

Entropy-Surgered Admissible Thick-to-Thin Reconstruction of the Yang–Mills Observable System: Compact-Semisimple Branchwise Reconstruction and Analytical Non-Vacuity

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Starting from a positive-thickness Yang–Mills Wilson-tube system on a controlled incised background, we prove a sector-preserving zero-thickness reconstruction on selected background–region pairs with positive incision clearance. The ordinary observable layer becomes a normal Haag–Kastler functor with incised isotony, causality, additivity, controlled-background covariance, causal dynamics, and time slice. The represented layer remains strict, scale-obstructed, line-twisted, or correspondence-valued according to its retained coherence data; no global physical Hilbert descent is assumed. Thin stiffness surgery is reconstructed as the inherited Čech-descended graph limit of positive-thickness conditional expectations, with Orlicz, reference-logarithm, and entropy estimates providing sharp descent, rooted lifting, and restart. Gauss–Ward and collar data pass through the same finite-diagram limit. Comparison–Mosco convergence yields a closed thin form with exact complete-ground kernel and a uniform positive off-ground threshold. Exact-dilation Friedrichs branches verify analytical non-vacuity for compact connected semisimple global forms, and the canonical physical $SU(3)$ family provides the explicit branchwise realization. A certified interface theorem discharges the primitive Paper-IV packages from the Paper-III pass-through certificates. The result constructs the thin observable and form-theoretic system but does not prove the Poincaré mass gap; spectral covariance, no-leakage, interaction, and invariant-mass transfer remain for Paper V.

Keywords: Yang–Mills; algebraic quantum field theory; thick-to-thin reconstruction; coefficient gerbes; quadratic forms; Gauss–Ward identities

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I. INTRODUCTION

A. The reconstruction problem

A positive-thickness Wilson-tube system contains more information than a family of local algebras indexed by a real parameter. The local-net and functorial-covariance language follows the Haag–Kastler and locally covariant frameworks, while the complete positive-thickness incised input is supplied by Paper III.^{1–4} The thickness parameter changes the local carrier, the Friedrichs domain, the collar geometry, the regional comparison maps, the selected spectral expectations, and the coefficient objects that record strict or twisted composition. The zero-thickness problem is therefore not a pointwise limit of fields and is not solved by writing down a formal operator at $\varepsilon = 0$. One must reconstruct, simultaneously and compatibly,

- (i) the bounded ordinary observable net;
- (ii) the sector-appropriate represented object;
- (iii) the state-space expectation graph used by entropy surgery;
- (iv) the Gauss–Ward, collar-charge, current-defect, and boundary-character data;
- (v) a closed thin form with the correct complete-ground kernel; and
- (vi) the coherence invariant that distinguishes strict, scale-obstructed, and fiber-twisted branches.

The principal difficulty is that the ordinary observable shadow may descend strictly even when the represented system does not. Observable products and normal-state restrictions can be single valued while the coefficient-line or correspondence lift retains a nontrivial associator. Accordingly,

$$\boxed{\text{global ordinary observable descent} \not\Rightarrow \text{one global physical represented Hilbert space.}} \quad (1)$$

This distinction is structural rather than cosmetic. It is the reason the article separates the ordinary Haag–Kastler theorem from the represented branch theorem.

B. Why entropy surgery is an expectation graph

At positive thickness, stiffness surgery is defined on normal states through source-selected conditional expectations. The correct thin object must preserve that provenance. At a finite good spectral window N , the limiting object is a normal conditional expectation

$$\mathbb{E}_{G,\xi;D}^{(N),\text{as}}$$

with fixed-point algebra

$$\mathcal{F}_{G,\xi;D}^{(N),\text{as}} = \text{Ran } \mathbb{E}_{G,\xi;D}^{(N),\text{as}}.$$

The full surgery is the graph limit

$$\boxed{\mathcal{S}_{G,\xi,0;D}^{\text{st,as},*} = \text{graph-lim}_{n \rightarrow \infty} \mathfrak{T}_{0n;D}^{G,\xi,*} \mathcal{S}_{G,\xi,n;D}^{\text{st},*} \mathfrak{T}_{n0;D}^{G,\xi,*}} \quad (2)$$

Complete atomic pinching of a bare thin operator may identify this graph under an eventual no-splitting criterion, but it is not the definition and is not needed for existence. This correction removes a hidden dependence on globally labelled thin eigenprojections and makes the construction compatible with chartwise or correspondence-valued represented descent.

C. Why a constructive witness is necessary

A reconstruction theorem with a long list of compactness and entropy assumptions can be formally correct yet mathematically uninformative if no branch verifies the assumptions. The article therefore proves non-vacuity inside the main theorem package. For every compact connected semisimple global form

$$G = \tilde{G}/\Gamma, \quad \tilde{G} = \prod_i G_i^{\text{sc}}, \quad \Gamma \leq Z(\tilde{G}),$$

we construct an exact-dilation Friedrichs comparison branch. Using the standard highest-weight and center-character conventions for compact connected groups,⁵ the quotient-compatible weight lattice

$$P_G = \{\lambda \in P : \chi_\lambda|_\Gamma = 1\}$$

satisfies

$$P_G/Q \cong \widehat{Z(G)}.$$

The identity coefficient system yields a strict Sector-I witness for every declared global form. A selected element $h \in Z(G)$ of order $n > 1$ yields a quotient-compatible irreducible Wilson channel on which h acts by a primitive n th root of unity and hence gives the two oriented cyclic Sector-III branches. The canonical physical $SU(3)$ family supplies the first fully worked twisted realization and, branch specifically, realizes all three represented sectors. Its fixed-fiber detector and quantitative controlled-scale detector are logically independent.

D. Main results

The article proves eight principal results.

Theorem A. A normal locally covariant Haag–Kastler observable net is reconstructed on the selected zero-thickness category.

Theorem B. The represented object descends in the strict, scale-obstructed, fiber-twisted, or correspondence target selected by its coherence class.

Theorem C. Corrected SLSP constructs the inherited expectation graph, proves sharp Čech descent, and provides one rooted joint lift for every finite diagram.

Theorem D. The thin Gauss–Ward and collar package is reconstructed and is compatible with inherited surgery.

Theorem E. The comparison–Mosco limit is closed, has exact complete-ground kernel, and inherits a positive off-ground threshold.

Theorem F. Root data and global-form descent produce strict and scalar twisted branches uniformly over compact connected semisimple groups.

Theorem G. A certified interface theorem translates the Paper-III pass-through certificates clause by clause into the primitive Paper-IV packages for the canonical physical branch. Separately, exact-dilation witnesses verify analytical non-vacuity for the wider abstract reconstruction class; the complete incised $SU(3)$ family gives the explicit physical specialization and branchwise detector table.

Theorem H. The complete branchwise object and an exact Paper-V handoff certificate are assembled without importing a spectrum or mass-gap conclusion.

E. Proof architecture

Reader’s roadmap. Paper IV performs one task in several typed layers: it receives a finite positive-thickness system from Paper III and reconstructs its zero-thickness observable, represented, state-surgery, Gauss–Ward, and closed-form shadows. The layers are coupled, but they are not interchangeable. In particular, strict descent of bounded observables does not force strict descent of represented coherence, and a positive thin stiffness threshold is not yet an invariant-mass statement. Figure 1 records the ownership boundary that governs the rest of the paper.

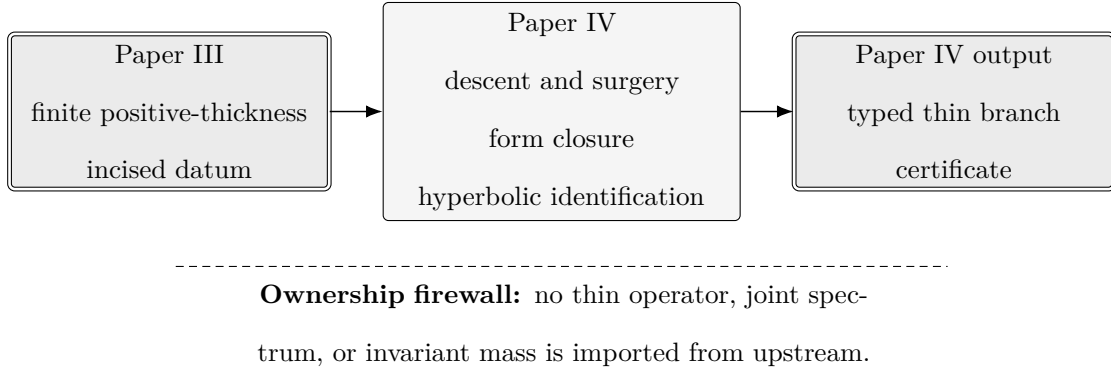


FIG. 1. Paper-IV ownership. The positive-thickness datum is inherited; every thin object is constructed in the present paper.

The logical order is

$$\begin{aligned}
 &\text{positive-thickness input} \implies \text{comparison and locality} \implies \text{expectation algebra,} \\
 &\quad \text{expectation algebra} \implies \text{Orlicz tails} \implies \text{sharp Cech descent,} \\
 &\quad \text{sharp Cech descent} \implies \text{rooted lifting} \implies \text{thin reconstruction.}
 \end{aligned} \tag{3}$$

The order of limits is equally important:

$$\begin{aligned}
 &\text{finite diagram and buffer fixed} \rightarrow \text{good window fixed} \\
 &\quad \rightarrow n \rightarrow \infty \rightarrow N \rightarrow \infty \rightarrow d \downarrow 0.
 \end{aligned} \tag{4}$$

The good window and smooth buffer are proof regulators. They are removed only after the thickness limit, using uniform mass, reference-logarithm, and entropy tails.

F. Series position and Paper-V boundary

Only the positive-thickness input is inherited from the preceding articles: finite-scale coercive and channel data from Paper I, branchwise scale-bordism and center-band selection from Paper II, and the complete incised positive-thickness spacetime system from Paper III.^{4,6,7} The imported Paper-III endpoint is its pass-through theorem III.PT (Theorem 24.26 in the frozen companion version), together with the positive-thickness certificates enumerated in Theorem IV.1. That theorem records the exact logical bridge into **OJ**, **RJ**, **SJ**, **GJ**, and **M** for the canonical physical branch. The present article performs the zero-thickness reconstruction. It does not prove an invariant vacuum state, a strongly continuous Poincare representation, self-adjoint translations, the forward-cone spectrum condition, physical zero-mode no-leakage, an interaction theorem, or a Lorentz-invariant mass gap. Those are independent obligations of Paper V.⁸ The present theorem applies unconditionally to the certified Paper-III physical branch through Theorem IV.1, and conditionally to any additional branch that independently verifies the five primitive packages. Noncoercive, nonadmissible, or uncertified branches are not silently included. In particular,

$$\boxed{\text{spec } K_0^{\text{thin}} \subseteq \{0\} \cup [\kappa_{\text{IV,thin}}, \infty) \text{ is not the Poincare mass-gap theorem.}} \quad (5)$$

II. CONTROLLED INCISED GEOMETRY AND ALGEBRAIC SETUP

A. Controlled incised backgrounds

A zero-thickness background is a controlled incised datum

$$\mathbb{M}_{0,\text{inc}} = (M, g, \mathfrak{o}, \mathfrak{t}, \Sigma, \mathfrak{C}_\Sigma, \mathfrak{U}_\Sigma, h, \mathfrak{R}^{\text{int}}, \mathfrak{R}^\partial, \mathfrak{C}^{\text{Cau}}). \quad (6)$$

The background-category and time-slice terminology is adapted from locally covariant QFT and standard globally hyperbolic propagation theory, with the incision and collar data added as part of the present selected category.^{3,4,9,10} Here $(M, g, \mathfrak{o}, \mathfrak{t})$ is an oriented and time-oriented globally hyperbolic ambient spacetime, $\Sigma \subset M$ is the declared controlled incision complex, $\mathfrak{C}_\Sigma$ is its selected collar system, $\mathfrak{U}_\Sigma$ is an adapted chart system, and h is a complete control metric used to measure distance from the incision. The final three entries specify the selected interior regions, collar-adapted regions, and admissible Cauchy data. The physical spacetime

on which the present reconstruction is performed is

$$M_{\text{inc}}(\mathbb{M}_{0,\text{inc}}) := M \setminus \Sigma.$$

The incision is part of the background datum. It is not filled, removed, or moved inside one fixed observable fiber.

An admissible interior object is a pair

$$X = (\mathbb{M}_{0,\text{inc}}, O), \quad O \in \mathfrak{R}^{\text{int}}(\mathbb{M}_{0,\text{inc}}),$$

with

$$O \subseteq M_{\text{inc}}(\mathbb{M}_{0,\text{inc}}), \quad \text{clr}_{\Sigma,h}(O) := \inf_{x \in O} d_h(x, \Sigma) > 0, \quad (7)$$

such that O is causally convex in a selected incision-adapted globally hyperbolic neighborhood and admits a positive thickening radius for which every selected thickening remains disjoint from Σ . Collar-adapted objects belong to the separate category $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm},\partial}$ and retain their collar orientation and quotient data. They are not silently included in the interior Einstein-causality theorem.

The category $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ has these controlled background–region pairs as objects. An arrow

$$\psi : (\mathbb{M}_{0,\text{inc}}, O) \longrightarrow (\mathbb{M}'_{0,\text{inc}}, O')$$

is induced by an admissible background embedding preserving the metric, space and time orientations, the incision stratification, the selected collar orientation, adapted charts, causal structure, testing covers, and Cauchy data, with $\psi(O) \subseteq O'$. If a spacetime transformation moves Σ , it is an arrow between two controlled backgrounds, not an automorphism of a single fixed fiber. The full Poincare group therefore need not act on one fixed incised background.

For two objects in one background, the notation

$$O_1 \perp_U O_2$$

means that O_1 and O_2 are spacelike separated inside one common incision-adapted admissible ambient region $U \subseteq M_{\text{inc}}(\mathbb{M}_{0,\text{inc}})$ with positive causal and incision margins. All locality statements below use this relative notion. Likewise, a Cauchy arrow is admissible only when it is a Cauchy embedding inside a selected incision-adapted globally hyperbolic subspacetime; no time-slice argument propagates through or across Σ .

B. Cofinal thickness corridors over the incision

Fix

$$\varepsilon_n = 2^{-n}\varepsilon_0, \quad n \in \mathbb{N}_0,$$

or a declared cofinal corridor equivalent to this dyadic model on every finite controlled test diagram. A corridor is attached to the same incised background datum and consists of incision-preserving positive-thickness thickenings, collars, support carriers, and comparison maps. Common refinement is required only among corridors carrying the same controlled incision data or related by a declared admissible background arrow; it does not assert independence from the incision itself.

For each object $X = (\mathbb{M}_{0,\text{inc}}, O)$ and each n , the positive-thickness branch supplies a bounded local observable algebra $\mathcal{A}_{G,\xi,n}^W(X)$, a selected normal-state packet, regional comparison maps, a represented carrier, complete-ground data, a local selected form, and a positive-thickness stiffness expectation. The thickness comparison is not one global unitary. It is a coherent family

$$\mathfrak{T}_{mn;D}^{G,\xi}$$

attached to finite regional, Čech, Cauchy, and background diagrams D in $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$. The maps are uniformly bounded on the selected packet norms and preserve the saturated common null ideals used in the observable quotient.

C. Finite Wilson-packet core and observable quotient

For $X = (\mathbb{M}_{0,\text{inc}}, O)$ let $\mathcal{W}_n(X)$ be the finite linear span of bounded gauge-invariant analytic Wilson packets whose selected carriers lie in O and remain disjoint from Σ throughout the admitted thickness tail. A thin testing section is a family

$$a = (a_n)_{n \geq n_0}, \quad a_n \in \mathcal{W}_n(X),$$

with uniformly bounded packet norm and vanishing comparison defects on every selected finite common refinement. The null space $\mathcal{N}_0(X)$ is required to be common to all admissibly comparable choices and saturated under left multiplication, right multiplication, and involution. The quotient and completion define $\mathcal{B}_0(X)$. Let $\mathfrak{S}_0(X)$ be the reconstructed separating

family of compatible thin states and let

$$\pi_X^{\text{sel}} := \bigoplus_{\rho \in \mathfrak{S}_0(X)} \pi_\rho, \quad H_X^{\text{sel}} := \bigoplus_{\rho \in \mathfrak{S}_0(X)} H_\rho.$$

Its selected normal envelope is

$$\mathcal{A}_{G,\xi,0}^W(X) := \pi_X^{\text{sel}}(\mathcal{B}_0(X))'' \cong z_X \mathcal{B}_0(X)^{**}, \quad (8)$$

where z_X is the central support of the representation π_X^{sel} , equivalently the complement of the central kernel projection of its normal extension to $\mathcal{B}_0(X)^{**}$. It is not the unproved join of ordinary state-support projections. The regional state-extension theorem makes $\pi_Y^{\text{sel}} \circ \alpha_\psi^0$ quasi-equivalent to π_X^{sel} and therefore supplies the normal unital arrow between the selected envelopes.

A packet-localizing cover $\{X_i \rightarrow X\}$ is incision compatible when every generator in the finite Wilson-packet core of X admits a finite subdivision into allowed packets carried by the X_i , with recombination that never fills or crosses Σ . Directed isotony alone does not imply this additivity statement.

D. Represented branch datum

A compact-semisimple branch datum over $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ is

$$\mathfrak{D}_{G,\xi} = \left(\mathbb{M}_{0,\text{inc}}, G = \tilde{G}/\Gamma, P_G, A_\xi, \Lambda_\xi, \mathfrak{R}_{G,\xi}, \kappa_\xi^{\text{fib}}, \hat{\kappa}_\xi^{\text{cs}} \right). \quad (9)$$

Here $A_\xi \leq Z(G)$ is the selected finite coefficient band, $\Lambda_\xi \subset P_G^+$ is a finite Wilson-channel frame, and $\mathfrak{R}_{G,\xi}$ is the incision-preserving regional exact-dilation comparison branch. The fixed-thickness coefficient class is κ_ξ^{fib} ; the total controlled chart-scale obstruction is $\hat{\kappa}_\xi^{\text{cs}}$. In the non-scalar case the coefficient objects are ground-pointed normal W^* -correspondences and the retained invariant is $\kappa_\xi^{\text{Corr}} \in \pi_0 \text{Assoc}_{\text{Corr}}$. All coefficient lines, correspondences, tensorators, mixed cells, and detectors are indexed over the same incision-preserving chart-regional-scale system.

E. Complete-ground data

At each positive thickness and controlled regional object the represented branch carries an abstract complete-ground pointing or pointed subobject $\hat{\Omega}_{G,\xi,n;X}^b$. Whenever the selected

component is realized on a Hilbert form carrier, the same ground face is represented by an actual orthogonal projection $P_{\Omega,G,\xi,n;X}$. The symbol $P_{\Omega,G,\xi,n;X}$ is not used for a line object or correspondence subobject. Complete-ground transport of $\hat{\Omega}^b$ is part of the represented branch datum; transport of P_{Ω} is the additional Hilbert-carrier statement used in TT1, form nullity, and exact-kernel recovery. Both are preserved by the selected comparisons in their proper target. Neither yields one global represented Hilbert space without an independent effective-descent theorem.

F. State surgery as a normal-state graph

For a normal state ρ in the selected positive-thickness packet,

$$\mathcal{S}_{G,\xi,n;D}^{\text{st},*}(\rho) = \mathbb{E}_{G,\xi,n;D,*}(\rho),$$

possibly after restriction to a good finite window. The star records that surgery acts on normal states. It is an idempotent state map at positive thickness and gives, after limiting reconstruction, a graph on a declared normal-state domain. It is not a linear operator on Hilbert vectors. The limiting graph is constructed in [section VII](#); its existence is not included in the primitive hypotheses below.

