Coarse Transport, Central Twist, and Finite-Scale Coercivity for Yang–Mills Wilson-Tube Hamiltonians: Physical Wilson-Channel Reduction and Exact Ultraviolet–Infrared Leakage

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We prove a resolved finite-scale coercivity criterion for canonical gauge-invariant Yang–Mills Wilson-tube Hamiltonians. The ground sector is treated through the complete spectral projection $P^{\varepsilon}_{n,\mathrm{vac}}$, with no distinguished vacuum vector or rank-one assumption. Starting from a finite center-sensitive Wilson graph, we remove every raw graph-excitation direction lying in the exact interacting ground sector, capture the surviving channel inside a finite spectral subspace of the vacuum-orthogonal electric form operator, and Gram-normalize it. Vacuum orthogonality and electric admissibility are therefore conclusions rather than hypotheses. Recompression of the canonical magnetic cells yields a physical infrared constant. The remaining ultraviolet–infrared magnetic block decomposes exactly into reference-band mixing, localized-transport associator leakage, and collar leakage. Strict subcriticality of this mixed block gives

$$H^{\varepsilon}_{n} \geq E^{\varepsilon}_{0,n} I + \widehat{\kappa}_{n,\varepsilon,\theta}(b)(I - P^{\varepsilon}_{n,\mathrm{vac}}), \qquad \widehat{\kappa}_{n,\varepsilon,\theta}(b) > 0.$$

A complete $j \leq 1$ $SU(2)$ Peter–Weyl Galerkin realization computes the ground projection, Gram operator, physical recompression, exact reference leakage, and the explicit bound $\widehat{\kappa}_G = 0.657490679870\ldots$. In addition, an exact fixed-scale $SU(3)$ triangle packet is constructed in the two nontrivial triality grades. Its complete ground projection is the neutral vacuum line and its exact selected-flux gap is

$$\Delta_{\triangle}^{SU(3)} = \frac{12 - \cos(2\pi/9)}{9},$$

which proves the retained lower bound $\kappa_{\mathrm{cof}} = \frac{1}{2}(\frac{17}{5} - \sqrt{\frac{557}{75}})$. The results are finite-scale and time-zero; no continuum or relativistic mass-gap claim is made.

Keywords: lattice gauge Hamiltonians; Wilson observables; central twist; magnetic graph Laplacian; quadratic-form coercivity; spectral projection; coarse transport

Reconstruction and augmentation status. The inherited article is a provenance-marked source reconstruction from the compiled 53-page reference and an older TeX chassis; it is not represented as recovery of the missing original source. The $SU(3)$ triangle packet in Section XII is new mathematical work, proved inside this package and independently certified by exact symbolic calculations. Unrecovered historical calculations are isolated rather than guessed.

I. INTRODUCTION

A. The finite-scale spectral problem

For a compact gauge group on a finite spatial complex, the regulated electric Hamiltonian has discrete representation spectrum, the Wilson magnetic potential is bounded, and the complete Hamiltonian has compact resolvent. Abstract discreteness does not by itself resolve the mechanisms producing separation above the interacting ground sector. The present criterion separates the lower bound into

$$\begin{gathered}\text{electric exclusion}\\ +\text{physicalized magnetic separation}\\ > \text{reference, transport, and collar leakage.}\end{gathered}$$

The resulting estimate is relative to the complete ground projection,

$$H_n^{\varepsilon} - E_{0,n}^{\varepsilon} I \geq \widehat{\kappa}_{n,\varepsilon,\theta}(b)(I - P_{n,\mathrm{vac}}^{\varepsilon}). \tag{I.1}$$

No normalized vector in the possibly degenerate ground eigenspace is selected.

B. Graph minima and physical vacua

Detected center phases define a finite magnetic graph operator $L_\theta = d_\theta^* d_\theta$. Its complete lowest spectral projection $P_{\theta,\min}$ acts on a coefficient space. By contrast, $P_{n,\mathrm{vac}}^{\varepsilon}$ acts on the gauge-invariant Hilbert space and is determined by the complete electric-plus-magnetic Hamiltonian. The two projections are not identified.

C. Physicalization of a raw Wilson channel

Let $F^W_{n,\theta} : C^0_n \to \mathcal{H}^\varepsilon_n(\Omega)$ be an isometric raw Wilson frame and set

$$E^W_{n,\theta} = \mathrm{Ran}(I - P_{\theta,\min}), \qquad T_{n,\theta} = (I - P^\varepsilon_{n,\mathrm{vac}})F^W_{n,\theta}|_{E^W_{n,\theta}}.$$

After deleting $\mathrm{Ker}\, T_{n,\theta}$, define

$$C^{\mathrm{phys}}_{n,\theta} = E^W_{n,\theta} \ominus \mathrm{Ker}\, T_{n,\theta}.$$

A finite spectral projection of the vacuum-orthogonal electric operator captures the image injectively. If J is the captured map and $G = J^*J$, then

$$F^{\mathrm{phys}}_{n,\theta} = JG^{-1/2}$$

is an isometry with $P^\varepsilon_{n,\mathrm{vac}}F^{\mathrm{phys}}_{n,\theta} = 0$ and $P^\varepsilon_{n,\mathrm{IR}}F^{\mathrm{phys}}_{n,\theta} = F^{\mathrm{phys}}_{n,\theta}$.

D. Exact mixed-block decomposition

With an interior–transported–collar partition, the magnetic ultraviolet–infrared block decomposes exactly as

$$P^\varepsilon_{n,\mathrm{UV}}H^\varepsilon_{n,\mathrm{mag}}P^\varepsilon_{n,\mathrm{IR}} = K^\varepsilon_{n,\mathrm{ref}} + K^\varepsilon_{n,\mathrm{tr}} + K^\varepsilon_{n,\mathrm{col}}. \tag{I.2}$$

The transport term measures the difference between two-step localized transport and a direct comparator; it is not the entire mixed block.

E. Principal constant and scope

If $\widehat{c}^\varepsilon_{n,\mathrm{UV}} > 0$, $\widehat{c}^\theta_{n,\mathrm{IR}}(b) > 0$, and $(\widehat{\epsilon}^\varepsilon_{n,\mathrm{mix}})^2 < \widehat{c}^\varepsilon_{n,\mathrm{UV}}\widehat{c}^\theta_{n,\mathrm{IR}}(b)$, then

$$\widehat{\kappa}_{n,\varepsilon,\theta}(b) = \frac{\widehat{c}^\varepsilon_{n,\mathrm{UV}} + \widehat{c}^\theta_{n,\mathrm{IR}}(b) - \sqrt{(\widehat{c}^\varepsilon_{n,\mathrm{UV}} - \widehat{c}^\theta_{n,\mathrm{IR}}(b))^2 + 4(\widehat{\epsilon}^\varepsilon_{n,\mathrm{mix}})^2}}{2} > 0. \tag{I.3}$$

The theorem is finite-scale and time-zero. It constructs no continuum measure, Haag–Kastler net, Poincaré-covariant vacuum representation, confinement theorem, or relativistic mass gap.

