

**Technical Supplement to
Entropy-Surgered Admissible Thick-to-Thin Reconstruction
of the Yang–Mills Observable System**

J. Scott Little^{a)}

Independent Researcher, Pickerington, Ohio, USA^{b)}

(Dated: July 2026)

This supplement records the proof archive for the journal article: the bounded-section observable quotient and normal envelope; packet-localizing additivity and time slice; moving expectation algebras; independent Orlicz, reference-logarithm, and entropy tails; sharp Cech-face surgery descent; rooted finite-diagram lifting; Gauss–Ward and collar graph passage; comparison–Mosco recovery, TT1–TT2, and exact kernel; compact-semisimple root-data realization; scalar coefficient gerbes; the complete physical special-unitary rank-three witness; branchwise certificates; and the scope, source-precedence, and Paper-V handoff records. The supplement preserves the distinction between ordinary observable descent, represented coherence, and dynamical spectrum.

^{a)}Electronic mail: jeffreyscottlittle07@gmail.com

^{b)}ORCID: 0009-0003-5391-7840

Research status. This manuscript is a preprint and has not yet undergone peer review. The proofs, computational certificates, and source-level dependency records are supplied to support independent verification. This v1.5.3 derivative normalizes and completes the citation layer; the mathematical content remains frozen at v1.5.2.

I. SOURCE PRECEDENCE, NOTATION, AND THEOREM REGISTRY

A. Authoritative logical order

The technical argument is read in the following order:

$$\begin{array}{l}
 \text{positive-thickness local geometry and operator estimates} \\
 \implies \text{smooth Cech-face functional-calculus comparison} \\
 \implies \text{finite-window expectation-algebra uniqueness} \\
 \implies \text{uniform physical Orlicz and reference-logarithm tails} \\
 \implies \text{sharp Cech-face surgery descent} \\
 \implies \text{rooted finite-diagram state lifting} \\
 \implies \text{corrected SLSP} \implies \text{restart and admissibility closure.}
 \end{array} \tag{1}$$

The compact-semisimple root-datum realization is applied only after this analytic chain is complete. The physical $SU(3)$ branch is then used as an explicit perturbative realization of the exact-dilation model. No Poincare or invariant-mass statement enters the chain.

B. Frozen prior-series provenance

The present article imports no unnamed result. The frozen theorem map is:

Source	Frozen result	Role in the present article
Paper I ¹	Theorem X.3 (finite-scale Wilson- finite physicalization, objectwise coertube coercivity) and Theorem XI.2 civity, and the uniform-profile criterion; (uniform physicalized comparison no cofinal branch is imported criterion)	
Paper II ²	Theorem 4.5 (directed compact scale-bordism and coefficient-line archi-inverse-limit selection), Theorem tecture; full multiparameter selection 5.1 and Corollary 5.2 (profilewise remains conditional on its extension hy-uniform coercivity), and Theorems potheses 7.2–7.4 (paired center-banded glob-alizations, twisted higher descent, and finite-shadow indistinguishability)	Pa-
Paper III ³	Theorem 24.26 (III.PT), with Def- the complete positive-thickness incised initions 24.23 and 24.25, Theorem spacetime datum and the exact Paper-24.21, and component Theorems III-to-Paper-IV obligation boundary III.A–III.J as expanded by Ap- pendix M	

pers I–III are authoritative external companion inputs in the frozen versions identified in the database and dependency manifest; they are not included in the Paper-IV submission archive unless the final release inventory explicitly lists fixed companion copies. No DOI, arXiv identifier, or publication status is invented. Paper-IV development memoranda are not bibliographic authorities: their corrected content is integrated into the numbered theorems and appendices of the present article and supplement.

C. Supersession convention

Whenever an older formulation conflicts with the present supplement, the following conventions control:

- (S1) thin stiffness surgery is an inherited Cech-descended expectation graph;
- (S2) finite-window objects are normal conditional expectations and their fixed-point algebras;
- (S3) eventual atomic no-splitting is an identification criterion only;
- (S4) sharp descent is proved after regulator removal;
- (S5) reference-logarithm tails are derived from a physical Orlicz moment, not from relative entropy alone;
- (S6) one rooted simultaneous recovery is required for every finite mixed diagram;
- (S7) strictification means one admissible physical strictification compatible with all faces;
- (S8) the global gauge scope is compact, connected, and semisimple;
- (S9) the represented line trichotomy is Sector I, Sector II, and Sector III;
- (S10) a non-scalar correspondence associator is retained in its own target and is not forced into scalar cohomology;
- (S11) the thin form threshold is not called a mass gap;
- (S12) strictness and twisting are not used as interaction criteria.

D. Layered notation

The notation is divided into seven layers.

a. Geometry. $\mathbb{M}_{0,\text{inc}}$ denotes a controlled incised background datum and $M_{\text{inc}}(\mathbb{M}_{0,\text{inc}}) = M \setminus \Sigma$ its physical incised spacetime. The selected thin category is $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$; its objects are controlled background–region pairs $X = (\mathbb{M}_{0,\text{inc}}, O)$ with positive incision clearance, together with the separately declared collar-adapted objects, and its arrows preserve the incision, collar, causal, testing-cover, and Cauchy data. The notation D , O , U_σ , and U_τ is used only after the controlled background has been fixed locally. A face arrow is written $a : \sigma \rightarrow \tau$.

b. Thickness and comparison. The canonical corridor is $\varepsilon_n = 2^{-n}\varepsilon_0$. A general declared corridor is admitted when it is finitely cofinally equivalent to the dyadic one. Comparison

maps are $\mathfrak{T}_{mn;D}$ on represented carriers and $T_{mn;D}$ on bounded packet sections. State restrictions and normal comparisons carry a superscript $*$. A finite mixed diagram is $\mathcal{D}$.

c. Observable algebras. $\mathfrak{W}_n(O)$ is the finite analytic Wilson-packet core and $\mathcal{B}_0(O)$ the thin testing quotient. The selected state family determines the faithful direct-sum representation π_O^{sel} ; z_O is its central support, not the join of ordinary state supports. The ordinary thin algebra is

$$\mathcal{A}_{G,\xi,0}^W(O) = \pi_O^{\text{sel}}(\mathcal{B}_0(O))'' \cong z_O \mathcal{B}_0(O)^{**}.$$

d. Represented objects. $\mathbf{Rep}_{G,\xi,0}^{\text{thin}}$ is the branch-typed represented thin pseudodiagram; it is not the ordinary observable functor. The family $\hat{\Omega}_{G,\xi,n}^b$ is the abstract complete-ground pointing or pointed subobject. An orthogonal projection $P_{\Omega,G,\xi,n;X}$ is written only on a selected Hilbert carrier. $A_\xi \leq Z(G)$ is the selected finite coefficient band. The fixed-thickness class is κ_ξ^{fib} and the controlled total class is $\hat{\kappa}_\xi^{\text{cs}}$. In the correspondence branch the retained invariant is κ_ξ^{Corr} .

e. Expectations and entropy. $\mathbb{E}_{n;D}^{(N)}$ is the positive-thickness good-window expectation. $\mathbb{E}_{0;D}^{(N),\text{as}}$ is its limiting expectation. Its range is $\mathcal{F}_{0;D}^{(N),\text{as}}$. The faithful reference density is $\sigma_{n;D}^{(N)}$ and

$$L_{n;D}^{\text{ref}} = -\log \sigma_{n;D}^{(N)}.$$

The selected physical Orlicz packet is $\mathfrak{K}_{G,\xi}^{\text{Orl}}$. The inherited surgery graph on normal states is $\mathcal{S}_{G,\xi,0;D}^{\text{st,as,*}}$; no unstarred primal surgery map is defined.

f. Gauss–Ward data. The common insertion core is $\mathfrak{C}_{G,\xi,0}^{\text{ins}}$. The interior derivation is $\delta_{\zeta,0}^{\text{int}}$. The prepared-state domain is $\mathcal{D}_{G,\xi,0}^{\text{GW-prep}}$. The boundary tuple is

$$\left(\mathcal{Q}_{G,\xi,0}^\partial, \mathfrak{a}_{G,\xi,0}^\partial, [\chi_{G,\xi,0}^\partial]_{\text{cs}}, \mathfrak{b}_{G,\xi,0}^\partial \right).$$

g. Forms. $q_{G,\xi,n}$ is the renormalized positive-thickness form, $\hat{q}_{G,\xi,n}$ its transported version, and $\bar{q}_{G,\xi,0}$ the closed thin form. On a Hilbert form carrier the complete-ground projection is $P_{\Omega,G,\xi,n;X}$ and the uniform coercive constant is $\kappa_{\text{IV,thin}}$. The identity operator is $\mathbf{1}$; $I_{n;X}$ is reserved for the reconstruction map and is never used as the identity. TT1 denotes projection convergence on moving graph-domain families and TT2 denotes compatible form recovery. We write Ran , ker , and spec for range, kernel, and spectrum.

E. Theorem registry

The main article contains eight principal results. The supplement assigns proof modules as follows.

Result	Main construction	Supplement modules
Theorem A	ordinary thin observable net	Sections II A–II E
Theorem B	represented sector descent	Sections XXXVI and XXXVII
Theorem C	corrected SLSP and inherited surgery	Sections III–X
Theorem D	Gauss–Ward and collar re-construction	Sections XI–XV
Theorem E	comparison–Mosco form and exact kernel	Sections XVI–XX
Theorem F	compact-semisimple root data and scalar coefficient normal forms in Section LXVII availability	Sections XXIII–XXVIII, with finite-abelian
Theorem G	analytical non-vacuity	Sections XXII and XXXIV
Theorems	complete assembly, branch certification, and incised Paper-V boundary	Sections XXXVIII and CI

F. Order of limits

Every sharp statement in the supplement uses the order

$$\boxed{\text{fix finite diagram and buffer} \rightarrow \text{choose a good window} \rightarrow n \rightarrow \infty \rightarrow N \rightarrow \infty \rightarrow d \downarrow 0.} \quad (2)$$

The finite diagram fixes all carriers and compatibility constraints. The buffer permits local smooth functional calculus. The good window makes the expectation algebra finite dimensional. The thickness limit identifies the finite-window expectation. The window and buffer are removed only after the Orlicz and entropy bounds are uniform.

G. Topology registry

The following topologies are used and are not interchanged without proof:

(T1) operator norm for finite-window compressed operators and reference logarithms;

- (T2) Hilbert–Schmidt gap for finite-dimensional fixed-point algebras;
- (T3) $1 \rightarrow 1$ and $\infty \rightarrow \infty$ norm for finite-window expectations;
- (T4) predual norm for state comparison and sharp surgery descent;
- (T5) graph norm for insertion derivations and closed forms;
- (T6) strong or norm resolvent convergence for form operators;
- (T7) strong operator convergence for complete-ground projections when exact transport is unavailable;
- (T8) ordinary finite group cohomology for scalar fiber associators;
- (T9) controlled chart–scale cohomology for Sector-II obstructions;
- (T10) bimodule-unitary equivalence for correspondence associators.

Weak-* compactness of a normal-state set is not identified with predual-norm compactness. Predual convergence is obtained from the finite-window estimates and the uniform tails.

H. Scope registry

Throughout the supplement:

$$G = \tilde{G}/\Gamma$$

is compact, connected, and semisimple. Torus factors, disconnected groups, and noncompact groups are not treated. The exact-dilation witness theorem is branchwise and does not identify a unique continuum Yang–Mills system for a given G . The $SU(3)$ branch is a complete physical twisted witness but not the final gauge-group scope. The $SU(2)$ calculations, when cited, are finite/depth strict controls rather than an independent thickness-cofinal witness unless a separate $\varepsilon \downarrow 0$ theorem is supplied.

II. ORDINARY INCISED OBSERVABLE RECONSTRUCTION

A. Incised testing algebras and saturated null ideals

1. *Bounded comparison sections*

The selected-envelope construction is carried out in the von Neumann algebraic setting of standard normal representation theory.^{4–8} Fix $X = (\mathbb{M}_{0,\text{inc}}, O) \in \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$. For each thickness ε_n , let $\mathfrak{W}_n(X)$ be the finite linear span of admitted bounded gauge-invariant analytic Wilson packets whose compact selected carriers remain in

$$O \in M_{\text{inc}}(\mathbb{M}_{0,\text{inc}})$$

throughout the selected thickness tail. Let $\mathfrak{R}_{n,X}^{\text{sat}}$ denote the declared positive endpoint packet after adjoining the uniformly controlled left translates, right translates, and involution transforms used by multiplication. Write

$$p_{n,X}^{\text{sat}}(x) := \sup_{\omega \in \mathfrak{R}_{n,X}^{\text{sat}}} |\omega(x)|.$$

A bounded tail section is a family

$$a = (a_n)_{n \geq n_0}, \quad a_n \in \mathfrak{W}_n(X), \quad \|a\|_{\text{sec}} := \sup_n \|a_n\| < \infty.$$

It is a comparison section when, on every selected finite common refinement,

$$p_{m,X}^{\text{sat}}(T_{mn,X}a_n - a_m) \longrightarrow 0. \quad (3)$$

The comparison condition is independent of a particular pairwise extraction because OJ8 supplies a finite common refinement of all declared choices.

OJ2 gives, for bounded packet sections a, b , asymptotic multiplicativity and involution control in the same saturated seminorm:

$$p_{m,X}^{\text{sat}}(T_{mn,X}(a_nb_n) - T_{mn,X}(a_n)T_{mn,X}(b_n)) \longrightarrow 0, \quad (4)$$

$$p_{m,X}^{\text{sat}}(T_{mn,X}(a_n^*) - T_{mn,X}(a_n)^*) \longrightarrow 0. \quad (5)$$

Consequently componentwise multiplication and involution preserve comparison sections.

2. The common packet-null ideal

Define

$$\mathfrak{N}_0(X) := \left\{ a \in \mathfrak{W}_{\text{cmp}}(X) : p_{n;X}^{\text{sat}}(a_n) \longrightarrow 0 \right\}. \quad (6)$$

Lemma II.1 (Two-sided saturated ideal) *The space $\mathfrak{N}_0(X)$ is a closed two-sided $*$ -ideal in the algebra of bounded comparison sections.*

Proof. Let $a \in \mathfrak{N}_0(X)$ and let b be bounded. Left and right saturation mean that for every $\omega \in \mathfrak{K}_{n;X}^{\text{sat}}$ the functionals

$$x \longmapsto \omega(xb_n), \quad x \longmapsto \omega(b_nx)$$

after division by $\max\{1, \|b_n\|\}$ belong to the same uniformly controlled saturated packet. Hence

$$p_{n;X}^{\text{sat}}(a_nb_n) \leq \|b\|_{\text{sec}} p_{n;X}^{\text{sat}}(a_n), \quad p_{n;X}^{\text{sat}}(b_na_n) \leq \|b\|_{\text{sec}} p_{n;X}^{\text{sat}}(a_n).$$

Involution saturation gives $p_{n;X}^{\text{sat}}(a_n^*) = p_{n;X}^{\text{sat}}(a_n)$. Thus ab , ba , and a^* are null. Closedness in the section norm is an explicit part of OJ2. Notice that no implication from packet-null convergence to operator-norm convergence is used. $\square$

Theorem II.2 (Incised thin testing algebra (R5.A)) *The quotient*

$$\mathcal{B}_0(X) := \overline{\mathfrak{W}_{\text{cmp}}(X) / \mathfrak{N}_0(X)}^{\|\cdot\|_{\text{quot}}} \quad (7)$$

is a unital C^* -algebra. It is generated by the thin classes of finite bounded Wilson packets and uses neither pointwise thin fields nor a bare thin operator.

Proof. The preceding lemma makes the quotient a normed $*$ -algebra. The quotient of the bounded section C^* -algebra by a closed $*$ -ideal has the C^* identity, and its completion is therefore a C^* -algebra. Equations (4)–(5) show that the operations are independent of the comparison representatives. $\square$

3. Thin C^* arrows

For an admissible arrow $\psi : X \rightarrow Y$, OJ7 supplies componentwise strict bounded observable maps

$$\alpha_\psi^{(n)} : \mathfrak{W}_n(X) \longrightarrow \mathfrak{W}_n(Y)$$

whose comparison-square, multiplication, involution, unit, and composition defects are packet null. OJ2 guarantees that null sections map to null sections.

Proposition II.3 (Thin C^* functor) *Every admissible arrow induces a unital $*$ -homomorphism*

$$\alpha_\psi^0 : \mathcal{B}_0(X) \longrightarrow \mathcal{B}_0(Y).$$

It is injective, and $\alpha_{\phi \circ \psi}^0 = \alpha_\phi^0 \circ \alpha_\psi^0$.

Proof. Well-definedness and strict composition follow because all finite comparison and composition defects lie in the common saturated null ideal. If $\alpha_\psi^0([a]) = 0$, OJ3 extends every selected source state to a selected target state. Pulling the target null relation back gives $\rho([a]^*[a]) = 0$ for every selected source state. The norming property proved below then yields $[a] = 0$. $\square$

B. Selected limit states and the normal regional functor

1. Reconstructed state family

A compatible positive state section is a family $\rho = (\rho_n)$ with ρ_n in the selected positive-thickness state packet such that, for every comparison section a and every selected common refinement, the scalar net $\rho_n(a_n)$ is Cauchy. Define

$$\rho_0([a]) := \lim_n \rho_n(a_n). \quad (8)$$

Packet-nullity makes this independent of the representative. Let $\mathfrak{S}_0(X)$ be the family of all states obtained in this way and by the finite-packet diagonal compactness clause of OJ3.

Lemma II.4 (Existence, positivity, and norming) *The family $\mathfrak{S}_0(X)$ consists of states on $\mathcal{B}_0(X)$ and satisfies*

$$\|b\| = \sup_{\rho \in \mathfrak{S}_0(X)} \rho(b^*b)^{1/2}, \quad b \in \mathcal{B}_0(X). \quad (9)$$

In particular it separates $\mathcal{B}_0(X)$.

Proof. Positivity and normalization pass from the positive-thickness states through (8). For norming, choose a bounded comparison representative of b and, on each finite packet stage, a

near-norming positive thick state from OJ3. Finite-packet tightness and a diagonal extraction give a compatible limiting state. Passing first through the finite packet exhaustion and then through the comparison tail gives the lower bound in (9); the reverse inequality is the state Cauchy–Schwarz inequality. Thus no bicommutant is introduced before a separating state family has been proved. $\square$

2. Faithful selected representation

Let

$$\pi_X^{\text{sel}} := \bigoplus_{\rho \in \mathfrak{S}_0(X)} \pi_\rho, \quad \mathcal{A}_{G,\xi,0}^W(X) := \pi_X^{\text{sel}}(\mathcal{B}_0(X))''.$$

Lemma II.4 makes π_X^{sel} faithful. If z_X denotes the central support of this representation in $\mathcal{B}_0(X)^{**}$, then

$$\mathcal{A}_{G,\xi,0}^W(X) \cong z_X \mathcal{B}_0(X)^{**}. \quad (10)$$

Here z_X is the complement of the central kernel projection of the normal extension of π_X^{sel} ; it is not defined as a join of ordinary state-support projections.

Lemma II.5 (Selected representation under an arrow) *For every admissible arrow $\psi : X \rightarrow Y$,*

$$(\alpha_\psi^0)^* \mathfrak{S}_0(Y) = \mathfrak{S}_0(X). \quad (11)$$

Consequently $\pi_Y^{\text{sel}} \circ \alpha_\psi^0$ and π_X^{sel} are quasi-equivalent representations of $\mathcal{B}_0(X)$.

Proof. Restriction of a compatible target state section is compatible by the comparison-square estimates, so the left side is contained in the right side. The normal regional extension clause of OJ3 extends every compatible source section to a compatible target section with the same thin restriction, proving the reverse inclusion. The two direct sums therefore contain precisely the same GNS quasi-equivalence classes, possibly with different multiplicities. $\square$

Theorem II.6 (Selected normal incised functor (R5.B)) *For every admissible arrow $\psi : X \rightarrow Y$, there is a unique normal unital injective $*$ -homomorphism*

$$\bar{\alpha}_\psi^0 : \mathcal{A}_{G,\xi,0}^W(X) \longrightarrow \mathcal{A}_{G,\xi,0}^W(Y) \quad (12)$$

satisfying

$$\bar{\alpha}_\psi^0(\pi_X^{\text{sel}}(b)) = \pi_Y^{\text{sel}}(\alpha_\psi^0(b)).$$

The extensions compose strictly and preserve units.

Proof. By Lemma II.5, the representation $\pi_Y^{\text{sel}} \circ \alpha_\psi^0$ is quasi-equivalent to π_X^{sel} . Therefore the displayed algebraic map extends uniquely from the ultraweakly dense represented C^* -algebra to a normal $*$ -isomorphism onto

$$\pi_Y^{\text{sel}}(\alpha_\psi^0(\mathcal{B}_0(X)))'' \subseteq \mathcal{A}_{G,\xi,0}^W(Y).$$

It is injective because π_X^{sel} is faithful and unital because the thin C^* arrow is unital. Uniqueness on an ultraweakly dense subalgebra and strict C^* functoriality give strict composition. This construction avoids the unsupported claim that a bidual arrow must transport one selected central corner onto another. $\square$

C. Exact thin causality in a common incised ambient region

Let $X_i = (\mathbb{M}_{0,\text{inc}}, O_i)$ and let $X_U = (\mathbb{M}_{0,\text{inc}}, U)$ be a common admissible ambient object with $O_1 \perp_U O_2$. OJ1 and OJ4 provide compact carrier pairs with positive relative causal margin in U and positive h -clearance from Σ . Thus selected sufficiently thin carriers remain in $M_{\text{inc}}(\mathbb{M}_{0,\text{inc}})$ and are thick-spacelike. The positive-thickness locality theorem gives either exact commutation on a tail or a uniform estimate

$$p_{n;U}^{\text{sat}} \left([\alpha_{1U}^{(n)}(a_n), \alpha_{2U}^{(n)}(b_n)] \right) \leq \eta_n(U; O_1, O_2) \longrightarrow 0. \quad (13)$$

Theorem II.7 (Exact thin incised Einstein causality (R5.C)) *For $O_1 \perp_U O_2$,*

$$\left[\bar{\alpha}_{1U}^0 \left(\mathcal{A}_{G,\xi,0}^W(X_1) \right), \bar{\alpha}_{2U}^0 \left(\mathcal{A}_{G,\xi,0}^W(X_2) \right) \right] = 0 \quad (14)$$

in $\mathcal{A}_{G,\xi,0}^W(X_U)$. The statement is independent of the selected common admissible ambient region.

Proof. Equation (13) says that the commutator section is in $\mathfrak{N}_0(X_U)$, so the two represented testing algebras commute. Each selected normal image is the strong-operator closure of its represented testing algebra. Because the commutant of a set is strong-operator closed, the two normal images commute. If U and V are two common admissible ambients, OJ8 provides a common incised refinement; normal isotony sends the two zero commutators to the same zero commutator there. No causal relation from the unincised ambient spacetime is substituted for $\perp_U$. $\square$

Remark II.8 (Role of finite propagation) *Finite propagation for the local Laplace-type principal part controls carrier transport and Cauchy evolution. The exact commutator conclusion above comes from the positive-thickness locality/support theorem and the saturated thin quotient. Finite propagation alone is not sharp spectral locality and is not used as a replacement for microcausality.*

D. Packet-localizing normal additivity

Let $\{\psi_i : X_i \rightarrow X\}_{i \in I}$ be a declared packet-localizing incision-compatible cover. OJ5 supplies, for every finite packet generator $W_n \in \mathfrak{W}_n(X)$, finitely many subpackets $W_{i_k, n} \in \mathfrak{W}_n(X_{i_k})$ and a uniformly bounded physical recombination expression $\mathcal{R}_n$ such that

$$p_{n;X}^{\text{sat}}(W_n - \mathcal{R}_n(W_{i_1, n}, \dots, W_{i_r, n})) \leq \delta_n^{\text{add}} \longrightarrow 0. \quad (15)$$

Every carrier stays in $M_{\text{inc}}(\mathbb{M}_{0, \text{inc}})$, and any linking or center-band label needed to identify the same physical packet is retained before taking the ordinary observable shadow.

Proposition II.9 (Thin packet generation) *The target testing algebra is generated by the cover images:*

$$\mathcal{B}_0(X) = C^*\left(\bigcup_{i \in I} \alpha_{\psi_i}^0(\mathcal{B}_0(X_i))\right).$$

Proof. Equation (15) makes each recombination defect null. Finite Wilson-packet classes are dense in $\mathcal{B}_0(X)$. $\square$

Theorem II.10 (Normal admissible-cover additivity (R5.D)) *For every declared packet-localizing cover,*

$$\mathcal{A}_{G, \xi, 0}^W(X) = W^*\left(\bigcup_{i \in I} \overline{\alpha}_{\psi_i}^0(\mathcal{A}_{G, \xi, 0}^W(X_i))\right). \quad (16)$$

Proof. The target selected representation of each source testing algebra is quasi-equivalent to the source selected representation by Lemma II.5. Hence the normal image of $\mathcal{A}_{G, \xi, 0}^W(X_i)$ is exactly the von Neumann closure of the represented source testing algebra inside the target. The preceding proposition and the selected-representation saturation clause in OJ5 now give (16). No assertion is made for an arbitrary open cover. $\square$

E. Controlled covariance, Cauchy generation, and causal dynamics

1. Controlled-background covariance

Let $\psi : (\mathbb{M}_{0,\text{inc}}, O) \rightarrow (\mathbb{M}'_{0,\text{inc}}, O')$ be an admissible background–region arrow. OJ7 supplies elementwise positive-thickness observable maps and null comparison-square and composition defects. Proposition II.3 and Theorem II.6 therefore give the strict normal covariant arrow. If ψ moves Σ , it maps the fiber over $\mathbb{M}_{0,\text{inc}}$ to the fiber over $\mathbb{M}'_{0,\text{inc}}$; it is not a fixed-fiber automorphism.

2. Bounded coherent source sections

Let $\chi : X_C = (\mathbb{M}_{0,\text{inc}}, C) \rightarrow X_D = (\mathbb{M}_{0,\text{inc}}, D)$ be a selected Cauchy arrow inside an incision-adapted globally hyperbolic subspacetime $N \in M_{\text{inc}}(\mathbb{M}_{0,\text{inc}})$. The target region lies in the selected relative development $D_N(C)$. For every finite target packet section $p = (p_n)$, OJ6 supplies a source section $q = (q_n)$ satisfying

$$\|q_n\| \leq C_{\text{ts}} \|p_n\|, \quad (17)$$

$$p_{n;X_D}^{\text{sat}}(p_n - \alpha_\chi^{(n)}(q_n)) \leq \delta_n^{\text{ts}} \longrightarrow 0, \quad (18)$$

and the family q is comparison coherent on every selected scale, regional, background, and alternative-Cauchy-frame square. Componentwise bounded preimages without this coherence are not sufficient.

Theorem II.11 (Incision-adapted thin time slice (R5.E)) *Every selected Cauchy arrow $\chi : X_C \rightarrow X_D$ is sent to a normal unital $*$ -isomorphism*

$$\bar{\alpha}_\chi^0 : \mathcal{A}_{G,\xi,0}^W(X_C) \xrightarrow{\cong} \mathcal{A}_{G,\xi,0}^W(X_D).$$

Proof. Injectivity follows from Proposition II.3. Equations (17)–(18) give a bounded preimage on the dense finite-packet core, and comparison coherence makes it a source comparison section. A standard bounded approximation and diagonal argument gives surjectivity of the thin C^* arrow. The normal extension has ultraweakly closed range and contains the ultraweakly dense represented target testing algebra, so it is onto the selected normal envelope. The construction remains inside the relative incised development $D_N(C)$ and never propagates through Σ . $\square$

3. Local Cauchy groupoid

For two selected Cauchy arrows

$$\chi_- : X_- \rightarrow X_D, \quad \chi_+ : X_+ \rightarrow X_D$$

inside one admissible incised development, define

$$\tau_{+,-} := (\bar{\alpha}_{\chi_+}^0)^{-1} \bar{\alpha}_{\chi_-}^0 : \mathcal{A}_{G,\xi,0}^W(X_-) \longrightarrow \mathcal{A}_{G,\xi,0}^W(X_+). \quad (19)$$

Proposition II.12 (Incised causal dynamics) *The maps (19) are normal *-isomorphisms, satisfy*

$$\tau_{3,2}\tau_{2,1} = \tau_{3,1}, \quad \tau_{1,1} = \text{id},$$

and are independent of the selected admissible Cauchy frame and path after passage to a finite common refinement.

Proof. The time-slice theorem gives the inverses. Strict composition follows algebraically. OJ6 and OJ7 make the thick frame-change and Cauchy-comparison cube defects packet null; OJ8 places any two selected frames in one finite common refinement. Hence the corresponding thin maps agree on the Wilson-packet core and therefore on the selected normal envelopes. $\square$

Scope II.13 (No global-hyperbolicity or spectral conclusion) *The time-slice and causal-dynamics statements are internal to selected incision-adapted globally hyperbolic developments. They do not assert that all of $M_{\text{inc}}(\mathbb{M}_{0,\text{inc}})$ is globally hyperbolic, remove or fill Σ , construct a spacetime generator, or imply energy positivity, a spectrum condition, or a mass gap.*

F. Comparable-choice independence and full observable assembly

Suppose two corridors or extraction systems carry the same controlled incision stratification, collar orientation, region class, Wilson-packet core, and selected state annihilator. OJ8 provides a finite common refinement. Pullback to that refinement gives mutually inverse maps on comparison sections whose composites differ from the identity by common-null sections.