III. PRIMITIVE JOURNAL HYPOTHESES

The reconstruction is organized by five input families. They are stated entirely at positive thickness or as quantitative comparison data. In particular, none of the following assumes sharp surgery descent, simultaneous rooted lifting, restart closure, Mosco convergence, TT1–TT2, the exact thin kernel, or the thin coercive estimate.

A. Ordinary observable inputs OJ1–OJ8

Criterion III.1 (Primitive incised observable package) *For every finite controlled diagram in $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ the positive-thickness observable shadow satisfies:*

- (OJ1) Controlled incised geometry. *Every object is a controlled background–region pair $X = (\mathbb{M}_{0,\text{inc}}, O)$ satisfying (7); every arrow preserves the incision stratification, collar*

orientation, causal and time orientations, adapted charts, packet-localizing covers, and selected Cauchy data. Every finite locality test admits a common incision-adapted globally hyperbolic ambient region with positive causal and incision margins.

- (OJ2) *Elementwise bounded observable comparison and saturated null ideals. Independently of the represented target, the branch supplies strict elementwise maps on the bounded Wilson-packet cores. These maps are uniformly bounded, asymptotically multiplicative and involutive, coherent on finite common refinements, and preserve one common closed left-, right-, and involution-saturated packet-null ideal.*
- (OJ3) *Norming limit states and normal regional extension. Positive thick state packets are finite-packet tight and near norming. Their compatible limits separate and norm the thin quotient. For every selected arrow $\psi : X \rightarrow Y$, pullback maps the selected target state family onto the selected source state family and restriction maps selected target states into the selected source family. Thus the selected direct-sum representations are quasi-equivalent under restriction and no normal summand is lost.*
- (OJ4) *Incised support calculus and locality margins. Compact packet carriers, subdivisions, and recombinations remain in $M_{\text{inc}}(\mathbb{M}_{0,\text{inc}})$. For $O_1 \perp_U O_2$, positive incision clearance and positive relative causal margin imply either exact thick commutation on a cofinal tail or a commutator defect tending to zero in the saturated ambient packet seminorm. Finite propagation is used only for carrier and Cauchy control, not as a substitute for microcausality.*
- (OJ5) *Packet-localizing additivity. Every declared cover is incision compatible and carries uniformly bounded finite subdivision and physical recombination maps whose defect is packet null. The selected representations of the cover images jointly saturate the target selected representation. Arbitrary-cover additivity is not assumed.*
- (OJ6) *Incision-adapted Cauchy generation. For each selected Cauchy arrow inside an incision-adapted globally hyperbolic development, finite target packets admit uniformly bounded, comparison-coherent source sections with vanishing reconstruction defect. The construction is compatible with regional restriction, controlled-background arrows, alternative selected Cauchy frames, and the selected state systems in both directions.*
- (OJ7) *Controlled-background covariance and causal coherence. Strict incision-preserving*

background maps act elementwise on the bounded observable cores; comparison-square and composition defects are packet null. Selected local Cauchy developments form a coherent groupoid after quotienting. A map moving Σ is an arrow between controlled backgrounds, never an automatic fixed-fiber automorphism.

- (OJ8) Complete comparable-choice control. *Declared cofinal corridors, packet systems, and extraction choices carrying the same controlled incision datum admit finite common refinements, preserve the same saturated null ideals and selected state annihilators, and induce mutually inverse maps only after passage to the common reconstructed quotient objects. No physical thickness arrow is inverted, and no comparison with a filled, removed, or noncomparable incision is asserted.*

These assumptions concern only the strict bounded observable shadow. They do not use represented Hilbert descent, entropy surgery, Gauss–Ward reconstruction, or the thin closed-form theorem.

B. Represented and coherence inputs RJ1–RJ8

Criterion III.2 (Primitive represented branch package) *Over the same controlled incised category the branch supplies:*

- (RJ1) Incised represented pseudodiagram. *Selected local-normal represented carriers and every chart, regional, scale, Cauchy, background, and mixed-cell arrow are indexed over one incision-preserving category $\mathfrak{C}_{G,\xi,\text{inc}}^{\text{rep}} \rightarrow \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$.*
- (RJ2) Selected supports and represented ground objects. *Central supports, selected sectors, and abstract complete-ground pointings or pointed subobjects $\hat{\Omega}_{n;X}^b$ are transported in the represented target, exactly or with a declared quantitative comparison defect on every selected arrow. Actual projections $P_{\Omega,n;X}$ are invoked only after a Hilbert form carrier has been selected.*
- (RJ3) Declared coefficient target. *The branch carries either scalar coefficient lines with tensorators and mixed cells, or self-dual normal correspondences with normalized Markov pointings and pointed two-cell transport. The strict observable and normal-state shadows are elementwise defined independently of the represented target.*

- (RJ4) Branch-specific detector package. *The branch declares either a discrete Hausdorff detector invariant under admissible rephasing and neutral stabilization, a quantitative obstruction with a uniform positive $\liminf$, one compactly controlled global strictifying primitive, or a correspondence component detector with proved closed separated target components.*
- (RJ5) Sector-I primitive. *A prospective Sector-I branch supplies one positive-thickness physical two-cochain on the complete incised chart–regional–scale system, compatible with all faces, mixed cells, support, ground, expectation, smooth-channel, Gauss–Ward prepared, collar, and stabilization data. Compatibility with the inherited thin graph is a conclusion.*
- (RJ6) Physically evaluated scalar obstruction. *A prospective Sector-II or Sector-III branch supplies the evaluated controlled total class or scalar fiber associator after selected-sector, central-support, and complete-ground compression, together with its positive-thickness comparison law.*
- (RJ7) Correspondence target and obstruction separation. *Scalar reduction is used only after a coherent one-dimensional central reduction on all incised objects, arrows, fusion cells, and associators. Correspondence-valued typing and nontriviality of $\kappa_{\xi}^{\text{Corr}}$ are distinct; the latter requires a separate physical input.*
- (RJ8) Common refinement and neutral stabilization. *Finite common refinements, admissible rephasings, and neutral stabilizations preserve the selected pointing, strict shadows, coefficient target, and declared detector.*

Abstract cocycle vanishing is not a substitute for a coherent physical strictification. Finite-face trivializations do not imply one global strictifying cochain, and local represented data do not imply one global physical Hilbert space.

C. Expectation, reference, and smooth-seam inputs SJ1–SJ7

Criterion III.3 (Primitive state and entropy package) *For every finite controlled diagram and every fixed buffer the branch supplies:*

- (SJ1) *source-defined positive-thickness finite-window spectral conditional expectations pre-*

serving the selected corner, sector, central support, and complete-ground face;

- (SJ2) *a branch-specific fixed-window expectation-germ certificate: exact transported stationarity, a summable one-step defect, or an explicit Cauchy/uniqueness estimate on a fixed finite carrier;*
- (SJ3) *coherent good-window compression and zero-extension identities, rather than merely inclusion of expectation ranges;*
- (SJ4) *faithful transported reference densities, a superlinear Young function Φ , uniform physical Orlicz moments, an independent reference growth/counting profile sufficient for logarithm and discarded-entropy tails, and spatial collar tightness;*
- (SJ5) *finite smooth Kraus-channel comparison across buffered Cech and regional arrows, uniformly on joint endpoint Orlicz packets;*
- (SJ6) *an exact unsurgered positive-thickness rooted state-shadow diagram with strict regional, Cauchy, background, and scale arrows and with the declared coefficient tensorators or correspondence compositors;*
- (SJ7) *packet stability under the positive-thickness expectations and a separately declared admissible fiber-thickness environment for the later restart theorem.*

These inputs do not assume the limiting expectation graph, sharp descent, a simultaneous surgically rooted lift, or restart closure. Relative entropy control alone is not used to infer reference-logarithm integrability.

D. Gauss–Ward and collar inputs GJ1–GJ8

Criterion III.4 (Primitive graph-domain constraint package) *For every controlled regional object and finite compatible diagram the branch supplies:*

- (GJ1) *positive-thickness Gauss graph forms and interior insertion derivations on transported graph-dense cores, with closability and gauge equivariance;*
- (GJ2) *the exact positive-thickness graph-form Ward identity on the prepared core, including the Gauss-form terms and observable insertion term;*

- (GJ3) *finite-window expectation equivariance with the interior derivations on the common core;*
- (GJ4) *prepared-state domains stable under the positive-thickness expectation and under the declared comparison maps;*
- (GJ5) *a collar gauge-parameter quotient and compatible positive-thickness collar-charge graph forms;*
- (GJ6) *separately typed comparison data for the two-parameter current-algebra defect $\mathfrak{a}^\partial$, the boundary-character class $[\chi^\partial]_{\text{cs}}$, and the first boundary-Ward defect $\mathfrak{b}^\partial$;*
- (GJ7) *complete-ground data lying in the positive-thickness interior Gauss kernels, with compatible regional and collar transport;*
- (GJ8) *exact unsurgered finite-diagram lifts satisfying the prepared-state, collar, current, and boundary-Ward constraints before surgery is applied.*

No anomaly cancellation, numerical boundary-charge expectation identity, or global self-adjoint Gauss generator is assumed.

E. Closed-form inputs M1–M6

Criterion III.5 (Primitive comparison–form package) *For each bounded controlled regional object there are closed nonnegative renormalized forms $\hat{q}_n$ on carriers H_n such that:*

- (M1) *Comparison carriers and domains. Uniformly bounded comparison maps $J_n : H_n \rightarrow H_0$ and reconstruction maps $I_n : H_0 \rightarrow H_n$ are asymptotically inverse on graph-bounded sets; their form domains have a common domain or graph-dense common images. Exact unitary straightening is allowed as a witness specialization.*
- (M2) *Local sesquilinear form control. On the common form core the full sesquilinear defect tends to zero uniformly in the graph norm, with compatible regional restrictions and lower semiboundedness.*
- (M3) *Comparison weak compactness. Graph-bounded families admit comparison-weakly convergent subfamilies and the comparison-Mazur or synchronized approximation mechanism required for a weak $\liminf$ proof.*

(M4) Primitive recovery data. *Core recovery, graph-density extension estimates, inclusion-compatible synchronization, and a monotone regional-exhaustion diagonal are supplied; the full TT2 statement is not assumed.*

(M5) Complete-ground form structure. *On each selected Hilbert form carrier, the orthogonal ground projection $P_{\Omega,n;X}$ preserves the form domain, is form null, is form orthogonal to the excitation part, and satisfies the moving graph-domain comparison estimates needed later for TT1. Abstract pointings and correspondence subobjects enter through RJ2 and are not substituted into projection formulas.*

(M6) Uniform off-ground coercivity. *One branchwise constant $\kappa_{\text{IV},\text{thin}} > 0$ satisfies*

$$\hat{q}_n[u] \geq \kappa_{\text{IV},\text{thin}} \|(\mathbf{1} - P_{\Omega,n})u\|^2$$

on a cofinal tail, with the corresponding banded formulation when no global represented Hilbert descent is available.

Mosco convergence, TT1, TT2, closedness of the relaxed thin form, the exact thin kernel, and thin coercivity are conclusions to be proved from M1–M6.

Remark III.6 (Umbrella notation and noncircularity) *For compact theorem statements we write*

$$\mathbf{OJ} := \text{OJ1–OJ8}, \quad \mathbf{RJ} := \text{RJ1–RJ8}, \quad \mathbf{SJ} := \text{SJ1–SJ7},$$

$$\mathbf{GJ} := \text{GJ1–GJ8}, \quad \mathbf{M} := \text{M1–M6}.$$

The reconstruction order is

$$\mathbf{OJ} \Rightarrow \text{ordinary incised net},$$

$$\mathbf{SJ} \Rightarrow \text{expectation germ and sharp state descent},$$

$$\mathbf{M} \Rightarrow \text{Mosco/TT/kernel/coercivity}.$$

*with represented and Gauss–Ward conclusions added from **RJ** and **GJ** only after the relevant state and form theorems have been established. The detailed source crosswalk appears in the technical supplement.*

IV. PAPER-III-TO-PAPER-IV INTERFACE THEOREM

Reader’s roadmap. The reconstruction begins only after the incoming datum has been typed. Paper III supplies a controlled incised site, a finite-Čech-compatible positive-thickness

system, branch-appropriate represented descent data, Gauss–Ward and collar information, source-defined entropy surgery, and uniform coercive control. The interface theorem records that package without importing any thin operator or spectral conclusion. Everything constructed below must be derived from this intake.

The abstract reconstruction theorems below are stated for any branch satisfying **OJ**, **RJ**, **SJ**, **GJ**, and **M**. For the canonical physical branch inherited from Paper III, however, these packages are not five new untracked assumptions. They are the clausewise unpacking of the Paper-III pass-through datum together with its named positive-thickness certificates.

Theorem IV.1 (Certified Paper-III-to-Paper-IV bridge) *Let $\mathfrak{D}_{\text{YM},\mathfrak{d}}^{\text{thick,fin}}$ be an admissible finite positive-thickness realization exported by the Paper-III pass-through theorem III.PT, with the same controlled incision datum, selected regional and Cauchy category, complete-ground packet, represented coefficient data, source-selected expectation packet, Gauss–Ward/collar packet, and closed positive-thickness comparison-form packet. Assume the accompanying Paper-III certificates assert, on every selected finite controlled diagram, the following five source blocks:*

- (III–IV1) *incised locality, packet comparison, selected-state extension, packet localization, controlled-background covariance, Cauchy generation, and comparable-choice control;*
- (III–IV2) *represented support and complete-ground transport, coefficient-line or correspondence typing, mixed-cell coherence, branch detector data, and neutral stabilization;*
- (III–IV3) *source-defined finite-window expectations, faithful transported references, the declared Orlicz/reference-growth/collar controls, smooth buffered comparison, rooted unsurgered state transport, and restart-environment stability;*
- (III–IV4) *positive-thickness Gauss graph forms, the exact prepared-core Ward identity, expectation equivariance, prepared-domain stability, collar quotient and charge forms, current and boundary-character data, complete-ground Gauss nullity, and unsurgered constrained lifts;*
- (III–IV5) *comparison and reconstruction carriers, common-core sesquilinear control, graph-bounded compactness and synchronization data, primitive recovery data, exact complete-ground form structure, and a uniform positive off-ground lower bound.*

Then the exported branch satisfies, with no additional physical input,

$$\mathbf{OJ}, \quad \mathbf{RJ}, \quad \mathbf{SJ}, \quad \mathbf{GJ}, \quad \mathbf{M}.$$

Consequently Theorems A–E apply to that branch. The limiting expectation graph, sharp Cech descent, simultaneous rooted surgical lift, restart closure, Mosco convergence, TT1–TT2, exact thin kernel, and thin coercive estimate are not imported from Paper III; they remain conclusions proved in Paper IV.

Proof. The assertion is a typed clause-by-clause translation. Source block (III–IV1) discharges OJ1–OJ8; source block (III–IV2) discharges RJ1–RJ8; source block (III–IV3) discharges SJ1–SJ7; source block (III–IV4) discharges GJ1–GJ8; and source block (III–IV5) discharges M1–M6. The translation preserves the incision, selected-region and selected-Cauchy quantifiers, represented target, complete-ground type, comparison direction, and constant ownership. None of the thin conclusions listed in the final sentence occurs among the source blocks, so the bridge does not assume any Paper-IV output. The complete source-location and theorem-to-clause ledger is retained in the technical supplement and dependency manifests. $\square$

What has now been established. The upstream and downstream responsibilities are now separated. Positive-thickness geometry and compatibility cells are certified inputs; the thin observable net, represented descent, inherited expectation graph, closed form, and physical hyperbolic realization remain Paper-IV obligations. This firewall prevents the reconstruction from assuming the objects it is meant to construct.

Scope IV.2 (General branches versus the certified physical branch) *For an arbitrary compact-semisimple comparison branch, $\mathbf{OJ}$, $\mathbf{RJ}$, $\mathbf{SJ}$, $\mathbf{GJ}$, and $\mathbf{M}$ remain explicit Paper-IV hypotheses and may be verified by the exact-dilation witnesses of Section XI. For the canonical physical $SU(3)$ branch, Theorem IV.1 and the branch certificate of Section XII discharge those hypotheses from the Paper-III source package. A model witness is therefore not silently identified with the physical branch.*

V. ORDINARY THIN OBSERVABLE RECONSTRUCTION

Reader's roadmap. The first limit is taken at the ordinary observable level. Regional algebras, overlap maps, and higher compatibility cells are carried through the prescribed limits as one finite diagram. Only after compatibility survives on the entire finite nerve is the ordinary thin net extracted. The crossed-out route in Figure 2 is the forbidden alternative: taking isolated chartwise limits and attempting to glue afterward.

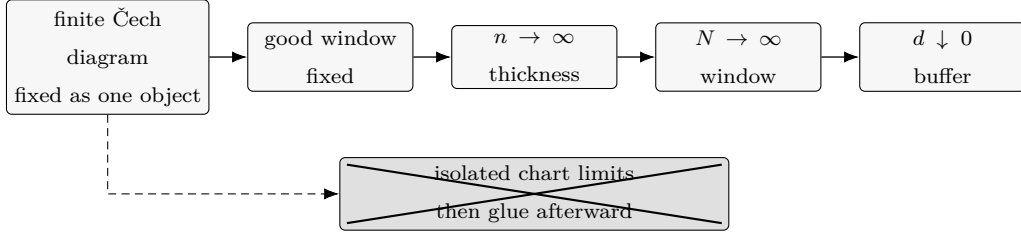


FIG. 2. Mandatory limit order. Objects, arrows, and higher cells pass through the limit together; chartwise limiting followed by later gluing is not admissible.

Interpretive lead-in. All local approximants now belong to one compatible finite system, and the comparison maps satisfy the required locality, covariance, and Cauchy-generation conditions. The next theorem reconstructs the algebraic thin shadow. Its conclusion is deliberately representation-free: represented branch data are retained for the following section rather than being forced into a single global Hilbert-space model.

Theorem V.1 (Incised thick-to-thin observable reconstruction (Theorem A))

*Assume the primitive incised observable package **OJ**. The terms isotony, causality, additivity, covariance, and time slice are used in the operator-algebraic and locally covariant senses, but all proofs below are carried out on the selected incised category rather than delegated to the standard frameworks.^{1–4} Then the bounded Wilson-packet comparison system defines a strict covariant functor*

$$\mathcal{A}_{G,\xi,0}^W : \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}} \longrightarrow W^*\mathbf{Alg}_{\mathbf{n}}$$

with the following properties.

(HK0) *For each $X = (\mathbb{M}_{0,\text{inc}}, O)$, $\mathcal{A}_{G,\xi,0}^W(X)$ is the selected faithful normal envelope of the thin testing algebra $\mathcal{B}_0(X)$.*

- (HK1) *Every admissible arrow is sent to a normal unital injective $*$ -homomorphism, and composition is strict.*
- (HK2) *If $O_1 \perp_U O_2$ in one common incision-adapted admissible ambient region U , then the two images commute in $\mathcal{A}_{G,\xi,0}^W(\mathbb{M}_{0,\text{inc}}, U)$.*
- (HK3) *Every declared incision-compatible packet-localizing cover normally generates the target algebra.*
- (HK4) *Admissible incision-preserving background embeddings act covariantly; a map moving Σ is an arrow between controlled backgrounds, not an automorphism of one fixed fiber.*
- (HK5) *Every selected incision-adapted Cauchy arrow is sent to a normal $*$ -isomorphism.*
- (HK6) *The induced local Cauchy evolution is path and frame independent on the selected incised Cauchy groupoid.*
- (HK7) *The functor is ultraweakly generated by finite bounded gauge-invariant Wilson packets. It is independent, up to a canonical normal natural isomorphism, of admissibly comparable corridors and extraction systems carrying the same controlled incision datum. It makes no assertion that Σ can be removed, filled, or changed without a declared admissible background arrow.*

Proof. Write $X = (\mathbb{M}_{0,\text{inc}}, O)$.