F. New fixed-scale $SU(3)$ closure

The reconstructed source contained an exact raw fundamental-channel $SU(3)$ triangle but no complete physicalized $SU(3)$ packet. Section XII supplies that missing fixed-scale example without importing a theorem from a later paper. The two nontrivial triality grades are realized by conjugate finite Peter–Weyl packets. Their complete ground projection is computed from the full packet Hamiltonian, the vacuum-removal and Gram steps are exact, and the selected-flux spectral gap is obtained symbolically. The result is a fixed-scale seed only; cofinal scale transport and downstream branch identity remain separate obligations.

II. CANONICAL FINITE-SCALE WILSON-TUBE SYSTEMS

A. Configuration space and Gauss law

Let $\Omega \subset \mathbb{R}^3$ be bounded and let $\mathcal{L}_n(\Omega)$ be a finite oriented cell complex with positively oriented edge set $E_n^+(\Omega)$, vertex set $V_n(\Omega)$, and magnetic-cell set $F_n(\Omega)$. For a compact connected Lie group G,

$$X_n(\Omega) = G^{E_n^+(\Omega)}, \qquad \mu_n = \bigotimes_{e \in E_n^+(\Omega)} dg_e.$$

The finite gauge group acts by $(h \cdot U)_e = h_{s(e)} U_e h_{t(e)}^{-1}$. Haar averaging gives the orthogonal Gauss projection Γ_n, and

$$\mathcal{H}_n(\Omega) = \operatorname{Ran} \Gamma_n = L^2(X_n, \mu_n)^{\mathcal{G}_n(\Omega)}. \tag{II.1}$$

B. Electric and magnetic Hamiltonians

For positive weights $\eta_{n,e}^{\mathrm{el}}$ and coupling g_n, define

$$\widetilde{H}_{n,\mathrm{el}} = \frac{g_n^2}{2} \sum_{e \in E_n^+(\Omega)} \eta_{n,e}^{\mathrm{el}} \Delta_e. \tag{II.2}$$

It commutes with Γ_n and restricts to the physical electric operator $H_{n,\mathrm{el}}$. For each magnetic cell choose a bounded gauge-invariant self-adjoint potential $V_{n,f}^{\varepsilon} \geq 0$ and set

$$h_{n,f}^{\varepsilon} = \omega_{n,f}^{\mathrm{mag}} M_{V_{n,f}^{\varepsilon}}, \qquad H_{n,\mathrm{mag}}^{\varepsilon} = \sum_{f \in F_n(\Omega)} h_{n,f}^{\varepsilon}.$$

Then

$$H_n^\varepsilon = H_{n,\mathrm{el}} + H_{n,\mathrm{mag}}^\varepsilon, \qquad D(H_n^\varepsilon) = D(H_{n,\mathrm{el}}). \tag{II.3}$$

Theorem II.1 (Finite-scale canonical Hamiltonian)**.** *The operator H_n^ε is nonnegative, self-adjoint, gauge invariant, and has compact resolvent on $\mathcal{H}_n(\Omega)$. Its lowest spectral value $E_{0,n}^\varepsilon$ is an eigenvalue of finite multiplicity, and*

$$P_{n,\mathrm{vac}}^\varepsilon = \mathbf{1}_{\{E_{0,n}^\varepsilon\}}(H_n^\varepsilon)$$

is a finite-rank projection.

Proof. Peter–Weyl theory gives the self-adjoint compact-resolvent electric operator. The magnetic term is bounded and self-adjoint. Bounded perturbation preserves self-adjointness, domain equality, and compact resolvent; the spectral assertions follow. □

C. Finite Wilson windows

A finite Wilson window is a finite-dimensional unital involution-invariant subspace $\mathcal{W}_n^\varepsilon(\Omega) \subset \mathfrak{O}_n^{\mathrm{gi}}(\Omega)$. These windows are comparison data and do not modify the canonical Hamiltonian.

III. EXACT COARSE TRANSPORT AND FINITE WILSON LOCALIZATION

A. Haar-compatible microscopic refinement

For $m \preceq n$, let $p_{mn} : X_n(\Omega) \to X_m(\Omega)$ be a measurable coarse-holonomy map satisfying exact composition, measure preservation, and gauge equivariance. Cylindrical pullback

$$\iota_{mn} A = A \circ p_{mn}$$

is an isometric unital $*$-monomorphism, and its L^2 adjoint $\mathbb{E}_{mn} = \iota_{mn}^*$ is fiberwise Haar conditional expectation.

Theorem III.1 (Exact microscopic coarse transport)**.** *The maps $\mathbb{E}_{mn}$ are unital, positive, adjoint preserving, and contractive in L^2 and L^∞. They satisfy*

$$\mathbb{E}_{\ell m}\mathbb{E}_{mn} = \mathbb{E}_{\ell n}.$$

B. Finite Wilson localization and associator leakage

Let L_n^Ω denote orthogonal localization onto the finite Wilson window and $\jmath_n^\Omega$ its inclusion. The effective Wilson transport is

$$\Lambda_{mn}^\Omega = L_m^\Omega \mathbb{E}_{mn} \jmath_n^\Omega.$$

For $\ell \preceq m \preceq n$, the failure of strict composition is an intermediate-scale localization error. In the source notation the associator has the exact factorization

$$\alpha_{\ell mn}^\Omega = L_\ell^\Omega \mathbb{E}_{\ell m} \Lambda_m^\Omega \mathbb{E}_{mn} \jmath_n^\Omega. \tag{III.1}$$

This identity is later used only to estimate the transported-cell correction; it does not make physical scale transport invertible.

IV. CENTRAL SECTOR DATA AND DETECTED MAGNETIC GRAPHS

A. Center-valued cocycles

Finite Wilson trivialization data determine a center-valued simplicial 2-cocycle $\zeta_n \in Z^2(N_n; Z(G))$. For a center character $\zeta_R : Z(G) \to \mathbb{T}$, detection sends cohomologous center cocycles to cohomologous circle-valued cocycles.

Proposition IV.1 (Detection commutes with refinement)**.** *For a refinement map $r_{mn} : N_n \to N_m$,*

$$\zeta_R(r_{mn}^* \zeta_m) = r_{mn}^* \zeta_R(\zeta_m).$$

Thus center detection commutes with simplicial pullback.

B. Detected graph realization

A detected graph realization consists of a finite connected weighted graph $\mathcal{G}_\theta = (V_\theta, E_\theta, a)$, a transgression rule to a phase cochain θ, and covariance under vertex rephasing. The magnetic graph operator is

$$L_\theta = d_\theta^* d_\theta = \sum_{\{v,w\} \in E_\theta} a_{vw} \ell_{vw}^\theta.$$

Let $\lambda_1(\theta)$ be its lowest eigenvalue, $P_{\theta,\min}$ the full lowest spectral projection, $m_1 = \operatorname{rank} P_{\theta,\min}$, and

$$\delta_{\sec}(\theta) = \lambda_{m_1+1}(\theta) - \lambda_1(\theta).$$

Then

$$L_\theta - \lambda_1(\theta) I \geq \delta_{\sec}(\theta)(I - P_{\theta,\min}). \tag{IV.1}$$

The theorem uses separation above the complete graph-minimum space, not the baseline eigenvalue alone.