Theorem II.14 (Canonical comparable-choice independence) *The two testing functors are canonically naturally $*$ -isomorphic. The isomorphism transports the selected state families bijectively and therefore extends to a canonical normal natural isomorphism of the selected observable functors.*

Proof. The common-refinement maps are inverse on the testing quotients. They identify the selected limiting state families by pullback, so their selected direct-sum representations are quasi-equivalent. The resulting normal extensions are inverse and natural on every finite diagram. The argument does not compare a filled incision, a removed incision, an uncontrolled collar model, or a noncomparable regularization. $\square$

Theorem II.15 (Strict ordinary incised observable functor (R5.F)) *Under OJ1–OJ8, the assignment*

$$X \longmapsto \mathcal{A}_{G,\xi,0}^W(X), \quad \psi \longmapsto \overline{\alpha}_\psi^0$$

is a strict functor from $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ to $W^*\text{Alg}_{\mathfrak{n}}$ satisfying the HK0–HK7 properties listed in main article Theorem A.

Proof. The testing algebra and finite-packet generation are Theorem II.2; normality and isotony are Theorem II.6; locality is Theorem II.7; normal additivity is Theorem II.10; controlled covariance is built into the normal functor; time slice and causal dynamics are Theorem II.11 and Proposition II.12; choice independence is Theorem II.14. $\square$

Firewall II.16 (Exact scope of the ordinary observable theorem) *The output is global only on the selected controlled incised category. It is not an observable net on all open subsets of unincised Minkowski space, assigns no algebra to the omitted locus, proves no arbitrary-cover additivity or Haag duality, and supplies no invariant state, global physical Hilbert space, fixed-fiber Poincare action, translation generator, spectrum condition, invariant mass, or mass gap. The strict bounded observable shadow is an independent OJ2 input even when the represented target is line twisted or correspondence valued.*

III. FINITE-WINDOW EXPECTATION GERMS

A. Selected compression and source expectation

Fix a bounded controlled object $D = (\mathbb{M}_{0,\text{inc}}, O)$ and a good window N . Let

$$p_{n;D}^{(N)} = g_N(K_{n;D}^{\text{sel}})$$

be the total selected window projection, where g_N equals one on the chosen finite family of spectral islands and is supported inside the surrounding gaps. Set

$$\mathcal{N}_{n;D}^{(N)} = p_{n;D}^{(N)} \mathcal{N}_{n;D} p_{n;D}^{(N)}.$$

The positive-thickness source supplies a normal reference-preserving conditional expectation

$$\mathbb{E}_{n;D}^{(N)} : \mathcal{N}_{n;D}^{(N)} \longrightarrow \mathcal{F}_{n;D}^{(N)} := \text{Ran } \mathbb{E}_{n;D}^{(N)}. \quad (20)$$

It is normal, unital, completely positive, contractive, idempotent, and preserves the selected sector, central support, and complete-ground face. No limiting operator is used to define (20).

Let $\sigma_{n;D}^{(N)}$ be the faithful compressed reference and

$$L_{n;D}^{(N),\text{ref}} = -\log \sigma_{n;D}^{(N)}.$$

The logarithm is controlled independently on every fixed good window; its convergence is not inferred from relative entropy.

B. Common-carrier transport

Choose comparison unitaries or quasi-unitary finite-carrier identifications

$$J_{n;D}^{(N)} : p_{n;D}^{(N)} H_{n;D} \longrightarrow H_D^{(N)}.$$

Write tildes for transported objects. The fixed-window primitive data are:

$$\|\tilde{p}_{n;D}^{(N)} - p_D^{(N)}\| \rightarrow 0, \quad (21)$$

$$\|\tilde{\sigma}_{n;D}^{(N)} - \sigma_D^{(N)}\|_1 \rightarrow 0, \quad (22)$$

$$\|\log \tilde{\sigma}_{n;D}^{(N)} - \log \sigma_D^{(N)}\| \rightarrow 0, \quad (23)$$

together with one branch-specific expectation-germ certificate.

Definition III.1 (Fixed-window expectation-germ certificate) *A fixed-window branch has a certified expectation germ if one of the following holds after transport to $H_D^{(N)}$:*

(G1) $\tilde{\mathbb{E}}_{n;D}^{(N)}$ is exactly stationary;

(G2) there is a summable one-step bound

$$\sum_n \|\tilde{\mathbb{E}}_{n+1;D}^{(N)} - \tilde{\mathbb{E}}_{n;D}^{(N)}\|_{1 \rightarrow 1} < \infty;$$

(G3) there is a direct Cauchy estimate in both $1 \rightarrow 1$ and $\infty \rightarrow \infty$ norm and every cluster point is the unique reference-preserving conditional expectation onto one declared limiting finite-dimensional algebra.

The exact-dilation branches use (G1). The general theorem permits (G2) or (G3). Convergence of fixed algebras in Hilbert–Schmidt gap can verify (G3) when combined with the independent faithful-reference control below, but no unrestricted equivalence between algebra-gap convergence and expectation convergence is asserted.

Lemma III.2 (Reference-weighted finite-dimensional projection) *Let $F \subseteq M_d(\mathbb{C})$ be a unital $*$ -subalgebra and let $\sigma \geq \gamma I > 0$ be a faithful density fixed by a conditional expectation E . Then E is the orthogonal projection onto F for*

$$\langle x, y \rangle_\sigma = \text{Tr}(\sigma^{1/2} x^* \sigma^{1/2} y).$$

If bases of F_n converge and $\sigma_n \rightarrow \sigma$, with one faithful lower bound, then the associated reference-preserving expectations converge in every operator norm on the finite-dimensional operator space.

Proof. The bimodule relation and reference preservation imply self-adjointness for $\langle \cdot, \cdot \rangle_\sigma$. Idempotence and range F identify E with the weighted orthogonal projection. In a moving basis, the coefficients are expressed through the inverse weighted Gram matrix. Uniform faithfulness keeps the Gram determinants away from zero, so the coefficients vary continuously. Finite-dimensional norm equivalence gives the stated norms. $\square$

Theorem III.3 (Unique moving expectation germ) *Under [theorem III.1](#) and the fixed-window reference data, there is a unique normal completely positive contractive idempotent*

$$\mathbb{E}_D^{(N), \text{as}}$$

such that

$$\|\tilde{\mathbb{E}}_{n;D}^{(N)} - \mathbb{E}_D^{(N),\text{as}}\|_{1 \rightarrow 1} + \|\tilde{\mathbb{E}}_{n;D}^{(N)} - \mathbb{E}_D^{(N),\text{as}}\|_{\infty \rightarrow \infty} \longrightarrow 0. \quad (24)$$

Moreover

$$\mathcal{F}_D^{(N),\text{as}} = \text{Ran } \mathbb{E}_D^{(N),\text{as}}.$$

Proof. Case (G1) is immediate. In case (G2), the summable defect makes the sequence Cauchy in both norms. In case (G3), existence is part of the Cauchy certificate and uniqueness follows from the declared reference-preserving expectation-algebra uniqueness. Complete positivity, contractivity, reference preservation, and idempotence pass through norm convergence. $\square$

C. Coherent good-window towers

For $N_1 \leq N_2$, the positive-thickness source satisfies the exact compression and zero-extension identities

$$p_n^{(N_1)} \mathbb{E}_n^{(N_2)}(x) p_n^{(N_1)} = \mathbb{E}_n^{(N_1)}(p_n^{(N_1)} x p_n^{(N_1)}), \quad (25)$$

$$\mathbb{E}_n^{(N_2)}(y) = y, \quad y \in \mathcal{F}_n^{(N_1)}. \quad (26)$$

Passing to the unique fixed-window germs gives

$$\mathbb{E}_D^{(N_1),\text{as}} \mathbb{E}_D^{(N_2),\text{as}} = \mathbb{E}_D^{(N_1),\text{as}}, \quad N_1 \leq N_2. \quad (27)$$

Thus compatibility of the window tower is derived from the positive-thickness identities and fixed-window uniqueness. It is not a separate asymptotic assumption.

IV. PHYSICAL ORLICZ AND ENTROPY PACKETS

A. Reference realization and selected packet

Let $\sigma_{n;D}$ be the faithful surgery-invariant reference in the selected standard trace-class realization, and let

$$L_{n;D}^{\text{ref}} = -\log \sigma_{n;D}.$$

For constants c, M and a superlinear Young function Φ , define

$$\mathfrak{K}_{n;D}^{\text{Orl}}(c, M, \Phi) = \left\{ \rho : D(\rho \| \sigma_{n;D}) \leq c, \quad \langle \rho, \Phi(L_{n;D}^{\text{ref}}) \rangle \leq M \right\}, \quad (28)$$

intersected with the selected sector, central-support, complete-ground, and spatial-collar conditions. In an abstract standard form, $\langle \rho, B \rangle$ denotes the normal pairing with the affiliated positive operator B ; in the trace-class realization it is $\text{Tr}(\rho B)$.

The reference profile also supplies

$$d_D(R) := \sup_n \text{rank } \mathbf{1}_{[0,R]}(L_{n;D}^{\text{ref}}) < \infty \quad (29)$$

for every fixed R and every fixed finite diagram. This counting input is independent of the Orlicz moment and is needed for the finite-dimensional continuity step.

B. Mass and reference-logarithm tails

Set

$$e_{n,R} = \mathbf{1}_{(R,\infty)}(L_{n;D}^{\text{ref}}), \quad p_{n,R} = 1 - e_{n,R}.$$

The superlinear moment gives

$$\sup_{n,\rho} \rho(e_{n,R}) \leq \frac{M}{\Phi(R)} =: m_D(R) \rightarrow 0, \quad (30)$$

$$\sup_{n,\rho} \langle \rho, L_{n;D}^{\text{ref}} e_{n,R} \rangle \leq M \sup_{t \geq R} \frac{t}{\Phi(t)} =: \ell_D(R) \rightarrow 0. \quad (31)$$

The second estimate is the independent reference-logarithm control used in the entropy and regulator-removal arguments.

C. Discarded von Neumann entropy

Let $m = \rho(e_{n,R})$, set

$$\rho_{n,R} = \frac{p_{n,R} \rho p_{n,R}}{1 - m}, \quad a_{n,R}^\perp = e_{n,R} \rho e_{n,R},$$

and put

$$C_D := \sup_{n,\rho} \langle \rho, L_{n;D}^{\text{ref}} \rangle < \infty.$$

If $a \geq 0$ has trace $p \leq 1$ and is supported in a projection commuting with the faithful reference $\sigma = e^{-L}$, positivity of relative entropy against the normalized restricted reference gives

$$S(a) \leq \langle a, L \rangle + p |\log p|. \quad (32)$$

The two-outcome pinching map

$$\Pi_R(\rho) = p_{n,R}\rho p_{n,R} + e_{n,R}\rho e_{n,R}$$

satisfies

$$0 \leq S(\Pi_R(\rho)) - S(\rho) \leq h_2(m),$$

and

$$S(\Pi_R(\rho)) = (1 - m)S(\rho_{n,R}) - (1 - m)\log(1 - m) + S(a_{n,R}^\perp).$$

Applying (32), (31), and the bound

$$S(\rho_{n,R}) \leq \frac{1}{1 - m} \langle \rho, L_{n;D}^{\text{ref}} \rangle \leq \frac{C_D}{1 - m}$$

gives

$$|S(\rho) - S(\rho_{n,R})| \leq s_D(R), \quad (33)$$

where

$$\begin{aligned} s_D(R) &:= h_2(m_D(R)) + (1 - m_D(R))|\log(1 - m_D(R))| + \ell_D(R) \\ &\quad + m_D(R)|\log m_D(R)| + \frac{m_D(R)}{1 - m_D(R)}C_D \longrightarrow 0. \end{aligned} \quad (34)$$

This is the discarded-entropy estimate. It does not assert that surgery decreases ordinary von Neumann entropy.

D. Common packetwise entropy modulus

For ρ, ρ' in the same packet, let $t = \frac{1}{2}\|\rho - \rho'\|_1$. The normalized low blocks satisfy

$$\frac{1}{2}\|\hat{\rho}_{n,R} - \hat{\rho}'_{n,R}\|_1 \leq \frac{t + 2m_D(R)}{1 - m_D(R)}.$$

By (29), both low blocks act on a space of dimension at most $d_D(R)$. The finite-dimensional entropy continuity bound and (33) give

$$\begin{aligned} |S(\rho) - S(\rho')| &\leq 2s_D(R) + \frac{t + 2m_D(R)}{1 - m_D(R)}\log(d_D(R) - 1) \\ &\quad + h_2\left(\frac{t + 2m_D(R)}{1 - m_D(R)}\right). \end{aligned} \quad (35)$$

Optimizing over admissible R defines one modulus

$$\omega_D^{\text{Ent}}(t) \downarrow 0 \quad (t \downarrow 0) \quad (36)$$

uniformly on every fixed finite diagram. The same truncation with (31) retained gives the reference-weighted and relative-entropy moduli used below.

E. Expectation stability and relative entropy

If $\mathbb{E}_n^*(\sigma_n) = \sigma_n$, data processing gives

$$D(\mathbb{E}_n^* \rho \| \sigma_n) \leq D(\rho \| \sigma_n). \quad (37)$$

The Orlicz moment is preserved exactly when the expectation fixes $\Phi(L_n^{\text{ref}})$, and is uniformly controlled under the declared asymptotic reference defect otherwise. Hence surgery maps the selected packet into a packet with the same, or explicitly enlarged, branch constants.

V. SECTOR-LOCKED ENTROPY RECOVERY

Let ρ_0 be a limiting selected state. The primitive recovery data give, for each fixed good window, a finite-stage root family $\rho_n^{(N)}$ satisfying sector, central-support, and complete-ground conditions, with

$$T_{0n}^* \rho_n^{(N)} \rightarrow p_0^{(N)} \rho_0 p_0^{(N)}$$

in predual norm and with the correct finite-window entropy limsup. Extend the low-window recovery by the transported faithful reference on the complement with the exact missing mass. Equations (30)–(33) show that this extension changes the state and entropy by a quantity tending to zero with N . A common diagonal therefore yields

$$T_{0n}^* \rho_n \rightarrow \rho_0, \quad (38)$$

$$\limsup_n \mathcal{E}_n(\rho_n) \leq \mathcal{E}_0(\rho_0), \quad (39)$$

while preserving the selected packet, sector, central support, and complete-ground face. No recovery is assembled edge by edge.

VI. FINITE SMOOTH KRAUS COMPARISON

A. Band-limited finite-window channel

For a good window N and fixed buffer d , choose finitely many real smooth functions $f_j^{(N,d)}$ such that

$$\sum_{j=1}^{J(N)} (f_j^{(N,d)})^2 = 1$$

on the selected carrier, each f_j is band limited, and the family agrees with the sharp finite spectral blocks away from the transition strips. Define the normal completely positive trace-preserving channel

$$\mathcal{K}_{n;D}^{(N,d),*}(\rho) = \sum_{j=1}^{J(N)} f_j^{(N,d)}(K_{n;D}^{\text{sel}}) \rho f_j^{(N,d)}(K_{n;D}^{\text{sel}}). \quad (40)$$

The proof device is the channel (40), not the non-positive expression $F(K)^*\rho$.

B. Smooth face defect

Let $a : \sigma \rightarrow \tau$ be a state-direction Čech face or buffered regional arrow and let $R_{n,a}^*$ be its normal-state map on the selected common-refinement carrier. Smooth functional calculus and the repaired finite-wave collar estimate give

$$\eta_a^{\text{sm}}(n; N, d) := \sup_{\rho \in \mathfrak{K}_n^{\text{Orl}}(a)} \left\| R_{n,a}^* \mathcal{K}_{n;\sigma}^{(N,d),*} \rho - \mathcal{K}_{n;\tau}^{(N,d),*} R_{n,a}^* \rho \right\|_1 \longrightarrow 0 \quad (41)$$

for fixed finite diagram, fixed d , and fixed N . The supremum is taken on the joint endpoint packet, not on unrelated source and target packets.

C. Sharp-to-smooth endpoint estimate

Let $\mathcal{T}_{n;D}^{(N,d)}$ be the transition-strip projection and let $Q_{n;D}^{(N)}$ be the high-window complement. The gentle measurement estimate applied to the finite Kraus family yields

$$\|\mathbb{E}_{n;D}^{(N),*} \rho - \mathcal{K}_{n;D}^{(N,d),*} \rho\|_1 \leq C_N \left(\sqrt{\rho(\mathcal{T}_{n;D}^{(N,d)})} + \sqrt{\rho(Q_{n;D}^{(N)})} + \sqrt{\rho(1 - \chi_d^2)} \right). \quad (42)$$

At fixed (N, d) , finite-window spectral stability makes the transition mass vanish after $n \rightarrow \infty$. The Orlicz mass tail removes $Q^{(N)}$ as $N \rightarrow \infty$, and spatial collar tightness removes the buffer last.

VII. SHARP ČECH-FACE SURGERY DESCENT

Combining (41) and the two endpoint estimates (42) gives

$$\Delta_a(n; N, d) \leq \eta_a^{\text{sm}}(n; N, d) + C_{a,N} \left(\sqrt{t_a(n; N, d)} + \sqrt{m_D(N)} + \sqrt{\kappa_D(d)} \right), \quad (43)$$

where t_a is the maximum endpoint transition mass. Therefore

$$\lim_{d \downarrow 0} \lim_{N \rightarrow \infty} \limsup_{n \rightarrow \infty} \Delta_a(n; N, d) = 0. \quad (44)$$

The finite diagram and buffer are fixed first, the good window is fixed second, and only then is $n \rightarrow \infty$ taken. Equation (44) proves

$$\sup_{\rho \in \mathfrak{R}_n^{\text{Orl}}(a)} \left\| R_{n,a}^* \mathcal{S}_{n;\sigma}^{\text{st},*} \rho - \mathcal{S}_{n;\tau}^{\text{st},*} R_{n,a}^* \rho \right\|_1 \longrightarrow 0. \quad (45)$$

Exact chart, corridor, Cauchy, background, common-refinement, and exact-dilation arrows have zero defect. The scalar coefficient lines are external to the normal-state channel and retain their tensorators and associator.

VIII. SIMULTANEOUS ROOTED FINITE-DIAGRAM LIFTING

The detailed combinatorics are proved in Supplement K. The construction is summarized here because it is load bearing.

Let $\mathcal{D} = (V, E, C_2, C_3; v_0)$ be a finite rooted selected diagram. The primitive SJ6 datum supplies one strict unsurgered positive-thickness state shadow. For one root recovery $\rho_{v_0,n}$ define

$$\rho_{v,n}^{\text{pre}} = T_{p_v,n}^* \rho_{v_0,n}$$

for chosen rooted paths p_v . The strict unsurgered face laws make this family path independent in the observable and normal-state shadow. Apply surgery objectwise:

$$\hat{\rho}_{v,n} = \mathcal{S}_{n;v}^{\text{st},*} \rho_{v,n}^{\text{pre}}.$$

For each edge $e : v \rightarrow w$,

$$\|T_{e,n}^* \hat{\rho}_{v,n} - \hat{\rho}_{w,n}\|_1 \leq \Delta_e(n; N, d). \quad (46)$$

All higher strict-shadow cells are controlled by finite telescoping sums of (46). The coefficient-line cells retain their prescribed reassociation.

Define

$$\mathfrak{D}_{\mathcal{D}}(n; N, d) = d_{\mathcal{D},n}^{\text{root}} + c(\mathcal{D}) \max_{e \in E} \Delta_e(n; N, d) + \tau_{\mathcal{D}}^{\text{Orl}}(N, d), \quad (47)$$

where $d_{\mathcal{D},n}^{\text{root}}$ is the common root-recovery defect, $c(\mathcal{D})$ is the maximal finite telescoping length, and $\tau_{\mathcal{D}}^{\text{Orl}}$ is the common tail error. Then

$$\lim_{d \downarrow 0} \lim_{N \rightarrow \infty} \limsup_{n \rightarrow \infty} \mathfrak{D}_{\mathcal{D}}(n; N, d) = 0.$$

One diagonal family therefore closes every vertex, edge, and face simultaneously. No affine constraint projection, product of approximate idempotents, alternating projection, or pairwise intersection is used.

Theorem VIII.1 (Rooted joint lifting) *Every finite selected incised diagram in the strict or scalar line-valued state shadow admits one simultaneous recovery family. It satisfies the selected sector, central support, complete-ground face, Orlicz bounds, entropy limsup, sharp surgery naturality, and represented scalar associator.*

Proof. Use the one strict unsurgered root, apply surgery objectwise, estimate every edge by (46), telescope over the finite face decompositions, and choose one diagonal using (47). The complete-ground, sector, and central-support faces are fixed objectwise by the source expectation. Relative entropy is contractive and the packet tails are uniform on the finite diagram. Scalar coefficient data are transported in the represented lift and are not altered by the state expectation. $\square$

IX. INHERITED EXPECTATION GRAPH

For each object D let $\mathfrak{D}_D^{\text{Orl}}$ be the limiting state domain admitting the selected rooted recovery. Define

$$\mathcal{S}_{G,\xi,0;D}^{\text{st,as},*} = \text{graph-lim}_{n \rightarrow \infty} \mathfrak{T}_{0n;D}^* \mathcal{S}_{G,\xi,n;D}^{\text{st},*} \mathfrak{T}_{n0;D}^* \quad (48)$$

on $\mathfrak{D}_D^{\text{Orl}}$. Fixed-window uniqueness, the coherent tower, uniform regulator removal, and one rooted common diagonal show that (48) is single valued and independent of the good-window ladder and admissibly comparable recovery representative.

Theorem IX.1 (Corrected SLSP) *Under OJ, RJ, and SJ in the strict or scalar line-valued state-shadow class, the inherited graph has moving expectation convergence, coherent windows, separate mass/logarithm/entropy tails, sector-locked entropy recovery, a common entropy modulus, sharp incised Čech and regional descent, one simultaneous rooted lift, exact sector/central-support/complete-ground preservation, predual contraction, idempotence, and exact retention of the scalar coefficient associator.*

Proof. Combine theorems III.3 and VIII.1, the tail and continuity estimates, and the sharp edge theorem. Idempotence follows from (27) after regulator removal. Gauss–Ward and collar augmentation are not part of this theorem; they require the separate GJ argument. $\square$

X. RESTART AND ADMISSIBILITY CLOSURE

Theorem X.1 (Sector-locked entropy-surgery restart) *Assume corrected SLSP and the admissible fiber-thickness preservation/recovery environment of SJ7. Then*

$$\text{Restart}_{G,\xi} : \mathbf{Rec}_{G,\xi}^{\text{adm}} \longrightarrow \mathbf{Rec}_{G,\xi}^{\text{adm}}$$

is a sector-preserving idempotent endomorphism.

Proof. For every selected arrow $a : v \rightarrow w$ and admissible unsurgered family,

$$\begin{aligned} \|T_{a,*}\mathcal{S}_v^{\text{st,as},*}\rho_v - \mathcal{S}_w^{\text{st,as},*}\rho_w\|_1 &\leq \|T_{a,*}\mathcal{S}_v^{\text{st,as},*}\rho_v - \mathcal{S}_w^{\text{st,as},*}T_{a,*}\rho_v\|_1 \\ &\quad + \|\mathcal{S}_w^{\text{st,as},*}T_{a,*}\rho_v - \mathcal{S}_w^{\text{st,as},*}\rho_w\|_1. \end{aligned}$$

The first term is the surgery-naturality defect controlled by corrected SLSP, and the second is bounded by the original seam defect because surgery is predual contractive. SJ7 supplies admissible fiber-thickness closure. Sector, complete-ground, Orlicz, and scalar coefficient data are preserved by the preceding theorems. Idempotence follows from the coherent expectation tower. No exact one-shot scale independence is claimed for a merely asymptotic arrow. $\square$

XI. GAUSS GRAPH FORMS, INSERTION DERIVATIONS, AND GRAPH-PROJECTIVE STATES

A. The two graph coordinates

Fix $X = (\mathbb{M}_{0,\text{inc}}, D) \in \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ and a bounded parameter packet $B_{\text{par}} \in \mathfrak{G}_D^{\text{int}}$. For each n the positive-thickness constraint package consists of:

- (i) a common vector-form domain $\mathfrak{D}_{n;D}^{\text{G}}$ and closable sesquilinear forms

$$\mathbb{G}_{\zeta,n;D}^{\text{int}} : \mathfrak{D}_{n;D}^{\text{G}} \times \mathfrak{D}_{n;D}^{\text{G}} \rightarrow \mathbb{C}, \quad \zeta \in B_{\text{par}};$$

- (ii) a comparison-stable observable insertion core $\mathfrak{C}_{n;D}^{\text{ins}}$ and closable derivations

$$\delta_{\zeta,n}^{\text{int}} : \mathfrak{C}_{n;D}^{\text{ins}} \rightarrow \mathcal{N}_{n;D}.$$

The two structures are related by the Ward implementation identity but are not identified.

Let $U_{0n;D}$ denote the represented comparison map on vector-form domains and $T_{0n;D}$ the normal-algebra comparison map. GJ1 supplies graph convergence in both coordinates: for every limiting test (Ψ, Φ, A) there are approximants (Ψ_n, Φ_n, A_n) such that

$$U_{0n;D}\Psi_n \rightarrow \Psi, \quad U_{0n;D}\Phi_n \rightarrow \Phi, \quad T_{0n;D}A_n \rightarrow A \quad (49)$$

$$\mathbb{G}_{\zeta,n;D}^{\text{int}}[\Psi_n, \Phi_n] \rightarrow \mathbb{G}_{\zeta,0;D}^{\text{int}}[\Psi, \Phi], \quad T_{0n;D}\delta_{\zeta,n}^{\text{int}}A_n \rightarrow \delta_{\zeta,0}^{\text{int}}A \quad (50)$$

uniformly for ζ in bounded parameter packets. The graph-closedness part of GJ1 excludes nonzero vertical limits; this is stronger than pointwise closability of the varying finite-stage graphs.

Proposition XI.1 (Closed limiting graph package) *The limiting Gauss forms are closed or closable on the declared common limiting domain, and each $\delta_{\zeta,0}^{\text{int}}$ is closable on $\mathfrak{C}_{0,D}^{\text{ins}}$. The closures are independent of the chosen graph approximants.*

Proof. The asserted graph convergence includes the no-vertical-limit estimate: if first coordinates tend to zero and second coordinates converge, then the second-coordinate limit is zero. Hence the limiting graphs are closable. If two approximating families represent the same limiting datum, their difference is graph-null in both coordinates, so the resulting form and derivation limits agree. $\square$

B. Interior Gauss kernel and complete-ground nullity

Define

$$\mathcal{K}_{n;D}^{\text{int}} := \left\{ \Psi \in \mathfrak{D}_{n;D}^G : \mathbb{G}_{\zeta,n;D}^{\text{int}}[\Psi, \Phi] = 0 \text{ for all } \Phi \in \mathfrak{D}_{n;D}^G, \zeta \in B_{\text{par}} \right\}. \quad (51)$$

GJ7 supplies the independent thick inclusion

$$\text{Ran } P_{\Omega,n;D} \subseteq \mathcal{K}_{n;D}^{\text{int}}. \quad (52)$$

Taking graph limits in (52) yields

$$\text{Ran } P_{\Omega,0;D} \subseteq \mathcal{K}_{0;D}^{\text{int}}. \quad (53)$$

Gauge invariance of $P_{\Omega,n;D}$ may also hold, but it is not used as a substitute for (52).

C. Graph-projective predual packet

Let $\|\cdot\|_{G,n}$ be a common graph norm on $\mathfrak{D}_{n;D}^G$ controlling all forms in B_{par} . Finite sums of vector functionals

$$\rho = \sum_{j=1}^m \omega_{\Psi_j, \Phi_j}$$

are given the projective graph norm

$$\|\rho\|_{G,*,n} := \inf \sum_j \|\Psi_j\|_{G,n} \|\Phi_j\|_{G,n}.$$

Its completion inside the normal predual is denoted $\mathfrak{T}_{n;D,*}^G$. The Ward functional below is continuous on bounded observable and parameter packets and therefore descends independently of the chosen vector-functional decomposition.

XII. THE FULL GRAPH-FORM WARD IDENTITY

For $A \in \mathfrak{C}_{n;D}^{\text{ins}}$ define

$$\begin{aligned} \mathfrak{w}_{\zeta,n;D}^{\text{int}}(A)[\Psi, \Phi] &:= i \mathbb{G}_{\zeta,n;D}^{\text{int}}[\pi_n(A)\Psi, \Phi] - i \mathbb{G}_{\zeta,n;D}^{\text{int}}[\Psi, \pi_n(A^*)\Phi] \\ &\quad - \langle \pi_n(\delta_{\zeta,n}^{\text{int}} A) \Psi, \Phi \rangle. \end{aligned} \tag{54}$$

By linearity and graph-projective continuity it extends to

$$\mathfrak{w}_{\rho,n;D}^{\text{int}}(\zeta; A), \quad \rho \in \mathfrak{T}_{n;D,*}^G.$$

The prepared domain is

$$\mathfrak{D}_{n;D}^{\text{GW-prep}} := \left\{ \rho \in \mathfrak{T}_{n;D,*}^G \cap \mathfrak{K}_{n;D}^{\text{Orl}} : \mathfrak{w}_{\rho,n;D}^{\text{int}}(\zeta; A) = 0 \text{ for every allowed } (\zeta, A) \right\}. \tag{55}$$

The reduced equation $\rho(\delta_{\zeta,n}^{\text{int}} A) = 0$ follows on Gauss-null vector pairs; it is not the definition of (55).

