet^{[9](#),[10](#)}, but none of those later operations enters the present proof. A source packet does not become a spacetime object by changing notation.

G. Nonclaims

The article proves no zero-thickness limit, continuum observable net, Haag–Kastler reconstruction, arbitrary Cauchy-slice reconstruction, Poincaré covariance, relativistic spectrum condition, confinement theorem, scattering theory, or the Yang–Mills existence and mass-gap conclusion formulated in the official problem description⁸. It does not identify physical scale arrows with equivalences, transport lower bounds through one-sided seams without independent endpoint certification, identify a bounded finite packet Hamiltonian with a local Friedrichs or Mosco object, or infer a spacetime incision from the marked source carrier.

The finite-scale theorem is deliberately modest in habitat and not modest in content. It proves exactly what the later reconstruction arguments require—and no more. Nothing is gained by calling the estimate a mass gap; the result is already strong enough when stated exactly.

SUPPLEMENTARY MATERIAL

See the supplementary material for the detailed finite-dimensional certificate ledgers, explicit $SU(2)$ and $SU(3)$ witness documentation, exact matrix and characteristic-polynomial checks, machine-readable outputs, reproduction instructions, and checksum manifests supporting the finite-scale calculations reported in this article.

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USE OF GENERATIVE ARTIFICIAL INTELLIGENCE

The author used several versions of OpenAI’s ChatGPT, ranging from GPT-4.5 to GPT-5.6, as interactive research, adversarial-review, and editorial tools during the development

of this manuscript and the broader five-paper series. Their use included assistance with organizing mathematical material, stress-testing formulations and proposed proof steps, improving exposition, checking notation and cross-references, preparing and reviewing LaTeX source, and supporting computational and document-audit workflows.

The conceptual framework, mathematical ontologies, definitions, conjectures, identification of the principal obstructions, load-bearing proof components, and overall proof strategies originated with the author. The generative-AI systems did not independently originate the mathematical program, establish any theorem, or certify the correctness of any proof.

All substantive mathematical decisions, claims, definitions, theorems, proofs, computations, interpretations, citations, and final wording were determined, reviewed, and approved by the author, who assumes full responsibility for the content of the manuscript. No generative artificial-intelligence system is credited as an author.

AUTHOR DECLARATIONS

Conflict of interest. The author has no conflicts to disclose.

Author contributions. Jeffrey S Little: Conceptualization; Formal analysis; Investigation; Methodology; Software; Validation; Writing—original draft; Writing—review and editing.

DATA AND CODE AVAILABILITY

The accompanying source, supplementary material, and computational reproducibility archive include machine-readable $SU(3)$ center projectors and graph matrices, exact characteristic-polynomial checks, cofinal three-sector profile certificates, Casimir and end-point checks, the reference–transport–collar mixing split, marked-source and Wilson-quotient interface data, compilation records, and checksum manifests. The complete historical $j \leq 1$ $SU(2)$ Galerkin witness, independent verification script, constants ledger, JSON certificate, and recorded outputs are included in the accompanying computational archive.

Appendix A: An Explicit Raw $SU(2)$ Witness

1. Scope and center detection

This appendix verifies raw representation-theoretic mechanisms before physical reduction, using standard compact-group character and Schur-orthogonality conventions^{6,7}. Let $G = SU(2)$ with $Z(G) = \{I, -I\}$. For spin j , $C_2(j) = j(j+1)$, so $C_2(0) = 0$, $C_2(1/2) = 3/4$, and $C_2(1) = 2$. The fundamental character satisfies

$$\int_G \chi_{1/2}(g) dg = 0, \quad \int_G |\chi_{1/2}(g)|^2 dg = 1, \quad \chi_{1/2}(-g) = -\chi_{1/2}(g).$$

2. Raw three-channel frame

On $L^2(G^3)^{\text{Ad } G}$ define $\phi_i(g_1, g_2, g_3) = \chi_{1/2}(g_i)$. Schur orthogonality gives $\langle \phi_i, \phi_j \rangle = \delta_{ij}$, hence $F_\theta^W e_i = \phi_i$ is an isometric raw frame.

For $\theta_{ij} \in \{\pm 1\}$ define

$$V_{ij}^\theta(g) = \beta_{\text{mag}} \left(1 - \frac{1}{2} \theta_{ij} \chi_{1/2}(g_i g_j^{-1}) \right). \quad (\text{A.1})$$

Proposition A.1 (Raw $SU(2)$ compression). *The compression is*

$$(F_\theta^W)^* M_{V_{ij}^\theta} F_\theta^W = \beta_{\text{mag}} \left[I - \frac{\theta_{ij}}{4} (E_{ij} + E_{ji}) \right].$$

The maximal raw coefficient for one edge is $\beta_{ij}^{W, \text{max}} = \frac{5}{8} \beta_{\text{mag}}$.