V. RAW WILSON CHANNELS AND MAGNETIC DOMINATION

A. Raw frame

Let $F^W_{n,\theta} : C^0_n \to \mathcal{H}^\varepsilon_n(\Omega)$ be an isometric frame of gauge-invariant Wilson observables. For each canonical cell define the raw compression

$$M^W_{n,f} = (F^W_{n,\theta})^* h^\varepsilon_{n,f} F^W_{n,\theta} \geq 0.$$

B. Disjoint positive edge extraction

Choose an active graph-edge set and a disjoint allocation of canonical magnetic cells. For each active edge define the largest coefficient for which the cell compression dominates the associated edge Laplacian. Summing over the disjoint allocation gives a global raw coefficient $b^{W,\max}_{n,\mathrm{ch}}$.

Proposition V.1 (Raw magnetic domination)**.** *If every required graph edge is active and* $b^{W,\max}_{n,\mathrm{ch}} > 0$*, then for every* $0 \leq b \leq b^{W,\max}_{n,\mathrm{ch}}$*,*

$$\sum_{f \in F_n(\Omega)} M^W_{n,f} \geq b L_\theta.$$

Scope Note V.2. *Positive extraction on the raw frame does not imply positive extraction after exact vacuum removal, finite electric capture, and Gram normalization. The canonical cell operators must be recompressed on the physicalized frame.*

VI. VACUUM-ORTHOGONAL ELECTRIC SPECTRAL REDUCTION

A. Restricted electric form

Set

$$\mathcal{V}^{\varepsilon}_{n,\mathrm{vac}} = \operatorname{Ran} P^{\varepsilon}_{n,\mathrm{vac}}, \qquad \mathcal{H}^{\varepsilon}_{n,\perp} = \operatorname{Ran}(I - P^{\varepsilon}_{n,\mathrm{vac}}).$$

The vacuum projection need not commute with $H_{n,\mathrm{el}}$. Define instead

$$D(Q^{\perp}_{n,\mathrm{el}}) = D(Q_{n,\mathrm{el}}) \cap \mathcal{H}^{\varepsilon}_{n,\perp}, \qquad Q^{\perp}_{n,\mathrm{el}}[\psi] = Q_{n,\mathrm{el}}[\psi].$$

Lemma VI.1 (Form-domain preservation)**.** *The finite-rank projection $P^{\varepsilon}_{n,\mathrm{vac}}$ maps $D(Q_{n,\mathrm{el}})$ into itself.*

Theorem VI.2 (Vacuum-orthogonal electric operator)**.** *The restricted form $Q^{\perp}_{n,\mathrm{el}}$ is densely defined, closed, and nonnegative on $\mathcal{H}^{\varepsilon}_{n,\perp}$. Its associated operator $H^{\perp}_{n,\mathrm{el}}$ is nonnegative, self-adjoint, and has compact resolvent.*

B. Finite electric capture

Let $\Pi^{\perp}_{n,\Lambda} = \mathbf{1}_{[0,\Lambda]}(H^{\perp}_{n,\mathrm{el}})$. Because the vacuum-removed raw map has finite-dimensional injective domain after kernel deletion, a sufficiently large finite cutoff captures it injectively. The captured map $J^{\varepsilon}_{n,\theta}$ has positive Gram operator

$$G^{\varepsilon}_{n,\theta} = (J^{\varepsilon}_{n,\theta})^{*} J^{\varepsilon}_{n,\theta} > 0,$$

and the normalized frame is $F^{\mathrm{phys}}_{n,\theta} = J^{\varepsilon}_{n,\theta}(G^{\varepsilon}_{n,\theta})^{-1/2}$.

C. Ultraviolet exclusion

The retained electric projection is $P^{\varepsilon}_{n,\mathrm{IR}} = \Pi^{\perp}_{n,\Lambda}$ on $\mathcal{H}^{\varepsilon}_{n,\perp}$, extended by zero on the vacuum space, and

$$P^{\varepsilon}_{n,\mathrm{UV}} = I - P^{\varepsilon}_{n,\mathrm{vac}} - P^{\varepsilon}_{n,\mathrm{IR}}.$$

The spectral separation above the retained electric band yields a vacuum-renormalized constant $\widehat{c}^{\varepsilon}_{n,\mathrm{UV}} > 0$ under the explicit source hypotheses.

VII. PHYSICAL WILSON-CHANNEL REDUCTION

A. Surviving coefficient space and Gram normalization

The surviving coefficient space is

$$C_{n,\theta}^{\mathrm{phys}} = \mathrm{Ran}(I - P_{\theta,\min}) \ominus \mathrm{Ker}\big((I - P_{n,\mathrm{vac}}^{\varepsilon})F_{n,\theta}^{W}\big).$$

The physical frame $F_{n,\theta}^{\mathrm{phys}}$ is isometric, vacuum orthogonal, and contained in the retained electric block.

B. Physicalized graph operator

Let $\mathsf{P}_{n,\theta}^{\mathrm{phys}}$ denote the coefficient projection. Define

$$A_{n,\theta}^{\mathrm{raw}} = \mathsf{P}_{n,\theta}^{\mathrm{phys}}(L_\theta - \lambda_1(\theta)I)\mathsf{P}_{n,\theta}^{\mathrm{phys}},$$

and

$$\widetilde{A}_{n,\theta}^{\varepsilon} = (G_{n,\theta}^{\varepsilon})^{-1/2} A_{n,\theta}^{\mathrm{raw}} (G_{n,\theta}^{\varepsilon})^{-1/2} \geq \delta_{\mathrm{sec}}(\theta)(G_{n,\theta}^{\varepsilon})^{-1}. \tag{VII.1}$$

C. Physical magnetic recompression

For each canonical cell recompute

$$M_{n,f}^{\mathrm{phys}} = (F_{n,\theta}^{\mathrm{phys}})^{*} h_{n,f}^{\varepsilon} F_{n,\theta}^{\mathrm{phys}} \geq 0.$$

A disjoint physical allocation gives a positive extraction coefficient when the corresponding finite matrix inequalities hold.

D. Channel coercivity

The exact finite-dimensional form identity has the source form

$$\widehat{H}_{n,\mathrm{ch}}^{\mathrm{phys}} = b\widetilde{A}_{n,\theta}^{\varepsilon} + R_{n,\mathrm{sec}}^{\mathrm{phys}}(b).$$

Set

$$r_{n,\mathrm{sec}}^{\mathrm{phys}}(b) = \|[R_{n,\mathrm{sec}}^{\mathrm{phys}}(b)]_{-}\|, \qquad c_{n,\mathrm{ch}}^{\mathrm{phys}}(b) = b\delta_{\mathrm{sec}}(\theta) - r_{n,\mathrm{sec}}^{\mathrm{phys}}(b).$$

Theorem VII.1 (Physical Wilson-channel coercivity)**.** *If* $c^{\mathrm{phys}}_{n,\mathrm{ch}}(b) > 0$*, then for every* $s \in \operatorname{Ran} F^{\mathrm{phys}}_{n,\theta}$*,*

$$\widehat{Q}^{\varepsilon}_{n}[s] \geq c^{\mathrm{phys}}_{n,\mathrm{ch}}(b)\|s\|^2.$$

E. Residual infrared sector

Decompose

$$\operatorname{Ran} P^{\varepsilon}_{n,\mathrm{IR}} = \mathcal{S}^{\mathrm{phys}}_{n,\theta} \oplus \mathcal{R}^{\mathrm{IR}}_{n,\theta}.$$

The residual diagonal constant and channel–residual mixing are combined by the same two-block estimate used later, yielding the complete infrared constant $\widehat{c}^{\theta}_{n,\mathrm{IR}}(b) > 0$ when the residual subcriticality condition holds. If the residual block vanishes, $\widehat{c}^{\theta}_{n,\mathrm{IR}}(b) = c^{\mathrm{phys}}_{n,\mathrm{ch}}(b)$.