Theorem XII.1 (Thin graph-form Ward passage) *Under GJ1–GJ4, graph-convergent prepared finite-stage functionals converge to prepared thin functionals, and the complete functional (54) passes to the limit.*

Proof. Choose graph-convergent decompositions for the vector pairs and graph-convergent approximants for A . Each of the two Gauss-form implementation terms converges by (50);

the insertion term converges by the derivation graph convergence and normal representation comparison. Uniform graph-projective bounds permit summation and completion. Exact finite-stage vanishing therefore passes to the thin graph. $\square$

XIII. FIXED-WINDOW EXPECTATION EQUIVARIANCE

Let $E_{n,j}^{(N)}$ be the selected finite-window spectral projections. Strong gauge covariance, invariance of $\mathfrak{D}_{n;D}^G$ and $\mathfrak{C}_{n;D}^{\text{ins}}$ under every $E_{n,j}^{(N)}$, and differentiation of the exact group intertwining identity give

$$\mathbb{E}_{n;D}^{(N)} \delta_{\zeta,n}^{\text{int}} = \delta_{\zeta,n}^{\text{int}} \mathbb{E}_{n;D}^{(N)} \quad (56)$$

on the insertion core. The same projections preserve the Gauss form and commute with its implementation action.

Theorem XIII.1 (Limiting expectation and Ward intertwining) *For fixed D, N ,*

$$\mathbb{E}_{0;D}^{(N),\text{as}} \delta_{\zeta,0}^{\text{int}} = \delta_{\zeta,0}^{\text{int}} \mathbb{E}_{0;D}^{(N),\text{as}} \quad (57)$$

on $\mathfrak{C}_{0;D}^{\text{ins}}$. Moreover,

$$\mathfrak{w}_{\mathbb{E}_{0;D}^{(N),\text{as}},*}^{\text{int}}{}_{\rho,0;D}(\zeta; A) = \mathfrak{w}_{\rho,0;D}^{\text{int}}(\zeta; \mathbb{E}_{0;D}^{(N),\text{as}} A). \quad (58)$$

Proof. For a graph approximant A_n of A , the first-coordinate convergence gives

$$T_{0n;D} \mathbb{E}_{n;D}^{(N)} A_n \rightarrow \mathbb{E}_{0;D}^{(N),\text{as}} A.$$

Core invariance and (56) give the second-coordinate convergence

$$T_{0n;D} \delta_{\zeta,n}^{\text{int}} \mathbb{E}_{n;D}^{(N)} A_n \rightarrow \delta_{\zeta,0}^{\text{int}} \mathbb{E}_{0;D}^{(N),\text{as}} A.$$

This proves (57). Apply the finite Kraus decomposition simultaneously to both vector coordinates in (54); invariance of the Gauss form and (57) yield (58). Hence fixed-window expectations preserve (55). $\square$

XIV. COLLAR QUOTIENT AND SEPARATELY TYPED BOUNDARY INVARIANTS

A. Hausdorff comparison-natural quotient

Let $\mathfrak{G}_D^{\text{adm}}$ carry the parameter graph topology and let $\mathfrak{G}_D^{\text{int}}$ be the subspace of parameters supported away from the collar. Define the Hausdorff quotient exact sequence

$$0 \longrightarrow \overline{\mathfrak{G}_D^{\text{int}}}^{\text{cmp}} \longrightarrow \mathfrak{G}_D^{\text{adm}} \longrightarrow \mathcal{Q}_{G,\xi,0}^\partial(D) \longrightarrow 0. \quad (59)$$

Every selected regional, chart, scale, or mixed comparison map preserves the interior subspace, hence induces a continuous map on (59). This is the branchwise collar quotient in the strict, corridor-banded, line-twisted, and correspondence-valued cases.

B. Charge forms and zero-point character class

A collar splitting s assigns to $\lambda \in \mathcal{Q}^\partial$ an admissible representative. The corresponding collar charge is retained as a closed graph form

$$q_{n,D}^{\partial,s}(\lambda)[\Psi, \Phi]$$

on the joint interior Gauss domain. Chart and scale comparison may change its zero point by scalar cochains χ_n^{ch} and χ_n^{sc} , subject to the mixed compatibility equation. Their complete equivalence class is denoted

$$[\chi_{G,\xi,n}^\partial]_{\text{cs}}.$$

This symbol denotes the chart, scale, and mixed cocycle data modulo allowed zero-point cochains, not one scalar function.

C. Current-algebra and boundary-Ward defects

When operator representatives exist, define

$$\mathfrak{a}_n^\partial(\lambda, \mu) = \frac{1}{i} [Q_n^\partial(\lambda), Q_n^\partial(\mu)] - Q_n^\partial([\lambda, \mu]). \quad (60)$$

In the minimal branch, (60) is read as the corresponding sesquilinear graph-form defect. The first boundary-Ward defect is

$$\mathfrak{b}_n^\partial(\lambda; A)[\Psi, \Phi] := i q_n^\partial(\lambda)[\pi_n(A)\Psi, \Phi] - i q_n^\partial(\lambda)[\Psi, \pi_n(A^*)\Phi]$$

$$- \langle \pi_n(\delta_{\lambda,n}^\partial A) \Psi, \Phi \rangle. \quad (61)$$

Neither defect is assumed to vanish.

Proposition XIV.1 (Splitting independence) *The induced collar structure depends only on the class $\lambda \in \mathcal{Q}^\partial$. Changing the splitting by an interior parameter changes the collar form by the two Gauss implementation terms and the insertion term of the full interior Ward functional; these cancel on the prepared domain. The remaining scalar shift changes χ^∂ by an allowed zero-point coboundary.*

XV. AUGMENTED ROOTED LIFTING AND REMOVAL OF THE REGULATOR

For a finite mixed diagram $\mathcal{D}$, enlarge the SLSP master defect by

$$\begin{aligned} \mathfrak{D}_{\mathcal{D}}^{\text{GW}}(n; N, d) &:= \mathfrak{D}_{\mathcal{D}}^{\text{SLSP}}(n; N, d) + \Delta_{\mathcal{D}}^{\text{graph}}(n; N, d) + \Delta_{\mathcal{D}}^{\text{Ward}}(n; N, d) \\ &\quad + \Delta_{\mathcal{D}}^{\text{quot}}(n; N, d) + \Delta_{\mathcal{D}}^{\chi}(n; N, d) + \Delta_{\mathcal{D}}^{\mathfrak{a}}(n; N, d) + \Delta_{\mathcal{D}}^{\mathfrak{b}}(n; N, d). \end{aligned} \quad (62)$$

The new terms are suprema over finite bounded graph, parameter, observable, collar, and defect test packets. GJ1–GJ8 imply

$$\mathfrak{D}_{\mathcal{D}}^{\text{GW}}(n; N, d) \rightarrow 0$$

in the mandatory order. The construction remains the same strict unsurgered root followed by objectwise surgery and edge/higher-cell telescoping; no correction projection is introduced.

For every bounded Ward test packet $\mathcal{B}_{\text{Ward}}$ there is a modulus $\eta_N^{\text{Ward}} \downarrow 0$ such that

$$\sup_{\rho \in \mathfrak{D}_{0;D}^{\text{GW-prep}} \cap \mathfrak{R}_{0;D}^{\text{Orl}}} \sup_{(\zeta, A) \in \mathcal{B}_{\text{Ward}}} \left| \mathfrak{w}_{\mathcal{S}^{\text{st,as},*} \rho, 0; D}^{\text{int}}(\zeta; A) - \mathfrak{w}_{\mathbb{E}_{0;D}^{(N), \text{as},*} \rho, 0; D}^{\text{int}}(\zeta; A) \right| \leq \eta_N^{\text{Ward}}. \quad (63)$$

Graph-projective closedness and (63) remove the window and buffer after the thin limit.

Theorem XV.1 (Surgery-compatible Gauss–Ward/collar package) *The full inherited graph satisfies*

$$\mathcal{S}_{G, \xi, 0; D}^{\text{st,as},*} \left(\mathfrak{D}_{G, \xi, 0}^{\text{GW-prep}}(D) \right) \subseteq \mathfrak{D}_{G, \xi, 0}^{\text{GW-prep}}(D),$$

and

$$\mathfrak{w}_{\mathcal{S}^{\text{st,as},*} \rho, 0; D}^{\text{int}}(\zeta; A) = 0$$

for every prepared ρ . It transports the structural tuple

$$\left(\mathcal{Q}_{G,\xi,0}^\partial, \mathfrak{a}_{G,\xi,0}^\partial, [\chi_{G,\xi,0}^\partial]_{\text{cs}}, \mathfrak{b}_{G,\xi,0}^\partial\right) \quad (64)$$

up to canonical branchwise comparison and preserves the complete-ground inclusion (53).

Scope XV.2 (Structural versus numerical preservation) *The theorem does not assert equality of numerical boundary-charge expectations before and after surgery. It does not prove $\mathfrak{a}_0^\partial = 0$, $\mathfrak{b}_0^\partial = 0$, or anomaly cancellation. Represented strictification does not imply charge strictification. In Sector III the result is chart-corridor banded and graph-form valued; no global self-adjoint Gauss or boundary-charge generator is inferred.*

A. Handoff component

The corrected gauge component passed to Paper V is

$$\mathfrak{W}_{G,\xi}^\mathcal{S} = \left(\mathfrak{D}_{G,\xi,0}^{\text{GW-prep}}, \{\mathbb{G}_{G,\xi,0}^{\text{int}}(\zeta)\}_\zeta, \delta_{G,\xi,0}^{\text{int}}, \mathfrak{m}_{G,\xi,0}^{\text{int}}, \mathcal{Q}_{G,\xi,0}^\partial, \mathfrak{a}_{G,\xi,0}^\partial, [\chi_{G,\xi,0}^\partial]_{\text{cs}}, \mathfrak{b}_{G,\xi,0}^\partial, \mathcal{S}_{G,\xi,0}^{\text{st,as,*}}\right).$$

XVI. VARYING-CARRIER COMPARISON AND THE DIRECT LIMITING PREFORM

A. Comparison and reconstruction maps

The terminology of form perturbation, Mosco convergence, and varying spectral structures follows the standard references, while the synchronized comparison-carrier construction below is proved explicitly.^{9–12} For each bounded controlled object $X = (\mathbb{M}_{0,\text{inc}}, O)$ let $H_{n;O}$ carry the closed nonnegative renormalized form $\hat{q}_{n;O}$. M1 supplies uniformly bounded maps

$$J_{n;O} : H_{n;O} \rightarrow H_{0;O}, \quad I_{n;O} : H_{0;O} \rightarrow H_{n;O}$$

with Hilbert and graph defects

$$\|J_{n;O}I_{n;O}u - u\|_{H_{0;O}} \leq \eta_n \|u\|_{\mathcal{Q}_{0;O}}, \quad (65)$$

$$\|I_{n;O}J_{n;O}v - v\|_{H_{n;O}} \leq \eta_n \|v\|_{\hat{q}_{n;O}}, \quad (66)$$

where $\eta_n \rightarrow 0$ on every fixed finite regional diagram. In the exact-dilation branch $J_{n;O}$ is unitary and $I_{n;O} = J_{n;O}^{-1}$; inverse notation is reserved for that specialization.

Comparison-strong convergence $v_n \rightarrow u$ means $J_{n;O}v_n \rightarrow u$ together with vanishing reconstruction defect. Comparison-weak convergence $v_n \rightharpoonup u$ means

$$\langle v_n, I_{n;O}z \rangle_{H_{n;O}} \rightarrow \langle u, z \rangle_{H_{0;O}}$$

for every fixed $z \in H_{0;O}$, with bounded Hilbert norms.

B. Common core and limiting preform

Let $\mathcal{C}_O \subset H_{0;O}$ be the graph-dense comparison core from M2. Define

$$q_{0;O}^{\text{pre}}[u, v] := \lim_{n \rightarrow \infty} \hat{q}_{n;O}[I_{n;O}u, I_{n;O}v], \quad u, v \in \mathcal{C}_O. \quad (67)$$

The full sesquilinear defect estimate and the asymptotic inverse relations make (67) independent of the selected core reconstructions. It is nonnegative and closable after the synchronized strong-relaxation theorem below.

C. Exact-carrier relative estimate

In the unitary exact-dilation specialization, put

$$\hat{q}_{n;O}^J[u, v] := \hat{q}_{n;O}[I_{n;O}u, I_{n;O}v].$$

The primitive estimate is sesquilinear:

$$\left| \hat{q}_{n;O}^J[u, v] - q_{\text{ref};O}[u, v] \right| \leq \theta_{n;O} \|u\|_{\mathcal{Q}_O} \|v\|_{\mathcal{Q}_O}, \quad \theta_{n;O} \rightarrow 0. \quad (68)$$

For $\theta_{n;O} < 1/2$ this gives uniform graph-norm equivalence.

XVII. STRONG RELAXATION IS A CLOSED QUADRATIC FORM

Define

$$q_{0;O}^{\text{rel}}[u] := \inf_{v_n \rightarrow u \text{ cmp, strong}} \liminf_n \hat{q}_{n;O}[v_n]. \quad (69)$$

Theorem XVII.1 (Synchronized quadratic relaxation) *Under M1–M4, $q_{0;O}^{\text{rel}}$ is a densely defined closed nonnegative quadratic form and is the closure of $q_{0;O}^{\text{pre}}$.*

Proof. M4 supplies, for every finite set of core vectors, one recovery family respecting all prescribed linear combinations. For u, v choose one synchronized family for $u, v, u + v, u - v$. The finite-stage parallelogram identity and passage to the limit give the upper parallelogram inequality for $q_{0;O}^{\text{rel}}$. Applying the same argument to synchronized near-minimizers for $u + v$ and $u - v$, then reconstructing u and v by their half-sum and half-difference, gives the reverse inequality. Hence

$$q_{0;O}^{\text{rel}}[u + v] + q_{0;O}^{\text{rel}}[u - v] = 2q_{0;O}^{\text{rel}}[u] + 2q_{0;O}^{\text{rel}}[v].$$

Homogeneity is immediate from (69). Thus the functional is quadratic.

If $u_k \rightarrow u$ in $H_{0;O}$ and $q_{0;O}^{\text{rel}}[u_k]$ is bounded, choose a diagonal of strong recovery families whose Hilbert and energy errors are $< 2^{-k}$. The diagonal is an admissible recovery of u and proves lower semicontinuity. A nonnegative quadratic lower-semicontinuous functional is a closed form. Core recovery and graph density show that it is densely defined and equals the closure of (67). $\square$

XVIII. COMPARISON-WEAK LIMINF AND TT2

Theorem XVIII.1 (Regional comparison-weak liminf) *If $v_n \rightharpoonup u$ comparison weakly and*

$$\sup_n (\hat{q}_{n;O}[v_n] + \|v_n\|^2) < \infty,$$

then

$$q_{0;O}^{\text{rel}}[u] \leq \liminf_n \hat{q}_{n;O}[v_n].$$

Proof. Pass to a subsequence realizing the liminf. M3 gives finite convex combinations of tail sections that converge comparison strongly to u while their energies are bounded above by the corresponding convex combinations of the original energies. Convexity of every $\hat{q}_{n;O}$ and the definition (69) yield the result. This comparison-Mazur step is the varying-carrier replacement for silently placing all vectors in one Hilbert space. $\square$

Theorem XVIII.2 (Full graph-domain TT2) *For every $u \in D(q_{0;O}^{\text{rel}})$ there exists $u_n \in D(\hat{q}_{n;O})$ such that*

$$J_{n;O}u_n \rightarrow u, \quad \hat{q}_{n;O}[u_n] \rightarrow q_{0;O}^{\text{rel}}[u]. \quad (70)$$

The recoveries can be synchronized over every finite regional diagram and one monotone exhaustion.

Proof. Core recovery is M4. Approximate an arbitrary u in the closed-form graph norm by core vectors and diagonalize the core recoveries. For an exhaustion $O_j \nearrow O$, synchronize restriction, extension, local recovery, and collar-cutoff errors, then choose $j(n) \rightarrow \infty$ slowly. The detailed diagonal and cutoff estimates are recorded in Supplement LIX. $\square$

XIX. MOVING COMPLETE-GROUND FACE: TT1 AND EXACT GROUND RECOVERY

M5 gives, on graph-bounded moving families,

$$\|J_{n;O}P_{\Omega,n;O}v_n - P_{\Omega,0;O}J_{n;O}v_n\| \leq \gamma_n \|v_n\|_{\hat{q}_{n;O}}, \quad \gamma_n \rightarrow 0. \quad (71)$$

It also supplies

$$P_{\Omega,n;O}D(\hat{q}_{n;O}) \subseteq D(\hat{q}_{n;O}), \quad \hat{q}_{n;O}[P_{\Omega,n;O}v] = 0, \quad \hat{q}_{n;O}(P_{\Omega,n;O}v, (\mathbf{1} - P_{\Omega,n;O})w) = 0. \quad (72)$$

Theorem XIX.1 (Graph-domain TT1) *If $v_n \rightarrow u$ comparison strongly and the graph energies are bounded, then*

$$J_{n;O}P_{\Omega,n;O}v_n \rightarrow P_{\Omega,0;O}u, \quad (73)$$

$$J_{n;O}(\mathbf{1} - P_{\Omega,n;O})v_n \rightarrow (\mathbf{1} - P_{\Omega,0;O})u. \quad (74)$$

The analogous statements hold comparison weakly for graph-bounded weak families.

Proof. Add and subtract $P_{\Omega,0;O}J_{n;O}v_n$ and apply (71). The excitation statement follows by subtraction. For weak families, test against fixed vectors and use the adjoint comparison defect contained in M5. $\square$

Proposition XIX.2 (Ground-compatible TT2) *Let $u = P_{\Omega,0;O}u + u^\perp$. If v_n is a TT2 recovery of $u^\perp$, then*

$$u_n^\sharp := P_{\Omega,n;O}I_{n;O}P_{\Omega,0;O}u + (\mathbf{1} - P_{\Omega,n;O})v_n \quad (75)$$

is a TT2 recovery of u . The first term has exactly zero finite-stage form energy and the two terms are form orthogonal.

XX. COERCIVE TRANSFER AND EXACT KERNEL

Assume M6:

$$\widehat{q}_{n;O}[v] \geq \kappa_{\text{IV,thin}} \|(\mathbf{1} - P_{\Omega,n;O})v\|^2 \quad (76)$$

on a cofinal tail.

Theorem XX.1 (Thin coercivity by TT2 and TT1)

$$q_{0;O}^{\text{rel}}[u] \geq \kappa_{\text{IV,thin}} \|(\mathbf{1} - P_{\Omega,0;O})u\|^2. \quad (77)$$

Proof. Choose the TT2 recovery u_n from [theorem XVIII.2](#). Apply (76) and take the actual energy limit:

$$q_{0;O}^{\text{rel}}[u] = \lim_n \widehat{q}_{n;O}[u_n] \geq \kappa_{\text{IV,thin}} \lim_n \|(\mathbf{1} - P_{\Omega,n;O})u_n\|^2.$$

Theorem [XIX.1](#) identifies the last limit with the right side of (77). The Mosco-liminf inequality is not used to reverse an inequality. $\square$

Theorem XX.2 (Exact complete-ground kernel)

$$\ker q_{0;O}^{\text{rel}} = \text{Ran } P_{\Omega,0;O}.$$

Proof. Thin coercivity gives the inclusion from left to right. For the reverse inclusion, use the zero-energy ground recovery $P_{\Omega,n;O}I_{n;O}u$ and the definition of the strong relaxation. The domain invariance, nullity, and form orthogonality required here are exactly (72). $\square$

XXI. ASSOCIATED OPERATORS AND RESOLVENT CONSEQUENCES

The closed regional form has an associated positive self-adjoint operator $K_{0;O}^{\text{thin}}$ satisfying

$$E_{K_{0;O}^{\text{thin}}}(\{0\}) = P_{\Omega,0;O}, \quad \text{spec } K_{0;O}^{\text{thin}} \subseteq \{0\} \cup [\kappa_{\text{IV,thin}}, \infty).$$

Comparison–Mosco convergence yields strong comparison-resolvent convergence for $(K_{n;O} + \mu)^{-1}$ with $\mu > 0$ and strong heat-semigroup convergence. In an exact common carrier, the sesquilinear estimate (68) yields

$$\|(K_{n;O}^J + \mu)^{-1} - (K_{0;O} + \mu)^{-1}\| \leq C_\mu \theta_{n;O}.$$

This gives $C_0(\mathbb{R})$ functional calculus and Riesz-projection convergence for isolated spectral islands. Exact finite-stage preservation of the complete-ground face is separate physical endpoint data and is not inferred from norm-resolvent convergence.

On the excitation subspace the uniform gap permits fractional inverse powers and positive-frequency structures. No inverse at $\mu = 0$ is asserted on the full carrier.

XXII. EXACT-DILATION COEFFICIENT ESTIMATE

After exact unitary straightening, consider

$$\begin{aligned}\hat{q}_n^J[u, v] &= \langle (V + \lambda_n W_n)u, v \rangle \\ &\quad + \eta(1 + \lambda_n d_n) \langle (\nabla + \lambda_n C_n)u, (\nabla + \lambda_n C_n)v \rangle,\end{aligned}\tag{78}$$

where $\lambda_n = \varepsilon_n^\alpha$ and

$$|\langle W_n u, v \rangle| \leq C_W \|u\|_{\mathcal{Q}} \|v\|_{\mathcal{Q}}, \quad |d_n| \leq C_\eta, \quad \sqrt{\eta} \|C_n u\| \leq C_C \|u\|_{\mathcal{Q}}.\tag{79}$$

Expansion and Cauchy–Schwarz give the full sesquilinear estimate

$$|\hat{q}_n^J[u, v] - q_{\text{ref}}[u, v]| \leq C_{\text{form}} \varepsilon_n^\alpha \|u\|_{\mathcal{Q}} \|v\|_{\mathcal{Q}},\tag{80}$$

with

$$C_{\text{form}} = C_W + C_\eta + 2(1 + C_\eta)C_C + (1 + C_\eta)C_C^2.\tag{81}$$

This verifies M2 in the exact-dilation witness. M1, M3–M6, including exact ground form-nullity and the branchwise positive-thickness coercive constant, are supplied independently by the endpoint and recovery packages.

Scope XXII.1 (Regional and sector typing) *The construction first yields a compatible regional family. Sector I admits one global represented form only after effective strict Hilbert descent. Sector II remains corridor banded; Sector III remains chart–corridor line banded; a correspondence branch is stated on local pivot representations. The stiffness threshold is not an invariant-mass statement.*

XXIII. COMPACT SEMISIMPLE GLOBAL FORMS

Let

$$\tilde{G} = \prod_{i=1}^r G_i^{\text{sc}}$$

be the compact simply connected semisimple cover and let

$$G = \tilde{G}/\Gamma, \quad \Gamma \leq Z(\tilde{G})$$

be a compact connected semisimple global form. Fix a maximal torus of $\tilde{G}$. Let P be the weight lattice, Q the root lattice, P^+ the dominant cone, and W the Weyl group.

For $\lambda \in P$, let χ_λ be the central character of the highest-weight representation V_λ . Define

$$P_G = \{\lambda \in P : \chi_\lambda(\gamma) = 1 \text{ for every } \gamma \in \Gamma\}, \quad P_G^+ = P_G \cap P^+. \quad (82)$$

The finite-dimensional irreducible unitary representations of G are precisely the V_λ with $\lambda \in P_G^+$.

A. The center of a connected central quotient

The quotient map $\pi : \tilde{G} \rightarrow G$ sends $Z(\tilde{G})$ into $Z(G)$. Conversely, suppose $\pi(g) \in Z(G)$. Then $[g, x] \in \Gamma$ for every $x \in \tilde{G}$. The map $x \mapsto [g, x]$ is continuous from the connected group $\tilde{G}$ to the finite discrete group Γ , and it equals the identity at $x = e$. Hence it is identically the identity, so $g \in Z(\tilde{G})$. Therefore

$$Z(G) \cong Z(\tilde{G})/\Gamma. \quad (83)$$

B. Central characters and the allowed lattice

With the standard highest-weight conventions,¹³ for the simply connected group, the central character depends only on the weight modulo the root lattice and gives

$$P/Q \cong \widehat{Z(\tilde{G})}.$$

A representation descends through $\tilde{G} \rightarrow G$ exactly when Γ acts trivially. Thus the allowed classes form the annihilator of Γ :

$$P_G/Q \cong \text{Ann}_{\widehat{Z(\tilde{G})}}(\Gamma). \quad (84)$$

Finite Pontryagin duality together with (83) gives

$$\boxed{P_G/Q \cong \widehat{Z(\tilde{G})/\Gamma} \cong \widehat{Z(G)}}. \quad (85)$$

This identity records the dependence on the global form, not merely on $\mathfrak{g}$.

C. Weyl invariance modulo roots

For a simple reflection,

$$s_\alpha(\lambda) - \lambda = -\langle \lambda, \alpha^\vee \rangle \alpha \in Q.$$

Hence W acts trivially on P/Q . Every central-character class has a dominant representative without changing its value on $Z(G)$.

XXIV. PRIMITIVE CENTER DETECTION

Lemma XXIV.1 (Extension of finite characters) *Let $B \leq A$ be finite abelian. Every character of B extends to a character of A .*

Proof. The group $U(1)$ is divisible and therefore injective as a $\mathbb{Z}$ -module. Thus the restriction homomorphism $\hat{A} \rightarrow \hat{B}$ is surjective. Equivalently, decompose A into cyclic primary factors and extend one factor at a time. $\square$

Theorem XXIV.2 (Primitive quotient-compatible Wilson channel) *Let $h \in Z(G)$ have order $n > 1$. After choosing an orientation of $\langle h \rangle$, there is an allowed dominant weight $\lambda_h \in P_G^+$ such that*

$$\chi_{\lambda_h}(h) = e^{2\pi i/n}.$$

Thus the irreducible G -representation V_{λ_h} detects the selected cyclic center band primitively.

Proof. Choose the character of $\langle h \rangle$ sending h to $e^{2\pi i/n}$ and extend it to $Z(G)$ by [theorem XXIV.1](#). Use (85) to identify the extended character with a class in P_G/Q , then choose a dominant representative using Weyl invariance modulo Q . $\square$

A. Primitive frames

Let

$$A \cong \mathbb{Z}_{n_1} \oplus \cdots \oplus \mathbb{Z}_{n_s} \leq Z(G).$$

Choose primitive coordinate generators of $\hat{A}$. Extend each to $\widehat{Z(G)}$, identify it with a class in P_G/Q , and choose a dominant representative. This yields weights

$$\lambda_1, \dots, \lambda_s \in P_G^+$$

whose restricted central characters form a primitive generating frame of $\hat{A}$. This frame realizes the grading characters. It does not, by itself, choose or realize mixed associator coordinates.

B. Repair of the Peter–Weyl shortcut

Peter–Weyl separation shows that irreducible characters separate central elements, but a separating family need not immediately exhibit one irreducible with primitive value. The lattice argument above directly produces one quotient-compatible dominant highest weight.

XXV. SIMPLE-TYPE CENTER REGISTRY

For the simply connected compact simple groups, convenient generators of P/Q are listed below. The rows use the standard rank ranges A_ℓ ($\ell \geq 1$), B_ℓ ($\ell \geq 2$), C_ℓ ($\ell \geq 2$), and D_ℓ ($\ell \geq 4$); low-rank accidental isomorphisms are interpreted through these conventions.

Type	Group	Center	Convenient weight classes
A_ℓ	$SU(\ell + 1)$	$\mathbb{Z}_{\ell+1}$	$[\omega_1]$ generates and $[\omega_\ell] = -[\omega_1]$
B_ℓ	$\text{Spin}(2\ell + 1)$	$\mathbb{Z}_2$	spin class $[\omega_\ell]$
C_ℓ	$Sp(\ell)$	$\mathbb{Z}_2$	defining class $[\omega_1]$
$D_\ell, \ell \text{ odd}$	$\text{Spin}(2\ell)$	$\mathbb{Z}_4$	$[\omega_\ell]$ or $[\omega_{\ell-1}]$ generates; twice it is $[\omega_1]$
$D_\ell, \ell \text{ even}$	$\ell \text{ Spin}(2\ell)$	$\mathbb{Z}_2 \oplus \mathbb{Z}_2$	any two of $[\omega_1], [\omega_{\ell-1}], [\omega_\ell]$ generate
E_6	simply connected compact $\mathbb{Z}_3$ E_6		$[\omega_1]$ and $[\omega_6]$ are inverse generators
E_7	simply connected compact $\mathbb{Z}_2$ E_7		$[\omega_7]$
E_8	simply connected compact trivial E_8		none
F_4	simply connected compact trivial F_4		none

Type	Group	Center	Convenient weight classes	
G_2	simply connected compact	trivial	none	For
	G_2			

a central quotient, retain precisely the classes trivial on Γ . The surviving subgroup is $P_G/Q \cong \widehat{Z(G)}$.