3. Triangle spectra

For the equal-weight triangle, the strict flux choice has spectrum $\{0, 3, 3\}$, while the frustrated choice $\theta_{12} = \theta_{23} = 1$, $\theta_{31} = -1$ has spectrum $\{1, 1, 4\}$. The summed raw magnetic compression is

$$(F_\theta^W)^* H_{\Delta, \text{mag}}^\theta F_\theta^W = \frac{5}{2} \beta_{\text{mag}} I + \frac{1}{4} \beta_{\text{mag}} L_\theta.$$

Long convolution and associator calculations from the compiled appendix remain to be recovered verbatim.

Appendix B: Exact Raw $SU(3)$ Fundamental-Channel Witness

1. Representations, center character, and triality

Let

$$G = SU(3), \quad Z(G) = \{I, \omega I, \omega^2 I\}, \quad \omega = e^{2\pi i/3}.$$

For the irreducible representation of highest weight (p, q) , with standard $SU(3)$ conventions^{6,7}, the center acts by

$$\omega I \longmapsto \omega^{p-q} I,$$

with exponent understood modulo 3. The fundamental representation $\mathbf{3} = (1, 0)$ has triality 1, while $\bar{\mathbf{3}} = (0, 1)$ has triality 2.

Fix $a \in \{1, 2\}$ and write

$$\chi_a = \begin{cases} \chi_{\mathbf{3}}, & a = 1, \\ \chi_{\bar{\mathbf{3}}}, & a = 2. \end{cases}$$

Then $\chi_a(zg) = z^a \chi_a(g)$ for $z \in Z(G)$.

2. Gauge-invariant three-channel frame

Interpret g_1, g_2, g_3 as based loop holonomies with residual diagonal conjugation

$$(g_1, g_2, g_3) \longmapsto (hg_1h^{-1}, hg_2h^{-1}, hg_3h^{-1}).$$

On $L^2(G^3)^{\text{Ad } G}$ define

$$\phi_i^{(a)}(g_1, g_2, g_3) = \chi_a(g_i), \quad i = 1, 2, 3.$$

Schur orthogonality gives

$$\langle \phi_i^{(a)}, \phi_j^{(a)} \rangle = \delta_{ij},$$

so

$$F_a^W : \mathbb{C}^3 \longrightarrow L^2(G^3)^{\text{Ad } G}, \quad F_a^W e_i = \phi_i^{(a)},$$

is an isometric raw Wilson frame. Its range has triality a under simultaneous central multiplication of the three loop variables.

3. Canonical pair cells and exact compression

For an oriented graph edge (i, j) choose $\theta_{ij} \in \{1, \omega, \omega^2\}$ and set

$$V_{ij}^{(a, \theta)}(g) = \beta_{\text{mag}} \left(1 - \frac{1}{3} \text{Re}[\theta_{ij} \chi_a(g_i g_j^{-1})] \right), \quad \beta_{\text{mag}} > 0. \quad (\text{B.1})$$

Because $|\chi_a(g)| \leq 3$, this is a bounded nonnegative gauge-invariant multiplication potential. Use the compression-aligned edge vector

$$r_{ij}^\theta = e_i - \bar{\theta}_{ij} e_j, \quad \ell_{ij}^\theta = |r_{ij}^\theta\rangle \langle r_{ij}^\theta|.$$

Proposition B.1 (Raw $SU(3)$ compression and phase-independent extraction). *The exact compression of (B.1) is*

$$M_{ij}^{W, \theta} := (F_a^W)^* M_{V_{ij}^{(a, \theta)}} F_a^W = \beta_{\text{mag}} \left[I - \frac{1}{18} (\theta_{ij} E_{ij} + \bar{\theta}_{ij} E_{ji}) \right]. \quad (\text{B.2})$$

Moreover,

$$M_{ij}^{W, \theta} - b \ell_{ij}^\theta \geq 0 \quad \Longleftrightarrow \quad 0 \leq b \leq \frac{19}{36} \beta_{\text{mag}}. \quad (\text{B.3})$$

The maximal extraction coefficient is independent of the edge phase.

Proof. The diagonal entries follow from character orthogonality. The only nonzero off-diagonal entries lie in the (i, j) block. The fundamental convolution identity contributes a factor $1/3$, and the real-part normalization and the coefficient $1/3$ in (B.1) supply the remaining factor $1/6$, giving (B.2).