VIII. EXACT ULTRAVIOLET–INFRARED MIXED BLOCK

A. Three-part decomposition

Let the canonical magnetic cells be partitioned into reference, transported, and collar classes. The exact mixed block is

$$K^{\varepsilon}_{n} = P^{\varepsilon}_{n,\mathrm{UV}} H^{\varepsilon}_{n,\mathrm{mag}} P^{\varepsilon}_{n,\mathrm{IR}} = K^{\varepsilon}_{n,\mathrm{ref}} + K^{\varepsilon}_{n,\mathrm{tr}} + K^{\varepsilon}_{n,\mathrm{col}}. \tag{VIII.1}$$

Define

$$\widehat{\epsilon}^{\varepsilon}_{n,\mathrm{mix}} = \|K^{\varepsilon}_{n}\|, \qquad \epsilon^{\varepsilon}_{n,\mathrm{ref}} = \|K^{\varepsilon}_{n,\mathrm{ref}}\|.$$

Exact reference-band closure is the special case $\epsilon^{\varepsilon}_{n,\mathrm{ref}} = 0$ and is not assumed.

B. Transport and collar majorants

Finite-window transport estimates give

$$\|K^{\varepsilon}_{n,\mathrm{tr}}\| \leq C^{\varepsilon}_{n,\mathrm{tr}} D^{\varepsilon}_{n,\mathrm{lax}}, \qquad \|K^{\varepsilon}_{n,\mathrm{col}}\| \leq C^{\varepsilon}_{n,\mathrm{col}} \Delta^{\varepsilon}_{n,\mathrm{col}}.$$

Hence the hybrid sufficient majorant

$$\epsilon^{\varepsilon}_{n,\mathrm{hyb}} = \epsilon^{\varepsilon}_{n,\mathrm{ref}} + C^{\varepsilon}_{n,\mathrm{tr}} D^{\varepsilon}_{n,\mathrm{lax}} + C^{\varepsilon}_{n,\mathrm{col}} \Delta^{\varepsilon}_{n,\mathrm{col}} \tag{VIII.2}$$

satisfies $\widehat{\epsilon}^{\varepsilon}_{n,\mathrm{mix}} \leq \epsilon^{\varepsilon}_{n,\mathrm{hyb}}$.

Theorem VIII.1 (Exact mixed-block estimate)**.** *The physical ultraviolet–infrared mixed norm is exactly* $\widehat{\epsilon}^{\varepsilon}_{n,\mathrm{mix}} = \|K^{\varepsilon}_{n,\mathrm{ref}} + K^{\varepsilon}_{n,\mathrm{tr}} + K^{\varepsilon}_{n,\mathrm{col}}\|$ *and is bounded above by the hybrid majorant in Equation (VIII.2). No interior or direct-comparator block-closure hypothesis is required.*

IX. COMPLETE VACUUM REDUCTION AND PHYSICAL EXCITATION BLOCKS

A. Vacuum-renormalized form

Set

$$\widehat{H}^{\varepsilon}_{n} = H^{\varepsilon}_{n} - E^{\varepsilon}_{0,n} I \geq 0.$$

Since $\operatorname{Ker} \widehat{H}^{\varepsilon}_{n} = \operatorname{Ran} P^{\varepsilon}_{n,\mathrm{vac}}$, the associated sesquilinear form satisfies

$$\widehat{q}^{\varepsilon}_{n}(v, \psi) = 0$$

for $v \in \operatorname{Ran} P^{\varepsilon}_{n,\mathrm{vac}}$ and $\psi \in D(\widehat{Q}^{\varepsilon}_{n})$.

B. Exact three-space decomposition

The projections satisfy

$$I = P^{\varepsilon}_{n,\mathrm{UV}} + P^{\varepsilon}_{n,\mathrm{IR}} + P^{\varepsilon}_{n,\mathrm{vac}}$$

orthogonally and preserve the form domain. For $\psi \in D(\widehat{Q}^{\varepsilon}_{n})$, set $u = P^{\varepsilon}_{n,\mathrm{UV}}\psi$, $w = P^{\varepsilon}_{n,\mathrm{IR}}\psi$, and $v = P^{\varepsilon}_{n,\mathrm{vac}}\psi$. Then

$$\widehat{Q}^{\varepsilon}_{n}[\psi] = \widehat{Q}^{\varepsilon}_{n}[u] + \widehat{Q}^{\varepsilon}_{n}[w] + 2\operatorname{Re} \widehat{q}^{\varepsilon}_{n}(u, w), \tag{IX.1}$$

and the electric cross term vanishes, so

$$\widehat{q}^{\varepsilon}_{n}(u, w) = \langle u, H^{\varepsilon}_{n,\mathrm{mag}} w\rangle = \langle u, K^{\varepsilon}_{n} w\rangle.$$

Theorem IX.1 (Exact excitation reduction)**.** *For every* $\psi \in D(\widehat{Q}^{\varepsilon}_{n})$*, the complete ground component drops out and the form reduces exactly to Equation (IX.1), with mixed term governed by* K^{ε}_{n}*.*

Remark IX.2. *The graph-minimum projection, surviving coefficient projection, retained electric projection, and physical vacuum projection act on different spaces or encode different structures and are never identified.*

X. PRINCIPAL FINITE-SCALE COERCIVITY THEOREM

A. Two-block lemma

Lemma X.1 (Hermitian two-block lower bound). *Let $\mathcal{K} = \mathcal{K}_1 \oplus \mathcal{K}_2$ and suppose a nonnegative quadratic form satisfies*

$$q[x+y] \geq c_1\|x\|^2 + c_2\|y\|^2 - 2\eta\|x\|\|y\|.$$

Then

$$q[z] \geq \kappa\|z\|^2, \qquad \kappa = \frac{c_1 + c_2 - \sqrt{(c_1 - c_2)^2 + 4\eta^2}}{2}.$$

In particular, $\kappa > 0$ when $\eta^2 < c_1 c_2$.

Corollary X.2 (Relative leakage bound). *If $\eta^2 \leq \rho c_1 c_2$ with $0 \leq \rho < 1$, then*

$$\kappa \geq \frac{1-\rho}{2}\min\{c_1, c_2\}.$$

B. Reduced physical comparison package

A verified fixed-scale package contains: the canonical finite Hamiltonian and complete ground projection; a raw Wilson frame and graph with nonzero excitation complement; nontrivial physical channel survival; finite electric capture and positive ultraviolet exclusion; positive physical magnetic extraction; positive channel calibration and residual infrared control; an interior–transported–collar partition; quantified reference leakage; exact transported-cell comparator identities; and unital involution-invariant finite Wilson windows.