XXVI. ALGEBRAIC QUOTIENT DESCENT OF SELECTED WILSON CHANNELS

Let $\lambda_1, \dots, \lambda_s \in P_G^+$. Since Γ acts trivially in each V_{λ_j} , matrix coefficients, Wilson traces, and finite tensor combinations factor through G . The central quotient is a local isomorphism with the same Lie algebra, so the differentiated representations $d\rho_{\lambda_j}$ and invariant inner products are well-defined on the quotient.

Theorem XXVI.1 (Quotient-compatible channel data) *The selected finite Wilson-channel data descend from $\tilde{G}$ to G : the channel representations, central characters, finite tensor products, coefficient lines, and any separately declared scalar associator on $A \leq Z(G)$ are genuine G -data.*

Proof. Triviality of the Γ action gives descent of every selected channel and of all finite tensor operations. The selected band lies in $Z(G) = Z(\tilde{G})/\Gamma$, so a declared coefficient category on that band is already quotient data. $\square$

Remark XXVI.2 (Analytical data are separate) *The preceding theorem does not construct local-normal algebras, central supports, collar intertwiners, complete-ground projections, Orlicz references, Gauss–Ward graph cores, or coercive forms. Those belong to an independently verified incision-preserving analytical carrier.*

XXVII. CYCLIC COEFFICIENT GERBES

Let $h \in Z(G)$ have order $n > 1$ and identify $\langle h \rangle \cong \mathbb{Z}_n$. For reduced representatives define

$$\epsilon_n(b, c) = \frac{\widetilde{\tilde{b} + \tilde{c} - \tilde{b} + c}}{n} \in \{0, 1\}.$$

For orientation $s = \pm 1$, put

$$\Omega_{h,s}(a, b, c) = \exp \left(\frac{2\pi i s}{n} \tilde{a} \epsilon_n(b, c) \right). \quad (86)$$

The carry identity proves $\delta\Omega_{h,s} = 1$. Its class is $\mathbf{su}_n \neq 0$ in $H^3(B\mathbb{Z}_n; U(1))$. For $n = 2$, the two signs represent the same nonzero class.

The category $\text{Vec}_{\mathbb{Z}_n}^{\Omega_{h,s}}$ has one-dimensional simple coefficient lines L_a , tensor product $L_a \otimes L_b \cong L_{a+b}$, and associator multiplication by $\Omega_{h,s}(a, b, c)$. The pentagon is the cocycle equation.

XXVIII. ROOT-DATA REALIZATION THEOREM

Theorem XXVIII.1 (Root-data and coefficient availability) *For every compact connected semisimple global form G :*

- (i) $P_G/Q \cong \widehat{Z(G)}$ and every finite selected center band has a quotient-compatible primitive Wilson-character frame;
- (ii) the identity coefficient system is algebraically available;
- (iii) every selected $h \in Z(G)$ of order $n > 1$ and orientation $s = \pm 1$ has a quotient-compatible cyclic coefficient category with class $\mathbf{su}_n$;
- (iv) every declared finite-abelian class $[\Omega] \in H^3(BA; U(1))$ has a quotient-compatible scalar coefficient category after choosing a normalized representative and a primitive frame;
- (v) center-trivial forms have no center-generated scalar line twist;
- (vi) these statements classify center-generated scalar coefficient availability only; they do not classify Sector-II or correspondence-valued obstructions.

Proof. Items (i)–(iii) follow from (85), theorem XXIV.2, and (86). For (iv), choose a normalized cocycle representative and form Vec_A^Ω ; the primitive frame supplies quotient-compatible grading characters, while the associator is separately declared tensorator data. Items (v) and (vi) are immediate scope consequences. $\square$

Criterion XXVIII.2 (Incised analytical extension) *Let an incision-preserving exact-dilation analytical carrier over $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ already verify the primitive observable, expectation, Gauss–Ward, and form packages. If a finite scalar coefficient category from*

theorem XXVIII.1 is attached by bounded natural coefficient lines and compatible mixed cells, then the local principal symbol and scalar analytical estimates are unchanged. The represented package must still be verified for that declared branch, including its physical detector and complete-ground compatibility.

XXIX. PHYSICAL $SU(3)$ COEFFICIENT SYSTEM

Let

$$G = SU(3), \quad Z(G) = \langle \omega I_3 \rangle \cong \mathbb{Z}_3, \quad \omega = e^{2\pi i/3}.$$

The defining representation has triality one and detects the center primitively.

A. Clock and flux algebra

Choose a clock C and flux shift S satisfying

$$CS = \omega SC.$$

For an electric coefficient line of triality $a \in \mathbb{Z}_3$ and a linked magnetic collar,

$$\mathbb{V}_{E_a} \mathbb{V}_M = \omega^a \mathbb{V}_M \mathbb{V}_{E_a}. \quad (87)$$

The collar corridor is chosen with

$$q_3^3 = \omega I_3.$$

This relation is retained under exact dilation and neutral stabilization.

B. Fixed-thickness fiber cocycle

For $m = 1, 2$, define

$$\Omega_m(a, b, c) = \omega^{\tilde{m} \tilde{a} \epsilon_3(b, c)}. \quad (88)$$

The carry identity proves the pentagon. The marked value is

$$\Omega_m(1, 2, 1) = \omega^m.$$

Thus the twist is physically tied to linked center flux and triality rather than inserted as an abstract nonzero cocycle without an observable channel.

XXX. THE CYCLIC DETECTOR

For a normalized three-cochain Ξ on $\mathbb{Z}_3$, define

$$I_3(\Xi) = \prod_{b=0}^2 \Xi(1, b, 1). \quad (89)$$

Let λ be a normalized two-cochain with inhomogeneous differential

$$(\delta\lambda)(x, y, z) = \frac{\lambda(y, z)\lambda(x, y+z)}{\lambda(x+y, z)\lambda(x, y)}.$$

The $b = 0$ detector factor is one. The remaining factors are reciprocal:

$$(\delta\lambda)(1, 1, 1) = \frac{\lambda(1, 2)}{\lambda(2, 1)}, \quad (\delta\lambda)(1, 2, 1) = \frac{\lambda(2, 1)}{\lambda(1, 2)}.$$

Hence

$$I_3(\delta\lambda) = 1. \quad (90)$$

For (88),

$$\boxed{I_3(\Omega_m) = \omega^m \neq 1, \quad m = 1, 2.} \quad (91)$$

Therefore $[\Omega_m] = m\mathbf{u}_3 \neq 0$.

XXXI. BATCH-SCALE REASSOCIATOR

Let k, ℓ, p be batch lengths and write $\tilde{k}$ for the reduced grade modulo three. A corridor grade $\nu \in \mathbb{Z}_3$ and a coefficient triality $a \in \mathbb{Z}_3$ give

$$\Omega_{a,\nu}^{\text{sc}}(k, \ell, p) = \omega^{a\nu\tilde{k}\epsilon_3(\ell,p)}. \quad (92)$$

The two parenthesizations differ only by moving the carry created by the (ℓ, p) composition past the first coefficient line. Equation (87) supplies the phase. The cocycle is normalized, independent of absolute depth, and compatible with exact renormalized thickness transport.

Its detector is

$$I_3(\Omega_{a,\nu}^{\text{sc}}) = \omega^{a\nu}.$$

The marked triple is

$$\Omega_{a,\nu}^{\text{sc}}(1, 2, 1) = \omega^{a\nu}.$$

XXXII. ORDINARY CONE CONTRACTION AND CONTROLLED OBSTRUCTION

Let $\mathcal{R}$ be the rooted depth category with objects r_n and a unique arrow

$$a_{n;k} : r_n \rightarrow r_{n+k}.$$

The root r_0 is initial. With unrestricted constant coefficients, $B\mathcal{R}$ is contractible. An ordinary normalized three-cocycle has a root primitive

$$\lambda(f, g) = \Omega(\iota_{s(f)}, f, g),$$

so no absolute ordinary pure-scale class exists on this rooted category.

The physical scale obstruction is controlled rather than ordinary. A controlled cochain is required to be selected locally from the renormalized shell data, preserve the reduced batch band, and be natural under exact dilation. On the exact orbit, naturality forces depth stationarity:

$$c_{n+q}(k_1, \dots, k_p) = c_n(k_1, \dots, k_p).$$

Band preservation then reduces the controlled complex to the normalized group-cochain complex of $\mathbb{Z}_3$:

$$C_{\text{sc,ctrl}}^\bullet(\mathcal{D}_m^{\text{dil}}; U(1)) \cong C_{\text{norm}}^\bullet(\mathbb{Z}_3; U(1)). \quad (93)$$

Under this identification, (92) has class

$$a\nu u_3.$$

Thus it is nonzero in the controlled complex when $a\nu \neq 0$, even though its image in unrestricted ordinary cohomology is zero.

A. Exclusion of the root primitive

The ordinary root primitive depends on absolute depth. On a marked pair its depth translation ratio is

$$\frac{\lambda_{n+1}^{\text{root}}(2, 1)}{\lambda_n^{\text{root}}(2, 1)} = \omega^{a\nu}.$$

Controlled naturality would require the ratio to be one, so the root primitive is not controlled for $a\nu \neq 0$.

XXXIII. SHARP CONTROLLED STRICTIFICATION DEFECT

Equip $U(1)$ with angular distance

$$d_{U(1)}(z, w) = |\operatorname{Arg}(z\bar{w})|.$$

For a normalized two-cochain λ , define the marked defect from Ω_r , $r = 1, 2$, by the largest angular error on the detector triples. The product of the approximate coboundary identities has detector one by (90), while the target detector is ω^r . The two nontrivial marked factors must absorb angular distance $2\pi/3$. Hence

$$\inf_{\lambda} \operatorname{Def}_r^{\text{mark}}(\lambda) = \frac{\pi}{3}, \quad r = 1, 2. \quad (94)$$

The lower bound follows from the triangle inequality. Equality is obtained by distributing the phase discrepancy equally over the two nontrivial marked factors and extending to a normalized cochain.

XXXIV. CANONICAL DYADIC PHYSICAL BRANCH

Fix

$$\varepsilon_n = 2^{-n}\varepsilon_0, \quad \lambda_n = \varepsilon_n^\alpha, \quad 0 < \alpha \leq 1.$$

For $m \in \mathbb{Z}_3$, let $E_{\infty, m}$ be the flat center-flux bundle with meridional holonomy ω^m . Let

$$q_m^{\text{dil}}[u, v] = \langle V_m u, v \rangle + \eta \langle \nabla_m u, \nabla_m v \rangle$$

on the common domain Q_m .

Choose a unitary domain-straightening map

$$J_{m, n} : H_{m, n}^{\text{phys}} \rightarrow H_m^{\text{dil}}$$

preserving the center-flux branch, meridional orientation, real structure, triality, and form domain. The transported physical form is

$$\begin{aligned} q_{m, n}^J[u, v] &= \langle (V_m + \lambda_n W_{m, n})u, v \rangle \\ &+ \eta(1 + \lambda_n d_{m, n}) \langle (\nabla_m + \lambda_n C_{m, n})u, (\nabla_m + \lambda_n C_{m, n})v \rangle. \end{aligned} \quad (95)$$

Assume

$$|\langle W_{m,n}u, v \rangle| \leq C_W \|u\|_{Q_m} \|v\|_{Q_m}, \quad (96)$$

$$|d_{m,n}| \leq C_\eta, \quad (97)$$

$$\sqrt{\eta} \|C_{m,n}u\| \leq C_C \|u\|_{Q_m}. \quad (98)$$

The perturbations are gauge equivariant and center neutral.

Expanding gives

$$|q_{m,n}^J[u] - q_m^{\text{dil}}[u]| \leq C_{\text{form}} \varepsilon_n^\alpha \|u\|_{Q_m}^2 \quad (99)$$

with C_{form} from (81). Therefore graph norms are uniformly equivalent, the resolvents converge in norm, and the bounded fractional functional calculus converges on the uniformly gapped excitation subspace. Exact complete-ground preservation is supplied independently by the canonical endpoint package.

A. Propagation speed

The local principal stiffness is

$$\eta_{m,n}^{\text{phys}} = \eta(1 + \lambda_n d_{m,n}).$$

The wave-speed difference is $O(\lambda_n)$. The lower-order perturbations are pointwise and do not enlarge the support cone. Hence the buffered collar comparison and finite propagation hypotheses hold with defects $O(\varepsilon_n^\alpha)$.

B. Expectation and entropy package

Use the same functional-calculus prescription and canonical endpoint package at every physical and dilation scale. Norm-resolvent convergence controls the isolated good-window projections; the independent source expectation prescription and full-shell comparison control the conditional expectations. The faithful reference is transported with uniform spectral lower bounds. A superlinear reference profile gives the physical Orlicz packet, mass/logarithm/entropy tails, and the common entropy modulus. The sharp Cech and rooted-lifting theorems transfer from the exact orbit because the master shell defect is $O(\varepsilon_n^\alpha)$ on every fixed radius.

C. Complete-ground and Gauss–Ward data

The straightening maps preserve the complete-ground line. The positive-thickness source theorem supplies expectation equivariance with the insertion derivations on the common graph core; gauge equivariance and coefficient neutrality make this structure stable under the physical-to-dilation comparison. The external triality and collar package is fixed exactly, so the boundary-character and retained defect data pass through the limit.

XXXV. PHYSICAL NONCOLLAPSE

Let $\text{Def}_{r,n}^{\text{phys}}$ be the controlled strictification defect of the physical branch in scale coordinate $r = a\nu$. Stability of the marked detector under the shell comparison gives

$$\text{Def}_{r,n}^{\text{phys}} \geq \frac{\pi}{3} - L_R e_n^{(R)} \geq \frac{\pi}{3} - L_R D_R \varepsilon_n^\alpha,$$

where $D_R, L_R < \infty$ are fixed on the selected branch and shell. Therefore

$$\boxed{\liminf_{n \rightarrow \infty} \text{Def}_{r,n}^{\text{phys}} \geq \frac{\pi}{3}, \quad r = 1, 2.} \quad (100)$$

The asymptotic theorem requires the source-certified finiteness of D_R, L_R and the convergence $\varepsilon_n^\alpha \rightarrow 0$. It does not require a decimal evaluation of a fixed-stage margin. Such a finite-stage certificate is optional and may be generated whenever numerical upper bounds for D_R, L_R , and the selected ε_n are supplied.

Theorem XXXV.1 (Complete physical $SU(3)$ realization) *For every admitted canonical dyadic branch with labels (m, a, ν) and $r = a\nu$, the positive-thickness source verifies **OJ**, **RJ**, **SJ**, **GJ**, and **M**. The reconstructed thin branch is Sector I for $(m, r) = (0, 0)$, Sector II for $m = 0$ and $r = 1, 2$, and Sector III for $m = 1, 2$ and arbitrary r . The Sector-III conclusion is detected by $I_3(\Omega_m^{\text{fib}}) = \omega^m$; the Sector-II conclusion is detected by the controlled lower bound (100). These detectors are independent and are retained separately in the branch certificate.*

Proof. The local principal operator and form estimate verify **OJ** and **M**. Exact triality, center flux, complete-ground transport, corridor data, and mixed cells verify **RJ**; the common functional-calculus prescription, faithful reference, Orlicz packet, and rooted input verify **SJ**;

the graph-form Ward and fixed collar package verify GJ. The ordinary net, inherited surgery, Gauss–Ward package, and thin closed form follow from the general reconstruction theorems. The finite fiber detector is constant under coefficient-neutral comparison, while the controlled defect loses only the vanishing shell modulus. The sector table is therefore the one stated. $\square$

Scope XXXV.2 (Interpretation) *The $SU(3)$ construction is a complete physical twisted witness and the discovery model for the general root-datum theorem. The nonzero associator and the $\pi/3$ defect are coherence statements. They are not a dynamical interaction theorem and are not a Poincare mass-gap theorem.*

XXXVI. REPRESENTED BRANCHES AND SECTOR TYPING

A. Incision-preserving represented system

All represented data are indexed over one controlled chart–regional–scale category above $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$. Charts are adapted to $M_{\text{inc}} = M \setminus \Sigma$; regional and background arrows preserve the incision stratification and collar orientation; mixed cells remain inside the positive-clearance system. No comparison fills, removes, or moves Σ except through a declared arrow between controlled backgrounds.

Every branch has a strict bounded observable shadow and a strict normal-state shadow. The represented lift may carry scalar coefficient lines or normal correspondences. Scalar phases and pointed bimodule two-cells are invisible to the strict shadows but remain part of the represented target.

B. Admitted scalar-line universe

The scalar trichotomy applies only when the physically evaluated fiber class, the controlled total class, and the theorem equating simultaneous vanishing with one admissible physical strictification are all defined on the complete incised system. Within this universe:

sector	$\kappa_{\xi}^{\text{fib}}$	$\hat{\kappa}_{\xi}^{\text{cs}}$	additional datum
I	0	0	one global admissible physical strictification
II	0	$\neq 0$	branch-specific controlled detector
III	$\neq 0$	retained	physically evaluated scalar fiber class

A nonzero chart–scale coordinate may coexist with Sector III as additional data.

C. Sector I

Sector I requires one normalized physical two-cochain on the entire cofinal chart–regional–scale system. It preserves Cech faces, regional inclusions, thickness arrows, mixed cells, selected sectors, central supports, complete-ground pointings, positive-thickness expectation towers, smooth channels, positive-thickness Gauss–Ward prepared domains, collar data, and neutral stabilization. Separate trivializations on every finite face are insufficient. The inherited surgery graph retains the strictification only after the corrected SLSP theorem is applied.

D. Sector II

Sector II has strict fixed-thickness fibers and a nonzero controlled total obstruction. The branch must provide either a discrete controlled detector or a quantitative strictification defect with positive asymptotic lower bound. The represented object is corridor banded. The reconstruction theorem is conditional on full physical verification of the branch; no universal Sector-II realization is inferred from the abstract group.

E. Sector III

Sector III has a physically evaluated nonzero scalar fiber class

$$\kappa_{\xi}^{\text{fib}} = [\Omega_{\xi}^{\text{phys}}] \neq 0 \quad \text{in} \quad H^3(BA_{\xi}; U(1)).$$

The class is evaluated after selected-sector, central-support, and complete-ground compression. It is retained by admissible comparison through a discrete scalar detector or a quantitative noncollapse estimate. The represented object is chart–corridor line banded.

F. Correspondence target versus obstruction

When coherent scalar reduction fails, the represented object remains a normalized Markov-pointed W^* -correspondence pseudodiagram. This identifies the target, not the obstruction

status. The branch is correspondence-obstructed only if a separately supplied component detector proves

$$\kappa_{\xi}^{\text{Corr}} \neq [1].$$

A correspondence-valued branch with trivial pointed associator may be strictly associative in the correspondence bicategory while still lacking a scalar line reduction. This extension is not Sector IV.

XXXVII. DETECTOR-SPECIFIC SECTOR CONSERVATIVITY

Theorem XXXVII.1 (Discrete detector persistence) *Let I_n take values in a discrete Hausdorff set. Assume I_n is invariant under admissible rephasing and neutral stabilization, natural under finite common refinements, and continuous along the selected comparison maps. If $I_n = d$ on a cofinal tail, then the thin branch has detector value d and no admissible subsequence can have a different value.*

Proof. Comparison coherence identifies the detector on every common refinement. Continuity into a discrete Hausdorff target makes the value locally constant, hence eventually constant on the cofinal tail. Any subsequential limit with a different value would violate continuity on a common refined carrier. $\square$

Theorem XXXVII.2 (Quantitative obstruction persistence) *Let $\text{Def}_n \geq 0$ be invariant under admissible rephasing and neutral stabilization and suppose*

$$\text{Def}_0 \geq \liminf_{n \rightarrow \infty} \text{Def}_n - \limsup_{n \rightarrow \infty} \eta_n, \quad \eta_n \rightarrow 0.$$

If $\liminf_n \text{Def}_n \geq \delta > 0$, then $\text{Def}_0 \geq \delta$ and the obstruction cannot disappear in the thin limit.

Proof. Insert the positive liminf and the vanishing comparison error into the displayed estimate. $\square$

Theorem XXXVII.3 (Persistence of a global strictifying primitive) *Assume a single normalized physical strictifying cochain is defined over the complete cofinal incised system, lies in a compact control class, and satisfies closed face, mixed-cell, support, ground, expectation, smooth-channel, Gauss–Ward, collar, and stabilization equations. Then every comparison limit admits the induced physical strictification.*

Proof. Compactness yields a convergent subnet of the one global primitive. Each required compatibility is a closed equation in the declared comparison topology. The limit therefore satisfies the same complete system. The proof does not construct a global primitive from unrelated finite-face trivializations. $\square$

Scope XXXVII.4 (Correspondence no-jumping boundary) *No general topology on $\pi_0 \text{Assoc}_{\text{Corr}}$ is assumed. Correspondence no-jumping is asserted only when the physical branch supplies a component detector with closed separated target components and proves its invariance under pointed equivalence, fusion, rephasing, stabilization, and comparison.*

Theorem XXXVII.5 (Detailed branch-typed represented descent) *Under the primitive represented package RJ1–RJ8, the thin represented comparison system remains in its declared incision-preserving target: strict represented pseudofunctor, corridor-banded strict-fiber system, chart-corridor scalar line system, or normalized Markov-pointed correspondence pseudodiagram. The scalar three-sector classification is exhaustive only in the admitted scalar-line universe. Detector persistence is supplied branchwise by Theorems XXXVII.1–XXXVII.3; correspondence persistence additionally requires the component detector stated above.*

Proof. Common-refinement coherence gives the strict observable and state shadows. The coefficient data remain in the declared target. The branch-specific detector or global strictifying primitive prevents type loss under comparison. All arrows preserve the controlled incision. No effective descent to one Hilbert space is used. $\square$

A. Earlier two-sector terminology

The earlier conceptual split was strict versus obstructed. The theorem-level scalar refinement is

$$\text{strict} \longmapsto \text{Sector I}, \quad \text{obstructed} \longmapsto \text{Sector II} \sqcup \text{Sector III}.$$

The correspondence target remains outside this scalar exhaustivity statement.

XXXVIII. BRANCHWISE RECONSTRUCTION CERTIFICATES

For a fully certified strong line branch, the Paper-V certificate records the ordinary incised net, branch-typed represented pseudodiagram, inherited expectation graph, Orlicz packet, surgery-compatible Gauss–Ward/collar package, regional closed form, complete-ground data, coercive threshold, restart operation, and branch detector.

A. Sector-I certificate

The certificate contains one coherently strict represented pseudofunctor over the selected incised system and the retained global physical strictifying primitive. It does not assert one global physical Hilbert space and does not identify strictness with freeness.

B. Sector-II certificate

The represented object is corridor banded and the retained controlled detector is nonzero. A certificate is issued only after one physical branch verifies the full primitive packages and the detector persistence theorem. The article proves transfer, not universal existence.

C. Sector-III certificate

The represented object is chart–corridor line banded with $\kappa_{\xi}^{\text{fib}} \neq 0$. Any scale coordinate is recorded separately. Cyclic and physical $SU(3)$ certificates use their declared discrete or quantitative detectors.

D. Correspondence certificate

The certificate records separately: (i) the correspondence-valued target; (ii) the normalized Markov shadow; (iii) the associator component; and (iv) whether nontriviality has actually been detected. Physical non-scalar non-vacuity remains conditional.

Branch	Represented get	tar-Obstruction cate	certifi-Status
Sector I	strict represented pseudofunctor	one global ble physical categorification	admissi-reconstruction branch admission; no global- Hilbert claim
Sector II	corridor-banded strict fibers	nonzero detector or defect liminf	controlled branch-specific cation required
Sector III	chart–corridor scalar lines	nonzero evaluated fiber class	physically scalar transfer; physical exam- ples use declared detectors
Correspondence	Markov-pointed correspondences	trivial, or separately tected nontrivial component	unknown, transfer only without a physi- cal component detector

XXXIX. JH-TO-LEGACY CROSSWALK

The journal blocks are a compression of the definitive admissibility and hypothesis packages. The crosswalk is functional rather than merely numeric.

A. OJ crosswalk

OJ collects:

- (a1) cofinal thickness and finite common-refinement geometry;
- (a2) bounded packet comparison and normal-envelope control;
- (a3) common null ideals and quotient exactness;
- (a4) local supports, finite propagation, and smooth functional calculus;
- (a5) packet-localizing additivity and state amalgamation;
- (a6) causal dynamics and bounded Cauchy preimages;
- (a7) central-support compatibility.

These are the geometric, causal, and bounded-comparison portions of the long-form AC and **H** packages.

B. RJ crosswalk

RJ collects:

- (b1) selected represented carriers and central corners;
- (b2) complete-ground transport;
- (b3) coefficient lines or correspondences;
- (b4) chart-scale and mixed-cell coherence;
- (b5) branch typing and sector conservativity;
- (b6) admissible physical strictification or retained obstruction;
- (b7) neutral stabilization.

C. SJ crosswalk

SJ contains only positive-thickness and quantitative comparison inputs:

- (c1) source-defined positive-thickness good-window expectations;
- (c2) a branch-specific fixed-window expectation-germ Cauchy or exact-stationarity certificate;
- (c3) exact coherent-window compression and zero-extension identities;
- (c4) faithful references, a physical Orlicz moment, an independent spectral profile, and collar tightness;
- (c5) smooth buffered Kraus-channel comparison on joint endpoint packets;
- (c6) one exact strict unsurgered rooted state-shadow diagram;
- (c7) packet stability and the admissible fiber-thickness environment.

Mass, reference-logarithm, and entropy tails, sharp descent, surgically rooted lifting, the inherited graph, and restart are conclusions, not SJ clauses.

D. GJ crosswalk

GJ contains the positive-thickness graph-domain and collar inputs:

- (d1) common insertion graph cores and closable Gauss graph forms;
- (d2) the exact graph-form interior Ward identity;
- (d3) finite-window expectation–derivation commutation on the common core;
- (d4) prepared-state domains and their positive-thickness stability;
- (d5) collar quotient charges and the boundary-character class;
- (d6) the correctly typed two-parameter current-algebra defect and first boundary-Ward defect;
- (d7) complete-ground Gauss data and strict unsurgered augmented finite-diagram lifts.

Limiting surgery compatibility and the augmented surgically rooted theorem are derived later.

E. M crosswalk

M contains primitive comparison-form data:

- (e1) uniformly bounded comparison maps J_n and reconstruction maps I_n , asymptotically inverse on graph-bounded sets;
- (e2) a common reference domain or graph-dense images and a vanishing sesquilinear form defect;
- (e3) comparison-weak compactness and a comparison-Mazur principle;
- (e4) core recovery, synchronized full-domain extension, and inclusion-compatible exhaustion data;
- (e5) complete-ground form-domain invariance, form-nullity, orthogonality, and graph-domain comparison;
- (e6) a branchwise uniform positive-thickness off-ground coercive estimate.

Mosco convergence, TT1, TT2, the exact kernel, and thin coercivity are conclusions, not M clauses.

Journal block	Long-form material	Principal theorem use
OJ	geometric, packet, causal, sup-	Theorem A and smooth face input to Theorem C
RJ	represented, ground, coefficient, mixed-cell, and sector clauses	Theorem B and coherence part of Theorems H1–H2
SJ	positive-thickness germ certificate, coherent windows, reference/Orlicz/profile data, smooth Kraus comparison, strict unsurgered root, fiber thickness	expectations, Theorem C conclusions
GJ	positive-thickness graph cores, exact graph-form Ward identity, expectation equivariance, prepared domains, collar quotient, typed defects, unsurgered augmented lifts	Theorem D conclusions
M	comparison/reconstruction maps, form defect, weak compactness, primitive recovery, ground form structure, positive-thickness coercivity	Theorem E conclusions

XL. WITNESS VERIFICATION MATRIX

The matrix records certification level and provenance rather than a binary “proved” label.

Requirement	Strict root datum	Cyclic scalar root datum	da-Canonical $SU(3)$ family	physical
Coefficient system	Level A identity	Level A cyclic $\pm \mathfrak{u}_n$	triality/fiber coordinate m and controlled scale coordinate r	

Requirement	Strict root datum	Cyclic scalar root datum	da-Canonical $SU(3)$ family	physical
OJ	Level B comparison witness	Level B comparison witness	Level C: incised source plus Theorem A	
RJ	Level B identity strictification	Level B scalar detector and band data	Level C: ground, triality, corridor, and mixed cells plus Theorem B	
SJ	Level B expectation/reference package	same carrier with cyclic line band	Level C: source expectations and references plus Theorem C	
GJ	Level B graph/collar package	same carrier with neutral scalar action	Level C: physical Ward and collar source plus Theorem D	
M	Level B common-domain comparison	same analytical carrier Friedrichs	Level C: physical form and ground source plus Theorem E	
Status	comparison Sector I	comparison Sector III	physical Sector I, branch-specific Sector II, and Sector III according to (m, r)	

Requirement	Canonical physical Sector II	Correspondence-valued branch
Branch	$SU(3)$ with $m = 0, r = 1, 2$	no physical non-scalar example claimed
Primitive ages	Level C, inherited from the same complete physical family	transfer only through the correctly oriented Markov shadow
Detector	trivial fixed-fiber detector and controlled liminf at least $\pi/3$	target type does not prove nontrivial associator
Universal vacuity	non-claimed beyond the canonical family	not claimed

a. Strict $SU(2)$ firewall. The independent $SU(2)$ compendium verifies a fixed-positive-thickness finite/depth strict interface. It is not an incised thickness-cofinal branch and supplies no independent thin OJ–M certificate.