Testing quotient. By OJ2 the bounded comparison sections form a $*$ -algebra and the common packet-null space is a closed left-, right-, and involution-saturated ideal. Thus multiplication descends in the saturated packet seminorm; no operator-norm convergence of representatives is inferred. The quotient completion is the thin testing algebra $\mathcal{B}_0(X)$; the detailed estimate is Technical Supplement Theorem R5.A.

Selected normal envelope and isotony. OJ3 reconstructs a separating norming state family $\mathfrak{S}_0(X)$. Its direct-sum GNS representation π_X^{sel} is faithful^{11,12}. For an arrow $\psi : X \rightarrow Y$, the selected restriction and extension theorem makes $\pi_Y^{\text{sel}} \circ \alpha_\psi^0$ quasi-equivalent to π_X^{sel} . Therefore the thin C^* arrow extends uniquely to a normal unital injective map

$$\bar{\alpha}_\psi^0 : \mathcal{A}_{G,\xi,0}^W(X) \longrightarrow \mathcal{A}_{G,\xi,0}^W(Y).$$

Finite-diagram comparison coherence makes these extensions strictly functorial. This is Technical Supplement Theorem R5.B.

Incised Einstein causality. Let $X_i = (\mathbb{M}_{0,\text{inc}}, O_i)$ and assume $O_1 \perp_U O_2$. OJ1 and OJ4 give compact carriers with positive relative causal margin and positive clearance from Σ . The positive-thickness locality theorem then gives exact commutation on a cofinal tail or a commutator defect tending to zero in the saturated ambient packet seminorm. Hence the thin testing images commute in $\mathcal{B}_0(\mathbb{M}_{0,\text{inc}}, U)$. Their selected von Neumann closures commute because commutants are strong-operator closed. The conclusion is independent of the chosen common admissible ambient region by normal isotony and complete comparable-choice control. Finite propagation is used only to control carriers and Cauchy evolution; it is not substituted for microcausality.

Packet-localizing normal additivity. OJ5 subdivides every finite Wilson packet into finitely many allowed subpackets and recombines them without crossing or filling Σ , with packet-null defect. This gives C^* generation. The selected-representation saturation clause identifies the source normal closures with the corresponding subalgebras of the target selected representation, hence

$$\mathcal{A}_{G,\xi,0}^W(X) = W^* \left(\bigcup_i \overline{\alpha}_{X_i \rightarrow X}^0 \left(\mathcal{A}_{G,\xi,0}^W(X_i) \right) \right).$$

No arbitrary-cover additivity is asserted.

Controlled covariance, time slice, and causal dynamics. OJ7 gives strict elementwise background maps on the bounded observable shadow; packet-null comparison-square and composition defects vanish in the quotient. OJ6 supplies uniformly bounded comparison-coherent source sections for every selected Cauchy arrow inside an incision-adapted globally hyperbolic development. The induced testing-algebra map is therefore surjective as well as injective, and the selected normal extension is a normal isomorphism. The resulting relative Cauchy maps compose strictly and are independent of the selected admissible Cauchy frame. No causal development propagates through the omitted locus.

Comparable-choice independence. OJ8 supplies finite common refinements whose two composites differ from the identity by common-null sections and whose state pullbacks identify the selected state families. The testing algebras are canonically $*$ -isomorphic and the selected representations are quasi-equivalent, so the isomorphism extends normally and naturally. The comparison preserves the same incision stratification, collar orientation, region class, packet core, and state annihilator. $\square$

Firewall V.2 (Observable reconstruction and the incision) *Theorem V.1 constructs*

an ordinary observable functor only on the selected controlled incised category. It does not construct a net on unincised Minkowski space, assign an algebra to Σ , prove arbitrary-cover additivity or Haag duality, produce one global physical Hilbert space, remove the incision, or yield a fixed-fiber Poincare action. In a twisted or correspondence-valued represented branch, OJ2 supplies an independent strict elementwise bounded observable shadow; correspondence transport is not applied directly to algebra elements.

VI. REPRESENTED DESCENT AND BRANCH CLASSIFICATION

Reader's roadmap. Once the ordinary thin net exists, the retained branch detector determines how it may be represented. Sector I descends strictly; Sector II remains controlled-scale obstructed; scalar Sector III retains coefficient-line twisting; and a genuinely non-scalar branch remains correspondence valued. The task is not to erase these distinctions but to preserve exactly the coherence type certified at positive thickness.

A. The admitted represented branch universe

Fix a declared branch (G, ξ) over the controlled incised category

$$\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}} = \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}, \quad M_{\text{inc}}(\mathbb{M}_{0,\text{inc}}) = M \setminus \Sigma.$$

The net-cohomological and relative-tensor-product literature supplies comparison language only; the branch classes and physical strictification criterion are proved here from the incised coefficient system.^{4,10,13} Its represented comparison datum is indexed by one incision-preserving chart-regional-scale category

$$\mathfrak{C}_{G,\xi,\text{inc}}^{\text{rep}} \longrightarrow \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}.$$

Objects carry selected local-normal represented carriers, central supports, complete-ground pointings, and either scalar coefficient lines or normal correspondences. Every chart refinement, regional arrow, thickness arrow, Cauchy arrow, background arrow, mixed cell, rephasing, and neutral stabilization preserves the same controlled incision, collar orientation, and selected support data.

The *admitted scalar-line universe* consists of branches for which the physically evaluated fiber class κ_ξ^{fib} , the controlled chart–scale class $\widehat{\kappa}_\xi^{\text{cs}}$, and the criterion equating simultaneous vanishing with one admissible physical strictification are all defined on the complete incised system. The trichotomy below is exhaustive only in that universe. A correspondence-valued target is a separate target extension and is not a fourth scalar sector.

B. Conceptual dichotomy and scalar trichotomy

At the coarsest level the represented branch is either coherently strictifiable or obstructed. The obstruction may live only in the total chart–scale system or already in each positive-thickness fiber:

$$\text{earlier strict} \longrightarrow \text{Sector I}, \quad \text{earlier obstructed} \longrightarrow \text{Sector II} \sqcup \text{Sector III}.$$

Sector type is a property of the complete branch datum (G, ξ) , not of the abstract group G alone.

Classification VI.1 (Sector I: coherently strictifiable) *A scalar line branch is Sector I when*

$$\kappa_\xi^{\text{fib}} = 0, \quad \widehat{\kappa}_\xi^{\text{cs}} = 0,$$

and there exists one admissible physical two-cochain strictification over the entire incised chart–regional–scale system. The strictification must preserve the selected sector, central support, complete-ground pointing, positive-thickness expectation tower, smooth comparison channels, mixed cells, Gauss–Ward prepared data, collar data, and neutral stabilization. Compatibility with the inherited thin surgery graph is a conclusion of Theorem VII.1, not part of the definition.

Classification VI.2 (Sector II: strict fibers and controlled scale obstruction) *A scalar line branch is Sector II when*

$$\kappa_\xi^{\text{fib}} = 0, \quad \widehat{\kappa}_\xi^{\text{cs}} \neq 0.$$

Each fixed-positive-thickness fiber is strict, but no admissible physical strictification exists over the complete controlled chart–scale system. The represented object remains corridor banded. Physical non-vacuity is branch specific.

Classification VI.3 (Sector III: twisted fibers) *A scalar line branch is Sector III when*

$$\kappa_{\xi}^{\text{fib}} = [\Omega_{\xi}^{\text{phys}}] \neq 0 \quad \text{in} \quad H^3(BA_{\xi}; U(1)).$$

The class is evaluated after the selected-sector, central-support, and complete-ground compression. It is already nonzero at fixed positive thickness. Any additional nonzero controlled chart-scale coordinate is retained as further branch data but does not change the Sector-III classification.

Remark VI.4 (Scalar sectors and structural roles) *The scalar trichotomy and the geometric three-object architecture are different classification axes. Sector I is the strict-bulk manifestation. Sector II is a scale-twisted bulk manifestation: every fixed fiber is strict, but controlled scale-natural strictification fails. Sector III is a fixed-fiber twisted bulk or coefficient-gerbe manifestation. The obstruction interface is a separate correspondence, bimodule, defect, or linking object carrying relative data between bulk realizations; it is not Sector III and is not a third ordinary bulk vacuum. In particular,*

$$\text{Sector III} \neq \text{obstruction interface}.$$

C. Correspondence-valued target and obstruction status

Classification VI.5 (Correspondence-valued extension) *When a coherent scalar line reduction has not been proved, the represented target is a pseudodiagram of normalized Markov-pointed normal W^* -correspondences. Its associator component*

$$\kappa_{\xi}^{\text{Corr}} \in \pi_0 \text{ASSOC}_{\text{Corr}}$$

is retained in the declared correspondence bicategory and is not forced into scalar group cohomology. Correspondence-valued typing alone does not imply $\kappa_{\xi}^{\text{Corr}} \neq [1]$. A branch may be correspondence valued yet pointed-pseudonaturally strictifiable. Nontrivial correspondence obstruction is claimed only when a separate physical detector proves

$$\kappa_{\xi}^{\text{Corr}} \neq [1].$$

This target extension is not called Sector IV.

D. Detector-specific persistence

Proposition VI.6 (Branch-specific no-jumping criterion) *Let a cofinal represented comparison family satisfy **RJ**. Sector or obstruction persistence holds under one of the following separately verified mechanisms:*

- (a) *a detector I with values in a discrete Hausdorff set, invariant under admissible rephasing and neutral stabilization and continuous along the selected comparison system, is eventually constant on the cofinal tail;*
- (b) *a quantitative obstruction Def_n satisfies $\liminf_n \text{Def}_n \geq \delta > 0$ and the comparison error tends to zero;*
- (c) *one normalized admissible strictifying cochain is defined on the complete cofinal system and is compact in the declared control topology, so every comparison limit carries the same global primitive;*
- (d) *a correspondence component detector with proved closed separated target components is supplied as part of the physical correspondence package.*

No universal finite-detector or compact-sector-locus principle is asserted.

Proof. In case (a), continuity into a discrete Hausdorff target and cofinal comparison coherence force eventual constancy; rephasing and neutral stabilization cannot alter the value. In case (b), a vanishing limiting obstruction would contradict the positive $\liminf$ after adding the vanishing comparison error. In case (c), the compact normalized control set supplies a convergent subnet of the single global cochain; closedness of the strictification, mixed-cell, support, ground, expectation, and smooth-channel equations gives an admissible limiting strictification. Case (d) is the same closed-component argument, but only after the topology and detector have been included in the branch data. Triviality on separately chosen finite faces is not used as a substitute for one coherent primitive. $\square$

Theorem VI.7 (Sector-appropriate represented descent (Theorem B)) *Assume **OJ** and **RJ** for a declared branch over $\mathfrak{C}_{G,\xi,\text{inc}}^{\text{rep}}$. Then the positive-thickness represented comparison system has a unique branch-typed thin pseudodiagram*

$$\text{Rep}_{G,\xi,0}^{\text{thin}}$$

on every finite controlled subdiagram and its cofinal compatible limit:

- (i) a strictly associative represented pseudofunctor over the selected incised system in Sector I;
- (ii) a corridor-banded strict-fiber represented system in Sector II;
- (iii) a chart-corridor scalar line-banded represented system in Sector III;
- (iv) a normalized Markov-pointed correspondence pseudodiagram when scalar reduction is unavailable.

The abstract complete-ground pointing or pointed subobject, selected sector, central support, coefficient target, and branch detector are retained under every admissible incision-preserving comparison. The three scalar sectors are pairwise disjoint and exhaustive only in the admitted scalar-line universe. The correspondence target and the correspondence obstruction status are recorded separately. The theorem does not assert effective descent to one global physical Hilbert space.

Proof. RJ1–RJ3 provide the incised represented pseudodiagram, selected supports and ground pointings, and the branch target. RJ5 gives one global positive-thickness physical strictification for a prospective Sector-I branch; RJ6 gives the physically evaluated scalar fiber or controlled class for prospective Sectors II and III. RJ4 and Proposition VI.6 give only the detector-specific persistence justified for the branch. RJ7 separates correspondence targeting, scalar reduction, and nontrivial correspondence obstruction. RJ8 supplies neutral stabilization and finite common-refinement coherence. Passing through the OJ quotient gives strict bounded observable and normal-state shadows, while the represented cells remain in their declared scalar-line or correspondence target. A map moving Σ is an arrow between controlled backgrounds and never identifies the two represented fibers as one fixed Hilbert-space representation. $\square$

What has now been established. The observable and represented outputs are now cleanly separated. Every branch has the same ordinary thin observable shadow, while its represented lift retains the strict, scale-obstructed, fiber-twisted, or correspondence-valued coherence appropriate to that branch. Figure 3 summarizes this common-shadow/different-lift structure.

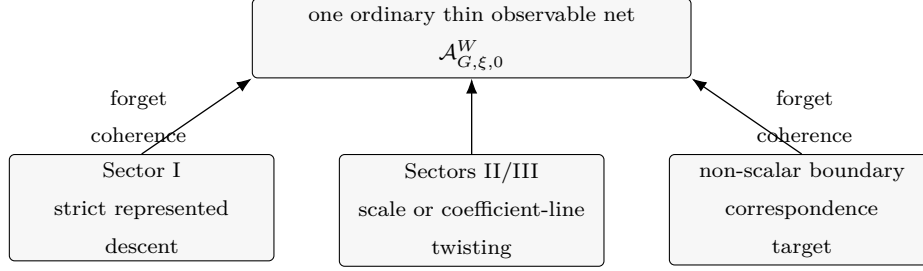


FIG. 3. A common ordinary observable shadow can support inequivalent represented coherence types. The reconstruction preserves rather than suppresses this branch information.

Firewall VI.8 (Strict represented descent is not global Hilbert descent) *A*

Sector-I branch is a coherently strict represented pseudofunctor over the selected incised regional-background system. It is not thereby one global physical Hilbert representation. Likewise,

$$\textit{strict} \neq \textit{free}, \quad \textit{twisted} \neq \textit{interacting}.$$

Both effective Hilbert descent and the interaction theorem require independent results.

VII. INHERITED ENTROPY SURGERY AND CORRECTED SLSP

Reader's roadmap. Entropy surgery is reconstructed as an inherited graph on normal states, not as a bare thin operator. The proof first constructs fixed-window expectation germs, then removes Orlicz, logarithmic, spectral, and collar regulators, and finally produces one rooted recovery family for the entire finite regional-chart-scale diagram. The simultaneous rooted lift is the point at which local convergence becomes descent-compatible reconstruction.

The positive-thickness surgery is the source-defined complete stiffness expectation. Standard relative-entropy, normal-expectation, and Orlicz-space results are used only as analytic tools; the expectation germ, uniform packet estimates, sharp descent, and rooted lift are proved in the present theorem chain.^{14–17} Smooth functional calculus enters only as a buffered comparison device. The thin object is therefore reconstructed as an inherited expectation graph, not as pinching by globally labelled spectral atoms of a bare thin operator.

A. Fixed-window expectation germs

Fix a bounded object D of the selected incised Čech–regional system and a good window N . On the selected corner let

$$\mathbb{E}_{G,\xi,n;D}^{(N)} : \mathcal{N}_{G,\xi,n}^{(N)}(D) \longrightarrow \mathcal{F}_{G,\xi,n;D}^{(N)}$$

be the compressed positive-thickness expectation. Transport the finite carrier, the faithful compressed reference, and the expectation to one fixed finite-dimensional carrier. The primitive certificate SJ2 is one of the following branchwise statements:

- (EG1) exact transported stationarity;
- (EG2) a summable one-step expectation defect;
- (EG3) an explicit Cauchy estimate in $1 \rightarrow 1$ and $\infty \rightarrow \infty$ norm, together with uniqueness of the reference-preserving limit.

It follows that there is one normal completely positive contractive idempotent

$$\mathbb{E}_{G,\xi;D}^{(N),\text{as}}$$

on the common finite carrier, with

$$\mathcal{F}_{G,\xi;D}^{(N),\text{as}} = \text{Ran } \mathbb{E}_{G,\xi;D}^{(N),\text{as}}.$$

For exact-dilation witnesses the convergence is equality after canonical transport. In a general comparison branch, convergence of the fixed algebras is a sufficient verification route only when the transported references and the reference-preserving expectations are controlled independently; it is not asserted to be automatically equivalent to operator convergence.

The exact compression and zero-extension identities in SJ3 imply a coherent good-window tower:

$$\mathbb{E}_{G,\xi;D}^{(N_1),\text{as}} \mathbb{E}_{G,\xi;D}^{(N_2),\text{as}} = \mathbb{E}_{G,\xi;D}^{(N_1),\text{as}}, \quad N_1 \leq N_2. \quad (10)$$

This identity is a conclusion of finite-stage window compatibility and expectation-germ uniqueness; it is not an additional limiting hypothesis.

B. Separate Orlicz, logarithm, and entropy control

Let $\sigma_{G,\xi,n;D}$ be the faithful reference state in the selected trace-class realization and put

$$L_{G,\xi,n;D}^{\text{ref}} = -\log \sigma_{G,\xi,n;D}.$$

For a superlinear Young function Φ , the selected packet satisfies

$$\sup_{n,\rho} \langle \rho, \Phi(L_{G,\xi,n;D}^{\text{ref}}) \rangle \leq M. \quad (11)$$

If $e_{n,R} = \mathbf{1}_{(R,\infty)}(L_{G,\xi,n;D}^{\text{ref}})$, then

$$\sup_{n,\rho} \rho(e_{n,R}) \leq \frac{M}{\Phi(R)} =: m_D(R) \longrightarrow 0, \quad (12)$$

$$\sup_{n,\rho} \langle \rho, L_{G,\xi,n;D}^{\text{ref}} e_{n,R} \rangle \leq M \sup_{t \geq R} \frac{t}{\Phi(t)} =: \ell_D(R) \longrightarrow 0. \quad (13)$$

These are independent mass and reference-logarithm estimates. Neither is inferred from a relative-entropy bound alone.

The reference profile in SJ4 also gives a uniform finite-rank bound for the low-reference subspace $p_{n,R} = 1 - e_{n,R}$ on each fixed finite diagram and a uniform first moment

$$C_D := \sup_{n,\rho} \langle \rho, L_{G,\xi,n;D}^{\text{ref}} \rangle < \infty.$$

For $m = \rho(e_{n,R})$ let

$$\rho_{n,R} = \frac{p_{n,R} \rho p_{n,R}}{1 - m}.$$

The entropy bound for the high block, the two-outcome pinching inequality, and positivity of relative entropy of $\rho_{n,R}$ against the normalized compressed reference give

$$|S(\rho) - S(\rho_{n,R})| \leq s_D(R), \quad (14)$$

where one may take

$$\begin{aligned} s_D(R) &:= h_2(m_D(R)) + (1 - m_D(R)) |\log(1 - m_D(R))| + \ell_D(R) \\ &\quad + m_D(R) |\log m_D(R)| + \frac{m_D(R)}{1 - m_D(R)} C_D \longrightarrow 0. \end{aligned} \quad (15)$$

Finite-dimensional continuity on $p_{n,R}$ and (14) then give one predual-norm modulus ω_D^{Ent} on the packet. The same truncation, with the independent logarithm tail retained, gives the corresponding reference-weighted and relative-entropy continuity statements.