Theorem X.3 (Finite-scale Wilson-tube coercivity). *Assume the reduced physical comparison package and*

$$(\widehat{c}^{\varepsilon}_{n,\mathrm{mix}})^2 < \widehat{c}^{\varepsilon}_{n,\mathrm{UV}}\widehat{c}^{\theta}_{n,\mathrm{IR}}(b). \tag{X.1}$$

Then

$$H^{\varepsilon}_n \geq E^{\varepsilon}_{0,n} I + \widehat{\kappa}_{n,\varepsilon,\theta}(b)(I - P^{\varepsilon}_{n,\mathrm{vac}}), \tag{X.2}$$

where

$$\widehat{\kappa}_{n,\varepsilon,\theta}(b) = \frac{\widehat{c}^{\varepsilon}_{n,\mathrm{UV}} + \widehat{c}^{\theta}_{n,\mathrm{IR}}(b) - \sqrt{(\widehat{c}^{\varepsilon}_{n,\mathrm{UV}} - \widehat{c}^{\theta}_{n,\mathrm{IR}}(b))^2 + 4(\widehat{c}^{\varepsilon}_{n,\mathrm{mix}})^2}}{2} > 0. \tag{X.3}$$

Equivalently,

$$\widehat{Q}_n^{\varepsilon}[\psi] \geq \widehat{\kappa}_{n,\varepsilon,\theta}(b)\|(I - P_{n,\mathrm{vac}}^{\varepsilon})\psi\|^2$$

for every $\psi \in D(\widehat{Q}_n^{\varepsilon})$.

Proof. Use the exact excitation reduction, the ultraviolet lower bound, the complete infrared lower bound, and the exact mixed norm. Apply Theorem X.1. □

Corollary X.4 (Hybrid sufficient condition)**.** *It is sufficient that* $(\epsilon_{n,\mathrm{hyb}}^{\varepsilon})^2 < \widehat{c}_{n,\mathrm{UV}}^{\varepsilon}\widehat{c}_{n,\mathrm{IR}}^{\theta}(b)$. *Replacing the exact mixed norm in Equation (X.3) by the hybrid majorant gives a valid, possibly weaker, constant.*

C. Finite-scale spectral consequence

Compact resolvent and Equation (X.2) imply

$$\mathrm{spec}(H_n^{\varepsilon}) \cap (E_{0,n}^{\varepsilon}, E_{0,n}^{\varepsilon} + \widehat{\kappa}_{n,\varepsilon,\theta}(b)) = \varnothing.$$

This is a resolved finite-scale gap above the complete ground spectral value, not a relativistic Yang–Mills mass gap.

XI. UNIFORM PHYSICALIZED COERCIVITY AND REFINEMENT HANDOFF

A. Directed finite-scale families

Let $(\mathfrak{G}, \preceq)$ be directed, and let γ index verified finite-scale packages. No common ambient Hilbert space is required for the numerical statement.

Theorem XI.1 (Uniform finite-scale coercivity)**.** *Suppose*

$$\widehat{c}_{\mathrm{UV},\gamma} \geq c_{\mathrm{UV},*} > 0, \qquad \widehat{c}_{\mathrm{IR},\gamma} \geq c_{\mathrm{IR},*} > 0,$$

and

$$\widehat{\epsilon}_{\mathrm{mix},\gamma}^2 \leq \rho\widehat{c}_{\mathrm{UV},\gamma}\widehat{c}_{\mathrm{IR},\gamma}, \qquad 0 \leq \rho < 1.$$

Then

$$\inf_{\gamma} \widehat{\kappa}_{\gamma} \geq \frac{1-\rho}{2}\min\{c_{\mathrm{UV},*}, c_{\mathrm{IR},*}\} > 0.$$

B. Uniform comparison criterion

Uniform graph separation, physical-rank survival, vacuum-angle control, electric capture, Gram conditioning, physical extraction and calibration, residual control, ultraviolet exclusion, and a uniform three-part leakage estimate imply Theorem XI.1.

C. Failure modes and handoff

The architecture separates collapse of graph separation, physical rank, vacuum angle, electric capture, Gram conditioning, physical extraction, calibration, residual infrared coercivity, reference-band control, transport or collar control, and final subcriticality. Uniform objectwise coercivity alone constructs no limiting Hilbert space or generator. A later theorem must provide comparison maps, convergence of forms and ground projections, and an explicit no-leakage or norm-recovery mechanism.

XII. A COMPLETE FIXED-SCALE $SU(3)$ TRIANGLE PACKET

A. Purpose and exact scope

We now close the raw $SU(3)$ triangle of Appendix B into an exact finite Peter–Weyl Galerkin packet. This is a fixed-scale, fixed-positive-thickness realization. It proves the complete-ground projection and a coercive estimate on the entire finite packet. It does not construct a cofinal scale family, transport a lower bound through a controlled seam, or identify the packet with a local Friedrichs or thin-limit Hamiltonian.

Fix $a \in \{1, 2\}$ and use the triality-a character χ_a of Appendix B. Choose the graph flux

$$\theta_{12} = \theta_{23} = 1, \qquad \theta_{31} = \omega^a.$$

Thus the fundamental and antifundamental packets carry the two nontrivial $\mathbb{Z}_3$ characters and conjugate central fluxes.

B. Finite packet and exact operator

Let

$$\Omega(g_1, g_2, g_3) = 1, \qquad \mathcal{S}_a = \operatorname{span}\{\phi_1^{(a)}, \phi_2^{(a)}, \phi_3^{(a)}\},$$

and define the four-dimensional gauge-invariant packet

$$\mathcal{K}_{\triangle,a} = \mathbb{C}\Omega \oplus \mathcal{S}_a \subset L^2(SU(3)^3)^{\mathrm{Ad}\, SU(3)}.$$

The displayed decomposition is orthogonal. The vacuum candidate has triality 0, while $\mathcal{S}_a$ has triality a.

Use the dimensionless normalization in which the electric energy of one fundamental or antifundamental character is its quadratic Casimir,

$$C_2(\mathbf{3}) = C_2(\overline{\mathbf{3}}) = \frac{4}{3},$$

and set $\beta_{\mathrm{mag}} = 1$. Let $P_{\triangle,a}$ be the orthogonal projection onto $\mathcal{K}_{\triangle,a}$. The finite packet Hamiltonian is the Galerkin compression

$$H^{\mathrm{G}}_{\triangle,a} = P_{\triangle,a} \left(H_{\mathrm{el}} + \sum_{\{i,j\}\in E(C_3)} M_{V_{ij}^{(a,\theta)}} \right) P_{\triangle,a}. \tag{XII.1}$$

This is a bounded self-adjoint operator on the full finite Hilbert space $\mathcal{K}_{\triangle,a}$; its associated quadratic form has domain $\mathcal{K}_{\triangle,a}$.