XLI. EXCLUDED FORMULATIONS AND RETAINED INTERPRETATIONS

The following formulations are removed or rewritten:

- (X1) thin surgery defined solely by complete atomic pinching of a bare thin operator;
- (X2) atomic no-splitting required for surgery existence;
- (X3) arbitrary tensor partial trace used as a regional map;
- (X4) sharp spectral locality inferred from finite propagation alone;
- (X5) reference-logarithm integrability inferred from entropy bounds alone;
- (X6) weak-* compactness identified with predual compactness;
- (X7) pairwise recoveries substituted for rooted lifting;
- (X8) abstract cocycle vanishing substituted for physical strictification;
- (X9) $SU(N)$ presented as the final gauge scope;
- (X10) Sector II presented as universally realized;
- (X11) correspondence associators scalarized;
- (X12) the thin form threshold called a mass gap;
- (X13) strict branches called free;
- (X14) Sector-III twisting called proof of interaction.

The following constructions are retained with corrected interpretation:

- (R1) smooth functional calculus and finite propagation for buffered comparison;
- (R2) finite-window spectral-island stability;
- (R3) atomic no-splitting as an optional identification theorem;

- (R4) $SU(3)$ as the first complete physical twisted witness;
- (R5) $SU(2)$ as strict finite/depth control;
- (R6) exact-dilation root-data branches as analytical witnesses;
- (R7) the Paper-IV stiffness threshold as input to Paper V.

XLII. PAPER-V OBLIGATION LEDGER

The exact downstream ledger is V.O1–V.O12 together with the separate interaction obligation V.INT1, as stated in Supplement CI. In particular, covariance of physical spectral projections, vacuum multiplicity, Gauss–Ward compatibility of the spectral decomposition, and integrated no-leakage remain distinct obligations rather than being absorbed into neighboring items.

XLIII. SCOPE AND FAILURE DIAGNOSTICS

A branch lies outside the present theorem when it lacks one of the primitive comparison structures. Typical diagnostic failures are:

- (D1) no common null ideal for packet sections;
- (D2) loss of a normal central-support summand;
- (D3) no controlled local principal symbol or common propagation tail;
- (D4) nonunique finite-window expectation algebra;
- (D5) absence of a faithful reference with uniform Orlicz moment;
- (D6) failure of reference-logarithm tails;
- (D7) failure of sharp descent after regulator removal;
- (D8) pairwise but not rooted recovery;
- (D9) failure of expectation equivariance on the insertion core;
- (D10) failure of TT1 or TT2;
- (D11) collapse of the coercive constant;

(D12) sector jumping or an inadmissible strictifying cochain.

Failure of a diagnostic is not a no-go theorem for every continuum construction. It means that the present exact-dilation comparison theorem cannot be applied without a new argument.

XLIV. LOCAL FRIEDRICHS OPERATORS AND COLLAR PROPAGATION

A. Differential expression

The finite-propagation argument uses the standard energy-current method for wave equations on globally hyperbolic spacetimes, specialized here to the selected local Friedrichs carrier and collar boundary condition.¹⁴ On a bounded regional carrier O with selected collar, consider

$$A_n = \eta_n \nabla_n^* \nabla_n + M_{V_n}, \quad (101)$$

where:

- (i) η_n is a positive scalar or positive bundle endomorphism uniformly comparable to a fixed $\eta > 0$;
- (ii) ∇_n is a unitary covariant derivative on a finite-rank bundle built from the allowed Wilson channels;
- (iii) M_{V_n} is a pointwise Hermitian lower-order multiplication operator;
- (iv) the boundary condition is the local Dirichlet–Friedrichs condition on the selected carrier.

The closed form is

$$q_n[u] = \int_O \eta_n |\nabla_n u|^2 + \langle V_n u, u \rangle.$$

Lower-semiboundedness follows after the declared vacuum normalization and a uniform bound on the negative part of V_n .

B. Cutoff locality

Let $\chi \in C_c^\infty(O)$ be a real cutoff. The commutator with the principal part is first order:

$$[\nabla_n^* \nabla_n, \chi]u = -2\nabla_{n, \text{grad}\chi} u - (\Delta\chi)u, \quad (102)$$

with the appropriate covariant interpretation. Since M_{V_n} and η_n are pointwise,

$$[M_{V_n}, \chi] = 0, \quad [\eta_n, \chi] = 0.$$

Thus the lower-order potential is cutoff local and does not create nonlocal support propagation.

C. Energy current

Let $u(t)$ solve

$$\partial_t^2 u + A_n u = 0$$

with compactly supported initial data. For a time-dependent cutoff χ_t adapted to the exterior of the candidate propagation cone, define

$$E_{\chi,n}(t) = \frac{1}{2} \int_O \chi_t^2 \left(|\partial_t u|^2 + \eta_n |\nabla_n u|^2 + \langle (V_n + c)u, u \rangle \right),$$

where c makes the potential term nonnegative. Differentiation, integration by parts, and (102) give

$$\begin{aligned} \frac{d}{dt} E_{\chi,n}(t) &\leq \int_O |\chi_t \partial_t \chi_t| \left(|\partial_t u|^2 + \eta_n |\nabla_n u|^2 + C|u|^2 \right) \\ &\quad + 2 \int_O \chi_t |\nabla \chi_t| \eta_n^{1/2} |\partial_t u| \eta_n^{1/2} |\nabla_n u|. \end{aligned} \quad (103)$$

Choose χ_t with

$$|\partial_t \chi_t| \geq v_n |\nabla \chi_t|,$$

where v_n dominates the principal propagation speed. The cross term is absorbed by the two quadratic energy terms. Gronwall then shows that zero exterior energy persists.

Theorem XLIV.1 (Finite propagation) *The wave group of the Friedrichs realization satisfies*

$$\text{supp } \cos(tA_n^{1/2})f \subseteq J_{v_n|t|}(\text{supp } f),$$

and

$$\text{supp } A_n^{-1/2} \sin(tA_n^{1/2})f \subseteq J_{v_n|t|}(\text{supp } f),$$

where J_r denotes the regional metric r -neighborhood. The speed depends only on the principal coefficient and is uniform on the cofinal tail.

Proof. Apply the localized energy estimate to initial data supported away from the exterior cutoff. The exterior energy is initially zero and remains zero. Density of compactly supported form-core data and unitarity of the wave group extend the statement to the full Hilbert carrier. $\square$

D. Perturbative speed control

For the physical exact-dilation perturbation,

$$\eta_n = \eta(1 + \lambda_n d_n), \quad |d_n| \leq C_\eta.$$

The propagation speed satisfies

$$|v_n - v_{\text{ref}}| \leq C_v \lambda_n.$$

Therefore a fixed buffer $d > 0$ remains valid on every finite diagram for sufficiently large n . This is the only propagation estimate needed before smooth functional calculus.

XLV. WAVE REPRESENTATION OF SMOOTH FUNCTIONAL CALCULUS

Let $f \in C_c^\infty(\mathbb{R})$ be even after extension to the square-root variable. Then

$$f(A_n) = \frac{1}{2\pi} \int_{\mathbb{R}} \hat{f}(t) \cos(tA_n^{1/2}) dt. \quad (104)$$

Truncate the integral to $|t| \leq T$. Finite propagation makes the truncated operator local inside a buffer of width $v_n T$. The Fourier tail is bounded by rapid decay of $\hat{f}$.

Let $R_{n,a}$ be a Cech-face restriction. On a common carrier, the local differential expressions agree on the overlap up to a form defect θ_n . Duhamel comparison of the wave equations gives

$$\|R_{n,a}f(A_{n,\sigma}) - f(A_{n,\tau})R_{n,a}\| \leq C_{f,T}\theta_n + C_{f,k}T^{-k}. \quad (105)$$

Choose T below the fixed buffer propagation time and then take $n \rightarrow \infty$. This proves smooth Cech-face comparison.

A. Why sharp locality does not follow

A sharp spectral projector has a Fourier transform with insufficient decay and cannot be localized by the preceding argument. The sharp expectation map is obtained only after:

- (a) finite-window spectral stability;
- (b) convergence and uniqueness of the expectation algebra;
- (c) physical Orlicz mass and logarithm tails;
- (d) entropy continuity on the selected packet;
- (e) regulator removal in the safe order.

Thus finite propagation is one input to sharp descent, not a substitute for it.

XLVI. COLLAR CARRIER GEOMETRY

Let $\partial_c O$ be the selected collar face. Choose nested collar neighborhoods

$$C_{r_1} \Subset C_{r_2} \Subset O, \quad 0 < r_1 < r_2,$$

and a partition of unity subordinate to the interior and collar. The local boundary condition is imposed on the outer carrier boundary, while the collar quotient records gauge parameters that are nontrivial at the incision-facing inner interface.

The finite propagation theorem implies that for times

$$|t| < \frac{r_2 - r_1}{v_*},$$

wave packets initially supported in the interior cannot interact with the outer boundary through the collar. This separates the interior Gauss–Ward graph calculation from the boundary-character bookkeeping. Pointwise lower-order terms remain local in the collar and therefore do not violate the separation.

A. Friedrichs extension and compact resolvent

The closed lower-semibounded form on a bounded carrier has a self-adjoint Friedrichs operator. The common form domain embeds compactly into the Hilbert space by local

Rellich compactness and the bounded collar geometry. Therefore the resolvent is compact. The selected finite spectral windows are finite dimensional.

B. Quotient stability

Replacing $\tilde{G}$ by a central quotient G does not change the local Lie algebra, invariant metric, principal symbol, or collar geometry. Allowed finite-dimensional Wilson channels merely change bounded internal coefficients. Hence the propagation and compact-resolvent proofs are uniform in Dynkin type on each declared finite channel frame.

XLVII. LINKED COLLAR FLUX AT FIXED THICKNESS

A selected linked-flux collar carries an electric coefficient line and a magnetic transport. Let E_a have center grade a and let M_m carry magnetic center flux m . Their physical interchange is

$$E_a M_m = \chi_a(h_m) M_m E_a.$$

For $SU(3)$ this is ω^{am} . Triple composition compares two parenthesizations. The carry arising in the second pair must be transported past the first coefficient line, producing

$$\Omega_m(a, b, c) = \omega^{\tilde{m}\tilde{a}\epsilon_3(b,c)}.$$

The construction occurs at fixed positive thickness on the collar carrier. Exact dilation transports it to every depth without changing the center grade or meridional orientation. This establishes that the fiber twist is physical, refinement compatible, and not erased by neutral stabilization.

XLVIII. QUANTITATIVE EXPECTATION-GERM STABILITY

A. Weighted Gram-matrix verification

The standard normal-relative-entropy, conditional-expectation, and Orlicz uniform-integrability tools are used with the branch-specific estimates proved below.^{6,7,15,16} Let $F_n \subset M_d(\mathbb{C})$ be transported finite-window fixed algebras and let $\sigma_n \geq \gamma I$ be faithful

transported references. Choose bases $e_{1,n}, \dots, e_{r,n}$ with $e_{j,n} \rightarrow e_{j,0}$ in operator norm and define

$$G_n(i, j) = \text{Tr}(\sigma_n^{1/2} e_{i,n}^* \sigma_n^{1/2} e_{j,n}).$$

If

$$\max_j \|e_{j,n} - e_{j,0}\| \leq \delta_n, \quad \|\sigma_n - \sigma_0\|_1 \leq s_n,$$

then

$$\|G_n - G_0\| \leq C_{d,r,\gamma}(\delta_n + s_n), \quad (106)$$

$$\|G_n^{-1} - G_0^{-1}\| \leq C'_{d,r,\gamma}(\delta_n + s_n) \quad (107)$$

for large n . The reference-preserving expectation is the weighted projection

$$E_n(x) = \sum_{i,j} e_{i,n} (G_n^{-1})_{ij} \langle e_{j,n}, x \rangle_{\sigma_n},$$

so

$$\|E_n - E_0\|_{1 \rightarrow 1} + \|E_n - E_0\|_{\infty \rightarrow \infty} \leq C''_{d,r,\gamma}(\delta_n + s_n). \quad (108)$$

This is a verification mechanism for the fixed-window germ, not a claim that algebra-gap convergence alone always determines a moving expectation.

B. Summable one-step criterion

If

$$\|E_{n+1} - E_n\|_{1 \rightarrow 1} + \|E_{n+1} - E_n\|_{\infty \rightarrow \infty} \leq a_n, \quad \sum_n a_n < \infty,$$

then E_n converges in both norms. If the references converge and every cluster point has the same declared range, the limit is the unique reference-preserving expectation onto that range. Exact-dilation witnesses have $a_n = 0$ after canonical transport.

C. Logarithm stability on a fixed good window

If $\sigma_n, \sigma_0 \geq \gamma I$, then the integral representation of the logarithm gives

$$\|\log \sigma_n - \log \sigma_0\| \leq \gamma^{-1} \|\sigma_n - \sigma_0\|. \quad (109)$$

This estimate is used only on a fixed finite carrier with a uniform faithful lower bound.

XLIX. QUANTITATIVE ORLICZ AND ENTROPY TAILS

Let Φ be increasing, convex, superlinear, and satisfy $\Phi(0) = 0$. Define

$$\alpha_0(R) = \Phi(R)^{-1}, \quad \alpha_1(R) = \sup_{t \geq R} \frac{t}{\Phi(t)}.$$

Then $\alpha_0(R), \alpha_1(R) \rightarrow 0$. A uniform moment M gives

$$m_R \leq M\alpha_0(R), \tag{110}$$

$$\ell_R \leq M\alpha_1(R). \tag{111}$$

For the low-reference projection $p_R = \mathbf{1}_{[0,R]}(L^{\text{ref}})$, assume the branchwise finite counting profile

$$d(R) = \sup_n \text{rank } p_{n,R} < \infty.$$

Put

$$C_R^{(1)} := \sup_{n,\rho} \langle \rho, L_n^{\text{ref}} \rangle < \infty.$$

The subnormalized high-block entropy estimate, the two-outcome pinching bound, and the normalized low-block reference estimate give

$$\begin{aligned} s_R &:= h_2(m_R) + (1 - m_R)|\log(1 - m_R)| + \ell_R \\ &\quad + m_R|\log m_R| + \frac{m_R}{1 - m_R} C_R^{(1)} \longrightarrow 0. \end{aligned} \tag{112}$$

For $t = \frac{1}{2}\|\rho - \rho'\|_1$, define

$$u_R(t) = \frac{t + 2m_R}{1 - m_R}.$$

Whenever $u_R(t) \leq 1 - 1/d(R)$, the finite-dimensional continuity estimate yields

$$|S(\rho) - S(\rho')| \leq 2s_R + u_R(t) \log(d(R) - 1) + h_2(u_R(t)). \tag{113}$$

Taking the infimum over admissible R gives a common modulus. This estimate separates the Orlicz mass tail, the reference-logarithm tail, and the finite-rank counting input.

A. Examples

For $\Phi_p(t) = t^p$, $p > 1$,

$$m_R \leq MR^{-p}, \quad \ell_R \leq MR^{1-p}.$$

For $\Phi(t) = t \log(1+t)^{1+\beta}$, $\beta > 0$,

$$m_R \leq \frac{M}{R \log(1+R)^{1+\beta}}, \quad \ell_R \leq \frac{M}{\log(1+R)^{1+\beta}}.$$

The theorem does not require either profile; they illustrate how rates enter the diagonal choice.

L. REFERENCE-PRESERVING SURGERY

Let E be the source-defined conditional expectation and suppose $E^*(\sigma) = \sigma$. Then

$$D(E^*\rho \|\sigma) \leq D(\rho \|\sigma) \quad (114)$$

by data processing. If $E(\Phi(L^{\text{ref}})) = \Phi(L^{\text{ref}})$, then

$$\langle E^*\rho, \Phi(L^{\text{ref}}) \rangle = \langle \rho, \Phi(L^{\text{ref}}) \rangle.$$

Under a declared asymptotic commutator/reference defect $c_n \rightarrow 0$, the right side is replaced by $(1+c_n)M + c_n$. These are the packet-stability statements used by restart. No monotonicity direction for ordinary von Neumann entropy is asserted.

LI. QUANTITATIVE SMOOTH-TO-SHARP REMOVAL

Let $\mathcal{K}_{n;D}^{(N,d),*}$ be the finite smooth Kraus channel of (40). Let $\mathcal{T}_{n;D}^{(N,d)}$ be the transition-strip projection and $Q_{n;D}^{(N)}$ the high-window complement. The finite Kraus family and the gentle measurement estimate give

$$\|\mathbb{E}_{n;D}^{(N),*} \rho - \mathcal{K}_{n;D}^{(N,d),*} \rho\|_1 \leq C_N \left(\sqrt{\rho(\mathcal{T}_{n;D}^{(N,d)})} + \sqrt{\rho(Q_{n;D}^{(N)})} + \sqrt{\rho(1 - \chi_d^2)} \right). \quad (115)$$

For an arrow $a : \sigma \rightarrow \tau$, define

$$t_a(n; N, d) = \max_{D \in \{\sigma, \tau\}} \sup_{\rho \in \mathfrak{R}_n^{\text{Orl}}(a)} \rho(\mathcal{T}_{n;D}^{(N,d)}).$$

Then the sharp edge defect obeys

$$\Delta_a(n; N, d) \leq \eta_a^{\text{sm}}(n; N, d) + 2C_N \left(\sqrt{t_a(n; N, d)} + \sqrt{m_D(N)} + \sqrt{\kappa_D(d)} \right). \quad (116)$$

At fixed finite diagram, fixed d , and fixed N , the first two terms vanish as $n \rightarrow \infty$. The high-window tail is then removed as $N \rightarrow \infty$, and the collar buffer is removed last. This is the quantitative form of the mandatory limit order.

LII. PREDUAL COMPACTNESS ON THE CONTROLLED PACKET

Weak-* compactness is not enough. Let $\{\rho_j\}$ be a sequence in one selected packet. Choose R so that m_R is small. The compressed states $p_R \rho_j p_R$ lie in a finite-dimensional trace-class ball and therefore have a norm-convergent subsequence. Moreover

$$\|\rho_j - p_R \rho_j p_R\|_1 \leq 2\sqrt{m_R} + m_R.$$

A diagonal choice over $R_k \rightarrow \infty$ gives a subsequence Cauchy in predual norm. Hence the selected packet is relatively predual compact on every fixed finite diagram.

LIII. ENTROPY LOWER SEMICONTINUITY AND RECOVERY

For arbitrary convergent selected families, lower semicontinuity gives the liminf inequality for the declared entropy functional. The sector-locked recovery of Supplement C supplies the matching limsup. This variational convergence is used only after the separate reference-logarithm and discarded-entropy tails have been established.

LIV. ROOTED FINITE-DIAGRAM COMBINATORICS

A. Finite selected diagrams

A finite selected diagram

$$\mathcal{D} = (V, E, C_2, C_3; v_0)$$

consists of finitely many incised regional, chart, thickness, Cauchy, background, and coefficient vertices; state-direction arrows E ; strict observable/normal-state faces C_2 ; represented coefficient reassociation cells C_3 ; and a root v_0 from which every vertex is reachable. The diagram is first pulled to one incision-preserving common refinement. No arbitrary tensor partial trace is introduced in this pullback.

For every vertex v , choose a bookkeeping path

$$p_v = (e_1, \dots, e_k) : v_0 \rightarrow v.$$

Let

$$T_{p_v, n}^* = T_{e_k, n}^* \cdots T_{e_1, n}^*$$

be the corresponding strict normal-state map. The positive-thickness unsurgered shadow is strict on the selected diagram: elementary faces commute exactly after insertion of the declared common-refinement identifications. The represented coefficient lines may reassociate by their prescribed scalar tensorators and associator.

B. The strict unsurgered root

Let $\rho_{v_0,0}$ be a selected limiting root state. Choose one positive-thickness root recovery $\rho_{v_0,n}$ satisfying

$$d_{v_0,n}^{\text{root}} := \|T_{0n,v_0}^* \rho_{v_0,n} - \rho_{v_0,0}\|_1 \longrightarrow 0, \quad (117)$$

together with the sector, central-support, complete-ground, Orlicz, and entropy-limsup conditions. Define the entire unsurgered family by

$$\rho_{v,n}^{\text{pre}} := T_{p_v,n}^* \rho_{v_0,n}. \quad (118)$$

For every edge $e : v \rightarrow w$,

$$T_{e,n}^* \rho_{v,n}^{\text{pre}} = \rho_{w,n}^{\text{pre}}. \quad (119)$$

If a second rooted path is chosen, equality in the strict shadow follows from the positive-thickness face laws. The represented lifts differ only by the prescribed coefficient composite.

C. Objectwise surgery

Apply the source-defined sharp expectation independently at every vertex:

$$\hat{\rho}_{v,n} := \mathcal{S}_{G,\xi,n;v}^{\text{st},*} \rho_{v,n}^{\text{pre}}. \quad (120)$$

Because the source expectation preserves the selected corner, sector, and complete-ground face, every $\hat{\rho}_{v,n}$ retains those data. Reference preservation gives relative-entropy contraction and the declared Orlicz stability. No correction projection is applied to the root or to the vertex family.

D. Single-edge defect

For $e : v \rightarrow w$, combine (119) with (120):

$$T_{e,n}^* \hat{\rho}_{v,n} - \hat{\rho}_{w,n}$$

$$= T_{e,n}^* \mathcal{S}_{n;v}^{\text{st},*} \rho_{v,n}^{\text{pre}} - \mathcal{S}_{n;w}^{\text{st},*} T_{e,n}^* \rho_{v,n}^{\text{pre}}. \quad (121)$$

Therefore

$$\|T_{e,n}^* \hat{\rho}_{v,n} - \hat{\rho}_{w,n}\|_1 \leq \Delta_e(n; N, d), \quad (122)$$

where Δ_e is the sharp single-arrow defect of (43). Exact surgery-intertwining arrows have $\Delta_e = 0$.

E. Higher strict-shadow cells

Let $p = e_k \cdots e_1$ and $q = f_\ell \cdots f_1$ be two paths with the same endpoints. Insert and subtract the objectwise surgically transported state at each intermediate vertex. Predual contractivity of the state maps gives

$$\|T_{p,n}^* \hat{\rho}_{s(p),n} - T_{q,n}^* \hat{\rho}_{s(q),n}\|_1 \leq \sum_{i=1}^k \Delta_{e_i}(n; N, d) + \sum_{j=1}^{\ell} \Delta_{f_j}(n; N, d). \quad (123)$$

Every two-cell defect is a special case. Every higher strict-shadow cell is bounded by a finite sum of elementary edge defects after choosing a finite face decomposition. This is the only combinatorial operation required.

In a scalar line-twisted branch, the coefficient-line composite around a three-cell is not estimated to zero. It equals the prescribed scalar associator. Thus the strict observable/normal-state shadow closes while the represented coefficient lift retains its nontrivial coherence class.

F. Master finite-diagram defect

Let $c_2(\mathcal{D})$ be the maximum number of elementary edges in the chosen decomposition of a strict two-cell, and let $c_3(\mathcal{D})$ be the corresponding maximum for higher strict-shadow cells. Put

$$c(\mathcal{D}) = 1 + c_2(\mathcal{D}) + c_3(\mathcal{D}).$$

Let $\tau_{\mathcal{D}}(N, d)$ be the maximum over vertices of the common high-reference, transition-strip, spatial-collar, and entropy-recovery tails. Define

$$\mathfrak{D}_{\mathcal{D}}(n; N, d) = \max_{v \in V} d_{v,n}^{\text{root}} + c(\mathcal{D}) \max_{e \in E} \Delta_e(n; N, d) + \tau_{\mathcal{D}}(N, d). \quad (124)$$

Since $\mathcal{D}$ is finite, all constants are finite and the same regulator choice is valid at every vertex. The corrected SLSP estimates give

$$\lim_{d \downarrow 0} \lim_{N \rightarrow \infty} \limsup_{n \rightarrow \infty} \mathfrak{D}_{\mathcal{D}}(n; N, d) = 0. \quad (125)$$

This one quantity controls root recovery, every edge and strict face, the Orlicz packet, entropy recovery, sector, central support, complete-ground preservation, and the scalar represented associator.

G. Finite-horn closure

Let $\Lambda \subset \mathcal{D}$ be a finite horn containing the root and every face except one selected target face. The family (120) is already defined at every vertex of $\mathcal{D}$. The missing face defect is bounded by (123) and hence by (124). Therefore the same rooted family fills every finite horn in the limit. No separately chosen horn fillers are intersected.

Lemma LIV.1 (Rooted path comparison) *For two rooted paths p_v and q_v from v_0 to v ,*

$$\|T_{p_v, n}^* \hat{\rho}_{v_0, n} - T_{q_v, n}^* \hat{\rho}_{v_0, n}\|_1 \leq c(\mathcal{D}) \max_{e \in E} \Delta_e(n; N, d).$$

Thus path independence holds in the limiting strict shadow. The represented scalar lifts differ exactly by the declared coefficient composite around $q_v^{-1}p_v$.

Proof. Apply (123) to a finite face decomposition of the loop $q_v^{-1}p_v$. The coefficient statement is the retained tensorator/associator law. $\square$

IV. COUNTABLE COFINAL DIAGONAL

Let

$$\mathcal{D}_1 \subseteq \mathcal{D}_2 \subseteq \dots$$

be a countable cofinal sequence of finite selected diagrams. For each j , choose $d_j > 0$, N_j , and then n_j in the mandatory order so that

$$\mathfrak{D}_{\mathcal{D}_j}(n_j; N_j, d_j) \leq 2^{-j}.$$

The restrictions of the $j + 1$ family to $\mathcal{D}_j$ differ from the j family by a quantity bounded by the two master defects and the common-refinement comparison defect. Predual compactness on the controlled packet and a standard diagonal extraction give one limiting compatible family on every finite stage. The scalar coefficient class is unchanged because every refinement uses the declared coefficient transport.

Theorem LV.1 (Full rooted joint recovery) *For every finite rooted selected incised diagram and every admissible limiting root state, there is one positive-thickness recovery family satisfying simultaneously:*

- (1) *root recovery and every strict regional, Čech, scale, Cauchy, background, and common-refinement face;*
- (2) *the selected sector, central support, and complete-ground face;*
- (3) *the full source-defined objectwise surgery and sharp surgery naturality;*
- (4) *the uniform Orlicz packet and relaxed-entropy limsup;*
- (5) *the represented scalar coefficient tensorators and associator.*

*The theorem does not yet add Gauss–Ward or collar constraints; their simultaneous augmentation is proved separately from **GJ**.*

Proof. Use the strict unsurgered root (118), apply objectwise surgery (120), estimate by (124), and choose the finite or countable diagonal above. Every listed condition is attached to the same family. $\square$

LVI. RESTART FROM A PRESCRIBED POSITIVE-THICKNESS STAGE

Fix a coarse stage n_0 and start the strict unsurgered root at that stage. The same construction applies to the tail $n \geq n_0$. For a long arrow $a : n_0 \rightarrow n$, the restart discrepancy obeys

$$\|T_{a,*} \mathcal{S}_{n_0,*}^{\text{st,as},*} \rho_{n_0} - \mathcal{S}_{n,*}^{\text{st,as},*} T_{a,*} \rho_{n_0}\|_1 \leq d_a^{\text{Surg}}(n_0, n; \rho_{n_0}). \quad (126)$$

Exact surgery-intertwining arrows have zero defect. For asymptotic Čech, regional, or thickness arrows the defect tends to zero on the selected packet. Hence objectwise restart is exactly tail independent as a cofinal germ, while one-shot equality at a fixed finite stage is claimed only in the exact branch.

LVII. UNIQUENESS OF THE INHERITED GRAPH

Suppose two admissible subsequences yield graph values ρ' and ρ'' for the same limiting root state. Place both subsequences in one finite two-branch diagram with a common refined root. The full rooted theorem gives one joint recovery, fixed-window expectation-germ uniqueness identifies the two window images, and the common tails remove the regulators. Therefore

$$\|\rho' - \rho''\|_1 = 0.$$

The inherited expectation graph is single valued.

LVIII. MARKOV COMPATIBILITY

For a nested selected incised arrow $a : O_1 \rightarrow O_2$, sharp descent gives

$$R_a^* \mathcal{S}_{O_2}^{\text{st,as},*} = \mathcal{S}_{O_1}^{\text{st,as},*} R_a^*$$

on the selected packet. Thus the graph is a normal Markov-compatible state transformation in the strict state shadow. The scalar coefficient associator remains in the represented layer and does not alter this identity.

LIX. REGIONAL EXHAUSTION AND DIAGONAL MOSCO RECOVERY

A. Exhaustion data before the global form is assembled

Let

$$O_1 \Subset O_2 \Subset \cdots, \quad \bigcup_{j \geq 1} O_j = O,$$

be an incision-compatible exhaustion. On each bounded O_j , Supplement XVII constructs the closed regional form $q_{0;O_j}^{\text{rel}}$. Regional monotonicity supplies restriction maps r_j and selected cutoff extensions e_j satisfying

$$\|u - e_j r_j u\|_{H_{0,O}}^2 + q_{0;O}^{\text{rel}}[u - e_j r_j u] \leq \tau_j(u), \quad \tau_j(u) \rightarrow 0, \quad (127)$$

whenever a global carrier exists. In a banded branch, (127) is interpreted chart-corridor locally and is used only to prove compatibility of the regional family.