A reference-preserving conditional expectation satisfies the data-processing inequality

$$D(\mathbb{E}^{(N),*} \rho \parallel \sigma) \leq D(\rho \parallel \sigma). \quad (16)$$

No monotonic decrease of ordinary von Neumann entropy is claimed; the theorem uses instead the uniform tail and recovery estimates above.

C. Sharp Čech-face descent

For a state-direction arrow $a : \sigma \rightarrow \tau$, choose at fixed (N, d) a finite smooth Kraus channel

$$\mathcal{K}_{n;D}^{(N,d),*}(\rho) = \sum_{j=1}^{J(N)} f_j^{(N,d)}(K_{n;D}^{\text{sel}}) \rho f_j^{(N,d)}(K_{n;D}^{\text{sel}}), \quad \sum_j (f_j^{(N,d)})^2 = \mathbf{1}.$$

The functions are band limited, agree with the selected spectral blocks away from the transition strip, and are subordinate to one fixed incision-compatible buffer. Smooth functional-calculus comparison and finite propagation give, on the joint endpoint packet,

$$\eta_a^{\text{sm}}(n; N, d) := \sup_{\rho} \left\| R_{n,a}^* \mathcal{K}_{n;\sigma}^{(N,d),*} \rho - \mathcal{K}_{n;\tau}^{(N,d),*} R_{n,a}^* \rho \right\|_1 \longrightarrow 0 \quad (17)$$

for fixed finite diagram, fixed buffer d , and fixed window N .

The sharp-to-smooth endpoint error is bounded by the transition-strip mass, the high-reference tail, and the spatial collar tail. Consequently

$$\Delta_a(n; N, d) \leq \eta_a^{\text{sm}}(n; N, d) + C_a \left(\sqrt{t_a(n; N, d)} + \sqrt{m_D(N)} + \sqrt{\kappa_D(d)} \right), \quad (18)$$

where the transition term vanishes after the thickness limit at fixed (N, d) . The legal order is

finite diagram and buffer fixed $\rightarrow$ good window fixed $\rightarrow n \rightarrow \infty \rightarrow N \rightarrow \infty \rightarrow d \downarrow 0$.

(19)

Thus

$$\sup_{\rho \in \mathfrak{K}_n^{\text{Orl}}(a)} \left\| R_{n,a}^* \mathcal{S}_{G,\xi,n;\sigma}^{\text{st},*} \rho - \mathcal{S}_{G,\xi,n;\tau}^{\text{st},*} R_{n,a}^* \rho \right\|_1 \longrightarrow 0. \quad (20)$$

Sharp descent is therefore obtained only after removal of the smooth and spectral regulators; it is not inferred from finite propagation alone.

D. One simultaneous rooted lift

Let $\mathcal{D} = (V, E, C_2, C_3; v_0)$ be a finite rooted selected regional-chart-scale diagram. By SJ6, one positive-thickness root recovery $\rho_{v_0, n}$ propagates through the strict unsurgered normal-state shadow:

$$\rho_{v, n}^{\text{pre}} = T_{p_v, n}^* \rho_{v_0, n},$$

where p_v is any chosen rooted path. Apply the source-defined sharp surgery objectwise,

$$\hat{\rho}_{v, n} = \mathcal{S}_{G, \xi, n; v}^{\text{st}, *} \rho_{v, n}^{\text{pre}}.$$

For an edge $e : v \rightarrow w$, strict unsurgered transport gives the exact identity $T_{e, n}^* \rho_{v, n}^{\text{pre}} = \rho_{w, n}^{\text{pre}}$, and hence

$$\|T_{e, n}^* \hat{\rho}_{v, n} - \hat{\rho}_{w, n}\|_1 \leq \Delta_e(n; N, d). \quad (21)$$

Every two-cell or higher strict-shadow defect is a finite telescoping sum of the edge defects. The coefficient-line lift is not strictified: its tensorators and associator are transported exactly in the represented layer.

Let $c(\mathcal{D})$ be the maximal number of edges occurring in the selected face decompositions. One master defect is

$$\mathfrak{D}_{\mathcal{D}}(n; N, d) := \max_{v \in V} d_{v, n}^{\text{root}} + c(\mathcal{D}) \max_{e \in E} \Delta_e(n; N, d) + \tau_{\mathcal{D}}^{\text{Orl}}(N, d), \quad (22)$$

where the first term is the common root recovery defect and the last collects the common finite-diagram tails. In the order (19),

$$\mathfrak{D}_{\mathcal{D}}(n; N, d) \longrightarrow 0.$$

This produces one family satisfying every vertex, edge, and face simultaneously. No product of approximate affine idempotents, alternating projection, averaging, or pairwise stitching is used.

Theorem VII.1 (Corrected SLSP and Markov theorem (Theorem C)) *Assume OJ, RJ, and SJ, and assume that the represented coefficient layer is strict or scalar line valued with a strict observable/normal-state shadow. Then, on every bounded object of $\mathbf{RegLoc}_{\text{inc}}^{0, \text{adm}}$ and every selected physical Orlicz packet:*

- (C1) *transported fixed-window expectations converge in $1 \rightarrow 1$ and $\infty \rightarrow \infty$ norm to a unique expectation germ and define a coherent good-window tower;*
- (C2) *compressed reference supports, densities, and logarithms converge on the common finite carriers;*
- (C3) *high-reference mass, reference-logarithm, and discarded-entropy tails vanish uniformly;*
- (C4) *every limiting selected state has a sector-locked relaxed-entropy recovery;*
- (C5) *the von Neumann and reference-weighted entropy functionals have common predual-continuity moduli on each packet; relative entropy is contractive under the reference-preserving surgery;*
- (C6) *sharp Čech-face and buffered regional surgery descent hold in predual norm after the limit order (19);*
- (C7) *every finite rooted regional-chart-scale-Cauchy-background diagram admits one simultaneous recovery family controlled by (22);*
- (C8) *selected central support, sector, complete-ground face, and scalar coefficient associator are retained;*
- (C9) *the inherited thin surgery is the single-valued normal-state graph*

$$\mathcal{S}_{G,\xi,0;D}^{\text{st,as},*} = \text{graph-lim}_{n \rightarrow \infty} \mathfrak{T}_{0n;D}^* \mathcal{S}_{G,\xi,n;D}^{\text{st},*} \mathfrak{T}_{n0;D}^*;$$

- (C10) *the graph is predual contractive, idempotent, and natural on the selected incised diagram category.*

The Gauss–Ward and collar augmentation is a separate conclusion of Theorem VIII.4 under GJ; it is not inserted into the present rooted theorem without that proof.

Proof. The fixed-window germ and tower follow from SJ1–SJ3. Equations (12)–(14) and the finite-rank reference profile give the separate tails and common moduli. The smooth Kraus estimate (17), endpoint removal, and the order (19) give sharp descent. Propagating one unsurgered root and applying surgery objectwise gives (21); finite telescoping gives the master defect and one joint diagonal for the entire diagram. Expectation-germ uniqueness and the coherent window tower give single-valuedness and idempotence of the graph. Sector,

central support, and complete-ground preservation are exact positive-thickness faces of the source expectation, while scalar coefficient data are external to the strict state shadow and are transported rather than altered. $\square$

What has now been established. The state-level reconstruction is now global on every selected finite diagram: one expectation graph descends sharply, one root controls all vertices and faces, and restart preserves the retained branch data. No pairwise stitching or post hoc averaging is required. This is the state-space mechanism that allows the later Gauss–Ward and form packages to pass through the same thin limit.

Corollary VII.2 (Retention of represented strictification and branch detectors)

*Under **RJ**, the inherited graph of Theorem VII.1 preserves the branch data supplied at positive thickness. In Sector I it intertwines the limiting physical strictification. In scalar Sectors II and III it preserves the declared discrete or quantitative detector. In a correspondence branch the same conclusion holds on the Markov shadow, while the represented associator is carried separately; nontriviality is retained only when a correspondence component detector is part of the branch package.*

Proof. The positive-thickness strictifying primitive and detectors commute with the source expectations and smooth comparison channels by RJ4–RJ8. Sharp descent and the single rooted master defect pass these identities to the inherited graph. Discrete and quantitative persistence are Proposition VI.6; the correspondence statement uses the correctly oriented Markov-shadow transfer proved in the technical supplement. $\square$

Corollary VII.3 (Sector-locked restart closure) *Assume in addition the admissible fiber-thickness preservation and recovery-category environment of SJ7. Then*

$$\text{Restart}_{G,\xi} : \mathbf{Rec}_{G,\xi}^{\text{adm}} \longrightarrow \mathbf{Rec}_{G,\xi}^{\text{adm}}$$

is an idempotent sector-preserving endomorphism. Objectwise restart is exactly tail independent as a cofinal transported germ; one-shot restart from a fixed coarse stage is exact only on exact surgery-intertwining arrows and is otherwise controlled by the corresponding long-arrow surgery defect.

Proof. The conditional restart theorem has the form

$$\text{admissible fiber-thickness preservation} + \text{corrected SLSP} \implies \text{closure under surgery}.$$

Theorem VII.1 supplies corrected SLSP, and SJ7 supplies the remaining antecedent. No restart hypothesis is added and no exact scale independence is inferred for a merely asymptotic arrow. $\square$

Firewall VII.4 (Atomic no-splitting) *Eventual atomic no-splitting is only an identification criterion under which the inherited graph may agree with pinching by a bare thin operator. It is not an existence hypothesis for Theorem VII.1.*

VIII. GAUSS–WARD, COLLAR, AND SURGERY COMPATIBILITY

A. Two graph structures and the Ward functional

Fix a controlled object $X = (\mathbb{M}_{0,\text{inc}}, D) \in \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ and a bounded packet of compactly supported interior gauge parameters. The constraint package contains two distinct graph structures:

- (i) a family of closed or closable sesquilinear Gauss forms

$$\mathbb{G}_{G,\xi,n;D}^{\text{int}}(\zeta)[\Psi, \Phi]$$

on a comparison-stable common vector-form domain $\mathfrak{D}_{G,\xi,n;D}^G$; and

- (ii) insertion derivations

$$\delta_{\zeta,n}^{\text{int}} : \mathfrak{C}_{G,\xi,n;D}^{\text{ins}} \longrightarrow \mathcal{N}_{G,\xi,n;D}$$

on a comparison-stable observable core.

The interior Gauss kernel is the joint form-annihilator

$$\mathcal{K}_{G,\xi,n;D}^{\text{int}} := \left\{ \Psi \in \mathfrak{D}_{G,\xi,n;D}^G : \mathbb{G}_{G,\xi,n;D}^{\text{int}}(\zeta)[\Psi, \Phi] = 0 \ \forall \Phi, \zeta \right\}.$$

The complete-ground input is the independent graph-form statement

$$\text{Ran } P_{\Omega,G,\xi,n;D} \subseteq \mathcal{K}_{G,\xi,n;D}^{\text{int}};$$

it is not inferred from gauge invariance of the projection.

For $A \in \mathfrak{C}_{G,\xi,n;D}^{\text{ins}}$ the full interior Ward functional is

$$\mathfrak{w}_{G,\xi,n;D}^{\text{int}}(\zeta; A)[\Psi, \Phi] := i \mathbb{G}_{G,\xi,n;D}^{\text{int}}(\zeta)[\pi_n(A)\Psi, \Phi]$$

$$\begin{aligned}
& -i \mathbb{G}_{G,\xi,n;D}^{\text{int}}(\zeta)[\Psi, \pi_n(A^*)\Phi] \\
& - \left\langle \pi_n(\delta_{\zeta,n}^{\text{int}} A) \Psi, \Phi \right\rangle.
\end{aligned} \tag{23}$$

Prepared vector pairs and their graph-projective normal-functionals are those on which (23) vanishes for every allowed test. The reduced relation $\rho(\delta_\zeta A) = 0$ is only a corollary on an invariant Gauss-null face.

Theorem VIII.1 (Limiting Gauss graph and Ward functional (Theorem D1))

*Assume **OJ**, **RJ**, and GJ1–GJ4. Then the transported Gauss graphs have a closed thin limit*

$$\mathbb{G}_{G,\xi,0;D}^{\text{int}}(\zeta)$$

on a common limiting form domain, the insertion derivations are closable on $\mathfrak{C}_{G,\xi,0;D}^{\text{ins}}$, and the full Ward functional obtained from (23) passes to the thin limit. Moreover,

$$\text{Ran } P_{\Omega,G,\xi,0;D} \subseteq \mathcal{K}_{G,\xi,0;D}^{\text{int}}.$$

All statements are branchwise and chart–corridor banded when no global represented Hilbert descent exists.

B. Fixed-window equivariance

Strong gauge covariance of the selected finite-window projections, invariance of both graph domains, and differentiation of the group intertwining identity give

$$\mathbb{E}_{n;D}^{(N)} \delta_{\zeta,n}^{\text{int}} = \delta_{\zeta,n}^{\text{int}} \mathbb{E}_{n;D}^{(N)}$$

on the positive-thickness insertion core. The same projections preserve the Gauss form domain and its left/right implementation terms.

Theorem VIII.2 (Fixed-window thin equivariance (Theorem D2)) *For every fixed good window N ,*

$$\boxed{\mathbb{E}_{G,\xi;D}^{(N),\text{as}} \delta_{\zeta,0}^{\text{int}} = \delta_{\zeta,0}^{\text{int}} \mathbb{E}_{G,\xi;D}^{(N),\text{as}}} \tag{24}$$

on $\mathfrak{C}_{G,\xi,0;D}^{\text{ins}}$. On the graph-projective prepared domain,

$$\mathfrak{w}_{\mathbb{E}^{(N),\text{as}},*,\rho,0}^{\text{int}}(\zeta; A) = \mathfrak{w}_{\rho,0}^{\text{int}}(\zeta; \mathbb{E}^{(N),\text{as}} A). \tag{25}$$

Consequently each fixed-window predual expectation preserves the prepared-state domain.

C. Collar quotient and boundary structure

Let $\mathfrak{G}_D^{\text{adm}}$ be the comparison-topological space of collar-admissible parameters and let $\mathfrak{G}_D^{\text{int}}$ be the closed subspace of interior parameters. The Hausdorff collar quotient is

$$\mathcal{Q}_{G,\xi,0}^\partial(D) = \mathfrak{G}_D^{\text{adm}} / \overline{\mathfrak{G}_D^{\text{int}}}^{\text{cmp}}.$$

Regional, chart, and scale maps preserve the interior subspaces and therefore induce quotient maps. The boundary package keeps four separately typed objects:

$$\left(\mathcal{Q}_{G,\xi,0}^\partial, \mathfrak{a}_{G,\xi,0}^\partial(\lambda, \mu), [\chi_{G,\xi,0}^\partial]_{\text{cs}}, \mathfrak{b}_{G,\xi,0}^\partial(\lambda; A) \right).$$

Here $\mathfrak{a}^\partial$ is the two-parameter current-algebra consistency defect, $[\chi^\partial]_{\text{cs}}$ is the complete chart–scale–mixed boundary-character class modulo allowed zero-point cochains, and $\mathfrak{b}^\partial$ is the first boundary-Ward defect. None is identified with a one-parameter affine response.

Theorem VIII.3 (Augmented rooted Gauss–Ward/collar lift (Theorem D3))

Assume Theorem VII.1 and GJ5–GJ8. For every finite mixed regional, chart, scale, Cauchy, and collar diagram there is one rooted family that simultaneously satisfies the thin state-shadow equations, the graph-form Ward identities, collar-quotient naturality, boundary-character cocycle relations, and transport of $\mathfrak{a}^\partial$ and $\mathfrak{b}^\partial$. The proof is obtained by adding these finitely many graph and collar errors to the SLSP master defect; no constraint projection, finite label register, or pairwise stitching is used.

Theorem VIII.4 (Gauss–Ward and collar reconstruction (Theorem D)) *Assume OJ, RJ, SJ, and GJ. Then the full inherited state-surgery graph satisfies*

$$\mathcal{S}_{G,\xi,0;D}^{\text{st,as,*}} \left(\mathfrak{D}_{G,\xi,0}^{\text{GW-prep}}(D) \right) \subseteq \mathfrak{D}_{G,\xi,0}^{\text{GW-prep}}(D),$$

and, for every prepared normal state ρ ,

$$\boxed{\mathfrak{w}_{\mathcal{S}_{G,\xi,0;D}^{\text{st,as,*}}}^{\text{int}}(\zeta; A) = 0.} \tag{26}$$

It preserves the complete-ground Gauss-null inclusion and transports, up to canonical branch-wise comparison, the structural tuple

$$\left(\mathcal{Q}_{G,\xi,0}^\partial, \mathfrak{a}_{G,\xi,0}^\partial, [\chi_{G,\xi,0}^\partial]_{\text{cs}}, \mathfrak{b}_{G,\xi,0}^\partial \right).$$

Proof. Theorem VIII.1 gives the limiting graphs and the full Ward functional; Theorem VIII.2 gives fixed-window intertwining. Theorem VIII.3 supplies one simultaneous rooted family with all graph and collar equations. On every bounded Ward test packet the difference between the full surgery and the N -window expectation has a uniform graph/Orlicz tail modulus tending to zero. Graph-projective closedness then removes the window and the buffer in the mandatory order

$$\text{finite diagram and buffer fixed} \rightarrow N \text{ fixed} \rightarrow n \rightarrow \infty \rightarrow N \rightarrow \infty \rightarrow d \downarrow 0.$$

Splitting independence follows from cancellation of both Gauss-form implementation terms and the insertion term in the full Ward functional. The collar quotient and the three boundary invariants are carried by the same augmented master defect. $\square$

Firewall VIII.5 (Boundary, charge, and anomaly claims) *Structural preservation does not imply numerical equality of boundary-charge expectations after surgery. It does not imply $\mathfrak{a}^\partial = 0$ or $\mathfrak{b}^\partial = 0$ and therefore does not prove anomaly cancellation. Coherent strictification of the represented vacuum system does not by itself strictify boundary charges or produce a global self-adjoint charge family. In twisted branches the unconditional result remains graph-form and chart-corridor banded; no global self-adjoint Gauss generator is inferred.*

IX. THIN CLOSED FORMS AND THE EXACT COMPLETE-GROUND KERNEL

Reader's roadmap. The algebraic thin net and inherited expectation graph do not yet provide the positive operator needed downstream. This section passes the positive-thickness forms through strong relaxation and comparison–Mosco convergence, transfers the coercive floor, and proves the complete-ground kernel exactly. The order is essential: the form is closed before its representing operator is interpreted, and kernel exactness is proved rather than inferred from a formal limit of ground vectors.

A. Varying carriers and the strong relaxed form

For each controlled bounded object $X = (\mathbb{M}_{0,\text{inc}}, O) \in \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$, let

$$J_{n;O} : H_{n;O} \rightarrow H_{0;O}, \quad I_{n;O} : H_{0;O} \rightarrow H_{n;O}$$

be the comparison and reconstruction maps supplied by M1. The proof uses standard closed-form perturbation and Mosco-convergence ideas, including their varying-Hilbert-space formulation, but establishes the comparison-carrier TT1–TT2 and exact-kernel statements directly.^{18–20} They are uniformly bounded and asymptotically inverse on graph-bounded sets. In the exact-dilation specialization they are mutual unitaries, but no inverse of $J_{n;O}$ is used in the general theorem.