Proposition XII.1 (Exact block matrix)**.** *Relative to* $\mathcal{K}_{\triangle,a} = \mathbb{C}\Omega \oplus \mathcal{S}_a$,

$$H^{\mathrm{G}}_{\triangle,a} = 3\, I_{\mathbb{C}\Omega} \oplus \left(\frac{4}{3} I_{\mathcal{S}_a} + \frac{26}{9} I_{\mathcal{S}_a} + \frac{1}{18} L_a \right). \tag{XII.2}$$

There are no vacuum–excitation matrix elements.

Proof. The electric operator annihilates Ω and acts by $4/3$ on every $\phi_i^{(a)}$. Each pair potential has vacuum expectation 1, because the nonconstant character term has Haar mean zero; the three cells therefore give vacuum energy 3. Vacuum–character matrix elements vanish by Haar orthogonality and integration in the unused loop variable. The excitation block is exactly (B.7) with $\beta_{\mathrm{mag}} = 1$, plus the electric Casimir $4/3$. □

C. Complete ground projection and exact gap

Let

$$P^0_{\triangle,a} = |\Omega\rangle\langle\Omega|.$$

The graph minimum is not used to define this projection.

Theorem XII.2 (Complete-ground coercivity of the selected $SU(3)$ packet)**.** *For $a = 1, 2$, the complete ground energy of $H^{\mathrm{G}}_{\triangle,a}$ is*

$$E_{\triangle,0} = 3,$$

the complete ground projection is exactly $P^0_{\triangle,a}$, and

$$H^{\mathrm{G}}_{\triangle,a} \geq 3I + \Delta^{SU(3)}_{\triangle} \left(I - P^0_{\triangle,a}\right), \tag{XII.3}$$

where the exact selected-flux gap is

$$\Delta^{SU(3)}_{\triangle} = \frac{12 - \cos(2\pi/9)}{9} = 1.2482172840978913294\ldots. \tag{XII.4}$$

In particular the ground space is one-dimensional and there are no additional zero modes.

Proof. If λ is an eigenvalue of L_a, the corresponding excitation energy above the vacuum is, by (XII.2),

$$\frac{4}{3} + \frac{26}{9} + \frac{\lambda}{18} - 3 = \frac{11}{9} + \frac{\lambda}{18}.$$

For $a = 1, 2$, Equation (B.6) gives $\lambda_{\min} = 2 - 2\cos(2\pi/9)$. Substitution yields (XII.4). Positivity of the gap proves both the ground-space identification and the operator inequality. □

Corollary XII.3 (Downstream-compatible retained constant)**.** *Define*

$$\kappa_{\mathrm{cof}} = \frac{1}{2}\left(\frac{17}{5} - \sqrt{\frac{557}{75}}\right) = 0.3374044376020202719\ldots. \tag{XII.5}$$

Then each selected packet satisfies the exact, nonsharp lower bound

$$H^{\mathrm{G}}_{\triangle,a} \geq 3I + \kappa_{\mathrm{cof}} \left(I - P^0_{\triangle,a}\right), \qquad a = 1, 2. \tag{XII.6}$$

Proof. Since

$$\frac{557}{75} - \left(\frac{13}{5}\right)^2 = \frac{2}{3} > 0,$$

we have $\kappa_{\mathrm{cof}} < 2/5$. Also $2/5 < 11/9 < \Delta^{SU(3)}_{\triangle}$. Combining this exact chain with Equation (XII.3) proves (XII.6). □

Remark XII.4 (Sharp gap versus retained profile)**.** *The exact packet gap is* (XII.4)*. The algebraic quantity κ_{cof} is a deliberately retained downstream-compatible lower bound, not a numerical fit and not the sharp eigenvalue of this four-dimensional packet. Both statements are displayed to prevent their provenance from being conflated.*

D. Physicalization data

Let $P_{a,\min}$ be the complete lowest spectral projection of L_a and set

$$E_a^W = \mathrm{Ran}(I - P_{a,\min}).$$

For the raw frame F_a^W,

$$P_{\triangle,a}^0 F_a^W = 0, \qquad T_a = (I - P_{\triangle,a}^0) F_a^W|_{E_a^W} = F_a^W|_{E_a^W}.$$

Thus T_a is injective. The vacuum-orthogonal electric operator acts as $4/3$ on the entire raw range, so the cutoff $\Lambda = 4/3$ captures it exactly. Consequently

$$J_a = T_a, \qquad G_a = J_a^* J_a = I_{E_a^W}, \qquad F_a^{\mathrm{phys}} = F_a^W|_{E_a^W}.$$

The graph-minimum line is retained as the residual infrared subspace, not identified with the physical ground. The finite packet has no ultraviolet block and hence no mixed, transport, or collar leakage:

$$P_{\mathrm{UV}} = 0, \qquad K_{\mathrm{ref}} = K_{\mathrm{tr}} = K_{\mathrm{col}} = 0.$$

This exact closure is why the complete packet can be diagonalized without a two-block leakage estimate. The nonzero-leakage mechanism remains independently tested by the $SU(2)$ Galerkin realization.

E. Typed fixed-scale export

For each fixed thickness label $\varepsilon > 0$ and $a \in \{1, 2\}$, this section constructs the typed finite datum

$$\mathfrak{P}_{\triangle,a}^{SU(3)}(\varepsilon) = \left(SU(3), \mathbb{Z}_3, a, \varepsilon, \mathcal{K}_{\triangle,a}, H_{\triangle,a}^{\mathrm{G}}, P_{\triangle,a}^0, \Delta_{\triangle}^{SU(3)}, \kappa_{\mathrm{cof}} \right).$$

The thickness is a fixed packet label; no thickness limit or cofinal transport theorem is proved here. A downstream identification requires a separate theorem matching this packet to the same regional, Wilson-quotient, scale, and carrier data used in Papers II and III.

XIII. INTERPRETATION AND SCOPE

A. Resolved mechanism

The finite-scale lower bound is resolved into

$$\widehat{c}^{\varepsilon}_{n,\mathrm{UV}} = \text{vacuum-orthogonal electric exclusion},$$

$$\widehat{c}^{\theta}_{n,\mathrm{IR}}(b) = \text{physical-channel plus residual infrared separation},$$

and

$$\widehat{\epsilon}^{\varepsilon}_{n,\mathrm{mix}} = \|K^{\varepsilon}_{n,\mathrm{ref}} + K^{\varepsilon}_{n,\mathrm{tr}} + K^{\varepsilon}_{n,\mathrm{col}}\|.$$

This has greater explanatory content than an appeal to compact resolvent alone, while remaining conditional on the explicit verification package.

B. Strict and twisted detected sectors

In a strict connected graph sector, the graph minimum consists of covariantly constant sections. In a frustrated detected sector the covariant kernel may vanish while the finite graph still has a nonzero lowest eigenspace. The comparison uses separation above the full lowest graph eigenspace. There is no universal ordering between final strict and twisted constants: vacuum survival, Gram conditioning, physical extraction, residual modes, and total leakage all contribute.

C. Physical role of the associator

The localization associator measures intermediate Wilson-window information loss. Its Hamiltonian contribution is only the transported-cell correction $K^{\varepsilon}_{n,\mathrm{tr}}$. Perfect associativity would not remove ordinary reference-band mixing $K^{\varepsilon}_{n,\mathrm{ref}}$, and unresolved collar geometry would leave $K^{\varepsilon}_{n,\mathrm{col}}$.