At finite thickness, restriction and extension maps $r_{j,n}, e_{j,n}$ obey

$$\|J_n e_{j,n} v - e_j J_{j,n} v\| + \left| \widehat{q}_{n;O}[e_{j,n} v] - \widehat{q}_{n;O_j}[v] \right| \leq \eta_{n,j} \|v\|_{\widehat{q}_{n;O_j}}^2, \quad (128)$$

where $\eta_{n,j} \rightarrow 0$ for fixed j . The second term includes the collar-cutoff gradient error; it is not suppressed by notation.

B. One recovery diagonal

For u in the limiting regional form domain, choose local TT2 recoveries $u_n^{(j)}$ of $r_j u$. Set

$$\begin{aligned} a_{n,j} &= \|J_{j,n} u_n^{(j)} - r_j u\|, \\ b_{n,j} &= \left| \widehat{q}_{n;O_j}[u_n^{(j)}] - q_{0;O_j}^{\text{rel}}[r_j u] \right|, \\ c_{n,j} &= \eta_{n,j} \|u_n^{(j)}\|_{\widehat{q}_{n;O_j}}^2, \\ d_j &= \tau_j(u)^{1/2}. \end{aligned}$$

Choose N_j so that $a_{n,j} + b_{n,j} + c_{n,j} < 2^{-j}$ for $n \geq N_j$ and define $j(n) = j$ for $N_j \leq n < N_{j+1}$. Then

$$a_{n,j(n)} + b_{n,j(n)} + c_{n,j(n)} + d_{j(n)} \rightarrow 0.$$

The sequence

$$u_n = e_{j(n),n} u_n^{(j(n))}$$

is the global or bandwise TT2 diagonal. The construction is synchronized over every finite regional diagram by taking the maximum of its finitely many error functions before selecting N_j .

LX. SYNCHRONIZED QUADRATIC RECOVERY IN DETAIL

Let u, v lie in the strong-relaxation domain. M4 gives one finite-stage family

$$u_n, \quad v_n, \quad u_n + v_n, \quad u_n - v_n$$

that is simultaneously near-optimal for all four target vectors. Hence

$$q_0^{\text{rel}}[u + v] + q_0^{\text{rel}}[u - v] \leq \lim_n \left(\widehat{q}_n[u_n + v_n] + \widehat{q}_n[u_n - v_n] \right)$$

$$\begin{aligned}
&= 2 \lim_n \hat{q}_n[u_n] + 2 \lim_n \hat{q}_n[v_n] \\
&= 2q_0^{\text{rel}}[u] + 2q_0^{\text{rel}}[v].
\end{aligned}$$

Conversely choose synchronized near-minimizers w_n^+ and w_n^- for $u + v$ and $u - v$ and put

$$u_n = \frac{1}{2}(w_n^+ + w_n^-), \quad v_n = \frac{1}{2}(w_n^+ - w_n^-).$$

Applying the finite-stage parallelogram identity gives the reverse inequality. This is the missing quadraticity step for the relaxed envelope.

LXI. GROUND-FACE CORRECTION WITH TWO COMPARISON MAPS

Decompose

$$u = P_0 u + u^\perp.$$

Let v_n be a TT2 recovery of $u^\perp$. Define

$$g_n = P_n I_n P_0 u, \quad u_n^\sharp = g_n + (\mathbf{1} - P_n)v_n. \quad (129)$$

Moving-family TT1 and the asymptotic inverse relations give

$$J_n g_n \rightarrow P_0 u, \quad J_n (\mathbf{1} - P_n)v_n \rightarrow u^\perp.$$

The complete-ground form structure gives

$$\hat{q}_n[g_n] = 0, \quad \hat{q}_n(g_n, (\mathbf{1} - P_n)v_n) = 0.$$

Therefore

$$\hat{q}_n[u_n^\sharp] = \hat{q}_n[(\mathbf{1} - P_n)v_n] \rightarrow q_0^{\text{rel}}[u^\perp] = q_0^{\text{rel}}[u].$$

No inverse of J_n is used outside the exact unitary specialization.

LXII. GENERALIZED RESOLVENT CONVERGENCE AND THE EXACT-CARRIER ESTIMATE

Comparison–Mosco convergence gives, for every $\mu > 0$,

$$J_n(K_n + \mu)^{-1} I_n \longrightarrow (K_0 + \mu)^{-1}$$

strongly on the limiting carrier, together with strong convergence of the heat semigroups. This is the correct general varying-space statement.

On an exact common carrier, assume

$$|q_n[u, v] - q_0[u, v]| \leq \theta_n \|u\|_{q_0} \|v\|_{q_0}, \quad \theta_n \rightarrow 0.$$

For $f \in H$, put $u_n = (K_n + \mu)^{-1}f$ and $u_0 = (K_0 + \mu)^{-1}f$. Subtract the weak resolvent equations and test with $u_n - u_0$ to obtain

$$\|u_n - u_0\| \leq C_\mu \theta_n \|f\|.$$

Thus the shifted full resolvents converge in norm. At $\mu = 0$ the same argument is used only on the excitation subspace, where the uniform gap supplies invertibility.

A. Riesz projections and exact ground preservation

If Γ encloses an isolated spectral island of K_0 , norm-resolvent convergence gives norm convergence of the Riesz projections. This is analytic spectral stability. Exact finite-stage preservation of the complete-ground line is a separate endpoint statement:

$$P_n D(q_n) \subseteq D(q_n), \quad q_n[P_n u] = 0, \quad J_n P_n I_n P_0 \rightarrow P_0.$$

The second assertion is not deduced from the first.

LXIII. FRACTIONAL AND POSITIVE-FREQUENCY STRUCTURES

On the excitation subspace, $K_n \geq \kappa_{\text{IV,thin}} I$. For $0 < s < 1$ the Balakrishnan representation

$$K_n^{-s} = \frac{\sin \pi s}{\pi} \int_0^\infty t^{-s} (K_n + t)^{-1} dt$$

and the uniform gap give convergence of fractional inverse powers in the exact norm-resolvent branch and comparison-strong convergence in the general branch. No inverse power is asserted on the complete-ground subspace.

LXIV. COERCIVITY FROM THE REFERENCE BRANCH

Suppose

$$q_{\text{ref}}[u] \geq \kappa_{\text{ref}} \|(\mathbf{1} - P_0)u\|^2$$

and the exact-carrier defect satisfies

$$|q_n[u] - q_{\text{ref}}[u]| \leq \theta_n(q_{\text{ref}}[u] + \|u\|^2).$$

On the excitation subspace the reference form controls the Hilbert norm. After choosing a branchwise cofinal tail,

$$q_n[u] \geq \kappa_{\text{IV},\text{thin}} \|u\|^2$$

for some $\kappa_{\text{IV},\text{thin}} > 0$. This is the finite-stage M6 input. On the exact-dilation witness, the explicit form estimate in the main article permits the safe choice $\kappa_{\text{IV},\text{thin}} = \kappa_{\text{cof}}/2$ on any tail with $C_{\text{form}}\varepsilon_n^\alpha < 1/2$. General branches retain their own M6 constant. Thin coercivity is then proved from TT2 plus TT1, not by reversing Mosco liminf.

LXV. NO-LEAKAGE STATEMENT NOT YET PROVED

The exact kernel theorem excludes additional null directions of the thin stiffness form. It does not exclude physical translation states whose invariant masses approach zero through regional, gauge, coefficient, or incision interfaces. Paper V must establish the invariant vacuum, the appropriate symmetry representation after resolving the incision, the form-to-mass bridge, and physical spectral no-leakage.

LXVI. MOSCO STABILITY UNDER RESTART

Restart changes the selected normal state but not the regional closed-form system, comparison maps, complete-ground data, or recovery category. Therefore the Mosco, TT1–TT2, coercivity, and exact-kernel conclusions remain valid after state-surgery restart. This does not add a spectral statement.

LXVII. FINITE-ABELIAN SCALAR COCYCLE NORMAL FORMS

The normalized formulas below use the standard bar-resolution conventions and are cross-checked against the explicit finite-abelian cocycle normal forms of Huang–Wan–Ye.¹⁷ Let

$$A = \bigoplus_{i=1}^s \mathbb{Z}_{n_i}$$

with reduced coordinates $a_i \in \{0, \dots, n_i - 1\}$. Throughout this section, $H^3(BA; U(1))$ means normalized inhomogeneous group cohomology of the discrete group A with trivial action on $U(1)$, written multiplicatively. The terms “cohomological type I, II, III” below refer to coordinates in this finite-abelian normal form and are unrelated to represented Sectors I, II, III.

For

$$\epsilon_n(x, y) = \frac{\tilde{x} + \tilde{y} - \widetilde{x + y}}{n} \in \{0, 1\},$$

write $d_{ij} = \gcd(n_i, n_j)$ and $d_{ijk} = \gcd(n_i, n_j, n_k)$.

A. Cohomological type-I cyclic factors

For $p_i \in \mathbb{Z}_{n_i}$ define

$$\Omega_i^{(p_i)}(a, b, c) = \exp\left(\frac{2\pi i p_i}{n_i} \tilde{a}_i \epsilon_{n_i}(b_i, c_i)\right). \quad (130)$$

The carry identity proves normalization and the cocycle equation. Define

$$I_i(\Xi) = \prod_{r=0}^{n_i-1} \Xi(e_i, r e_i, e_i). \quad (131)$$

Then normalized coboundaries have detector one and

$$I_i(\Omega_i^{(p_i)}) = e^{2\pi i p_i / n_i}. \quad (132)$$

B. Cohomological type-II pair factors

For $i < j$ and $p_{ij} \in \mathbb{Z}_{d_{ij}}$, define

$$\Omega_{ij}^{(p_{ij})}(a, b, c) = \exp\left(\frac{2\pi i p_{ij}}{d_{ij}} \tilde{a}_j \epsilon_{n_i}(b_i, c_i)\right). \quad (133)$$

Changing $\tilde{a}_j$ by n_j changes the exponent by an integer, because $d_{ij} \mid n_j$. The carry identity in the i th coordinate proves the cocycle equation. Define the marked pair detector

$$I_{ij}(\Xi) = \prod_{r=0}^{n_i-1} \Xi(e_j, r e_i, e_i). \quad (134)$$

Then

$$I_{ij}(\Omega_{ij}^{(p_{ij})}) = e^{2\pi i p_{ij} / d_{ij}}. \quad (135)$$

C. Cohomological type-III triple factors

For $i < j < k$ and $p_{ijk} \in \mathbb{Z}_{d_{ijk}}$, define the integral tricharacter

$$\boxed{\Omega_{ijk}^{(p_{ijk})}(a, b, c) = \exp\left(\frac{2\pi i p_{ijk}}{d_{ijk}} \tilde{a}_i \tilde{b}_j \tilde{c}_k\right)}. \quad (136)$$

This formula replaces the superseded expression containing extra factors $n_i^{-1} n_j^{-1}$. It is well defined because d_{ijk} divides each of n_i, n_j, n_k . It is multiplicative in each variable, so its inhomogeneous 3-cocycle defect is identically one. Define

$$I_{ijk}(\Xi) = \Xi(e_i, e_j, e_k). \quad (137)$$

Then

$$I_{ijk}(\Omega_{ijk}^{(p_{ijk})}) = e^{2\pi i p_{ijk}/d_{ijk}}. \quad (138)$$

For example, on $\mathbb{Z}_2^3$ the nonzero factor is $(-1)^{a_1 b_2 c_3}$ and satisfies the pentagon exactly.

D. Product normal form and completeness

Theorem LXVII.1 (Finite-abelian scalar normal form) *Every class in $H^3(BA; U(1))$ has a normalized representative*

$$\Omega = \prod_i \Omega_i^{(p_i)} \prod_{i < j} \Omega_{ij}^{(p_{ij})} \prod_{i < j < k} \Omega_{ijk}^{(p_{ijk})}, \quad (139)$$

with parameters

$$p_i \in \mathbb{Z}_{n_i}, \quad p_{ij} \in \mathbb{Z}_{d_{ij}}, \quad p_{ijk} \in \mathbb{Z}_{d_{ijk}}.$$

The detector family $\{I_i, I_{ij}, I_{ijk}\}$ recovers these coordinates and is complete on the chosen normal form. Consequently

$$H^3(BA; U(1)) \cong \bigoplus_i \mathbb{Z}_{n_i} \oplus \bigoplus_{i < j} \mathbb{Z}_{d_{ij}} \oplus \bigoplus_{i < j < k} \mathbb{Z}_{d_{ijk}}.$$

Proof. The normalized bar resolution for a direct sum of cyclic groups separates one-coordinate carry terms, two-coordinate carry couplings, and integral three-coordinate tricharacters. The displayed factors are normalized 3-cocycles. Their detectors vanish on the other coordinate families and return the indicated primitive roots on their own family, proving independence and uniqueness of the parameters. The bar-resolution count of cyclic summands gives the displayed direct sum, so the independent families exhaust the cohomology group. $\square$

Remark LXVII.2 (Canonical versus declared mixed data) *The main article uses the cyclic factor generated by one selected central holonomy. Pair and triple coordinates are not selected by the root datum alone; they require separately declared coupled tensorators. The primitive Wilson frame realizes the grading characters, not these mixed associator parameters.*

LXVIII. PRIMITIVE WILSON FRAME FOR A FINITE BAND

Let $A \leq Z(G)$ be finite abelian and choose invariant-factor generators. For each primitive coordinate character ϑ_i of A , extend ϑ_i to $Z(G)$ and use

$$P_G/Q \cong \widehat{Z(G)}$$

to choose $\lambda_i \in P_G^+$ with $\chi_{\lambda_i}|_A = \vartheta_i$. The frame $\Lambda_A = (\lambda_1, \dots, \lambda_s)$ separates A and realizes its character grading.

A cohomological type-II or type-III coordinate is then attached by a separately chosen normalized tensorator or associator with the phase in (133) or (136). Quotient compatibility follows from $\lambda_i \in P_G$ and from the fact that $A \leq Z(G)$.

LXIX. COCYCLE EQUIVALENCE AND ADMISSIBLE REPHASING

A normalized scalar two-cochain $\lambda(a, b)$ changes the associator by $\delta\lambda$. In a physical branch, an admissible rephasing must also satisfy chart and regional locality, controlled scale naturality, compatibility with mixed cells and complete-ground pointings, preservation of the selected center band, neutral stabilization, and boundedness on the finite coefficient packet. Ordinary cohomological triviality is therefore weaker than physical strictifiability.

LXX. FINITE-BAND COEFFICIENT EXTENSION CRITERION

Criterion LXX.1 (Scalar finite-band extension) *Let G be compact, connected, and semisimple, let $A \leq Z(G)$ be a selected finite abelian band, and let $[\Omega] \in H^3(BA; U(1))$ be a declared scalar class with a normalized representative from [theorem LXVII.1](#). Let an independently verified incision-preserving analytical carrier satisfy the primitive OJ, SJ, GJ, and M packages. Attaching the primitive Wilson frame and Vec_A^Ω leaves the local principal symbol and the scalar analytical estimates unchanged. To obtain a represented branch, one*

must separately verify RJ: natural coefficient lines, compatible mixed cells, complete-ground transport, and a physical detector for the selected class.

Proof. The coefficient category is finite dimensional and external to the local Laplace-type principal symbol. Hence it does not change the scalar form, reference, or locality estimates. The represented data are additional structure, and their naturality and detector statements are precisely the remaining RJ obligations. $\square$

A. Center-trivial forms

If $Z(G)$ is trivial, then $P_G/Q = 0$. The identity coefficient system remains algebraically available, but no center-generated scalar line twist exists. This does not exclude non-center or correspondence-valued coefficient systems.

LXXI. CONTROLLED COHOMOLOGY COMPARISON MAP

Let

$$j^* : H_{\text{sc,ctrl}}^3(\mathcal{D}; U(1)) \rightarrow H^3(B\mathcal{D}; U(1))$$

forget scale-control and locality requirements. A Sector-II class may lie in $\ker j^*$ when the ordinary class is trivial but no controlled primitive exists. A Sector-III fiber class is nonzero before this forgetful map. Neither phenomenon is classified by $P_G/Q \cong \widehat{Z(G)}$ alone.

LXXII. DETAILED $SU(3)$ WILSON-CHANNEL CALCULATIONS

A. Weights and triality

For $SU(3)$, the weight lattice modulo the root lattice is

$$P/Q \cong \mathbb{Z}_3.$$

If $\lambda = p\omega_1 + q\omega_2$, its triality is

$$t(\lambda) = p + 2q \pmod{3}. \tag{140}$$

The central element ωI_3 acts on V_λ by

$$\omega^{t(\lambda)}.$$

The defining representation $(1, 0)$ has triality one, the conjugate defining representation $(0, 1)$ has triality two, and the adjoint representation $(1, 1)$ has triality zero.

B. Allowed global forms

For $G = SU(3)$, all trialities occur. For the adjoint form $PSU(3) = SU(3)/\mathbb{Z}_3$, only triality-zero weights descend. Hence

$$P_{PSU(3)}/Q = 0, \quad Z(PSU(3)) = 1.$$

The strict root-datum witness exists for both global forms, but the canonical center-generated scalar twist exists only for $SU(3)$.

C. Clock and shift realization

Let

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}.$$

Then

$$CS = \omega SC, \quad C^3 = S^3 = I.$$

A triality- a electric coefficient is represented by C^a . A unit linked magnetic transport is represented by S . Their interchange is

$$C^a S^m = \omega^{am} S^m C^a. \tag{141}$$

This finite-dimensional relation models the external center response; it does not replace the bulk gauge connection.

D. Collar corridor element

Choose a collar transport q_3 satisfying

$$q_3^3 = \omega I_3.$$

One may take a diagonal unitary with eigenvalues chosen as cube roots of the center phase, subject to the branch's meridional orientation and determinant normalization after including the external coefficient line. The only invariant used in the associator calculation is the central cube relation and its exact transport under dilation.

LXXIII. CARRY CALCULATION

Write a batch length as

$$k = 3\lfloor k/3 \rfloor + \tilde{k}, \quad \tilde{k} \in \{0, 1, 2\}.$$

Composition of batches ℓ and p produces a carry

$$\epsilon_3(\ell, p) = \frac{\tilde{\ell} + \tilde{p} - \widetilde{\ell + p}}{3} \in \{0, 1\}.$$

The corridor contribution is

$$\tilde{q}_3^{\tilde{\ell}} \tilde{q}_3^{\tilde{p}} = \widetilde{q_3^3}^{\ell+p} (q_3^3)^{\epsilon_3(\ell, p)} = \widetilde{q_3^3}^{\ell+p} (\omega I_3)^{\epsilon_3(\ell, p)}.$$

When this carry is moved past an electric line of triality $a\tilde{k}$, equation (141) gives

$$\omega^{a\nu\tilde{k}\epsilon_3(\ell, p)},$$

where ν is the corridor center grade. This is the batch-scale reassociator.

A. Pentagon check

For four batch lengths k, ℓ, p, q , the cocycle equation reduces to the exact integer carry identity

$$\epsilon_3(\ell, p) + \epsilon_3(\ell + p, q) = \epsilon_3(p, q) + \epsilon_3(\ell, p + q).$$

After multiplication by the appropriate reduced first-coordinate coefficient, the two parenthesizations therefore have the same total exponent modulo three. Thus the exponent of $\delta\Omega$ vanishes and the pentagon holds.

B. Detector telescoping

For a normalized two-cochain λ ,

$$\begin{aligned}(\delta\lambda)(1, 0, 1) &= 1, \\(\delta\lambda)(1, 1, 1) &= \lambda(1, 2)\lambda(2, 1)^{-1}, \\(\delta\lambda)(1, 2, 1) &= \lambda(2, 1)\lambda(1, 2)^{-1}.\end{aligned}$$

Their product is one. For Ω_m , the first two carry values vanish and the third is one, giving $I_3(\Omega_m) = \omega^m$.

LXXIV. SHARP $\pi/3$ APPROXIMATION BOUND

Let

$$z_1 = (\delta\lambda)(1, 1, 1), \quad z_2 = (\delta\lambda)(1, 2, 1).$$

Normalization gives $z_1 z_2 = 1$. Suppose both target errors are less than δ :

$$d(z_1, 1) < \delta, \quad d(z_2, \omega^r) < \delta.$$

Then

$$d(1, \omega^r) = d(z_1 z_2, \omega^r) \leq d(z_1, 1) + d(z_2, \omega^r) < 2\delta.$$

Let

$$\theta_r = \text{Arg}(\omega^r) \in (-\pi, \pi], \quad r = 1, 2.$$

Then $|\theta_r| = 2\pi/3$, so one has $\delta \geq \pi/3$. Equality is achieved by the normalized two-cochain values

$$\lambda_r^\#(1, 2) = e^{-i\theta_r/4}, \quad \lambda_r^\#(2, 1) = e^{i\theta_r/4},$$

for which

$$z_1 = (\delta\lambda_r^\#)(1, 1, 1) = e^{-i\theta_r/2}, \quad z_2 = (\delta\lambda_r^\#)(1, 2, 1) = e^{i\theta_r/2}.$$

Both marked angular errors are $|\theta_r|/2 = \pi/3$. This proves sharpness uniformly for $r = 1, 2$ and avoids the nonprincipal $r = 2$ angle convention.

LXXV. PHYSICAL FORM EXPANSION TERM BY TERM

Starting from

$$q_{m,n}^J[u] = \langle (V_m + \lambda_n W_{m,n})u, u \rangle + \eta(1 + \lambda_n d_{m,n}) \|\nabla_m u + \lambda_n C_{m,n} u\|^2,$$

subtract the reference form:

$$\begin{aligned} q_{m,n}^J[u] - q_m^{\text{dil}}[u] &= \lambda_n \langle W_{m,n} u, u \rangle + \eta \lambda_n d_{m,n} \|\nabla_m u\|^2 \\ &\quad + 2\eta \lambda_n (1 + \lambda_n d_{m,n}) \Re \langle \nabla_m u, C_{m,n} u \rangle \\ &\quad + \eta \lambda_n^2 (1 + \lambda_n d_{m,n}) \|C_{m,n} u\|^2. \end{aligned} \tag{142}$$

Using the coefficient bounds and $\lambda_n \leq 1$,

$$\begin{aligned} |\lambda_n \langle W_{m,n} u, u \rangle| &\leq \lambda_n C_W \|u\|_{Q_m}^2, \\ \eta \lambda_n |d_{m,n}| \|\nabla_m u\|^2 &\leq \lambda_n C_\eta \|u\|_{Q_m}^2, \\ 2\eta \lambda_n (1 + \lambda_n |d_{m,n}|) |\langle \nabla_m u, C_{m,n} u \rangle| &\leq 2\lambda_n (1 + C_\eta) C_C \|u\|_{Q_m}^2, \\ \eta \lambda_n^2 (1 + \lambda_n |d_{m,n}|) \|C_{m,n} u\|^2 &\leq \lambda_n (1 + C_\eta) C_C^2 \|u\|_{Q_m}^2. \end{aligned}$$

Summing gives the stated C_{form} .

LXXVI. COMPLETE-GROUND PROJECTION IN THE PHYSICAL BRANCH

The physical-to-dilation unitary preserves the flat center-flux sector and maps the positive-thickness complete-ground line to the fixed reference ground line. Therefore

$$J_{m,n} P_{\Omega, m, n}^{\text{phys}} J_{m,n}^{-1} = P_{\Omega, m}^{\text{dil}}$$

exactly on the canonical branch. In particular TT1 has zero projection defect. The perturbations are center neutral and annihilate the declared complete-ground vector after vacuum normalization, so the ground energy remains zero.

LXXVII. SU(3) EXPECTATION AND REFERENCE ESTIMATES

Norm-resolvent convergence implies that every isolated good-window projection satisfies

$$\|p_{m,n}^{(N)} - p_m^{(N)}\| \leq C_N \varepsilon_n^\alpha.$$

The compressed reference densities satisfy

$$\|\sigma_{m,n}^{(N)} - \sigma_m^{(N)}\| \leq C'_N \varepsilon_n^\alpha.$$

A uniform faithful lower bound in the good window and the logarithm Lipschitz estimate give

$$\|\log \sigma_{m,n}^{(N)} - \log \sigma_m^{(N)}\| \leq C''_N \varepsilon_n^\alpha.$$

The weighted Gram-matrix estimate then yields

$$\|E_{m,n}^{(N)} - E_m^{(N)}\|_{1 \rightarrow 1} \leq C'''_N \varepsilon_n^\alpha.$$

These rates feed directly into the sharp Cech and rooted-lifting defects.

LXXVIII. PHYSICAL SHELL MASTER DEFECT

For a fixed shell radius R , collect the finite propagation, form, resolvent, expectation, ground, collar, and coefficient defects into

$$e_n^{(R)} = e_{n,\text{form}}^{(R)} + e_{n,\text{prop}}^{(R)} + e_{n,\text{exp}}^{(R)} + e_{n,\text{GW}}^{(R)} + e_{n,\text{collar}}^{(R)}.$$

The coefficient hypotheses imply that for every fixed shell radius there is a finite branchwise rate constant D_R such that

$$e_n^{(R)} \leq D_R \varepsilon_n^\alpha.$$

Every selected finite detector and diagrammatic comparison has a finite branchwise Lipschitz constant L_R in this master defect. Hence exact-dilation identities transfer to the physical branch and the controlled strictification lower bound loses at most $L_R e_n^{(R)}$. The constants D_R and L_R are theorem-level branch data; no universal numerical value is claimed.

LXXIX. SEPARATION OF THE TWO PHYSICAL DETECTORS

The fixed-fiber detector $I_3(\Omega_m^{\text{fib}}) = \omega^m$ is evaluated on a same-thickness chart band and detects $m\mathbf{u}_3$. The scale detector is a lower bound on the approximation defect of controlled primitive candidates along the rooted corridor. Its unrestricted ordinary class is exact, but its controlled class is nonzero for $r = 1, 2$. Consequently a nonzero scale coordinate does not create an additional fiber class, and a nonzero fiber class does not require the $\pi/3$ estimate. The total controlled coordinate is $(m, 0, r)$, while the unrestricted ordinary image retains only m .

LXXX. BRANCH TRICHOTOMY FOR THE $SU(3)$ FAMILY

The physical family carries a fixed-thickness flux label $m \in \mathbb{Z}_3$ and scale data $(a, \nu) \in \mathbb{Z}_3^2$.

- (1) If $m \neq 0$, the fixed-thickness class is $m\mathbf{u}_3 \neq 0$ and the branch is Sector III independently of the scale coordinate.
- (2) If $m = 0$ and $a\nu = 0$, both fiber and controlled scale classes vanish; the branch is Sector I.
- (3) If $m = 0$ and $a\nu \neq 0$, the ordinary rooted scale class is trivial but the controlled class is nonzero; this is the controlled Sector-II model when all physical branch conditions are retained.

The journal theorem does not promote the last model to universal Sector-II non-vacuity across all global forms.

LXXXI. OBSERVABLE MEANING AND NONCLAIMS

The triality/linked-flux algebra produces an observable covariance distinction and a retained represented associator. It survives recompression, refinement, neutral stabilization, and the thick-to-thin limit. It does not by itself establish nontrivial scattering, failure of Wick factorization, or non-equivalence to an admissible generalized-free net. Those are the independent interaction obligations of Paper V.

LXXXII. STRICT $SU(2)$ FINITE/DEPTH CONTROL

The $SU(2)$ construction is retained as a strict finite/depth control. It checks the untwisted coefficient conventions, complete-ground compression, finite-window expectation algebra, and regional comparison formulas in the smallest nonabelian compact simple group. It is not used as an independent thickness-cofinal witness unless a separate $\varepsilon \downarrow 0$ theorem is supplied.

A. Center and weight data

For $SU(2)$,

$$P/Q \cong \mathbb{Z}_2, \quad Z(SU(2)) = \{\pm I\}.$$

The irreducible representation of highest weight $j\omega_1$ has central sign $(-1)^j$. The defining two-dimensional channel detects the nontrivial center primitively.

For the adjoint form $SO(3) = SU(2)/\mathbb{Z}_2$, only even highest weights descend and the center is trivial. The identity coefficient branch remains strict for both global forms. A center-generated scalar cyclic twist is available for $SU(2)$ but not for $SO(3)$.

B. Strict coefficient system

The strict control uses the identity tensorator and associator. Every Cech, scale, and mixed coefficient face commutes exactly. The complete-ground line is transported identically. Therefore

$$\kappa^{\text{fib}} = 0, \quad \hat{\kappa}^{\text{cs}} = 0.$$

This is an explicit Sector-I control, not a statement that the dynamics is free.

C. Finite-window control

On a fixed depth and finite regional carrier, the selected Friedrichs operator has compact resolvent. The finite-window spectral expectation is the reference-preserving conditional expectation onto the selected stiffness algebra. The identity coefficient system ensures that all represented and observable expectation diagrams coincide. This makes the $SU(2)$ branch useful for testing:

- (i) central-support compression;
- (ii) complete-ground preservation;
- (iii) expectation-algebra uniqueness;
- (iv) Gauss–Ward equivariance;
- (v) strict Cech and scale composition;
- (vi) the absence of coefficient-line phase defects.

D. Depth versus thickness

A finite/depth control verifies all identities at each fixed positive thickness and across a declared finite depth diagram. A thickness-cofinal witness additionally requires:

- (a) a cofinal $\varepsilon_n \downarrow 0$ corridor;
- (b) uniform form and propagation comparison;
- (c) uniform Orlicz and reference-logarithm tails;
- (d) sharp descent after regulator removal;
- (e) rooted joint lifting on the cofinal tail;
- (f) comparison–Mosco recovery and a uniform coercive constant.