A family $u_n \in H_{n;O}$ converges comparison strongly to $u \in H_{0;O}$ when $J_{n;O}u_n \rightarrow u$ and its reconstruction defect tends to zero; comparison weak convergence is defined by testing against reconstructed fixed vectors. Define the strong relaxed functional

$$q_{0;O}^{\text{rel}}[u] := \inf \left\{ \liminf_{n \rightarrow \infty} \widehat{q}_{n;O}[u_n] : u_n \rightarrow u \text{ comparison strongly} \right\}. \quad (27)$$

Theorem IX.1 (Closed strong relaxation (Theorem E1)) *Under M1–M4, the functional (27) is a densely defined closed nonnegative quadratic form. It agrees with the closure of the direct limiting preform on the common comparison core.*

Proof. M2 gives a well-defined sesquilinear preform on the comparison core. M4 supplies synchronized recoveries for every finite family $u, v, u + v, u - v$. Applying the finite-stage parallelogram identity to one synchronized family and using the weak liminf for the reverse inequalities gives

$$q_{0;O}^{\text{rel}}[u + v] + q_{0;O}^{\text{rel}}[u - v] = 2q_{0;O}^{\text{rel}}[u] + 2q_{0;O}^{\text{rel}}[v].$$

Thus the relaxed functional is quadratic. Its sublevel sets are comparison-strongly closed by a diagonal selection, hence it is lower semicontinuous and therefore closed. Density follows from the graph-dense core recovery in M4. $\square$

B. Mosco liminf, TT2, and TT1

Theorem IX.2 (Comparison–Mosco reconstruction (Theorem E2)) *Under M1–M5, for every bounded controlled region:*

(i) if $u_n \rightharpoonup u$ comparison weakly and the graph energies are bounded, then

$$q_{0;O}^{\text{rel}}[u] \leq \liminf_n \hat{q}_{n;O}[u_n];$$

(ii) for every $u \in D(q_{0;O}^{\text{rel}})$ there is a comparison-strong TT2 recovery u_n satisfying

$$\hat{q}_{n;O}[u_n] \rightarrow q_{0;O}^{\text{rel}}[u];$$

(iii) graph-domain TT1 holds for moving families:

$$J_{n;O}P_{\Omega,n;O}u_n \rightarrow P_{\Omega,0;O}u, \quad J_{n;O}(\mathbf{1} - P_{\Omega,n;O})u_n \rightarrow (\mathbf{1} - P_{\Omega,0;O})u$$

for comparison-strong, graph-bounded families. The analogous comparison-weak statement holds for graph-bounded weak families.

Hence the regional forms converge in the comparison–Mosco sense.

Proof. M3 gives synchronized comparison-Mazur convexifications of every graph-bounded weak family; the definition of the strong relaxation then yields the weak liminf. M4 gives core recovery, graph-density extension, finite-diagram synchronization, and one monotone-exhaustion diagonal, proving TT2. For TT1, M5 controls

$$J_{n;O}P_{\Omega,n;O}u_n - P_{\Omega,0;O}J_{n;O}u_n$$

on graph-bounded moving families; adding the convergence of $J_{n;O}u_n$ proves the strong statement, and dual testing proves the weak statement. $\square$

C. Coercivity and the exact complete-ground kernel

Theorem IX.3 (Coercive transfer (Theorem E3)) *Under M1–M6,*

$$q_{0;O}^{\text{rel}}[u] \geq \kappa_{\text{IV,thin}} \|(\mathbf{1} - P_{\Omega,0;O})u\|^2. \quad (28)$$

Proof. Choose the TT2 recovery sequence from Theorem IX.2. Then

$$q_{0;O}^{\text{rel}}[u] = \lim_n \hat{q}_{n;O}[u_n] \geq \kappa_{\text{IV,thin}} \lim_n \|(\mathbf{1} - P_{\Omega,n;O})u_n\|^2.$$

Moving-family TT1 identifies the last limit with $\|(\mathbf{1} - P_{\Omega,0;O})u\|^2$. No lower bound is inferred from the Mosco-liminf inequality in the wrong direction. $\square$

Theorem IX.4 (Exact complete-ground kernel (Theorem E4)) *The limiting form satisfies*

$$\ker q_{0;O}^{\text{rel}} = \text{Ran } P_{\Omega,0;O}. \quad (29)$$

Proof. Equation (28) gives $\ker q_{0;O}^{\text{rel}} \subseteq \text{Ran } P_{\Omega,0;O}$. Conversely, M5 states that the finite-stage ground ranges preserve the form domains, are form-null, and are form-orthogonal to the excitation parts. For $u = P_{\Omega,0;O}u$, the ground recovery

$$g_n = P_{\Omega,n;O} I_{n;O} u$$

converges comparison strongly to u and has $\hat{q}_{n;O}[g_n] = 0$. The strong relaxation therefore gives $q_{0;O}^{\text{rel}}[u] = 0$. $\square$

Theorem IX.5 (Closed-form reconstruction (Theorem E)) *Assume $\mathbf{M}$ and the branch-appropriate complete-ground data. The positive-thickness forms reconstruct a unique compatible regional family of closed nonnegative forms*

$$\bar{q}_{G,\xi,0;O} := q_{0;O}^{\text{rel}}$$

satisfying comparison–Mosco convergence, graph-domain TT1–TT2, (28), and (29). Each regional pivot representation has an associated positive self-adjoint operator $K_{G,\xi,0;O}^{\text{thin}}$ with

$$E_{K_{G,\xi,0;O}^{\text{thin}}}(\{0\}) = P_{\Omega,G,\xi,0;O}, \quad \text{spec } K_{G,\xi,0;O}^{\text{thin}} \subseteq \{0\} \cup [\kappa_{\text{IV,thin}}, \infty).$$

Strong comparison-resolvent and heat-semigroup convergence follow regionally. Norm-resolvent convergence is a separate exact-carrier corollary under a uniform sesquilinear form-unit-ball defect.

Proof. Theorems IX.1–IX.4 give the regional result. M4 supplies compatibility under selected regional inclusions and exhaustion. A single global represented form/operator is assembled only in branches with effective Hilbert descent: Sector I after coherent strict descent, and any other branch only under its additional descent theorem. Sector II retains a corridor-banded family; Sector III retains chart–corridor line-banded local forms and operators; the correspondence branch is stated in local pivot representations. $\square$

What has now been established. The thin stiffness form is now closed, coercive off the complete-ground projection, and equipped with an exact kernel. These are branchwise

form and operator statements on the certified thin carriers. They do not yet supply a joint translation spectrum or invariant mass. Figure 4 shows the remaining Paper-IV hyperbolic identification and the precise point at which Paper V begins.

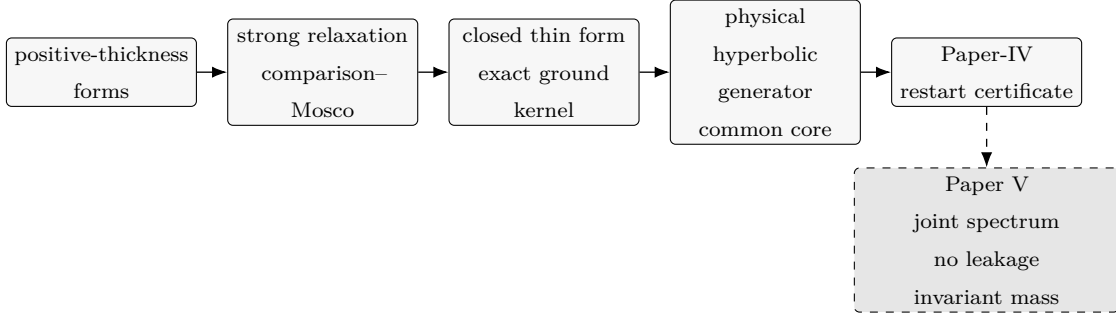


FIG. 4. Closed-form and hyperbolic reconstruction. Paper IV reaches a selected physical energy threshold; the joint spectral and invariant-mass upgrade begins only in Paper V.

Firewall IX.6 (Form threshold versus invariant mass) *The regional or branchwise operator in Theorem IX.5 is, at that stage, only a thin stiffness operator. The later descent-compatible hyperbolic ownership theorem identifies its positive square root with the selected physical timelike generator on the same thin carrier. Paper IV still does not construct a full Poincare translation representation or invariant mass. Joint-spectrum covariance, null-orbit exclusion, physical no-leakage, and the invariant-mass threshold remain Paper-V obligations.*

X. PHYSICAL THIN HYPERBOLIC REALIZATION AND ENERGY-STIFFNESS DOMINATION

Reader’s roadmap. The closed stiffness operator becomes physically useful only after its square root is identified with the selected timelike evolution on the same descent-compatible carrier. This section performs that identification and proves a positive selected-frame energy threshold on a common invariant core. It does not construct the full commuting translation family or its joint spectral measure.

Scope X.1 (Status and branch typing) *This section extracts a theorem already latent in the proved Paper-IV architecture. It introduces no second comparison Hilbert space and no new physical hypothesis. The theorem is asserted on each selected represented Hilbert carrier*

for which the Paper-IV Cauchy presentation is compatible with the admissible descent data. It is global in a branch only when effective Hilbert descent has separately been proved; otherwise it is objectwise, corridorwise, or coefficient-line banded exactly as in the represented-descent theorem.

Fix one such selected physical thin carrier and suppress its branch, chart, corridor, and coefficient-line labels. Write

$$\left(\mathfrak{H}_{\text{thin}}, \Omega_{\text{thin}}, P_{\text{gr}}, \bar{\mathfrak{q}}_{\text{thin}} \right)$$

for the represented carrier, a selected cyclic ground-pointing vector, the complete-ground projection, and the closed nonnegative thin stiffness form. The vector satisfies

$$\Omega_{\text{thin}} \in \text{Ran } P_{\text{gr}}, \quad (30)$$

but no rank-one assertion is made. Let $K_{\text{thin}} \geq 0$ be the unique self-adjoint operator represented by the form:

$$D(\bar{\mathfrak{q}}_{\text{thin}}) = D(K_{\text{thin}}^{1/2}), \quad \bar{\mathfrak{q}}_{\text{thin}}[\Psi, \Phi] = \langle K_{\text{thin}}^{1/2} \Psi, K_{\text{thin}}^{1/2} \Phi \rangle. \quad (31)$$

Definition X.2 (Descent-compatible selected Cauchy presentation) *A selected Paper-IV Cauchy presentation is called descent compatible when the already declared Cauchy comparison cells preserve the physical quotient, the common Cauchy–Gauss–Ward core, the collar condition, and the positive-frequency propagation, and when the comparison–Mosco limit of the positive-thickness spatial Friedrichs forms is $\bar{\mathfrak{q}}_{\text{thin}}$. Equivalently, compatibility means that the Cauchy propagation and the closed-form descent are two faces of the same selected admissible comparison diagram. No arbitrary Cauchy surface and no unrestricted scale arrow is included.*

Theorem X.3 (Descent-compatible hyperbolic ownership) *On every descent-compatible selected Cauchy presentation, the selected thin positive-frequency evolution is*

$$U_{\text{hyp}}(t) = e^{-itK_{\text{thin}}^{1/2}}, \quad t \in \mathbb{R}. \quad (32)$$

Consequently it is a strongly continuous unitary realization of the selected timelike one-parameter group on the physical thin carrier. It implements the descended thin observable dynamics and preserves the inherited gerbe, seam, Gauss–Ward, collar, and complete-ground data at their exact branchwise scope.

Proof. Let $K_n \geq 0$ denote the positive-thickness spatial Friedrichs operators on the selected Cauchy presentations after transport to the common comparison graph. The comparison–Mosco and TT2 theorem, together with the Paper-IV comparison-resolvent result, gives

$$K_n \longrightarrow K_{\text{thin}}$$

in the strong-resolvent sense. Nonnegativity and the continuous functional calculus imply

$$K_n^{1/2} \longrightarrow K_{\text{thin}}^{1/2}$$

in the strong-resolvent sense. The Trotter–Kato/Stone convergence theorem^{18,21} therefore gives, for each $t \in \mathbb{R}$,

$$e^{-itK_n^{1/2}}\Psi \longrightarrow e^{-itK_{\text{thin}}^{1/2}}\Psi \quad (33)$$

for every transported physical vector Ψ , uniformly for t in compact intervals. At positive thickness, $e^{-itK_n^{1/2}}$ is precisely the selected positive-frequency Cauchy evolution. The Paper-IV propagation comparison and time-slice theorem identify the descended thin evolution as the strong limit of those same transported Cauchy propagators on the common Cauchy–Gauss–Ward core. Uniqueness of the strong limit in (33) gives (32) on that core and hence on the completed carrier.

The positive-thickness propagation cells preserve the physical quotient, ground pointing, Gauss graph, Ward insertion graph, collar condition, and exact represented seams. Their limiting identities pass through the same rooted admissible descent diagram. Controlled nonunitary seams are not inverted and are not used to transport a lower bound; the theorem is asserted independently at each certified endpoint. This proves the stated compatibility. $\square$

Definition X.4 (Physical thin hyperbolic generator) *Set*

$$H_{\text{hyp}} := K_{\text{thin}}^{1/2}. \quad (34)$$

Then $U_{\text{hyp}}(t) = e^{-itH_{\text{hyp}}}$. This is the generator of the selected timelike one-parameter group; no full translation or Poincare representation is asserted in Paper IV.

Theorem X.5 (Thin hyperbolic realization and common invariant core) *The group U_{hyp} is strongly continuous and its Stone generator is the nonnegative self-adjoint operator*

$$H_{\text{hyp}} \geq 0.$$

Let $\mathcal{D}_{\text{Cau,GW}}$ be the descended common Cauchy–Gauss–Ward form core and define

$$\mathcal{C}_{\text{hyp}} := \text{span} \{ \chi(H_{\text{hyp}})\Phi : \Phi \in \mathcal{D}_{\text{Cau,GW}}, \quad \chi \in C_c^\infty([0, \infty)) \}. \quad (35)$$

Then

$$\mathcal{C}_{\text{hyp}} \subset D(H_{\text{hyp}}) \cap D(\bar{\mathfrak{q}}_{\text{thin}}), \quad U_{\text{hyp}}(t)\mathcal{C}_{\text{hyp}} \subseteq \mathcal{C}_{\text{hyp}}. \quad (36)$$

It is a graph core for H_{hyp} and a form core for $\bar{\mathfrak{q}}_{\text{thin}}$, and it preserves the inherited Gauss–Ward, collar, seam, and observable-insertion structures.

Proof. Strong continuity, unitarity, self-adjointness, and positivity follow from the spectral calculus and Stone’s theorem^{21,22}. Compactly supported spectral multipliers commute with $U_{\text{hyp}}(t)$, preserve every exact seam intertwining K_{thin} , and preserve the descended constraint core by the compatibility theorem.

Choose $\chi_N \in C_c^\infty([0, \infty))$ with $0 \leq \chi_N \leq 1$, $\chi_N = 1$ on $[0, N]$, and $\chi_N \rightarrow 1$ pointwise. Spectral dominated convergence gives

$$\chi_N(H_{\text{hyp}})\Psi \rightarrow \Psi, \quad H_{\text{hyp}}\chi_N(H_{\text{hyp}})\Psi \rightarrow H_{\text{hyp}}\Psi$$

for every $\Psi \in D(H_{\text{hyp}})$. Combining these truncations with the already proved form-core recovery from $\mathcal{D}_{\text{Cau,GW}}$ proves the graph-core and form-core assertions. $\square$

Theorem X.6 (Ground compatibility and exact energy–stiffness identity) *The complete-ground projection satisfies*

$$H_{\text{hyp}}P_{\text{gr}} = 0. \quad (37)$$

Moreover,

$$D(H_{\text{hyp}}) = D(\bar{\mathfrak{q}}_{\text{thin}}) \quad (38)$$

and

$$\|H_{\text{hyp}}\Psi\|^2 = \bar{\mathfrak{q}}_{\text{thin}}[\Psi], \quad \Psi \in D(H_{\text{hyp}}). \quad (39)$$

Thus the energy–stiffness domination holds on $\mathcal{C}_{\text{hyp}}$ and on the full closed domain with the proved dimensionless constant

$$\boxed{c_E = 1.} \quad (40)$$

Proof. Paper IV already proves

$$\ker \bar{\mathfrak{q}}_{\text{thin}} = \ker K_{\text{thin}} = \text{Ran } P_{\text{gr}}. \quad (41)$$

Functional calculus therefore gives $K_{\text{thin}}^{1/2} P_{\text{gr}} = 0$, which is (37). Equations (31) and (34) give the domain equality and the exact form identity directly.

For the explicit closure passage, let

$$\Psi_N = \mathbf{1}_{[0,N]}(H_{\text{hyp}})\Psi.$$

Then $\Psi_N \rightarrow \Psi$ and $H_{\text{hyp}}\Psi_N \rightarrow H_{\text{hyp}}\Psi$. Hence Ψ_N is Cauchy in the closed form norm because

$$\bar{\mathfrak{q}}_{\text{thin}}[\Psi_N - \Psi_M] = \|H_{\text{hyp}}(\Psi_N - \Psi_M)\|^2.$$

This proves the full-domain extension by graph/form closure. The constant c_E is not defined by a quotient infimum; it is the exact normalization in the squared-frequency identity. $\square$

Corollary X.7 (Exact zero-energy kernel) *One has*

$$\ker H_{\text{hyp}} = \text{Ran } P_{\text{gr}}, \quad P_{\text{gr}} = \mathbf{1}_{\{0\}}(H_{\text{hyp}}). \quad (42)$$

Furthermore,

$$U_{\text{hyp}}(t)\Omega_{\text{thin}} = \Omega_{\text{thin}}. \quad (43)$$

Proof. The kernels of a nonnegative self-adjoint operator and its positive square root coincide, so (41) gives (42). Equation (30) then implies $H_{\text{hyp}}\Omega_{\text{thin}} = 0$, and spectral calculus gives (43). $\square$

Theorem X.8 (Physical energy threshold) *Let $\kappa_{\text{IV},\text{thin}} > 0$ be the already proved Paper-IV stiffness floor:*

$$\bar{\mathfrak{q}}_{\text{thin}}[\Psi] \geq \kappa_{\text{IV},\text{thin}} \|(1 - P_{\text{gr}})\Psi\|^2. \quad (44)$$

Then

$$H_{\text{hyp}}^2 \geq c_E \kappa_{\text{IV},\text{thin}} (1 - P_{\text{gr}}) \quad (45)$$

in closed-form sense. Since $H_{\text{hyp}} \geq 0$,

$$H_{\text{hyp}} \geq \sqrt{c_E \kappa_{\text{IV},\text{thin}}} (1 - P_{\text{gr}}). \quad (46)$$

In the canonical normalization $c_E = 1$. The quantity

$$\Delta_{E,IV}^2 := c_E \kappa_{IV,\text{thin}}$$

is the Paper-IV physical energy-squared threshold. It is not yet the Paper-V invariant-mass threshold.