On $\text{span}\{e_i, e_j\}$ the matrices are

$$M_\theta = \beta_{\text{mag}} \begin{pmatrix} 1 & -\theta/18 \\ -\bar{\theta}/18 & 1 \end{pmatrix}, \quad r_\theta = \begin{pmatrix} 1 \\ -\bar{\theta} \end{pmatrix}.$$

The vector r_θ has eigenvalue $19\beta_{\text{mag}}/18$, while its orthogonal complement has eigenvalue $17\beta_{\text{mag}}/18$. Since $\|r_\theta\|^2 = 2$, subtraction of $b|r_\theta\rangle\langle r_\theta|$ is positive exactly when $19\beta_{\text{mag}}/18 - 2b \geq 0$. $\square$

Remark B.2 (Conjugation convention). *The conjugate on the second coefficient of r_{ij}^θ is load bearing. It aligns the rank-one graph form with the phase occurring in the compression matrix. Replacing it by $e_i - \theta_{ij} e_j$ compares the compression with the conjugate graph connection and can create an artificial phase dependence when $\theta_{ij}^2 \neq 1$.*

4. Triangle flux and magnetic graph spectrum

Choose the gauge representative

$$\theta_{12} = 1, \quad \theta_{23} = 1, \quad \theta_{31} = \omega^m, \quad m \in \{0, 1, 2\},$$

so the gauge-invariant flux is $\Phi_m = \omega^m$. Let

$$L_m = \ell_{12}^\theta + \ell_{23}^\theta + \ell_{31}^\theta.$$

For the strict flux $m = 0$,

$$\text{spec}(L_0) = \{0, 3, 3\}. \quad (\text{B.4})$$

For $m = 1, 2$, the characteristic polynomial is

$$\det(\lambda I - L_m) = \lambda^3 - 6\lambda^2 + 9\lambda - 3, \quad (\text{B.5})$$

and the spectrum is

$$\left\{ 2 - 2 \cos \frac{2\pi}{9}, 2 - 2 \cos \frac{4\pi}{9}, 2 - 2 \cos \frac{8\pi}{9} \right\}. \quad (\text{B.6})$$

In particular, the twisted graph has no covariantly constant vector and has a one-dimensional lowest eigenspace.

Summing the three pair-cell compressions gives the exact affine graph identity

$$(F_a^W)^* H_{\Delta, \text{mag}}^{(a, m)} F_a^W = \frac{26}{9} \beta_{\text{mag}} I + \frac{1}{18} \beta_{\text{mag}} L_m. \quad (\text{B.7})$$

Scope Note B.3. *Equations (B.2)–(B.7) are the raw representation-theoretic layer. The complete-ground projection and finite-packet coercive statement are proved separately in [Section XV](#); they are not inferred by identifying the graph-minimum space with the physical vacuum.*

Appendix C: Complete Physicalized $j \leq 1$ $SU(2)$ Galerkin Verification

The compiled article states that a complete $j \leq 1$ Peter–Weyl Galerkin realization computes the interacting ground projection, Gram operator, physical recompression, exact reference leakage, and the explicit resolved lower bound

$$\widehat{\kappa}_G = 0.657490679870 \dots$$

The exact matrices, basis ordering, constants ledger, high-precision output, and independent verification script are supplied in the accompanying computational reproducibility archive. The archive retains both the direct computation and an independent verification path.

The verification ledger for this appendix is:

- (C1) recover the exact finite basis and matrix entries;
- (C2) identify the complete ground projection directly from the full Galerkin Hamiltonian;
- (C3) reproduce vacuum removal and the surviving coefficient rank;
- (C4) reconstruct the captured electric projection and Gram matrix;
- (C5) recompute every physical magnetic cell;
- (C6) reproduce the exact reference, transport, and collar blocks;
- (C7) verify the final eigenvalue bound with exact or interval arithmetic.

Appendix D: Finite-Dimensional Comparison Lemmas

1. Schur-complement form of the two-block bound

For the Hermitian matrix

$$\mathbf{K} = \begin{pmatrix} c_1 & -\eta \\ -\eta & c_2 \end{pmatrix},$$

the smaller eigenvalue is

$$\lambda_-(\mathbf{K}) = \frac{c_1 + c_2 - \sqrt{(c_1 - c_2)^2 + 4\eta^2}}{2}.$$

It is positive exactly when $c_1 > 0$, $c_2 > 0$, and $\eta^2 < c_1 c_2$.

2. Finite capture lemma

If a finite-dimensional subspace injects into the domain of a nonnegative compact-resolvent operator after removal of a finite-dimensional kernel, then a sufficiently large spectral cutoff captures that subspace injectively. This is the finite-rank compactness step used to construct $J_{n,\theta}^\varepsilon$.

3. Gram normalization

For an injective map $J : C \rightarrow \mathcal{H}$ from a finite-dimensional Hilbert space, $G = J^*J$ is positive definite and $JG^{-1/2}$ is an isometry. All compressed operators must be recomputed under this normalized frame.

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