The strict root-datum exact-dilation theorem supplies this general cofinal witness. The finite/depth $SU(2)$ calculation remains an independent control, not the logical basis of the general theorem.

E. Optional $\mathbb{Z}_2$ scalar cocycle

For the nontrivial center element $h = -I$, the cyclic carry cocycle is

$$\Omega_{h,s}(a, b, c) = (-1)^{\widetilde{s}a\epsilon_2(b,c)}, \quad s = \pm 1.$$

The two orientations coincide as classes because $-1 = 1$ in $\mathbb{Z}_2$ orientation reversal. The primitive defining channel realizes the center response. This is covered by the general cyclic root-datum theorem and is not needed for the strict control.

Proposition LXXXII.1 (Role of the $SU(2)$ control) *The strict $SU(2)$ finite/depth construction validates the identity coefficient conventions and finite-stage operator/state diagrams. Constructive thickness-cofinal non-vacuity for Sector I is supplied by the compact-semisimple exact-dilation theorem, not by the finite/depth control alone.*

LXXXIII. COMPARISON WITH THE $SU(3)$ WITNESS

The two controls play different roles:

Feature	Strict $SU(2)$ control	Physical $SU(3)$ witness
Coefficient class	identity	nonzero triality carry class
Primary purpose	finite/depth strict check	complete physical twisted non-vacuity
Thickness	cofinal- not used without extra theorem	canonical dyadic physical branch
ity		
Detector	trivial	$I_3(\Omega_m) = \omega^m$
Controlled defect	zero	liminf at least $\pi/3$
Sector conclusion	Sector I control	Sector III for $m = 1, 2$
Interaction conclu-	not claimed	not claimed
sion		
Mass-gap conclu-	not claimed	not claimed
sion		

LXXXIV. ATOMIC NO-SPLITTING AS AN IDENTIFICATION THEOREM

The inherited expectation graph exists without a bare thin operator. Under an additional eventual atomic no-splitting hypothesis, it may be identified with complete atomic pinching of a thin operator. This section states the optional criterion and separates it from existence.

A. Bare thin operator data

Assume there is a self-adjoint thin operator K_0^{bare} on a fixed represented carrier with pure point spectral decomposition on the selected central corner:

$$K_0^{\text{bare}} = \sum_{j \in J} \lambda_j P_j.$$

The complete atomic pinching on normal states is

$$\mathbb{P}_{K_0}^*(\rho) = \sum_{j \in J} P_j \rho P_j, \quad (143)$$

where the sum converges in trace norm on the selected packet.

B. Eventual no-splitting

For every finite limiting spectral atom P_j and every sufficiently large good window, suppose there is one positive-thickness spectral island $P_{j,n}$ such that:

- (i) $P_{j,n}$ converges strongly or in norm to P_j on the selected carrier;
- (ii) distinct limiting atoms correspond to disjoint positive-thickness islands;
- (iii) no one positive-thickness island splits into several limiting atoms;
- (iv) no two distinct islands merge into one limiting atom;
- (v) the source expectation fixes precisely the block algebra generated by the $P_{j,n}$.

This is eventual atomic no-splitting.

Theorem LXXXIV.1 (Atomic identification criterion) *If the inherited graph exists by corrected SLSP and the bare thin operator satisfies eventual atomic no-splitting, then*

$$\mathcal{S}_{G,\xi,0}^{\text{st,as},*} = \mathbb{P}_{K_0}^*$$

on the selected normal-state domain.

Proof. At every good window, no-splitting identifies the limiting fixed-point algebra with the block-diagonal algebra generated by the limiting atoms. The faithful-reference-preserving expectation onto this algebra is the finite-window atomic pinching. Expectation uniqueness therefore identifies the inherited finite-window expectation with the bare pinching. Uniform tail removal passes the equality to the full graph. $\square$

C. Failure of the criterion

If an atom splits, merges, or cannot be globally labelled, the inherited graph remains well defined by the expectation tower. The bare pinching may be ambiguous or represent a different state transformation. This is precisely why no-splitting is not an existence hypothesis.

LXXXV. GRAPH PROPERTIES

The inherited surgery graph has:

- (G1) a selected normal-state domain containing the physical Orlicz packet;
- (G2) one value for every domain state;
- (G3) trace preservation and positivity;
- (G4) affine Markov behavior on convex state packets;
- (G5) idempotence;
- (G6) normal regional and Cech compatibility;
- (G7) complete-ground preservation;
- (G8) Gauss–Ward prepared-state invariance;
- (G9) sector and coefficient coherence retention;
- (G10) restart closure.

It is not treated as a bounded linear operator on a global physical Hilbert space.

A. Graph convergence definition

For graphs $\Gamma_n \subset X_n \times X_n$ transported to a limiting predual space X_0 , graph convergence means:

- (a) every bounded sequence $(\rho_n, \rho'_n) \in \Gamma_n$ with $\rho_n \rightarrow \rho$ has every convergent output subsequence tending to the unique ρ' with $(\rho, \rho') \in \Gamma_0$;
- (b) every $(\rho, \rho') \in \Gamma_0$ has a rooted recovery $(\rho_n, \rho'_n) \in \Gamma_n$ converging jointly.

Corrected SLSP proves both clauses on the selected packet. The second clause is why rooted joint lifting is required.

LXXXVI. SOURCE-DEFINED VERSUS OPERATOR-DEFINED SURGERY

The source-defined expectation depends on the selected positive-thickness spectral construction, faithful reference, regional chart data, and complete-ground pointing. A bare

thin operator may forget this provenance. The inherited graph preserves it through the comparison system. When atomic identification holds, the two descriptions coincide; when it does not, the source-defined graph remains the authoritative object.

LXXXVII. CORRESPONDENCE-VALUED REPRESENTED DESCENT

A. Normal correspondences and Markov pointings

We use the standard relative tensor-product convention for von Neumann correspondences.¹⁸ A normal M – N W^* -correspondence is a self-dual right N -module ${}_M\mathcal{X}_N$ with normal unital left action

$$\lambda_{\mathcal{X}} : M \longrightarrow \mathcal{L}_N(\mathcal{X}).$$

A normalized Markov pointing is a vector $\xi \in \mathcal{X}$ satisfying

$$\langle \xi, \xi \rangle_N = 1_N.$$

It defines the normal unital completely positive map

$$\Phi_{\mathcal{X},\xi} : M \longrightarrow N, \quad \Phi_{\mathcal{X},\xi}(x) = \langle \xi, \lambda_{\mathcal{X}}(x)\xi \rangle_N. \quad (144)$$

The preadjoint transports normal states in the opposite direction,

$$\Phi_{\mathcal{X},\xi,*} : N_* \longrightarrow M_*.$$

A bare correspondence does not canonically determine this channel; the pointing or an independently selected normal ucp map is part of the branch datum.

B. Fusion and strict Markov shadow

Let ${}_M\mathcal{X}_N$ and ${}_N\mathcal{Y}_P$ be normalized pointed correspondences with pointings ξ and η . Their Connes fusion carries the fused pointing $\xi \bar{\otimes} \eta$, and

$$\Phi_{\mathcal{X} \bar{\otimes}_N \mathcal{Y}, \xi \bar{\otimes} \eta} = \Phi_{\mathcal{Y},\eta} \circ \Phi_{\mathcal{X},\xi} : M \longrightarrow P. \quad (145)$$

Consequently the predual state maps compose as

$$\Phi_{\mathcal{X} \bar{\otimes}_N \mathcal{Y},*} = \Phi_{\mathcal{X},*} \circ \Phi_{\mathcal{Y},*} : P_* \longrightarrow M_*.$$

Proof. For $x \in M$, the P -valued inner product on the relative tensor product gives

$$\langle \xi \otimes \eta, \lambda_{\mathcal{X} \otimes \mathcal{Y}}(x)(\xi \otimes \eta) \rangle_P = \langle \eta, \lambda_{\mathcal{Y}}(\langle \xi, \lambda_{\mathcal{X}}(x)\xi \rangle_N) \eta \rangle_P,$$

which is exactly the right-hand side of (145). $\square$

A ground-preserving pointed bimodule-unitary two-cell leaves the Markov map unchanged. Thus a non-scalar represented associator can coexist with a strict observable and normal-state shadow.

C. Associator and obstruction status

Threefold fusion has the canonical comparison unitary

$$\alpha_{\mathcal{X}, \mathcal{Y}, \mathcal{Z}} : (\mathcal{X} \overline{\otimes} \mathcal{Y}) \overline{\otimes} \mathcal{Z} \longrightarrow \mathcal{X} \overline{\otimes} (\mathcal{Y} \overline{\otimes} \mathcal{Z}).$$

A selected branch may modify this by a ground-preserving bimodule unitary. Its component is retained as

$$\kappa_{\xi}^{\text{Corr}} \in \pi_0 \text{Assoc}_{\text{Corr}}.$$

There are two independent questions:

- (i) is the represented target correspondence valued because no coherent scalar line reduction has been proved?
- (ii) has a physical detector proved $\kappa_{\xi}^{\text{Corr}} \neq [1]$?

The first does not imply the second. A pointed-pseudonaturally strictifiable correspondence system may have trivial associator component without admitting scalar reduction.

D. Coherent scalar reduction criterion

A scalar reduction requires more than a one-dimensional endomorphism algebra at each object. One must prove a coherent family of one-dimensional central complete-ground summands on every incised object, compatibility with all regional, scale, Cauchy, background, and mixed arrows, compatibility with fusion and pointed two-cells, and scalar action of the full associator. Only then may one pass to a scalar line system and define a class in $H^3(BA; U(1))$.

LXXXVIII. PRIMITIVE CORRESPONDENCE TRANSFER PACKAGE

The correspondence-shadow transfer theorem uses the following primitive data:

- (CTA1) an incision-preserving pseudodiagram of self-dual normal correspondences;
- (CTA2) normalized Markov pointings and normal ucp shadows (144);
- (CTA3) pointed pseudonatural comparison maps and ground-preserving bimodule-unitary two-cells;
- (CTA4) exact fusion-shadow composition (145);
- (CTA5) selected state packets stable under the predual Markov maps;
- (CTA6) objectwise expectation germs, reference/Orlicz control, sharp surgery covariance, and rooted master-defect estimates on the Markov shadow;
- (CTA7) complete-ground, Gauss–Ward prepared, collar, and comparison–Mosco compatibility;
- (CTA8) a separately carried associator component and, when obstruction is claimed, a physical component detector stable under equivalence, fusion, rephasing, stabilization, and thickness comparison.

Theorem LXXXVIII.1 (Correspondence-shadow transfer) *Under CTA1–CTA7, the ordinary observable net, normal-state expectation graph, Orlicz packet, sharp descent, rooted lifting, Gauss–Ward/collar package, and comparison–Mosco form reconstruct through the strict Markov shadow. The represented object remains correspondence valued and carries its bimodule associator. Under CTA8, a detected nontrivial component is retained. Without CTA8, no nontriviality claim is made.*

Proof. Apply the ordinary and state-space reconstruction theorems to the strict bounded observable shadow and the predual Markov maps. Equation (145) gives the correct contravariant state-map composition on every rooted path. Pointed bimodule-unitary two-cells leave the shadow unchanged, while the represented correspondences and associator are carried separately. CTA6 and CTA7 supply the analytic and graph-domain hypotheses. CTA8, when present, supplies detector-specific no-jumping. No scalarization is used. $\square$

LXXXIX. CORRESPONDENCE PHYSICAL NON-VACUITY BOUNDARY

A complete physical non-scalar Yang–Mills witness would require explicit incised positive-thickness correspondences, normalized pointings, fusion and two-cell maps, full analytic comparison, and a detector proving a nontrivial correspondence component. Paper IV proves the transfer theorem but does not claim such a general physical witness.

XC. RELATION TO THE SCALAR TRICHOTOMY

If a correspondence system admits a coherent scalar line reduction, the resulting scalar branch enters Sector I, II, or III according to its physically evaluated fiber and controlled total classes. Without that reduction it remains correspondence valued. It is correspondence-obstructed only when nontriviality is independently detected.

XCI. PRIMITIVE-HYPOTHESIS CROSSWALK AND DEPENDENCY AUDIT

A. Scope of this crosswalk

The main article uses five primitive input families: **OJ**, **RJ**, **SJ**, **GJ**, and **M**. This supplement records how they replace the earlier compressed JH blocks. The replacement is logical, not merely notational: all conclusion-like clauses have been removed.

B. Old-to-new crosswalk

Earlier journal block	Retained primitive content	Removed from the hypotheses and derived later
JH1	OJ1–OJ8: controlled incision, packet comparison, corridor independence saturated null ideal, normal support, locality margins, covers, Cauchy data, covariance	ordinary thin net and comparable-
JH2	RJ1–RJ8: represented carriers, represented thin descent, no-pointings, coefficient objects, de-jumping, branch classification tectors, physical strictification data, nonzero branch classes, correspondence typing, stabilization	
JH3	SJ1–SJ7: positive-thickness limiting expectation graph, sharp de-expectations, expectation-germ scent, surgically rooted joint lifting, certificate, coherent windows, restart closure reference/Orlicz/profile data, smooth Kraus comparison, strict unsurgered rooted diagram, fiber-thickness environment	
JH4	GJ1–GJ8: graph cores, exact limiting Gauss–Ward package, aug-graph-form Ward identity, expected surgically rooted lift, surgery tation equivariance, prepared do-preservation of the structural tuple mains, collar quotient, correctly typed defects, ground Gauss data, unsurgered augmented lifts	
JH5	M1–M6: compari-closed relaxed form, Mosco conversion/reconstruction maps, gence, TT1, TT2, exact kernel, thin sesquilinear defects, weak compactness, primitive recovery data, ground form structure, positive-thickness coercivity	

Earlier journal Retained primitive content block	Removed from the hypotheses and derived later
--	---

C. Observable crosswalk

Input	Source-level role	Journal theorem use
OJ1	controlled incised backgrounds, domain and geometric typing of Theorem adapted ambients	positive-clearance regions, com-rem A
OJ2	elementwise bounded observable thin testing C^* functor comparison and saturated packet-null ideal	
OJ3	tight norming states, exact faithful normal envelopes and normal regional extension/restriction, isotony quasi-equivalent selected representations	
OJ4	incised support calculus, relative exact thin Einstein causality causal margins, positive-thickness locality defect	
OJ5	packet-localizing covers, physical normal additivity recombination, representation saturation	
OJ6	comparison-coherent thick time slice and causal dynamics Cauchy source sections and frame comparison	
OJ7	strict controlled-background ob-local covariance and Cauchy groupoid servable maps and null composition cubes	
OJ8	finite common refinements pre-canonical comparable-choice independence incision, null ideal, and dence selected states	

D. State and seam crosswalk

Input	Source-level role	Journal theorem use
SJ1	source-defined complete spectral finite-window surgery provenance expectations	
SJ2	exact stationarity or expectation- germ Cauchy/uniqueness estimate	C1 expectation germ
SJ3	coherent window compression and coherent tower extension	
SJ4	faithful reference, Orlicz moment, C2 mass/log/entropy/spatial tails spectral profile, collar tightness	
SJ5	smooth Kraus comparison on joint arrow packets	C3 sharp descent after regulator removal
SJ6	strict unsurgered rooted finite diagram	C4 root/objectwise-surgery/master-defect proof
SJ7	packet stability and thickness environment	fiber-restart corollary after C5

E. Gauss–Ward crosswalk

Input	Source-level role	Journal theorem use
GJ1	graph-domain compactness and limiting Gauss forms core transport	
GJ2	exact positive-thickness graph- exact limiting Ward functional form Ward identity	
GJ3	finite-window expecta- expectation equivariance tion/derivation commutation	
GJ4	prepared-domain comparison and limiting prepared-domain stability positive-thickness invariance	
GJ5	collar quotient and charge graph thin collar charge forms	
GJ6	two-parameter current defect, retained structural tuple character class, first Ward defect	
GJ7	complete-ground Gauss kernel physical ground constraint	
GJ8	unsurgered augmented root data later augmented rooted theorem	

F. Closed-form crosswalk

Input	Source-level role	Journal theorem use
M1	comparison and reconstruction maps, common graph topology	varying-space setup
M2	sesquilinear form comparison	limiting preform and graph-norm control
M3	weak compactness and synchronized approximation	liminf theorem
M4	core recovery and exhaustion chronization	syn-TT2
M5	ground-nullity, orthogonality, graph-domain comparison	TT1 and exact kernel
M6	positive-thickness off-ground coercivity	thin coercive threshold via TT2 and TT1

G. Circularity audit

The following implications are prohibited:

sharp descent as an SJ clause,
rooted lifting as an SJ clause,
restart closure as an SJ clause.

Mosco liminf or recovery as an M clause,
TT1 or TT2 as an M clause,
exact thin kernel as an M clause.

The allowed deduction order is

SJ1–SJ3 $\Rightarrow$ expectation germ,
 SJ4 $\Rightarrow$ separate tails and entropy modulus,
 SJ5 + tails $\Rightarrow$ sharp descent,
 SJ6 + objectwise surgery $\Rightarrow$ preliminary rooted family,
 preliminary rooted family + sharp descent $\Rightarrow$ one rooted lift,
 corrected SLSP + SJ7 $\Rightarrow$ restart,

and

M1–M6 $\Rightarrow$ closed relaxed form,
 closed relaxed form $\Rightarrow$ Mosco liminf and TT2,
 TT2 and ground-face comparison $\Rightarrow$ TT1,
 TT1–TT2 $\Rightarrow$ coercivity and exact kernel.

H. Incision firewall

Every crosswalk row is understood over $X = (\mathbb{M}_{0,\text{inc}}, O) \in \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$. Locality requires a common incision-adapted ambient region, time slice requires an incision-adapted Cauchy arrow, and comparable-corridor independence preserves the same controlled incision data. No row implies that Σ is removable or that the full Poincare group acts on a fixed incised fiber.

XCII. THEOREM-BY-THEOREM DEPENDENCY MAP

A. Theorem A

- a. Inputs.* OJ1–OJ8.
- b. Construction.* Bounded comparison sections, saturated common-null quotient, norming limit states, faithful direct-sum representation, and normal regional extension.
- c. Noncircularity.* The proof does not assume a thin algebra or thin state representation. It starts from positive-thickness packet sections and normal endpoint functionals.
- d. Outputs.* Normal isotony, Einstein causality, packet-localizing normal additivity, local covariance, causal dynamics, time slice, and finite packet generation.

e. Excluded shortcuts. No arbitrary tensor partial trace, full Haag duality, or global physical Hilbert space is used.

B. Theorem B

- a. Inputs.* RJ and the ordinary net from Theorem A.
- b. Construction.* Finite represented chart objects, coefficient arrows, mixed cells, complete-ground pointings, and branchwise path comparison.
- c. Noncircularity.* The sector is selected at positive thickness and shown conservative; it is not inferred from the existence of the thin represented object.
- d. Outputs.* Global strict object in Sector I, corridor-banded object in Sector II, chart–corridor line object in Sector III, and correspondence object in the weak branch.

C. Theorem C

- a. Inputs.* OJ smooth locality, RJ branch typing and ground data, and SJ expectation/entropy/recovery data.
- b. Construction.* A good-window expectation tower and moving expectation limit; separate Orlicz, logarithm, and entropy tails; smooth-to-sharp descent; strict-root/objectwise-surgery lifting; the graph limit; and restart under SJ7.
- c. Noncircularity.* The full thin surgery is not assumed and no bare thin operator is required. Finite-window expectations are source-defined at positive thickness.
- d. Outputs.* A single-valued inherited graph, sector and ground preservation, Markov compatibility, and conditional restart closure under SJ7.

D. Theorem D

- a. Inputs.* GJ and the rooted lifting theorem.
- b. Construction.* Graph-limit insertion derivations, exact Ward functional, collar quotient, boundary character, retained current and Ward defects.
- c. Noncircularity.* No global self-adjoint Gauss generator is assumed. The derivation is reconstructed graphwise on a common insertion core.
- d. Outputs.* Gauss–Ward and collar reconstruction compatible with the inherited graph.

E. Theorem E

- a. Inputs.* $\mathbf{M}$ plus the complete-ground pointing from $\mathbf{RJ}$.
- b. Construction.* Mosco $\liminf$, local and diagonal recovery, $\mathbf{TT1}$, $\mathbf{TT2}$, coercivity transfer, exact kernel.
- c. Noncircularity.* No global thin Hamiltonian, translation generator, or mass operator is assumed. The associated operator is defined only after the closed form is constructed.
- d. Outputs.* Closed thin form, exact complete-ground kernel, and positive off-ground stiffness threshold.

F. Theorem F

- a. Inputs.* Compact connected semisimple global form, root and weight lattices, finite center.
- b. Construction.* Annihilator identity, connected-quotient center proof, primitive character extension, dominant representatives, quotient-compatible grading frames, normalized cyclic and finite-abelian scalar coefficient categories.
- c. Noncircularity.* The root datum supplies allowed channel characters only. Mixed pair and triple associators are separately declared normalized tensorator data. No local-normal algebra, reference state, Gauss–Ward graph, complete-ground projection, or coercive form is inferred from the lattice theorem.
- d. Outputs.* Global-form representation theory and center-generated scalar coefficient availability. Analytical witness certification is Theorem G and remains a separate proof layer.

G. Theorem G

- a. Inputs.* Local exact-dilation Friedrichs branch and root-datum coefficient system.
- b. Construction.* Line-by-line verification of $\mathbf{OJ}$, $\mathbf{RJ}$, $\mathbf{SJ}$, $\mathbf{GJ}$, and $\mathbf{M}$, plus physical $SU(3)$ perturbation estimates.
- c. Noncircularity.* The witness does not use the reconstruction theorem to verify its own hypotheses. The local form, expectation, reference, and coefficient data are explicit before applying Theorems A–E.

d. Outputs. Constructive non-vacuity of Sector I and scalar Sector III; complete physical $SU(3)$ witness.

H. Theorems H1–H3 and Firewall H4

a. Inputs. Theorems A–G, cofinal independence, restart, detector-specific branch certificates, comparison shells, and provenance ledgers.

b. Construction. H1 assembles the ordinary net, branch-typed represented object, expectation graph, Gauss–Ward/collar package, branchwise form/kernel family, and global-form data. H2 discharges the assembly for Level-B comparison witnesses and the Level-C physical $SU(3)$ family with exact conditionality. H3 passes a data core, certification shell, and unresolved obligation ledger. H4 prohibits a physical-spectrum conclusion.

c. Noncircularity. The incision decision, candidate invariant state, V.O1–V.O12, and V.INT1 are outputs demanded of Paper V, not hypotheses used by Paper IV. Branchwise form and ground data remain banded unless effective descent is independently proved.

XCIII. CIRCULARITY EXCLUSIONS

A. Expectation graph

The moving expectation theorem uses finite-window operator/reference convergence and algebra-gap convergence. Sharp Cech descent uses the expectation theorem and independent tails. Rooted lifting uses sharp descent. The full graph uses rooted lifting. No step uses the full graph to prove an earlier finite-window premise.

B. Gauss–Ward

Positive-thickness equivariance is an input. Limiting equivariance uses moving expectation convergence and graph convergence of derivations. Surgery compatibility uses limiting equivariance. No surgery compatibility premise is used to prove expectation convergence.

C. Closed forms

The relative form estimate gives graph norm control and resolvent convergence. Mosco liminf and recovery construct the limiting form. Coercivity transfer and exact kernel are proved afterward. The positive threshold is not used to define the limiting form.

D. Root data

The identity $P_G/Q \cong \widehat{Z(G)}$ gives quotient-compatible grading characters and primitive channels. A normalized cocycle separately defines the scalar coefficient category. An independently verified incised analytical carrier is then required, and the represented package must still verify complete-ground compatibility, mixed-cell naturality, and a physical detector. No analytical conclusion is derived from root data alone.

E. Paper V

No ambient incision-removal theorem, invariant state, Poincare or stabilizer representation, or physical spectral decomposition is used in Paper IV. The form threshold is passed as branchwise data. Paper V must first resolve the incision continuation and then discharge V.O1–V.O12 before interpreting the threshold spectrally; V.INT1 remains independent.

XCIV. CHALLENGE AND RESOLUTION MATRIX

Challenge	Potential failure	Article response
The theorem is vacuous	no branch verifies all hypotheses	exact-dilation strict and scalar cyclic witnesses; physical $SU(3)$ specialization
The hypotheses restate assumptions	thin objects	JH blocks are positive-thickness comparison, integrability, and recovery data
Finite propagation is sharp	projector treated as local	only smooth wave calculus is local; sharpness follows from tails
Entropy control is complete	in-log tail inferred from entropy	separate physical Orlicz estimate proves log tail
State recovery is only pairwise	incompatible shared vertices	one strict-root/objectwise-surgery master-defect construction
The twist is formal	abstract cocycle not physically detected	primitive Wilson channel, linked flux, cyclic detector, noncollapse estimate
The gauge scope is too narrow or too broad	only $SU(N)$ or all compact groups claimed	exact compact connected semisimple global-form scope

Challenge	Potential failure	Article response
The form threshold is missing Poincare spectrum called a mass gap		explicit Paper-V firewall
Strict means free	coherence confused with dy-interaction theorem deferred namics	and independently specified
Correspondence data multiplicity information lost are scalarized		bimodule associator retained in correspondence target

XCV. PROOF DELEGATION MAP

Every principal theorem is stated once in the main article. The supplement proves subordinate lemmas, estimates, and criteria. The article may cite the supplement for:

- (1) the complete bounded-section ideal proof;
- (2) the quantitative finite propagation and smooth wave calculus;
- (3) Gram-matrix expectation estimates;
- (4) full Orlicz and entropy constants;
- (5) rooted lifting combinatorics;
- (6) full graph-domain Gauss–Ward passage;
- (7) diagonal Mosco recovery and resolvent estimates;
- (8) finite-abelian cocycle normal forms;
- (9) detailed $SU(3)$ matrix and detector calculations;
- (10) correspondence Markov-shadow transfer;
- (11) strict $SU(2)$ finite/depth controls;
- (12) all crosswalk, supersession, and circularity ledgers.

No principal theorem is delegated only to an unnumbered informal note.

XCVI. EXCLUDED FORMULATIONS AND SCOPE BOUNDARIES

This appendix records the exact replacement rule for every formulation that could alter the logical content of the journal theorem. Each entry identifies the rejected shortcut, the mathematical defect, the definitive replacement, and the audit test used in the compiled manuscript.

A. Bare atomic pinching as the definition of thin surgery

a. Excluded formulation. A bare thin operator need not exist in the represented branch, its atoms need not carry coherent labels, and complete pinching does not encode the source regional maps.

b. Adopted formulation. At each good finite window use the transported normal conditional expectation $\mathbb{E}^{(N),\text{as}}$ and its range algebra. Define the full surgery as the graph limit $\mathcal{S}^{\text{st,as},*}$ after the prescribed removal of the window and buffer.

c. Verification criterion. Every existence statement for thin surgery points to the expectation tower or the inherited graph, never to an autonomous thin spectral decomposition.

B. Atomic no-splitting as an existence assumption

a. Excluded formulation. This confuses a sufficient identification criterion with the construction and makes existence depend on eventual spectral labeling.

b. Adopted formulation. Expectation-algebra uniqueness supplies existence and uniqueness. Eventual atomic no-splitting is retained only in Supplement P as a theorem identifying the inherited graph with a particular bare pinching when the additional operator exists.

c. Verification criterion. No principal theorem assumes no-splitting.

C. Arbitrary tensor partial traces as regional maps

a. Excluded formulation. A tensor factorization is not canonical in gauge-constrained local algebras and may violate central supports, selected sectors, and Gauss constraints.

b. Adopted formulation. Use source-defined normal conditional expectations or algebraized observable restriction maps compatible with the selected normal envelopes.

c. Verification criterion. The observable quotient and SLSP modules use only declared regional maps and common-carrier expectations.

D. Sharp spectral locality from finite propagation alone

a. Excluded formulation. Finite propagation applies to compactly supported wave calculus and smooth functional calculus, not to discontinuous spectral projectors.

b. Adopted formulation. First prove smooth buffered descent from the local Laplace-type principal part and pointwise lower-order terms. Then remove the spectral and spatial regulators using separate mass, reference-logarithm, and entropy tails.

c. Verification criterion. The limit order is always finite diagram and buffer, good window, thickness limit, window removal, then buffer removal.

E. Reference-logarithm integrability inferred from entropy

a. Excluded formulation. Relative entropy can remain bounded while the reference logarithm has uncontrolled tails.

b. Adopted formulation. Assume and verify a physical Orlicz moment for L^{ref} and derive the logarithm tail independently by de la Vallée–Poussin uniform integrability.

c. Verification criterion. The logarithm-tail lemma is separate from the entropy-continuity lemma.

F. Weak-* compactness identified with predual compactness

a. Excluded formulation. Weak-* compactness of a state set does not provide the uniform predual convergence needed by a single-valued expectation graph.

b. Adopted formulation. Combine finite-window algebra convergence, explicit uniform integrability, and a common entropy modulus to obtain $\|\cdot\|_*$ estimates on the selected packets.

c. Verification criterion. Every sharp descent statement is uniform in predual norm.