Proof. Combine (39) with (44). Because $P_{\text{gr}} = \mathbf{1}_{\{0\}}(H_{\text{hyp}})$, the scalar inequality

$$\lambda^2 \geq c_E \kappa_{IV,\text{thin}} \quad \text{on } \text{spec}(H_{\text{hyp}}) \setminus \{0\}$$

and the nonnegativity of H_{hyp} imply (46) by Borel functional calculus. $\square$

Firewall X.9 (What remains for Paper V) *Paper IV has constructed only the selected timelike hyperbolic group and its physical energy threshold. It has not constructed the full joint translation representation, proved the forward-cone spectrum condition for that representation, formed the invariant mass, or excluded a non-ground future-null sector. Those remain Paper-V obligations.*

XI. COMPACT-SEMISIMPLE ANALYTICAL WITNESSES

A. Global forms and allowed weights

The representation-theoretic conventions in this subsection follow the standard compact-group highest-weight classification.⁵ Let

$$\tilde{G} = \prod_{i=1}^r G_i^{\text{sc}}, \quad G = \tilde{G}/\Gamma, \quad \Gamma \leq Z(\tilde{G}),$$

with G compact, connected, and semisimple. Fix a maximal torus of $\tilde{G}$ and let P, Q, P^+ be its weight, root, and dominant-weight lattices. For $\lambda \in P$, let χ_λ be the central character.

Definition XI.1 (Global-form weight lattice)

$$P_G = \{\lambda \in P : \chi_\lambda(\gamma) = 1 \text{ for every } \gamma \in \Gamma\}, \quad P_G^+ = P_G \cap P^+.$$

Proposition XI.2 (Central characters of a semisimple global form) *There are canonical identifications*

$$P/Q \cong \widehat{Z(\tilde{G})}, \quad P_G/Q \cong \text{Ann}_{\widehat{Z(\tilde{G})}}(\Gamma) \cong \widehat{Z(\tilde{G})/\Gamma} \cong \widehat{Z(G)}.$$

The irreducible unitary representations of G are exactly the highest-weight representations V_λ with $\lambda \in P_G^+$.

Proof. For the simply connected cover, the central character of V_λ depends only on λ modulo Q , and the resulting map $P/Q \rightarrow \widehat{Z(\tilde{G})}$ is an isomorphism. A representation descends through $\tilde{G} \rightarrow G$ exactly when Γ acts trivially, which is the defining condition for P_G . Thus P_G/Q is the annihilator of Γ . To identify the quotient center, note that if the class of $g \in \tilde{G}$ is central in G , then $x \mapsto [g, x]$ is a continuous map from connected $\tilde{G}$ to finite Γ and is therefore identically the identity. Hence $g \in Z(\tilde{G})$ and $Z(G) = Z(\tilde{G})/\Gamma$. Finite Pontryagin duality gives the stated identification, and highest-weight classification gives the final statement. $\square$

Lemma XI.3 (Extension of center characters) *Let A be a finite abelian group and $B \leq A$. Every character of B extends to a character of A .*

Proof. The group $U(1)$ is divisible and hence injective as a $\mathbb{Z}$ -module, so the restriction map $\hat{A} \rightarrow \hat{B}$ is surjective. Equivalently, decompose A into cyclic primary factors and extend on each factor. $\square$

Theorem XI.4 (Primitive central-character lemma) *Let $h \in Z(G)$ have order $n > 1$. After choosing the orientation of the generator of $\langle h \rangle$, there exists a dominant allowed weight $\lambda_h \in P_G^+$ such that*

$$\chi_{\lambda_h}(h) = e^{2\pi i/n}.$$

Hence the irreducible Wilson channel V_{λ_h} detects the selected cyclic center band primitively.

Proof. Choose the primitive character of $\langle h \rangle$ sending h to $e^{2\pi i/n}$ and extend it to $Z(G)$. By [theorem XI.2](#), the extension is represented by a class in P_G/Q . The Weyl group acts trivially on P/Q because $s_\alpha(\lambda) - \lambda = -\langle \lambda, \alpha^\vee \rangle \alpha \in Q$. A Weyl-conjugate dominant representative therefore has the same central character. $\square$

B. Witness hierarchy and certification levels

The journal distinguishes three logically different forms of non-vacuity.

Level A — coefficient availability.:

The global form and its allowed weights supply quotient-compatible Wilson characters and declared scalar coefficient categories. This is the algebraic content of Theorem XI.6; it contains no local-normal, entropy, Gauss–Ward, ground, or form conclusion.

Level B — analytical comparison witness.:

A declared branch over $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ supplies the complete incised exact-dilation Friedrichs carrier, source expectations, reference packets, graph cores, complete-ground data, and comparison forms. Such a branch proves nonemptiness of the primitive packages **OJ**, **RJ**, **SJ**, **GJ**, and **M** inside the stated comparison class. It is not thereby identified with a unique continuum Yang–Mills system.

Level C — fully physical branch.:

A positive-thickness Yang–Mills carrier constructed independently of the root-data model is shown, theorem by theorem, to enter the Level-B comparison class and to carry a physical sector detector. The canonical dyadic $SU(3)$ branch is the present Level-C witness.

The three levels must never be collapsed. In particular,

$$\begin{aligned} \text{root data} &\not\Rightarrow \text{analytical carrier,} \\ \text{analytical carrier} &\not\Rightarrow \text{unique physical Yang–Mills branch.} \end{aligned}$$

C. Exact-dilation Friedrichs comparison carrier

Fix an object $X = (\mathbb{M}_{0,\text{inc}}, O) \in \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$. Set $\varepsilon_n = 2^{-n}\varepsilon_0$ and choose incision-preserving regional straightening maps $J_{G,\xi,n;X}$, compatible with the selected collars and support carriers, such that

$$J_{G,\xi,n;X} D(\hat{q}_{G,\xi,n;X}) = \mathcal{Q}_{G,\xi;X}.$$

The local positive operator on the incision-adapted carrier of X is the Dirichlet–Friedrichs realization^{9,18,23} of

$$A_{G,\xi,n} = \eta \nabla_{G,\xi,n}^* \nabla_{G,\xi,n} + M_{V_{G,\xi,n}}, \quad (47)$$

where the principal symbol is Laplace type and the lower-order term is pointwise Hermitian. Bounded regional carriers give compact local resolvent. Finite propagation of the wave

calculus depends on the principal symbol and pointwise locality of the lower-order term; no type- (A) matrix identity is used.

The renormalized physical operator is

$$\hat{A}_{G,\xi,n} = \varepsilon_n^2 A_{G,\xi,n}^{\text{phys}}.$$

For the zero-defect orbit the transported forms and finite-window expectation algebras are stationary up to positive affine rescaling. More generally, the declared comparison class uses the primitive estimate

$$|\hat{q}_{G,\xi,n}[u] - q_{G,\xi,\text{ref}}[u]| \leq C_{G,\xi,X} \varepsilon_n^\alpha \|u\|_{\mathcal{Q}_{G,\xi;X}}^2. \quad (48)$$

The constants may depend on the global form, finite Wilson-channel frame, invariant metric, regional exhaustion, reference profile, and branch, but are uniform on that branch's cofinal tail. The comparison and reconstruction maps, exact complete-ground endpoint data, source expectations, faithful reference profile, physical Orlicz packet, graph-form Ward package, collar quotient, and primitive recovery family are part of the declared carrier; they are not inferred from root data or from norm-resolvent convergence alone.

Theorem XI.5 (Compact-semisimple exact-dilation comparison witness) *For every compact connected semisimple global form G and every finite allowed Wilson-channel frame, there exists a declared incision-preserving exact-dilation Friedrichs comparison branch satisfying the primitive analytical packages **OJ**, **SJ**, **GJ**, and **M**. After choosing either the identity scalar coefficient system or one of the declared scalar coefficient categories of Theorem XI.6, the represented package **RJ** is satisfied as well.*

The conclusion is branchwise. It proves analytical comparison non-vacuity and does not assert rank-uniform constants, uniqueness of the branch, or identification with every independently produced physical refinement.

Proof. The invariant inner product on the compact semisimple Lie algebra and the finite-dimensional unitary Wilson frame change only bounded finite-rank coefficient terms; the Laplace-type principal symbol and the incised collar geometry are unchanged. The local Friedrichs form is therefore closed and lower semibounded on the common regional Sobolev core, with compact local resolvent on bounded carriers and finite-wave collar propagation from the local principal part. Exact dilation gives affine/unitary transport, stationary

expectation algebras, coherent good windows, exact complete-ground transport, and the strict unsurgered rooted shadow. A faithful spectral reference with a declared superlinear profile gives the separate Orlicz, reference-logarithm, and entropy controls. The positive-thickness gauge-equivariant graph core and fixed collar package supply **GJ**, while the common form domain, comparison/reconstruction maps, primitive recovery data, ground form structure, and positive-thickness coercivity supply **M**. Theorems [V.1](#), [VII.1](#), [VIII.4](#), and [IX.5](#) then convert these primitive data into the corresponding thin conclusions; none of those conclusions is reused as an input here. $\square$

D. Strict and scalar twisted comparison branches

The identity coefficient gerbe and identity chart-scale tensorators give

$$\kappa_{\xi}^{\text{fib}} = 0, \quad \hat{\kappa}_{\xi}^{\text{cs}} = 0.$$

For a selected $h \in Z(G)$ of order $n > 1$, identify $\langle h \rangle \cong \mathbb{Z}_n$. If $\tilde{a} \in \{0, \dots, n-1\}$ and

$$\epsilon_n(b, c) = \frac{\tilde{b} + \tilde{c} - \widetilde{b+c}}{n} \in \{0, 1\},$$

define

$$\Omega_{h,s}(a, b, c) = \exp\left(\frac{2\pi i s}{n} \tilde{a} \epsilon_n(b, c)\right), \quad s = \pm 1. \quad (49)$$

This is a normalized three-cocycle with class $\mathbf{su}_n \neq 0$ in $H^3(B\mathbb{Z}_n; U(1))$. The primitive Wilson channel from [theorem XI.4](#) realizes the selected center character algebraically. [Theorem XI.5](#) supplies the independent analytical carrier.

Theorem XI.6 (Root-data and scalar-coefficient availability (Theorem F)) *For every compact connected semisimple global form G :*

- (i) $P_G/Q \cong \widehat{Z(G)}$, and every finite selected center band admits a quotient-compatible primitive Wilson-character frame;
- (ii) the identity scalar coefficient system is algebraically available;
- (iii) every selected $h \in Z(G)$ of order $n > 1$ and orientation $s = \pm 1$ admits the quotient-compatible cyclic coefficient category $\text{Vec}_{\mathbb{Z}_n}^{\Omega_{h,s}}$ with class $\mathbf{su}_n$;

- (iv) *every declared finite-abelian band $A \leq Z(G)$ and declared class $[\Omega] \in H^3(BA; U(1))$ admit a quotient-compatible scalar coefficient category after choosing a normalized representative and a primitive frame;*
- (v) *center-trivial global forms have no center-generated scalar line twist;*
- (vi) *these conclusions classify center-generated scalar coefficient availability only and do not classify Sector-II or correspondence-valued obstructions.*

No analytical OJ, SJ, GJ, or M conclusion follows from root data alone.

Proof. The lattice identity and primitive-channel theorem prove (i). The identity system gives (ii). Equation (49) gives (iii). For (iv), choose a normalized cocycle representative and form Vec_A^Ω ; the primitive frame supplies quotient-compatible grading characters, while mixed pair and triple associator coordinates are separately declared tensorator data. Items (v) and (vi) are scope consequences of $P_G/Q \cong \widehat{Z(G)}$. The absence of analytical conclusions is deliberate: local-normal algebras, references, graph cores, complete-ground transport, and coercive forms belong to the independent analytical carrier. $\square$

Theorem XI.7 (Three-level analytical non-vacuity certificate (Theorem G))

The witness hierarchy has the following certified content.

- (G1) *For every compact connected semisimple global form, the identity coefficient system attached to Theorem XI.5 gives a Level-B strict Sector-I exact-dilation comparison witness.*
- (G2) *For every selected central element h of order $n > 1$ and orientation $s = \pm 1$, the cyclic category $\text{Vec}_{\mathbb{Z}_n}^{\Omega_{h,s}}$ attached to the same carrier gives a Level-B scalar Sector-III exact-dilation comparison witness.*
- (G3) *Every admitted canonical dyadic physical $SU(3)$ branch $\xi_{m,a,\nu}$ is Level C. With $r = a\nu$, the branches $(m, r) = (0, 0), (0, 1), (0, 2)$, and $(1, 2) \times \mathbb{Z}_3$ realize Sectors I, II, and III, respectively, after application of Theorems V.1–IX.5.*
- (G4) *The physical $SU(3)$ certificate uses two independent detectors: $I_3(\Omega_m^{\text{fb}}) = \omega^m$ for fixed-fiber twisting and a controlled strictification defect with $\liminf$ at least $\pi/3$ for $r = 1, 2$.*

(G5) *The physical Sector-II rows are branch specific. No universal Sector-II theorem is inferred for arbitrary compact semisimple groups or independently selected carriers.*

(G6) *The independent strict $SU(2)$ construction is a fixed-positive-thickness finite/depth control only and is not used as a thickness-cofinal witness.*

The Level-B branches prove nonemptiness of the journal hypothesis class. Level C records a fully physical positive-thickness carrier whose complete incised branch tuple and detector data are certified in Theorem XII.1.

Proof. Items (G1)–(G2) combine Theorems XI.6 and XI.5. The physical $SU(3)$ source supplies the complete positive-thickness local-normal, Cauchy, complete-ground, collar, surgery, form, and coefficient data for the canonical dyadic branch class. The observable, represented, surgery, constraint, form, and witness theorems identify those data with the primitive OJ/RJ/SJ/GJ/M packages. The fixed-fiber detector, controlled shell estimate, and Proposition VI.6 yield the branch table in Theorem XII.1. The $SU(2)$ limitation follows because its independent construction varies depth at one positive thickness and supplies no $\varepsilon_n \downarrow 0$ corridor or TT1–TT2 theorem. $\square$

E. Journal witness matrix

The compact table records certification level rather than merely writing “proved.” Detailed theorem provenance is given in the supplement.

	Strict root datum	Cyclic scalar	Canonical physical $SU(3)$	Correspondence
Coefficient level	L-A identity	L-A cyclic	fiber m and controlled r	non-scalar target
Primitive packages	L-B proved	L-B proved	L-C certified for every admitted transfer only (m, a, ν)	
Detector	trivial	cyclic fiber class	$I_3(\Omega_m^{\text{fib}})$ and independent con- trolled defect	component detector re- quired
Physical status	comparison Sector I	comparison Sector III	physical Sectors I, II, III according to (m, r)	not claimed
Universal scope	every declared global form	every selected cyclic center band	one canonical $SU(3)$ family; Sector II remains branch specific	not claimed

XII. THE COMPLETE INCISED PHYSICAL $SU(3)$ FAMILY

Let

$$G = SU(3), \quad Z(G) = \langle \omega I_3 \rangle \cong \mathbb{Z}_3, \quad \omega = e^{2\pi i/3}.$$

The physical family carries a fixed-thickness center-flux label $m \in \mathbb{Z}_3$, an electric triality $a \in \mathbb{Z}_3$, and a corridor grade $\nu \in \mathbb{Z}_3$. Put

$$r = a\nu \pmod{3}.$$

The branch is denoted $\xi_{m,a,\nu}$; its sector depends on the pair (m, r) , while the full triple is retained in the physical branch data.

A. The complete incised branch tuple

For every admitted canonical dyadic branch define

$$\begin{aligned} \mathfrak{B}_{m,a,\nu}^{\text{phys}} := & \left(\mathbb{M}_{0,\text{inc}}, \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}, \{\varepsilon_n\}_{n \geq N}, \{\mathcal{M}_{m,n}(X), \mathcal{H}_{m,n}, P_{\Omega,m,n}\}_{X,n}, \{J_{m,n;X}, I_{m,n;X}\}_{X,n}, \right. \\ & \{q_{m,n;X}\}_{X,n}, \{\mathbb{E}_{m,n;D}^{(N_0)}\}_{N_0,D,n}, \{L_{m,n;D}^{\text{ref}}, \mathfrak{K}_m^{\text{Orl}}\}_{D,n}, \{\mathfrak{GW}_{m,n}, \mathfrak{C}_{m,n}^\partial\}_n, \\ & \left. \Omega_m^{\text{fib}}, \Omega_{a,\nu}^{\text{sc}}, \Omega_{m,r}^{\text{tot}}, \text{Restart}_{m,r}, \mathfrak{D}_m^{\text{fib}}, \mathfrak{D}_r^{\text{ctrl}} \right). \end{aligned} \quad (50)$$

Here $X = (\mathbb{M}_{0,\text{inc}}, O)$ ranges over the controlled incised category, the comparison/reconstruction maps preserve meridional orientation, triality, collar data, and the complete-ground face, and $\mathfrak{C}_{m,n}^\partial$ abbreviates the collar quotient, current defect, boundary-character class, and boundary-Ward defect. The tuple contains no asserted global vacuum vector and no unincised Poincare action.

B. Fixed-fiber cocycle and discrete detector

At each fixed positive thickness set

$$\Omega_m^{\text{fib}}(b, c, d) = \omega^{m\widetilde{b}\epsilon_3(c,d)}, \quad \epsilon_3(c, d) = \frac{\widetilde{c} + \widetilde{d} - c + d}{3}.$$

For a normalized three-cochain Ξ on $\mathbb{Z}_3$, define

$$I_3(\Xi) = \prod_{c=0}^2 \Xi(1, c, 1). \quad (51)$$

The inhomogeneous coboundary factors telescope, so $I_3(\delta\lambda) = 1$ for every normalized two-cochain λ . Since only the $c = 2$ carry is nonzero,

$$\boxed{\mathfrak{D}_m^{\text{fib}} := I_3(\Omega_m^{\text{fib}}) = \omega^m.} \quad (52)$$

Thus $m = 1, 2$ gives $[\Omega_m^{\text{fib}}] = m\mathbf{u}_3 \neq 0$ at every positive thickness. This detector is a same-thickness fiber detector; it uses no scale arrow.

C. Controlled scale cocycle and quantitative detector

The reduced corridor reassociator is

$$\Omega_{a,\nu}^{\text{sc}}(k, \ell, p) = \omega^{a\nu\tilde{k}\epsilon_3(\ell,p)}.$$

Its unrestricted class on the rooted depth category is exact. Its class in the stationary, local, band-preserving, scale-natural controlled complex is ru_3 . Let $\text{Def}_{r,n}^{\text{phys}}(\lambda)$ be the marked angular defect of an admissible controlled primitive candidate. The physical shell comparison gives

$$\text{Def}_{r,n}^{\text{phys}}(\lambda) \geq \max \left\{ 0, \frac{\pi}{3} - L_\lambda \tau_{R_\lambda}^{\text{full}}(n) \right\}.$$

Consequently

$$\mathfrak{D}_r^{\text{ctrl}} := \liminf_{n \rightarrow \infty} \inf_{\lambda \in C_{\text{adm}}^2} \text{Def}_{r,n}^{\text{phys}}(\lambda) \begin{cases} = 0, & r = 0, \\ \geq \pi/3, & r = 1, 2. \end{cases} \quad (53)$$

The discrete fiber detector (52) and the quantitative scale detector (53) are independent: the first detects a fixed-thickness gerbe class, while the second excludes uniformly controlled strictification of an ordinarily exact rooted-scale cocycle.