D. Complete ground sector rather than a vacuum vector

All statements use the complete spectral projection $P^{\varepsilon}_{n,\mathrm{vac}}$. A basis may be chosen for finite computation, but no vector is physically distinguished. Continuum vacuum uniqueness

is outside the scope of the fixed-scale theorem.

E. Nonclaims

The article proves no zero-thickness limit, continuum observable net, Poincaré covariance, relativistic spectrum condition, confinement theorem, scattering theory, or Yang–Mills mass gap. It does not identify physical scale arrows with equivalences and does not transport lower bounds through one-sided seams without independent endpoint certification.

F. Status of the new $SU(3)$ example

The $SU(3)$ triangle packet is a complete finite Galerkin Hamiltonian at one declared scale. It proves center grading, a nontrivial detected flux, an independently identified complete ground projection, exact physicalization of the graph-excitation channel, and a finite-packet coercive bound. It is not a proof that the same packet exists on a cofinal family of geometric scales, and it does not identify the packet with the local Friedrichs or represented objects of later papers. Those are typed interface theorems, not consequences of the finite matrix alone.

Appendix A: An Explicit Raw $SU(2)$ Witness

1. Scope and center detection

This appendix verifies raw representation-theoretic mechanisms before physical reduction. Let $G = SU(2)$ with $Z(G) = \{I, -I\}$. For spin j, $C_2(j) = j(j+1)$, so $C_2(0) = 0$, $C_2(1/2) = 3/4$, and $C_2(1) = 2$. The fundamental character satisfies

$$\int_G \chi_{1/2}(g)\, dg = 0, \qquad \int_G |\chi_{1/2}(g)|^2\, dg = 1, \qquad \chi_{1/2}(-g) = -\chi_{1/2}(g).$$

2. Raw three-channel frame

On $L^2(G^3)^{\mathrm{Ad}\, G}$ define $\phi_i(g_1, g_2, g_3) = \chi_{1/2}(g_i)$. Schur orthogonality gives $\langle \phi_i, \phi_j \rangle = \delta_{ij}$, hence $F_\theta^W e_i = \phi_i$ is an isometric raw frame.

For $\theta_{ij} \in \{\pm 1\}$ define

$$V_{ij}^{\theta}(g) = \beta_{\mathrm{mag}} \left(1 - \frac{1}{2}\theta_{ij}\chi_{1/2}(g_i g_j^{-1})\right). \tag{A.1}$$

Proposition A.1 (Raw $SU(2)$ compression)**.** *The compression is*

$$(F_\theta^W)^* M_{V_{ij}^\theta} F_\theta^W = \beta_{\mathrm{mag}} \left[I - \frac{\theta_{ij}}{4}(E_{ij} + E_{ji})\right].$$

The maximal raw coefficient for one edge is $\beta_{ij}^{W,\max} = \frac{5}{8}\beta_{\mathrm{mag}}$.

3. Triangle spectra

For the equal-weight triangle, the strict flux choice has spectrum $\{0, 3, 3\}$, while the frustrated choice $\theta_{12} = \theta_{23} = 1$, $\theta_{31} = -1$ has spectrum $\{1, 1, 4\}$. The summed raw magnetic compression is

$$(F_\theta^W)^* H_{\triangle,\mathrm{mag}}^\theta F_\theta^W = \frac{5}{2}\beta_{\mathrm{mag}} I + \frac{1}{4}\beta_{\mathrm{mag}} L_\theta.$$

Long convolution and associator calculations from the compiled appendix remain to be recovered verbatim.

Appendix B: Exact Raw $SU(3)$ Fundamental-Channel Witness

1. Representations, center character, and triality

Let

$$G = SU(3), \qquad Z(G) = \{I, \omega I, \omega^2 I\}, \qquad \omega = e^{2\pi i/3}.$$

For the irreducible representation of highest weight (p, q), the center acts by

$$\omega I \longmapsto \omega^{p-q} I,$$

with exponent understood modulo 3. The fundamental representation $\mathbf{3} = (1, 0)$ has triality 1, while $\overline{\mathbf{3}} = (0, 1)$ has triality 2.

Fix $a \in \{1, 2\}$ and write

$$\chi_a = \begin{cases} \chi_{\mathbf{3}}, & a = 1, \\ \chi_{\overline{\mathbf{3}}}, & a = 2. \end{cases}$$

Then $\chi_a(zg) = z^a \chi_a(g)$ for $z \in Z(G)$.

2. Gauge-invariant three-channel frame

Interpret g_1, g_2, g_3 as based loop holonomies with residual diagonal conjugation

$$(g_1, g_2, g_3) \longmapsto (hg_1h^{-1}, hg_2h^{-1}, hg_3h^{-1}).$$

On $L^2(G^3)^{\mathrm{Ad}\,G}$ define

$$\phi_i^{(a)}(g_1, g_2, g_3) = \chi_a(g_i), \qquad i = 1, 2, 3.$$

Schur orthogonality gives

$$\langle \phi_i^{(a)}, \phi_j^{(a)} \rangle = \delta_{ij},$$

so

$$F_a^W : \mathbb{C}^3 \longrightarrow L^2(G^3)^{\mathrm{Ad}\,G}, \qquad F_a^W e_i = \phi_i^{(a)},$$

is an isometric raw Wilson frame. Its range has triality a under simultaneous central multiplication of the three loop variables.

3. Canonical pair cells and exact compression

For an oriented graph edge (i, j) choose $\theta_{ij} \in \{1, \omega, \omega^2\}$ and set

$$V_{ij}^{(a,\theta)}(g) = \beta_{\mathrm{mag}} \left(1 - \frac{1}{3} \mathrm{Re}\big[\theta_{ij} \chi_a(g_i g_j^{-1})\big]\right), \qquad \beta_{\mathrm{mag}} > 0. \tag{B.1}$$

Because $|\chi_a(g)| \le 3$, this is a bounded nonnegative gauge-invariant multiplication potential. Use the compression-aligned edge vector

$$r_{ij}^{\theta} = e_i - \overline{\theta}_{ij} e_j, \qquad \ell_{ij}^{\theta} = |r_{ij}^{\theta}\rangle\langle r_{ij}^{\theta}|.$$

Proposition B.1 (Raw $SU(3)$ compression and phase-independent extraction)**.** *The exact compression of* (B.1) *is*

$$M_{ij}^{W,\theta} := (F_a^W)^* M_{V_{ij}^{(a,\theta)}} F_a^W = \beta_{\mathrm{mag}} \left[I - \frac{1}{18} \left(\theta_{ij} E_{ij} + \overline{\theta}_{ij} E_{ji} \right) \right]. \tag{B.2}$$

Moreover,

$$M_{ij}^{W,\theta} - b\ell_{ij}^{\theta} \ge 0 \quad \Longleftrightarrow \quad 0 \le b \le \frac{19}{36} \beta_{\mathrm{mag}}. \tag{B.3}$$

The maximal extraction coefficient is independent of the edge phase.