G. Pairwise recoveries substituted for one rooted lift

a. Excluded formulation. Pairwise compatible choices can disagree at shared vertices and may break composition around a finite diagram.

b. Adopted formulation. Choose one root recovery, propagate it through the strict unsurgered normal-state shadow, apply the source-defined surgery objectwise, and control every edge and higher cell by one telescoping master defect.

c. Verification criterion. Supplement K constructs one strict unsurgered rooted family, applies surgery objectwise, and proves simultaneous finite-diagram closure through the master defect.

H. Abstract cocycle vanishing substituted for physical strictification

a. Excluded formulation. A class can vanish through a primitive that violates locality, stationarity, ground pointing, selected sectors, or neutral stabilization.

b. Adopted formulation. Sector I requires one admissible physical two-cochain strictification preserving the complete chart–regional–scale package.

c. Verification criterion. The strict-transfer theorem cites compatible strictification, not merely abstract cohomology.

I. $SU(N)$ presented as final gauge scope

a. Excluded formulation. The analytic argument is Lie-type independent and the Wilson channels depend on the global-form weight lattice.

b. Adopted formulation. Use compact connected semisimple $G = \tilde{G}/\Gamma$, define P_G , and prove $P_G/Q \cong \widehat{Z(G)}$.

c. Verification criterion. The root-datum module includes all simple types, products, diagonal quotients, and center-trivial forms.

J. Universal Sector-II non-vacuity

a. Excluded formulation. A controlled scale obstruction is branch data and is not forced by the abstract group or by a nontrivial center.

b. Adopted formulation. Sector-II closure is a transfer theorem conditional on a physically supplied nonzero controlled chart–scale class.

c. Verification criterion. The witness matrix marks Sector II “conditional on branch realization.”

K. Scalarization of correspondence associators

a. Excluded formulation. A multiplicity-bearing bimodule associator need not reduce to a scalar class in $H^3(BA; U(1))$.

b. Adopted formulation. Retain κ_ξ^{Corr} in the correspondence associator space and use only the normal Markov shadow for the analytic reconstruction.

c. Verification criterion. Supplement Q separates the shadow theorem from the retained bimodule invariant.

L. Thin stiffness threshold called a mass gap

a. Excluded formulation. The reconstructed form is not yet the invariant mass operator and Paper IV has no Poincaré spectrum condition.

b. Adopted formulation. Call $\kappa_{\text{IV,thin}}$ an off-ground thin stiffness threshold and pass it to Paper V.

c. Verification criterion. Every theorem containing $\kappa_{\text{IV,thin}}$ carries the mass-gap firewall.

M. Strict branch called dynamically free

a. Excluded formulation. Strictness is a coherence property and says nothing about Gaussian factorization, scattering, or free-net equivalence.

b. Adopted formulation. Defer the interaction/free-net non-equivalence theorem to Paper V.

c. Verification criterion. The discussion never uses “strict” as a synonym for “free.”

N. Sector-III twisting called proof of interaction

a. Excluded formulation. A nonzero associator detects represented coherence, not connected correlation functions or dynamical non-equivalence.

b. Adopted formulation. Use the twist only to classify represented descent; require a separate observable interaction invariant in Paper V.

c. Verification criterion. The branch record retains $\kappa_{\xi}^{\text{fib}}$ and makes no dynamical inference.

XCVII. RETAINED CONSTRUCTIONS AND THEIR RESTRICTED INTERPRETATION

Retained object	Legitimate role	Forbidden inference
Smooth functional calculus	cal- local finite-propagation comparison and buffered Cech de-scent	com- locality of a discontinuous projector without tail removal
Finite spectral windows	win- compact expectation-comparison and finite-dimensional reference logs	algebra existence of globally labelled thin eigenvectors
Atomic no-splitting	optional identification of the existence or uniqueness of thin inherited graph with a bare surgery pinching	
Physical $SU(3)$ branch	first complete center-twisted physical witness and quanti-tative discovery model	endpoint of gauge-group scope or universal Sector-II realization
Strict $SU(2)$ control	finite/depth check of strict thickness-cofinal witness with-estimates and normalization out an independent $\varepsilon \downarrow 0$ theorems	
Exact-dilation data branch	root-explicit nonempty comparison class verifying OJ , RJ , Yang–Mills observable system SJ , GJ , and M	uniqueness of the continuum
Comparison–Mosco convergence	construction of a closed thin form and resol-vent/semigroup limits	construction of spacetime trans-lations or invariant mass
Coherence class	classification of represented interaction, descent and strictifiability	confinement, or scattering by itself

XCVIII. SCOPE BOUNDARIES

A. Gauge-group boundary

The theorem is restricted to compact connected semisimple global forms. Reductive torus factors would add continuous charge lattices and infrared zero-mode issues. Disconnected groups would require component-group actions and crossed-product descent. Noncompact groups would require a different operator, measure, and entropy package. None of these extensions is silently included.

B. Regional boundary

The ordinary observable functor is constructed on the selected category $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ of controlled background–region objects and incision-preserving arrows. Regions, backgrounds, or arrows outside that category require separate geometric, local-generation, and state-amalgamation theorems. The term Haag–Kastler refers to the axioms on this declared incised category; it neither fills nor removes Σ and does not assert an unrestricted extension to every open subset.

C. Represented boundary

A global ordinary observable net does not imply a global physical Hilbert descent. Sector I admits a coherent strict represented object. Sector II and Sector III retain corridor or chart–corridor banding. The correspondence branch remains in its bimodule target. Observable homomorphisms and represented arrows are never conflated.

D. Analytic-witness boundary

The exact-dilation branches prove non-vacuity of the journal hypotheses. They do not prove that every lattice, Galerkin, or continuum regularization lies in the same comparison class. A new branch must verify the common-domain, expectation, Orlicz, Gauss–Ward, and coherence hypotheses rather than merely exhibit pointwise coefficient convergence.

E. Dynamical boundary

Paper IV does not construct the invariant vacuum, a strongly continuous Poincaré representation, translation generators, the forward spectrum condition, an invariant mass operator, or physical no-leakage. The interaction theorem is likewise absent. These are mathematically independent bridges reserved for Paper V.

XCIX. BRANCH-HONESTY LEDGER

Branch	Constructive status	Retained invariant	Prohibited over-claim
Sector I	explicit for every $\kappa_{\xi}^{\text{fib}} = 0$, $\hat{\kappa}_{\xi}^{\text{cs}} = 0$, compatible strictification declared compact connected semisimple global form		free or noninteracting theory
Sector II	analytic transfer zero fiber class and universal existence proved; physical real-nonzero controlled chart—for every G ization branch-specific scale class		
Sector III	scalar cyclic witness for $[\Omega_{\xi}] \neq 0$ in interaction or con- every selected nontriv- $H^3(BA_{\xi}; U(1))$ finement theorem ial finite center band; mixed classes when de- clared		
Correspondence	Markov-shadow trans- $\kappa_{\xi}^{\text{Corr}}$ in the correspon- fer proved	dence associator space	scalar H^3 classification or physical non-vacuity

C. FINAL SCOPE CERTIFICATE

Theorem C.1 (Supersession closure) *Every principal theorem in the journal article is derivable using only the definitive formulations listed in this appendix. Replacing any of them by a superseded shortcut would either strengthen an input without proof, erase branch data, or*

cross the Paper-V spectral firewall. Therefore the definitive expectation-graph, rooted-lifting, physical-strictification, compact-semisimple, and form-threshold formulations are mandatory parts of the theorem statement.

CI. EXACT INCISED PAPER-V HANDOFF CONTRACT

This appendix expands Theorem H3 into a componentwise contract and records its downstream interface with Paper V.¹⁹ The Paper-IV output is a branch-typed certificate; it is neither an ambient spacetime theory nor a preliminary spectrum theorem.

A. Two-layer Paper-IV certificate

Definition CI.1 (Branchwise data core) *For any fully certified branch, let*

$$\mathfrak{D}_{\text{IV}}^{G,\xi} = \left(\mathbb{M}_{0,\text{inc}}, \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}, \mathcal{A}_{G,\xi,0}^W, \mathbf{Rep}_{G,\xi,0}^{\text{thin}}, \hat{\Omega}_{G,\xi,0}^b, \mathcal{S}_{G,\xi,0}^{\text{st,as,*}}, \mathfrak{K}_{G,\xi}^{\text{Orl}}, \mathfrak{W}_{G,\xi}^{\mathcal{S}}, \bar{q}_{G,\xi,0}, \mathbf{P}_{\Omega,G,\xi,0}, \kappa_{\text{IV,thin}}, \text{Restart}_{G,\xi}, \kappa_{\xi}, G, \Gamma, P_G, \mathcal{W}_{G,\xi} \right).$$

The ordinary functor $\mathcal{A}^W$ and the represented pseudodiagram $\mathbf{Rep}^{\text{thin}}$ have distinct targets. The abstract ground family $\hat{\Omega}^b$ is not the projection family $\mathbf{P}_{\Omega}$, and $\mathcal{S}^{\text{st,as,*}}$ is a normal-state graph. The bold entries are branchwise families. Sector III retains chart-corridor line transitions; a correspondence branch retains its pointed correspondences and Markov shadows; only an independently strictified effective descent may replace these families by one global represented Hilbert-space object.

Definition CI.2 (Certification shell) *Set*

$$\mathfrak{C}_{\text{IV}}^{G,\xi} = \left(\mathbf{Rec}_{G,\xi}^{\text{adm}}, \mathfrak{C}_{\varepsilon \rightarrow 0}^{G,\xi}, \mathfrak{G}_{\varepsilon \rightarrow 0}^{G,\xi}, \mathfrak{F}_{\varepsilon \rightarrow 0}^{G,\xi}, \mathfrak{S}_{\varepsilon \rightarrow 0}^{G,\xi}, \text{ChoiceInd}_{G,\xi}, \text{Prov}_{G,\xi} \right).$$

The shell retains the admissible recovery category, all comparison moduli, complete-ground transport, form/Mosco and sector/covariance/time-slice certificates, branchwise choice independence, and theorem provenance.

B. Componentwise provenance

Component	Paper-IV source	Exact meaning at handoff
$\mathbb{M}_{0,\text{inc}}, \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$	incised setup and Theo-rem A	controlled incision, collars, adapted charts, testing covers, Cauchy data, and incision-preserving arrows
$\mathcal{A}_{G,\xi,0}^W$	Theorem A	normal ordinary incised Haag-Kastler functor
$\mathbf{Rep}^{\text{thin}}$	Theorem B	branch-typed represented pseudodiagram, not the ordinary observable net
$\hat{\Omega}^b$	Theorem B	abstract branchwise complete-ground pointing or pointed subobject, not yet an invariant vacuum
$\mathcal{S}_{G,\xi,0}^{\text{st,as,*}}$	corrected SLSP	inherited Cech-descended graph on selected normal states
$\mathcal{R}_{G,\xi}^{\text{Orl}}$	entropy/reference analysis	selected packet, separate mass/logarithm/entropy controls, and common continuity modulus
$\mathfrak{W}_{G,\xi}^{\mathcal{L}}$	Theorem D	graph-form Ward, prepared domain, collar quotient, boundary class, and retained defects
$\overline{q}, P_{\Omega}$	Theorem E	branchwise closed stiffness forms and actual Hilbert-carrier projection family with exact kernel
$\kappa_{\text{IV},\text{thin}}$	coercivity transfer	positive off-ground stiffness constant, not invariant mass
$\text{Restart}_{G,\xi}$	corrected SLSP closure	admissibility after state selection and rooted lifting
κ_{ξ}	Theorem B and the se-lected physical $SU(3)$ the-orem	branch-typed scalar or correspon-dence coherence data and detectors

Component	Paper-IV source	Exact meaning at handoff
$(G, \Gamma, P_G, \mathscr{W})$	Theorem F	global form, allowed weights, and selected Wilson channels
$\mathfrak{C}_{\text{IV}}^{G, \xi}$	closure certificates	exact comparison, recovery, ground, form, sector, choice-independence, and provenance shell

C. Incision bifurcation

The first Paper-V theorem must resolve the geometry.

Auxiliary incision.:

Construct an unincised ambient net and coherent restriction equivalences for the complete incised family. Prove that arrows moving Σ arise from ambient automorphisms. Do not introduce an independent quantum degree of freedom for the position of the incision.

Physical incision.:

Retain the full controlled defect datum and work with its stabilizer. Replace the standard Lorentz-invariant mass claim by a precisely stated stabilizer-covariant energy or defect-spectrum claim.

Only the auxiliary-incision continuation can lead to the standard full-Poincare Yang–Mills mass theorem.

D. State obligation

After the symmetry target is constructed, produce a candidate locally normal state invariant under that target action. Then form its GNS representation, prove unitary implementation and strong continuity, and only afterward use the word vacuum in its spectrum-theoretic sense. The Paper-IV complete-ground pointing is not itself this conclusion.

E. Authoritative twelve spectral obligations

Let

$$\mathfrak{D}_V^{\text{spec}} = \{\text{V.O1}, \dots, \text{V.O12}\}.$$

Item	Required theorem	Firewall
V.O1	sector-appropriate continuous spacetime representation	strongly full Poincare is allowed only after ambient incision repre-descent
V.O2	commuting self-adjoint generator reconstruction and joint translation generator spectral measure	the stiffness operator is not substituted for a
V.O3	forward-cone or stabilizer-positivity of $\bar{\mathbf{q}}$ is not a spectrum condition appropriate spectrum condition	
V.O4	form-to-generator or invariant-mon physical core	no equality is inferred from notation, scaling, or mass comparison on a com- dimensions
V.O5	physical non-leakage at zero exact stiffness kernel alone is insufficient spectral value	
V.O6	threshold transfer with explicit constants	ex- $\kappa_{IV,\text{thin}}$ may require a comparison factor

Item	Required theorem	Firewall
V.O7	regional, Cauchy, corridor, pairwise local gaps do not automatically globalize and sector spectral-gluings stability	
V.O8	covariance of ground and positive spectral projections	local covariance alone does not imply spectral covariance
V.O9	branchwise vacuum invariant-ground multiplicity	or complete-ground rank is not automatically vacuum multiplicity
V.O10	Gauss–Ward compatibility of the physical spectral decomposition	structural Ward preservation is not yet spectral compatibility
V.O11	integrated end-to-end leakage theorem	no- all comparison, ground, form, sector, and gluings certificates must enter
V.O12	final branchwise mass-gap theorem and scope two incision continuations are not conflated	or uncertified branches remain conditional and the

F. Independent interaction obligation

The interaction theorem is not one of V.O1–V.O12. It is

$$\text{V.INT1 : } \quad \mathcal{A}_{G,\xi,0}^W \not\approx \mathcal{F} \quad \text{for every } \mathcal{F} \in \mathbf{Free}_{G,\xi}^{\text{adm}},$$

proved using a bounded gauge-invariant observable invariant such as a connected Wilson-packet cumulant or another explicitly declared dynamical quantity. The theorem must apply to strict and twisted branches alike.

G. Branchwise handoff matrix

Branch	Paper-IV certificate	Paper-V starting object	Status
Strict root-datum complete Level-B com- exact-dilation	parison shell	incised Sector-I comparison branch and ordinary net	analytical wit- ness
Cyclic scalar exact- complete Level-B com- dilation	parison shell	incised chart-corridor Sector-III line branch	analytical wit- ness

Branch	Paper-IV certificate	Paper-V starting object	Status
Physical $SU(3)$ Sec- tor I	complete C branch $(m, r) = (0, 0)$	Level-explicit incised at strict branch	physical physical witness
Physical $SU(3)$ Sec- tor II	complete controlled branch at $m = 0, r = 1, 2$	Level-C con-strict fibers with nonzero controlled scale detector	branch- specific physical witness
Physical $SU(3)$ Sec- tor III	complete branch at $m = 1, 2$ and arbitrary r	Level-C explicit triality/linked-flux line branch	physical witness
Generic Sector II	conditional assembly af- ter full branch verifica- tion	supplied fiber/nonzero-scale branch	strict-conditional
Correspondence target	Markov-shadow trans- fer plus supplied family physical correspon- dence data	pointed and ordinary vacuity shadow	physical condi- tional
Strict $SU(2)$ con- trol	fixed-positive- thickness finite/depth control	no thin Paper-V without an additional co- final theorem	branch control only

H. Complete pass-through theorem

Theorem CI.3 (Complete incised pass-through) *For every branch certified by the Paper-IV assembly theorem,*

$$\hat{\mathfrak{P}}_{IV \rightarrow V}^{G, \xi} = \left(\mathfrak{D}_{IV}^{G, \xi}, \mathfrak{C}_{IV}^{G, \xi}, \mathfrak{D}_V^{\text{inc}} \sqcup \mathfrak{D}_V^{\text{state}} \sqcup \mathfrak{D}_V^{\text{spec}} \sqcup \{\text{V.INT1}\} \right)$$

is a complete Paper-IV handoff. It contains every declared Paper-IV input to the downstream program and none of the downstream conclusions.

Proposition CI.4 (No hidden Paper-V premise) *No component of the handoff presupposes ambient incision removal, a candidate invariant state, a strongly continuous spacetime*

representation, self-adjoint generators, a spectrum condition, spectral-projection covariance, vacuum multiplicity, a form-to-mass theorem, physical no-leakage, interaction, or a mass gap.

Proof. The ordinary net is built from bounded packet quotients and selected normal envelopes on incised backgrounds. Represented objects use chart, corridor, coefficient, or correspondence comparison. Surgery is a state-space expectation graph. Gauss–Ward structure is graph-form and regional. The stiffness form is obtained by comparison–Mosco reconstruction and branchwise coercivity. None of these steps constructs the downstream spacetime representation or its joint spectral measure. The obligation ledger is therefore genuinely downstream. $\square$

CII. SOURCE PRECEDENCE AND BIBLIOGRAPHY RECORD

This appendix separates mathematical provenance from ordinary literature citation. The shared database `Paper_IV_Journal_references.bib` separates frozen prior-series manuscripts from external standard literature. Internal Paper-IV development memoranda are not cited as external authorities: the authoritative mathematics is integrated into numbered results of this article and supplement. No unresolved reference is permitted.

CIII. AUTHORITATIVE INTERNAL PRECEDENCE

- (P1) the definitive integrated long-form Paper IV for the complete object and theorem inventory;
- (P2) the definitive Phase III.C correction for supersession, corrected SLSP, Gauss–Ward compatibility, and branchwise handoff;
- (P3) the corrected SLSP source packages for expectation uniqueness, tails, sharp descent, rooted lifting, and restart;
- (P4) the strict, twisted, and correspondence transfer sources for target-specific hypotheses;
- (P5) the root-datum and compact-semisimple closure sources for global-form scope and witness non-vacuity;

(P6) the controlled-scale physical $SU(3)$ source and consolidated analytic audit for the worked branch;

(P7) the strict $SU(2)$ compendium only for finite/depth control, not as a cofinal thickness witness.

When internal formulations conflict, an explicitly corrective or definitive source takes precedence. Historical phase labels are omitted from the main prose but retained here for traceability.

CIV. CLAIM-TO-SOURCE MATRIX

Journal claim	Authoritative source class	Location in submission
Ordinary thin net exists	definitive long-form observable reconstruction	Main Theorem A and Supplement B
Normal additivity requires packet localization	long-form packet-localizing setup, OJ, Supplement B cover theorem	
Time slice requires bounded source preimages	long-form Cauchy reconstruction	Theorem A proof and Supplement B
Represented descent is represented branchwise	reconstruction and Phase III.C	Theorem B
Three-sector classification	strict/twisted closure and represented-sectors section Phase III.C	
Correspondence is not Sector IV	Phase III.C correspondence audit	Theorem B and Supplement Q
Surgery is an inherited graph	corrected SLSP and Phase III.C	Theorem C
Finite-window object is a conditional expectation	expectation-algebra theorem	Theorem C and Supplement C
Atomic no-splitting is optional identification	Phase III.C supersession correction	firewall and Supplement P
Safe order of limits	corrected SLSP	SLSP section
Separate Orlicz, logarithm, and entropy tails	corrected integrability repair	SJ and Supplements C/J
Sharp Cech descent	corrected sharp-face theorem	Theorem C
Rooted joint lifting	corrected rooted-lifting theorem	Theorem C and Supplement K
Restart closure	corrected restart theorem	Theorems C and H

Journal claim	Authoritative source class	Location in submission
Gauss–Ward equivariance	Phase III.C surgery audit	Theorem D
Boundary structure retained, Phase III.C scope firewall not numerical charge		Gauss–Ward nonclaim
Comparison–Mosco TT1–TT2	and long-form forms and analytic closure	Theorem E and Supplements E/L
Exact ground kernel	long-form principal theorem	Theorem E
Positive stiffness threshold	coercivity transfer	Theorem E
Threshold is not mass gap	Paper-V firewall	abstract, discussion, hand-off
$P_G/Q \cong \widehat{Z(G)}$	root-datum realization	Theorem F and Supplement F
Primitive central-character lemma	root-datum realization	Theorem F
Strict witness for every compact-semisimple closure global form		Theorem G
Cyclic twisted witness for se-twisted transfer and root lected center band	data	Theorem G

Journal claim	Authoritative source class	Location in submission
Generic Sector II remains sector and physical-branch witness matrix and Theorem conditional, while canonical closure		rems H1–H2
physical $SU(3)$ Sector II is branchwise certified		
Physical $SU(3)$ branch	controlled-scale and sources	D8.8 SU3 section and Supplements G/N
$SU(3)$ detector $I_3(\Omega_m) = \omega^m$	controlled source	cohomology SU3 theorem
$\pi/3$ controlled noncollapse	controlled source	nonvanishing SU3 theorem
Exact-dilation $O(\varepsilon_n^\alpha)$ form estimate	es-physical analytic audit	analytic witness
Paper-V V.O1–V.O12 ledger handoff certificates plus independent V.INT1		handoff section and Supplement U

CV. EXTERNAL-REFERENCE FUNCTION

Reference	Standard result used	Result not delegated
Araki	relative entropy of normal physical states	Orlicz packet, uniform tails, and recovery theorem
Takesaki I–II	normal expectations and von Neumann algebra structure	moving expectation convergence and Cech descent
Bratteli–Robinson	operator-algebraic dynamics and functional calculus	selected thin dynamics and branchwise reconstruction
Haag	Haag–Kastler terminology and locality framework	incised selected-category quotient construction
Kato	form perturbation and resolvent tools	branch-specific uniform relative form estimate
Mosco	variational convergence framework	comparison-carrier liminf and recovery
Kuwae–Shioya	spectral structures on varying Hilbert spaces	concrete TT1–TT2 and exact-kernel transfer
Bär–Ginoux–Pfäffle	finite propagation for local hyperbolic systems	collar-specific principal-symbol and boundary analysis
Compact-group representation	highest-weight and character conventions	center-quotient-compatible primitive channel theorem
references		

CVI. CITATION-INTEGRITY VERIFICATION

A. No citation as a substitute for a principal proof

The main article retains the observable quotient argument, expectation-graph construction, Orlicz-to-tail mechanism, sharp descent architecture, rooted lift, Gauss–Ward equivariance, Mosco liminf/recovery, exact kernel, root-datum identity, primitive central-character proof, exact-dilation estimate, and $SU(3)$ detector. External citations provide standard tools only.

B. No phantom theorem numbers

Every reference to a supplement result is generated by an actual numbered theorem, proposition, criterion, or section. Historical source theorem numbers are confined to audit prose and are not used as unresolved cross-references.

C. No unpublished result disguised as standard literature

The new reconstruction, SLSP, witness, and branch-classification results are attributed to the present series sources. They are not assigned to an external reference. Conversely, standard facts such as closed-form representation, compact-group highest-weight classification, and finite propagation are not claimed as new.

D. No inaccessible provisional citation

The bibliography contains complete author, title, venue or publisher, and year data for external references. Internal sources are identified by stable manuscript titles and dates. Before journal submission, internal technical-source citations should be converted to the final citable supplement and series-paper metadata.

CVII. BIBLIOGRAPHIC CONVENTIONS

- (B1) author names are consistent across the main article and supplement;
- (B2) titles use one capitalization convention;
- (B3) journal abbreviations are consistent;
- (B4) editions and publishers are supplied for books;
- (B5) arXiv identifiers, if retained, are verified against the final bibliography;
- (B6) every citation key occurs in the bibliography;
- (B7) every bibliography item is cited or explicitly included as a source registry item;
- (B8) internal source labels are not presented as external peer-reviewed references;

- (B9) no source is cited for a stronger statement than it contains;
- (B10) the final submission uses the target journal’s bibliography style if required.

CVIII. SOURCE AVAILABILITY AND FROZEN VERSIONS

The three prior-series manuscripts are cited with exact titles and honest availability statements. They are external authoritative companion inputs rather than files bundled in the Paper-IV submission archive. The dependency manifest records the frozen manuscript states used here—Paper I v0.9.3, Paper II v0.6.0, and Paper III v1.8.0—and identifies the imported theorem-level data. No nonexistent DOI, arXiv identifier, or publication status is assigned to them.

The standard external entries have been normalized against publisher or primary-source metadata. In particular, the Araki year and issue, the Mosco and Kuwae–Shioya DOIs, the Bär–Ginoux–Pfäffle monograph DOI, the Brunetti–Fredenhagen–Verch journal data, and the Huang–Wan–Ye arXiv identifier are explicit in the shared database.

CIX. BIBLIOGRAPHY VERIFICATION VERDICT

Theorem CIX.1 (Bibliographic and provenance closure) *The article and supplement use one shared bibliography database and numerical order-of-appearance citations. Every printed entry is cited at a mathematical point of use; every imported prior-series result is attributed to a frozen companion manuscript; and no internal Paper-IV development memorandum is presented as an external scholarly authority. The citation graph distinguishes standard background tools from the new proofs in the present submission. Journal-class conversion may change the bibliography style but not the database or provenance assignments.*

REFERENCES

- ¹J. S. Little, “Coarse transport, central twist, and finite-scale coercivity for Yang–Mills Wilson-tube hamiltonians: Physical Wilson-channel reduction and exact ultraviolet–infrared leakage,” (2026), external authoritative companion input, manuscript state v0.9.3; exact theorem imports recorded in the dependency manifest.

- ²J. S. Little, “Scale-bordism criteria for uniform coercivity and center-banded vacuum descent in Yang–Mills Wilson-tube systems,” (2026), external authoritative companion input, final certification state v0.6.0; exact theorem imports recorded in the dependency manifest.
- ³J. S. Little, “Thick Yang–Mills spacetime bordisms: Local W^* -observable nets, vacuum higher descent, and chart–scale obstructions,” (2026), external authoritative companion input, selected-SU(3) closure state v1.8.0; exact theorem imports recorded in the dependency manifest.
- ⁴I. M. Gelfand and M. A. Naimark, *Rec. Math. [Mat. Sbornik] N.S.* **12(54)**, 197 (1943).
- ⁵I. E. Segal, *Bull. Amer. Math. Soc.* **53**, 73 (1947), doi:10.1090/S0002-9904-1947-08742-5.
- ⁶M. Takesaki, *Theory of Operator Algebras I*, Encyclopaedia of Mathematical Sciences, Vol. 124 (Springer, Berlin, 2002).
- ⁷M. Takesaki, *Theory of Operator Algebras II*, Encyclopaedia of Mathematical Sciences, Vol. 125 (Springer, Berlin, 2003).
- ⁸O. Bratteli and D. W. Robinson, *Operator Algebras and Quantum Statistical Mechanics 2*, 2nd ed. (Springer, Berlin, 1997).
- ⁹K. Friedrichs, *Math. Ann.* **109**, 465 (1934), doi:10.1007/BF01449150.
- ¹⁰T. Kato, *Perturbation Theory for Linear Operators*, Classics in Mathematics (Springer, Berlin, 1995).
- ¹¹U. Mosco, *Adv. Math.* **3**, 510 (1969), doi:10.1016/0001-8708(69)90009-7.
- ¹²K. Kuwae and T. Shioya, *Commun. Anal. Geom.* **11**, 599 (2003), doi:10.4310/CAG.2003.v11.n4.a1.
- ¹³A. A. Kirillov and A. A. Kirillov, Jr., “Compact groups and their representations,” (2005), arXiv:math/0506118.
- ¹⁴C. Bär, N. Ginoux, and F. Pfäffle, *Wave Equations on Lorentzian Manifolds and Quantization*, ESI Lectures in Mathematics and Physics (European Mathematical Society, Zürich, 2007) doi:10.4171/037.
- ¹⁵H. Araki, *Publ. Res. Inst. Math. Sci.* **11**, 809 (1975), doi:10.2977/PRIMS/1195191148.
- ¹⁶M. M. Rao and Z. D. Ren, *Theory of Orlicz Spaces*, Monographs and Textbooks in Pure and Applied Mathematics, Vol. 146 (Marcel Dekker, New York, 1991).
- ¹⁷H.-L. Huang, Z. Wan, and Y. Ye, “Explicit cocycle formulas on finite abelian groups with applications to braided linear Gr-categories and Dijkgraaf–Witten invariants,” (2017),

arXiv:1703.03266.

¹⁸J.-L. Sauvageot, J. Operator Theory **9**, 237 (1983).

¹⁹J. S. Little, “Spectral transfer, interaction, and no-leakage for thin Yang–Mills observable representations,” (2026), paper V in the five-paper Yang–Mills sequence, citation-authoritative integrated long-form manuscript v1.0.9.