D. Canonical dyadic physical comparison

On the common straightened form domain Q_m , write

$$\begin{aligned} \hat{\mathfrak{a}}_{m,n}^J[u, v] &= \langle (V_m + \lambda_n W_{m,n})u, v \rangle \\ &\quad + \eta(1 + \lambda_n d_{m,n}) \langle (\nabla_m + \lambda_n C_{m,n})u, (\nabla_m + \lambda_n C_{m,n})v \rangle, \end{aligned} \quad (54)$$

where $\lambda_n = \varepsilon_n^\alpha$, $0 < \alpha \leq 1$, and

$$|\langle W_{m,n}u, v \rangle| \leq C_W \|u\|_{Q_m} \|v\|_{Q_m}, \quad |d_{m,n}| \leq C_\eta, \quad \sqrt{\eta} \|C_{m,n}u\| \leq C_C \|u\|_{Q_m}.$$

The perturbations are gauge equivariant and center neutral. With

$$C_{\text{form}} = C_W + C_\eta + 2(1 + C_\eta)C_C + (1 + C_\eta)C_C^2,$$

one has

$$|\hat{\mathfrak{a}}_{m,n}^J[u] - \hat{\mathfrak{a}}_m^{\text{dil}}[u]| \leq C_{\text{form}} \varepsilon_n^\alpha \|u\|_{Q_m}^2.$$

By Theorem IV.1, the Paper-III source package supplies exact complete-ground transport, incision-adapted propagation and Cauchy comparison, expectation/reference control, graph-form Ward and collar data, and full decorated-shell convergence. The Paper-IV closure theorems therefore apply to every admitted branch tuple (50).

E. Theorem-level provenance

TABLE I. Primitive-package provenance for the physical $SU(3)$ family.

Package	Positive-thickness physical source	Paper-IV closure
OJ	incised local-normal, causal, Cauchy, propagation, Theorem V.1 and the observable reconstruction certificate and full-shell comparison	
RJ	exact complete-ground, triality, corridor, coefficient multiplication, and mixed cells	Theorem VI.7 and the represented-sector certificate
SJ	source expectations, faithful references, shell convergence, and rooted recovery input	Theorem VII.1 and corrected R.1–R.8
GJ	gauge equivariance, common graph cores, coefficient neutrality, Ward and collar compatibility	Theorem VIII.4 and the Gauss–Ward/collar certificate
M	common domain, form rate, resolvent/fractional comparison, exact ground transport, and threshold	Theorem IX.5 and the comparison–Mosco/form certificate
Detectors	fixed-fiber triality cocycle and controlled marked defect estimate	Proposition VI.6 and the theorem below

Theorem XII.1 (Physical $SU(3)$ branch certificate) *Fix an admitted canonical dyadic branch $\xi_{m,a,\nu}$. The tuple in (50) verifies all five primitive packages:*

$$\text{OJ}, \quad \text{RJ}, \quad \text{SJ}, \quad \text{GJ}, \quad \text{M}.$$

Its zero-thickness reconstruction exists on $\text{RegLoc}_{\text{inc}}^{0,\text{adm}}$, and its sector is exactly

m	r	Retained detector	Thin type
0	0	$\mathfrak{D}_0^{\text{fib}} = 1$ and $\mathfrak{D}_0^{\text{ctrl}} = 0$	Sector I
0	1, 2	$\mathfrak{D}_0^{\text{fib}} = 1$ and $\mathfrak{D}_r^{\text{ctrl}} \geq \pi/3$	Sector II
1, 2	0, 1, 2	$\mathfrak{D}_m^{\text{fib}} = \omega^m \neq 1$	Sector III

In the Sector-III rows a nonzero controlled scale coordinate is retained as additional structure but does not change the sector. The ordinary image of the total class is

$$[\Omega_{m,r}^{\text{tot}}]_{\text{ordinary}} = \text{pr}_{\text{fib}}^*(m\mathbf{u}_3),$$

whereas the complete controlled coordinate is $(m, 0, r)$. The $m = 0, r = 1, 2$ rows are branch-specific physical Sector-II witnesses; they do not imply universal Sector-II non-vacuity for other gauge groups or carriers.

Proof. The Paper-III pass-through realization supplies all entries of (50); Theorem IV.1 translates its certified positive-thickness source blocks into the five primitive packages, with the package provenance summarized in table I. Theorems V.1–IX.5 reconstruct the ordinary net, branch-typed represented system, inherited surgery graph, Gauss–Ward/collar package, and closed form. The fixed-fiber detector is finite valued and is unchanged by complete-ground compression, coefficient- neutral expectations, rooted lifting, and restart. The scale estimate is stable under full-shell comparison and loses only a term tending to zero; Proposition VI.6 therefore retains its positive liminf. The sector table follows from the defining trichotomy: nonzero fiber class gives Sector III; zero fiber class and nonzero controlled obstruction give Sector II; simultaneous vanishing plus the canonical coherent strictification gives Sector I. $\square$

Scope XII.2 (Interpretation) *The $m = 1, 2$ rows are the first complete physical twisted witnesses and the discovery model for the general root-datum theorem. The neutral and controlled-scale rows additionally realize Sectors I and II within this canonical family. Neither detector proves dynamical interaction, removes the incision, constructs a fixed-fiber Poincare representation, or proves a mass gap.*

XIII. COMPLETE BRANCHWISE RECONSTRUCTION THEOREM

Interpretive lead-in. The individual reconstruction layers have now been proved in their correct targets: ordinary observables, represented coherence, inherited state surgery, Gauss–Ward/collar data, closed forms, analytical witnesses, and the selected hyperbolic generator. The next theorem assembles them without collapsing their types or enlarging their claims.

Theorem XIII.1 (Conditional complete assembly (Theorem H1)) *Let $G = \tilde{G}/\Gamma$ be compact, connected, and semisimple, and let ξ be a declared branch over the selected controlled incised category $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$ and satisfying the primitive packages **OJ**, **RJ**, **SJ**, **GJ**, and **M**. Then the following branchwise reconstruction is defined.*

(H1.1) *The ordinary thin observable system is a normal controlled-background-covariant functor*

$$\mathcal{A}_{G,\xi,0}^W : \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}} \longrightarrow W^* \text{Alg}_n$$

on the selected incised background–region category, with causality relative to common incision-adapted ambient regions, packet-localizing normal additivity, causal dynamics, and the incision-adapted time-slice property.

(H1.2) *The represented thin output is the branch-typed pseudodiagram $\mathbf{Rep}_{G,\xi,0}^{\text{thin}}$ of Theorem VI.7. Scalar-line sectors retain $(\kappa_\xi^{\text{fib}}, \widehat{\kappa}_\xi^{\text{cs}})$ and any admissible physical strictification; correspondence targeting and κ_ξ^{Corr} are recorded separately.*

(H1.3) *The inherited stiffness surgery*

$$\mathcal{S}_{G,\xi,0}^{\text{st,as,*}}$$

is the unique Cech-descended expectation graph on the selected normal-state packet, is simultaneously rooted on every finite admissible diagram, and is restart compatible.

(H1.4) *The limiting Gauss graph, graph-form Ward functional, collar quotient, boundary-character class, current-algebra defect, and boundary-Ward defect are reconstructed on their prepared domains and are structurally compatible with inherited surgery.*

(H1.5) *The thin stiffness form is a branch-appropriate regional or chart–corridor family $\overline{\mathbf{q}}_{G,\xi,0}$ with a family $\mathbf{P}_{\Omega,G,\xi,0}$ of actual orthogonal projections on its Hilbert form carriers satisfying*

$$\overline{\mathbf{q}}_{G,\xi,0} \geq \kappa_{\text{IV,thin}}(\mathbf{1} - \mathbf{P}_{\Omega,G,\xi,0}), \quad \ker \overline{\mathbf{q}}_{G,\xi,0} = \text{Ran } \mathbf{P}_{\Omega,G,\xi,0}$$

componentwise. These inequalities are componentwise. A single global operator or projection is asserted only when the Hilbert form carriers admit the required effective descent; the abstract represented ground family remains $\widehat{\Omega}_{G,\xi,0}^b$.

(H1.6) *The global-form datum (G, Γ, P_G) , selected Wilson-channel frame, comparison and recovery structures, choice-independence certificate, restart data, and theorem provenance are retained.*

Proof. The ordinary observable assertion is Theorem V.1; represented typing is Theorem VI.7; the expectation graph, sharp descent, rooted lifting, and restart are Theorem VII.1 and its

closure corollaries; the graph-form Gauss–Ward and collar statement is Theorem VIII.4; and the branchwise closed-form and exact-kernel statement is Theorem IX.5. The global-form and coefficient data are supplied by Theorem XI.6. Each component is constructed over the same controlled incision and admitted comparison system, so the outputs assemble without changing target category or branch type. $\square$

What has now been established. Paper IV has now completed the admissible thick-to-thin reconstruction. The output is not a single undifferentiated operator model: it is a typed branch certificate containing an ordinary thin net, a branch-appropriate represented system, an inherited expectation graph, constrained boundary data, a closed form with exact ground kernel, and selected hyperbolic evolution. The remaining task is to export only the data needed for the independent spectral analysis of Paper V.

Theorem XIII.2 (Branchwise certification (Theorem H2)) *The conditional assembly in Theorem XIII.1 is discharged in the following precisely typed cases.*

- (H2.1) *For every declared compact connected semisimple global form, the identity-coefficient exact-dilation Friedrichs branch is a Level-B Sector-I analytical comparison witness.*
- (H2.2) *For every selected nontrivial cyclic center band and orientation, the cyclic scalar exact-dilation branch is a Level-B Sector-III analytical comparison witness.*
- (H2.3) *The canonical incised physical $SU(3)$ family is Level C. It realizes Sector I at $(m, r) = (0, 0)$, branch-specific controlled Sector II at $m = 0$ and $r = 1, 2$, and Sector III at $m = 1, 2$ with arbitrary r . Its fixed-fiber and controlled-scale detectors remain distinct.*
- (H2.4) *Generic Sector-II branches outside the canonical physical $SU(3)$ family are certified only after a branch-specific nonzero controlled scale class and all primitive analytical packages are supplied.*
- (H2.5) *A correspondence-valued branch receives the complete transfer certificate only after a physical pointed correspondence system, its Markov shadow, its branch comparison data, and any claimed nontrivial correspondence component detector are supplied. Correspondence targeting alone does not certify $\kappa_{\xi}^{\text{Corr}} \neq [1]$.*

Proof. Items (1) and (2) are the Level-B witness statements of Theorem XI.7. Item (3) is the complete physical branch theorem in theorem XII.1. Items (4) and (5) restate the exact

conditionality of the sector and correspondence transfer theorems and do not promote an abstract target or coefficient class into a physical realization. $\square$

Corollary XIII.3 (Journal-facing principal statement) *Every declared branch satisfying the primitive packages admits a sector-preserving zero-thickness reconstruction over the controlled incised category. The ordinary observable shadow is a normal Haag–Kastler functor; the represented object retains its strict, scale-obstructed, scalar-twisted, or correspondence target; inherited surgery is a Cech-descended expectation graph compatible with the graph-form Gauss–Ward and collar package; and the branchwise thin form has exact complete-ground kernel and a uniform positive off-ground stiffness threshold. The theorem is analytically non-vacuous for every declared compact connected semisimple global form and is physically non-vacuous through the complete incised $SU(3)$ family.*

Firewall XIII.4 (No physical-spectrum conclusion (H4)) *Theorems [XIII.1](#) and [XIII.2](#), together with the hyperbolic extension of [section X](#), do not remove the incision, construct a full fixed-fiber Poincare action, identify the complete-ground object with a spacetime-invariant vacuum, construct the full commuting translation generators, impose a joint spectrum condition, identify the selected energy threshold with invariant mass, prove physical no-leakage, prove interaction, or establish a Yang–Mills mass gap. Those bridges are the independently typed obligations of [section XIV](#).*

XIV. EXACT INCISED PAPER-V HANDOFF

Reader’s roadmap. The full Paper-IV theorem package is larger than the datum required downstream. This section extracts the exact restart certificate: the thin observable and represented systems, the inherited surgery graph, Gauss–Ward and collar compatibility, the closed form and exact ground kernel, the physical hyperbolic generator and common core, branch seams, comparison data, and the retained positive threshold. No joint spectral measure, forward-cone theorem, no-leakage result, or invariant-mass operator is included.

The Paper-IV output is a typed branch certificate, not a preliminary spacetime-spectrum theorem⁸. The handoff therefore has three layers: a branchwise data core, a certification shell, and an explicit unresolved-obligation ledger.

A. Branchwise data core

For any branch certified by Theorem XIII.1, define

$$\mathfrak{D}_{\text{IV}}^{G,\xi} = \left(\begin{array}{c} \mathbb{M}_{0,\text{inc}}, \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}, \mathcal{A}_{G,\xi,0}^W, \mathbf{Rep}_{G,\xi,0}^{\text{thin}}, \hat{\Omega}_{G,\xi,0}^b, \mathcal{S}_{G,\xi,0}^{\text{st,as,*}}, \mathfrak{K}_{G,\xi}^{\text{Orl}}, \mathfrak{W}_{G,\xi}^{\mathcal{S}}, \\ \bar{\mathbf{q}}_{G,\xi,0}, \mathbf{P}_{\Omega,G,\xi,0}, \mathcal{C}_{\text{hyp}}, U_{\text{hyp}}, H_{\text{hyp}}, c_E, \kappa_{\text{IV,thin}}, \text{Restart}_{G,\xi}, \kappa_{\xi}, G, \Gamma, P_G, \mathcal{W}_{G,\xi} \end{array} \right). \quad (55)$$

Here $\mathcal{W}_{G,\xi}$ is the selected quotient-compatible Wilson-channel frame. The functor $\mathcal{A}_{G,\xi,0}^W$ and the represented pseudodiagram $\mathbf{Rep}_{G,\xi,0}^{\text{thin}}$ have different targets. The family $\hat{\Omega}^b$ is the abstract complete-ground pointing or pointed subobject, whereas $\mathbf{P}_{\Omega}$ is only the family of orthogonal projections on Hilbert form carriers. The symbol $\mathcal{S}^{\text{st,as,*}}$ is a graph on normal states. The entries $\mathcal{C}_{\text{hyp}}$, U_{hyp} , H_{hyp} , and $c_E = 1$ are the direct physical hyperbolic output of theorems X.3, X.5 and X.6. Bold symbols denote branch-appropriate indexed families; they become one global represented Hilbert-space object only after an independent effective-descent theorem. For scalar branches,

$$\kappa_{\xi} = (\kappa_{\xi}^{\text{fib}}, \hat{\kappa}_{\xi}^{\text{cs}}),$$

whereas a correspondence-valued branch retains $\kappa_{\xi}^{\text{Corr}}$ separately.

Firewall XIV.1 (Closed-form and global-carrier handoff) *Paper V receives the closed branchwise regional form $\bar{\mathbf{q}}_{G,\xi,0}$, not a preform-only promise, together with the selected physical hyperbolic generator $H_{\text{hyp}} = K_{\text{thin}}^{1/2}$ on every descent-compatible certified carrier. Any Paper-V preform must be a declared core restriction of this same closed form. Paper V must separately construct the full joint translation representation on the global or glued represented carrier, prove that its selected time-axis restriction agrees with U_{hyp} , assemble the regional complete-ground projections, and prove the invariant-mass orbit theorem. Until those theorems are established,*

$$\bar{\mathbf{q}}_{G,\xi,0} \not\equiv \{K_{G,\xi,0;O}^{\text{thin}}\}_O \not\equiv M_{\text{thin}}^2$$

as typed objects, and no second competing “thin form” is introduced by notation alone.

B. Certification shell

Define

$$\mathfrak{C}_{\text{IV}}^{G,\xi} = \left(\mathbf{Rec}_{G,\xi}^{\text{adm}}, \mathfrak{C}_{\varepsilon \rightarrow 0}^{G,\xi}, \mathfrak{G}_{\varepsilon \rightarrow 0}^{G,\xi}, \mathfrak{F}_{\varepsilon \rightarrow 0}^{G,\xi}, \mathfrak{S}_{\varepsilon \rightarrow 0}^{G,\xi}, \text{ChoiceInd}_{G,\xi}, \text{Prov}_{G,\xi} \right). \quad (56)$$

The entries record, respectively, the admissible recovery category, comparison certificate, complete-ground input certificate, comparison–Mosco certificate, sector/covariance/time-slice certificate, comparable-choice independence within the same incision and branch data, and theorem-level provenance.

C. Incision decision and obligation ledger

Before any invariant-mass statement is well typed, Paper V must choose and prove one of two geometric continuations.

- (I1) *Auxiliary-incision continuation.* Construct one ambient unincised observable net $\mathcal{A}_{G,\xi}^{W,\text{amb}}$ and coherent restriction isomorphisms from the Paper-IV incised family, prove that incision-moving arrows are restrictions of ambient spacetime automorphisms, and only then seek a full Poincare representation and invariant mass operator.
- (I2) *Physical-defect continuation.* Retain Σ as physical structure, use the stabilizer of the complete controlled incision datum as the spacetime symmetry group, and formulate a stabilizer-covariant defect-energy or defect-spectrum theorem rather than the standard Lorentz-invariant Yang–Mills mass gap.

A transformation moving Σ is currently a morphism between controlled backgrounds, not an automorphism of one fixed physical fiber. Quantizing the orbit of incision positions is not substituted for ambient descent.

Write the unresolved Paper-V ledger as

$$\mathfrak{D}_V = \mathfrak{D}_V^{\text{inc}} \sqcup \mathfrak{D}_V^{\text{state}} \sqcup \mathfrak{D}_V^{\text{spec}} \sqcup \mathfrak{D}_V^{\text{int}}. \quad (57)$$

The geometric ledger $\mathfrak{D}_V^{\text{inc}}$ contains the preceding auxiliary-incision versus physical-defect decision and its proof. After the corresponding algebraic symmetry action has been constructed, $\mathfrak{D}_V^{\text{state}}$ requires a candidate locally normal state invariant under that action, followed by the GNS implementation and its continuity properties.

The authoritative spectral ledger is

$$\mathfrak{D}_V^{\text{spec}} = \{\text{V.O1}, \dots, \text{V.O12}\},$$

with the following independent obligations.

- V.O1 Construct the sector-appropriate strongly continuous spacetime representation: full Poincare only after auxiliary-incision descent, or the declared stabilizer representation in the physical-defect continuation.
- V.O2 Reconstruct the relevant commuting self-adjoint generators and their joint spectral measure.
- V.O3 Prove the forward-cone spectrum condition in the ambient Poincare continuation, or state and prove the correct stabilizer-appropriate spectral condition in the defect continuation.
- V.O4 Prove that the Paper-V time-axis subgroup agrees with the exported U_{hyp} on the same branch-appropriate carrier and then use the joint translation spectrum to identify the invariant-mass threshold.
- V.O5 Prove physical non-leakage from the complete-ground face into nonvacuum zero spectral modes.
- V.O6 Transfer the Paper-IV off-ground stiffness threshold to the physical spectral quantity with every comparison factor explicit.
- V.O7 Prove regional, Cauchy, chart-corridor, and sector-gluing stability of the spectral exclusion.
- V.O8 Prove covariance of the ground and positive-spectrum spectral projections under the constructed spacetime representation.
- V.O9 Determine the multiplicity of the invariant ground or vacuum sector branchwise.
- V.O10 Prove compatibility of the physical spectral decomposition with the Gauss–Ward prepared domain, interior constraints, and retained collar data.
- V.O11 Prove the final end-to-end no-leakage theorem using the comparison, ground, form, sector, and gluing certificates together.
- V.O12 State the final branchwise mass-gap or defect-gap theorem with exact scope and conditionality.

The dynamical ledger is separate:

$$\mathfrak{D}_V^{\text{int}} = \{\text{V.INT1}\},$$

where V.INT1 must prove observable non-equivalence to every admissible free or generalized-free comparator using a bounded gauge-invariant dynamical invariant. Strictness and twisting are not interaction criteria.