Proof. The diagonal entries follow from character orthogonality. The only nonzero off-diagonal entries lie in the (i, j) block. The fundamental convolution identity contributes a factor $1/3$, and the real-part normalization and the coefficient $1/3$ in (B.1) supply the remaining factor $1/6$, giving (B.2).

On $\mathrm{span}\{e_i, e_j\}$ the matrices are

$$M_\theta = \beta_{\mathrm{mag}} \begin{pmatrix} 1 & -\theta/18 \\ -\overline{\theta}/18 & 1 \end{pmatrix}, \qquad r_\theta = \begin{pmatrix} 1 \\ -\overline{\theta} \end{pmatrix}.$$

The vector r_θ has eigenvalue $19\beta_{\mathrm{mag}}/18$, while its orthogonal complement has eigenvalue $17\beta_{\mathrm{mag}}/18$. Since $\|r_\theta\|^2 = 2$, subtraction of $b|r_\theta\rangle\langle r_\theta|$ is positive exactly when $19\beta_{\mathrm{mag}}/18 - 2b \geq 0$. □

Remark B.2 (Conjugation convention)**.** *The conjugate on the second coefficient of r_{ij}^θ is load bearing. It aligns the rank-one graph form with the phase occurring in the compression matrix. Replacing it by $e_i - \theta_{ij} e_j$ compares the compression with the conjugate graph connection and can create an artificial phase dependence when $\theta_{ij}^2 \neq 1$.*

4. Triangle flux and magnetic graph spectrum

Choose the gauge representative

$$\theta_{12} = 1, \qquad \theta_{23} = 1, \qquad \theta_{31} = \omega^m, \qquad m \in \{0, 1, 2\},$$

so the gauge-invariant flux is $\Phi_m = \omega^m$. Let

$$L_m = \ell_{12}^\theta + \ell_{23}^\theta + \ell_{31}^\theta.$$

For the strict flux $m = 0$,

$$\mathrm{spec}(L_0) = \{0, 3, 3\}. \tag{B.4}$$

For $m = 1, 2$, the characteristic polynomial is

$$\det(\lambda I - L_m) = \lambda^3 - 6\lambda^2 + 9\lambda - 3, \tag{B.5}$$

and the spectrum is

$$\left\{ 2 - 2\cos\frac{2\pi}{9}, 2 - 2\cos\frac{4\pi}{9}, 2 - 2\cos\frac{8\pi}{9} \right\}. \tag{B.6}$$

In particular, the twisted graph has no covariantly constant vector and has a one-dimensional lowest eigenspace.

Summing the three pair-cell compressions gives the exact affine graph identity

$$(F_a^W)^* H_{\triangle,\mathrm{mag}}^{(a,m)} F_a^W = \frac{26}{9}\beta_{\mathrm{mag}} I + \frac{1}{18}\beta_{\mathrm{mag}} L_m. \tag{B.7}$$

Scope Note B.3. *Equations* (B.2)–(B.7) *are the raw representation-theoretic layer. The complete-ground projection and finite-packet coercive statement are proved separately in Section XII; they are not inferred by identifying the graph-minimum space with the physical vacuum.*

Appendix C: Complete Physicalized $j \leq 1$ $SU(2)$ Galerkin Verification

The compiled article states that a complete $j \leq 1$ Peter–Weyl Galerkin realization computes the interacting ground projection, Gram operator, physical recompression, exact reference leakage, and the explicit resolved lower bound

$$\widehat{\kappa}_G = 0.657490679870\ldots.$$

The exact matrices, basis ordering, high-precision output, and independent verification script were supplied in supplementary material that is not presently mounted. They are therefore not recreated from inference.

The reconstruction gate for this appendix is:

(C1) recover the exact finite basis and matrix entries;

(C2) identify the complete ground projection directly from the full Galerkin Hamiltonian;

(C3) reproduce vacuum removal and the surviving coefficient rank;

(C4) reconstruct the captured electric projection and Gram matrix;

(C5) recompute every physical magnetic cell;

(C6) reproduce the exact reference, transport, and collar blocks;

(C7) verify the final eigenvalue bound with exact or interval arithmetic.

Appendix D: Finite-Dimensional Comparison Lemmas

1. Schur-complement form of the two-block bound

For the Hermitian matrix

$$\mathsf{K} = \begin{pmatrix} c_1 & -\eta \\ -\eta & c_2 \end{pmatrix},$$

the smaller eigenvalue is

$$\lambda_-(\mathsf{K}) = \frac{c_1 + c_2 - \sqrt{(c_1 - c_2)^2 + 4\eta^2}}{2}.$$

It is positive exactly when $c_1 > 0$, $c_2 > 0$, and $\eta^2 < c_1 c_2$.

2. Finite capture lemma

If a finite-dimensional subspace injects into the domain of a nonnegative compact-resolvent operator after removal of a finite-dimensional kernel, then a sufficiently large spectral cutoff captures that subspace injectively. This is the finite-rank compactness step used to construct $J^{\varepsilon}_{n,\theta}$.

3. Gram normalization

For an injective map $J : C \to \mathcal{H}$ from a finite-dimensional Hilbert space, $G = J^*J$ is positive definite and $JG^{-1/2}$ is an isometry. All compressed operators must be recomputed under this normalized frame.

Appendix E: Reconstruction Provenance and Completion Gate

This appendix is part of the working reconstruction package and is not intended for the final journal article.

Compiled reference.: Located in the File Library; 53 pages; not mounted as a writable binary.

Older TeX chassis.: Located in the File Library; it does not match the current title or theorem architecture.

Inherited Sections I–XI and scope section.: Mechanism, principal definitions, and load-bearing theorem statements reconstructed.

Long inherited proofs.: Partially reconstructed; verbatim collation remains pending direct attachment of the original source package.

Appendix A.: Principal raw $SU(2)$ formulas recovered; long historical calculations pending.

Appendix B.: The exact raw $SU(3)$ compression, phase-independent extraction, and triangle spectra are restored from the compiled reference.

Section XII.: New fixed-scale $SU(3)$ theorem augmentation, with exact matrices and an independently runnable symbolic certificate.

Earlier complete $SU(2)$ realization.: Final constant and role recovered; its exact historical matrices and scripts remain pending.

Bibliography.: Minimal provisional list only; the exact active bibliography of the missing source tree remains pending.

Original labels, macros, and comments.: Not recoverable from the compiled PDF; semantic replacements are used.

The inherited portion becomes a source-equivalent baseline only after page-by-page comparison against a directly mounted copy of the compiled reference and recovery of the missing historical supplement. The new $SU(3)$ theorem does not depend on that recovery: its proof and certificate are contained in the present package.

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AUTHOR DECLARATIONS

Conflict of interest. The author has no conflicts to disclose.

Author contributions. J. Scott Little: Conceptualization; Formal analysis; Investigation; Methodology; Software; Validation; Writing–original draft; Writing–review and editing.

DATA AND CODE AVAILABILITY

The reconstruction package records every recoverable source constant and theorem-level certificate. The new $SU(3)$ triangle packet is accompanied by machine-readable exact matrices, characteristic-polynomial checks, and an independent symbolic verification script. Exact matrices and scripts for the earlier complete $SU(2)$ Galerkin witness remain unrecovered from the missing source package and are listed explicitly as reconstruction blockers.

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