D. Complete handoff certificate

Theorem XIV.2 (Exact incised Paper-IV to Paper-V handoff (Theorem H3))

For every branch certified by Theorem XIII.1, Paper IV passes

$$\boxed{\widehat{\mathfrak{P}}_{IV \rightarrow V}^{G,\xi} = \left(\mathfrak{D}_{IV}^{G,\xi}, \mathfrak{C}_{IV}^{G,\xi}, \mathfrak{D}_V \right).} \quad (58)$$

This certificate, augmented by theorem XV.1, contains every Paper-IV geometric, observable, represented, expectation/entropy, Gauss–Ward/collar, form, selected hyperbolic, ground, threshold, restart, coherence, comparison, choice-independence, and provenance datum declared as input to the Paper-V program. It contains none of the conclusions in $\mathfrak{D}_V$.

Proof. The data core collects the outputs of Theorems V.1–XI.7 and the physical branch certificate where applicable. The certification shell records the comparison, ground, form, sector, covariance, time-slice, choice-independence, and provenance structures used to establish those outputs. The obligation ledger names every geometric, state, spectral, and interaction bridge not constructed in Paper IV. Because the incision is retained as part of both $\mathfrak{D}_{IV}^{G,\xi}$ and $\mathfrak{C}_{IV}^{G,\xi}$, no full fixed-fiber Poincare action is silently imported. $\square$

Firewall XIV.3 (Interaction, coherence, spectrum, and incision) *A nonzero associator is a coherence obstruction, not a dynamical interaction theorem. A coherently strict Sector-I branch is not thereby free. The identity*

$$\ker \bar{\mathbf{q}}_{G,\xi,0} = \text{Ran } \mathbf{P}_{\Omega,G,\xi,0}$$

is a branchwise stiffness-kernel statement, not physical spectral no-leakage. A controlled-background-covariant incised functor is not one fixed-fiber Poincare-covariant theory. Paper V must prove every item of $\mathfrak{D}_V$ independently.

XV. HYPERBOLIC AUGMENTATION OF THE PAPER-IV-TO-PAPER-V HANDOFF

Theorem XV.1 (Direct physical hyperbolic handoff) *At the branchwise scope of Section X, Paper IV exports directly*

$$(\mathfrak{H}_{\text{thin}}, \Omega_{\text{thin}}, P_{\text{gr}}, \bar{\mathfrak{q}}_{\text{thin}}, \mathcal{C}_{\text{hyp}}, U_{\text{hyp}}, H_{\text{hyp}}, c_E, \kappa_{\text{IV,thin}}), \quad (59)$$

where

$$U_{\text{hyp}}(t) = e^{-itH_{\text{hyp}}}, \quad H_{\text{hyp}} \geq 0, \quad (60)$$

$$D(H_{\text{hyp}}) = D(\bar{\mathfrak{q}}_{\text{thin}}), \quad \|H_{\text{hyp}}\Psi\|^2 = c_E \bar{\mathfrak{q}}_{\text{thin}}[\Psi], \quad (61)$$

$$c_E = 1, \quad P_{\text{gr}} = \mathbf{1}_{\{0\}}(H_{\text{hyp}}), \quad (62)$$

$$H_{\text{hyp}}^2 \geq c_E \kappa_{\text{IV,thin}}(1 - P_{\text{gr}}). \quad (63)$$

The complete-ground projection may have admissible multiplicity.

Scope XV.2 (Supersession of the former bridge boundary) *Theorem XV.1 supersedes only the former statement that Paper V must obtain the selected physical generator/form bridge from an external comparison representation. Paper V now consumes the physical thin dynamics on the physical thin carrier. With the Paper-V Fourier convention $U(a) = e^{iP \cdot a}$, it must prove that its full translation representation restricts on the selected time axis as*

$$U_{\text{trans}}(-te_0) = U_{\text{hyp}}(t),$$

and must separately prove the joint spectrum, spectrum condition, invariant-mass construction, non-ground null-orbit exclusion, physical no-light-cone leakage, and final invariant-mass threshold.

Firewall XV.3 (Constant ownership) *The Paper-IV constant*

$$\Delta_{E,\text{IV}}^2 = c_E \kappa_{\text{IV,thin}}$$

is an energy-squared threshold in the selected frame. The Paper-V invariant-mass threshold remains a separately named theorem output even when a later orbit theorem proves numerical equality.

XVI. DISCUSSION AND SCOPE

A. Unconditional and branch-conditional conclusions

The reconstruction theorem is conditional only on the primitive packages **OJ**, **RJ**, **SJ**, **GJ**, and **M**. Analytical comparison non-vacuity is unconditional for the declared strict root-datum branch of every compact connected semisimple global form and for the scalar cyclic Sector-III branch of every selected nontrivial center band. These are Level-B witnesses, not claims of unique physical Yang–Mills realization. The canonical physical $SU(3)$ branches verify the full primitive package by theorem-level provenance. They give Level-C Sector I at $(m, r) = (0, 0)$, branch-specific Level-C Sector II for $m = 0, r = 1, 2$, and Level-C Sector III for $m = 1, 2$ with arbitrary r . Sector II is not universalized beyond that declared family. The correspondence-valued Markov-shadow theorem is complete as a transfer statement with the channel typed $M \rightarrow N$ and the predual state map $N_* \rightarrow M_*$. Correspondence-valued targeting does not itself prove a nontrivial associator; a genuinely non-scalar physical witness and component detector remain open.

B. Why the global form matters

Two compact groups with the same Lie algebra can have different allowed highest weights and different centers. The quotient-compatible lattice P_G therefore changes the Wilson channels that exist as genuine G -representations and changes the selected center bands available for scalar twisting. The branch certificate must retain (G, Γ, P_G) ; suppressing the global form would identify physically different observable systems.

C. What the construction avoids

The theorem does not use pointwise thin gauge fields, a bare global thin Hamiltonian, globally labelled thin eigenvectors, arbitrary tensor partial traces, full Haag duality, or an assumed global physical Hilbert descent. The use of bounded packet sections and expectation graphs is not a technical detour: it is what permits a global observable net to coexist with twisted or correspondence-valued represented descent.

D. The threshold and the remaining spectral bridge

The exact kernel theorem eliminates additional null directions in the thin stiffness form and proves a positive off-ground form threshold on the admissible coercive reconstruction class. The hyperbolic extension identifies the positive square root of that stiffness operator with the selected physical timelike evolution on every descent-compatible certified carrier. This is the strengthened endpoint of Paper IV. To convert the selected-frame energy threshold into a mass gap, Paper V must still complete the incision continuation, construct the full Poincare representation where available, prove the forward-cone and Lorentz-orbit statements, and exclude leakage through the gluing, gauge, or sector interfaces. Only after that orbit bridge is proved can $c_{EKIV, \text{thin}}$ be interpreted as the invariant-mass-squared threshold⁸.

E. Final perspective

The result separates three logically different phenomena that are often conflated: bounded observable descent, represented coherence, and dynamical spectrum. The first is a selected incised realization of the Haag–Kastler and locally covariant paradigms, not an assertion of a universal unincised theory.^{1–3} Concretely, it is an ordinary functor on the selected controlled incised category. The second may be strict, scale obstructed, fiber twisted, or correspondence valued. The third is not addressed until the Poincare and interaction structures are constructed. This separation is the mechanism that makes the thick-to-thin theorem both general and constructively testable without overclaiming the mass gap.

Scope reminder. Paper IV ends with a sector-preserving, descent-compatible thin reconstruction and a positive selected-frame threshold. It does not identify a Poincare-invariant state, construct the full translation spectrum, prove physical no leakage, establish interaction, or form the invariant mass operator. Those are separate Paper-V obligations and are not consequences of the reader-interface additions made here.

SUPPLEMENTARY MATERIAL

See the supplementary material for the extended observable and represented descent modules, SLSP and Gauss–Ward details, closed-form and entropy-surgery estimates, $SU(2)$

and $SU(3)$ branch certificates, theorem and scope audits, Paper-V handoff documentation, and supporting reproducibility material used in this article.

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USE OF GENERATIVE ARTIFICIAL INTELLIGENCE

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The conceptual framework, ontologies, definitions, conjectures, identification of the principal obstructions, load-bearing components of the proofs, and overall proof strategies originated with and remain solely those of the author.

All mathematical claims, definitions, theorems, proofs, computations, interpretations, citations, and final wording were reviewed and approved by the author, who assumes full responsibility for the content of the manuscript. No generative artificial-intelligence system is credited as an author.

AUTHOR DECLARATIONS

Conflict of Interest

The author has no conflicts to disclose.

Author Contributions

Jeffrey S Little: Conceptualization; Formal analysis; Investigation; Methodology; Software; Validation; Writing–original draft; Writing–review and editing.

DATA AVAILABILITY

The data and code that support the findings of this study are available within the article, its supplementary material, and the accompanying fixed-version Paper-IV reproducibility package. The package contains the source, machine-readable theorem and claim maps, independent verification programs, generated certificates, build documentation, and SHA-256 manifests. Reconstructed verification code is labeled as such and is not represented as recovered original source.

Appendix A: Detailed proof modules

This section records the proof modules that are repeatedly invoked in the principal theorems. They are included in the journal article because each module contains a load-bearing passage that cannot be replaced by a formal appeal to compactness or functoriality. All statements are made inside a fixed declared branch and on a fixed finite regional–Čech–scale diagram unless otherwise indicated.

1. Bounded testing sections and the selected normal functor

For a controlled object $X = (\mathbb{M}_{0,\text{inc}}, O)$, bounded comparison sections are evaluated in the common left-, right-, and involution-saturated packet seminorm

$$p_{n;X}^{\text{sat}}(x) := \sup_{\rho \in \mathfrak{R}_{n;X}^{\text{sat}}} |\rho(x)|.$$

The null sections form a closed two-sided $*$ -ideal because saturation gives

$$p_{n;X}^{\text{sat}}(a_n b_n), p_{n;X}^{\text{sat}}(b_n a_n) \leq \|b\|_{\text{sec}} p_{n;X}^{\text{sat}}(a_n).$$

Thus

$$\mathcal{B}_0(X) = \overline{\mathfrak{W}_{\text{cmp}}(X) / \mathfrak{N}_0(X)}$$

is a well-defined C^* -algebra without any inference from packet-null convergence to operator-norm convergence.

OJ3 reconstructs a separating norming family $\mathfrak{S}_0(X)$ of thin states. The selected faithful representation and normal envelope are

$$\pi_X^{\text{sel}} = \bigoplus_{\rho \in \mathfrak{S}_0(X)} \pi_\rho, \quad \mathcal{A}_{G,\xi,0}^W(X) = \pi_X^{\text{sel}}(\mathcal{B}_0(X))''.$$

For an admissible arrow $\psi : X \rightarrow Y$, the exact state restriction/extension relation

$$(\alpha_\psi^0)^* \mathfrak{S}_0(Y) = \mathfrak{S}_0(X)$$

makes $\pi_Y^{\text{sel}} \circ \alpha_\psi^0$ quasi-equivalent to π_X^{sel} . Hence the testing arrow has a unique normal unital injective extension. This proves normal isotony without assuming that a bidual map carries one selected central corner onto another.

2. Incised locality, packet additivity, and time slice

Locality is tested only for $O_1 \perp_U O_2$ in a common incision-adapted ambient object. Positive relative causal margin and positive clearance from Σ keep all sufficiently thin carriers both spacelike and inside $M_{\text{inc}}(\mathbb{M}_{0,\text{inc}})$. The positive-thickness locality theorem makes the commutator packet null; the two testing images commute, and so do their selected von Neumann closures.

For a declared packet-localizing cover, finite subdivision and physical recombination stay inside the incised category and have null defect. This gives C^* generation, while OJ3 and the selected-representation saturation clause in OJ5 identify the source normal closures inside the target, giving normal additivity.

For a selected Cauchy arrow inside an incision-adapted globally hyperbolic development, OJ6 supplies bounded comparison-coherent source sections, not merely componentwise preimages. They make the thin testing arrow surjective. Its normal extension is therefore an isomorphism, and the resulting local Cauchy maps form a strict groupoid after the OJ7–OJ8 comparison cubes are quotiented. No propagation through or across the incision occurs.

3. Finite-window expectation algebras

Fix a bounded selected region D and a good finite window N . Let

$$K_{n,D}^{(N)} = p_{n,D}^{(N)} K_{n,D}^{\text{sel}} p_{n,D}^{(N)}$$

be the selected finite-dimensional compression, and let $\sigma_{n,D}^{(N)}$ be the faithful compressed reference density. The positive-thickness conditional expectation is the unique normal map

$$\mathbb{E}_{n,D}^{(N)} : \mathcal{N}_{n,D}^{(N)} \rightarrow \mathcal{F}_{n,D}^{(N)}$$

that fixes the finite-window algebra $\mathcal{F}_{n,D}^{(N)}$ and preserves the reference state. In finite dimension it is the orthogonal projection onto $\mathcal{F}_{n,D}^{(N)}$ for the weighted inner product

$$\langle x, y \rangle_{\sigma_n} = \text{Tr}(\sigma_n^{1/2} x^* \sigma_n^{1/2} y).$$

Lemma A.1 (Expectation convergence) *Work on one common finite dimensional carrier. Assume*

$$d_{\text{gap}}(\mathcal{F}_{n,D}^{(N)}, \mathcal{F}_{0,D}^{(N)}) \rightarrow 0, \tag{A1}$$

$$\|\sigma_{n,D}^{(N)} - \sigma_{0,D}^{(N)}\| \rightarrow 0, \quad \|\log \sigma_{n,D}^{(N)} - \log \sigma_{0,D}^{(N)}\| \rightarrow 0. \tag{A2}$$

Then

$$\|\mathbb{E}_{n,D}^{(N)} - \mathbb{E}_{0,D}^{(N)}\|_{1 \rightarrow 1} + \|\mathbb{E}_{n,D}^{(N)} - \mathbb{E}_{0,D}^{(N)}\|_{\infty \rightarrow \infty} \longrightarrow 0.$$

Proof. The faithful densities make the weighted Hilbert norms uniformly equivalent to the Hilbert–Schmidt norm. Equation (A2) makes the weighted Gram matrices converge. Equation (A1) gives convergence of bases for the fixed-point subspaces. The matrix formula for the weighted orthogonal projection therefore converges in operator norm. Finite-dimensional norm equivalence converts this to the stated $1 \rightarrow 1$ and $\infty \rightarrow \infty$ convergence. $\square$

Lemma A.2 (Expectation-algebra uniqueness) *Every convergent subsequence of transported good-window expectations has the same limit. Hence the limiting expectation and fixed-point algebra are independent of subsequence.*

Proof. Smooth functional-calculus comparison determines the limit on a generating family of spectral functions. Any subsequential limit is a normal idempotent preserving the limiting

faithful reference and fixing the same limiting algebra. By the uniqueness of the reference-preserving conditional expectation onto a fixed finite-dimensional algebra, the subsequential limits agree. The full family therefore converges. $\square$

If $N_1 < N_2$ are coherent good windows, the tower condition is

$$\mathbb{E}^{(N_1),\text{as}} \mathbb{E}^{(N_2),\text{as}} = \mathbb{E}^{(N_1),\text{as}} = \mathbb{E}^{(N_2),\text{as}} \mathbb{E}^{(N_1),\text{as}}.$$

This follows because the larger expectation fixes the smaller fixed-point algebra and both preserve the compatible reference. Thus the finite-window graphs form a directed tower rather than a collection of unrelated pinches.

4. Orlicz control and regulator removal

Let

$$L_n^{\text{ref}} = -\log \sigma_n$$

on the faithful reference support, and suppose

$$\sup_{n, \rho \in \mathfrak{K}_n} \text{Tr } \rho \Phi(L_n^{\text{ref}}) \leq M \tag{A3}$$

for a superlinear Young function Φ . Put

$$p_{n,R} = \mathbf{1}_{[0,R]}(L_n^{\text{ref}}), \quad m_R = \sup_{n, \rho} \text{Tr } \rho(1 - p_{n,R}).$$

Lemma A.3 (Uniform mass and logarithm tails) *Under (A3),*

$$m_R \leq \frac{M}{\Phi(R)} \longrightarrow 0, \tag{A4}$$

$$\ell_R := \sup_{n, \rho} \text{Tr } \rho L_n^{\text{ref}}(1 - p_{n,R}) \leq M \sup_{t \geq R} \frac{t}{\Phi(t)} \longrightarrow 0. \tag{A5}$$

Proof. On $\{L_n^{\text{ref}} > R\}$, $\Phi(L_n^{\text{ref}}) \geq \Phi(R)$, giving (A4). Also

$$L_n^{\text{ref}} \mathbf{1}_{(R,\infty)}(L_n^{\text{ref}}) \leq \left(\sup_{t \geq R} \frac{t}{\Phi(t)} \right) \Phi(L_n^{\text{ref}}),$$

which gives (A5). Superlinearity makes the supremum tend to zero. $\square$

The discarded entropy is controlled separately. Set

$$\rho_R = p_R \rho p_R, \quad \hat{\rho}_R = \rho_R / \text{Tr } \rho_R.$$

A block-decomposition estimate gives

$$|S(\rho) - S(\rho_R)| \leq h_2(m_R) + m_R R + \ell_R + r_R, \quad (\text{A6})$$

where h_2 is the binary entropy and $r_R \rightarrow 0$ controls the normalized compression and off-diagonal block. The important logical point is that (A5) precedes (A6); relative entropy alone is not used to manufacture reference-logarithm integrability.

Lemma A.4 (Common entropy modulus) *On a fixed packet satisfying (A3), there is a modulus $\omega_{M,\Phi}$, independent of n on the finite diagram, such that*

$$|\mathcal{E}_n(\rho) - \mathcal{E}_n(\rho')| \leq \omega_{M,\Phi}(\|\rho - \rho'\|_1), \quad \omega_{M,\Phi}(t) \downarrow 0 \quad (t \downarrow 0).$$

Proof. Choose R so that the two entropy tails in (A6) are below a prescribed tolerance. On the finite-dimensional carrier $p_R \mathcal{H}$, apply a finite-dimensional continuity bound to the normalized compressions. The normalization error is controlled by m_R , uniformly in n . First choose R and then the trace-norm tolerance. This yields one modulus for the full finite diagram. $\square$

5. Sharp Cech descent

For a face arrow $a : \sigma \rightarrow \tau$, let $R_{n,a}^*$ be the normal-state restriction. A smooth buffered selector $f_{N,d}$ satisfies

$$\sup_{\rho \in \mathfrak{K}_n} \|R_{n,a}^* f_{N,d}(K_{n,\sigma})^* \rho - f_{N,d}(K_{n,\tau})^* R_{n,a}^* \rho\|_1 \leq \eta_{n,N,d}, \quad (\text{A7})$$

with $\eta_{n,N,d} \rightarrow 0$ for fixed N, d . This estimate uses the local Laplace-type principal symbol, the fixed buffer, and common-carrier functional calculus. It is not sharp spectral locality.

Let $\mathbb{E}_{n,\sigma}^{(N)}$ and $\mathbb{E}_{n,\tau}^{(N)}$ be the sharp good-window expectations. The difference between a sharp expectation and its smooth approximation is concentrated in the transition strip and above the window. By theorems A.3 and A.4,

$$\sup_{n, \rho \in \mathfrak{K}_n} \|\mathbb{E}_n^{(N),*} \rho - f_{N,d}(K_n)^* \rho\|_1 \leq \alpha_N + \beta_d, \quad (\text{A8})$$

where $\alpha_N \rightarrow 0$ as $N \rightarrow \infty$ and $\beta_d \rightarrow 0$ as $d \downarrow 0$, after the thickness limit has been taken.

Combining (A7) and (A8) gives

$$\limsup_{n \rightarrow \infty} \sup_{\rho \in \mathfrak{K}_n} \|R_{n,a}^* \mathbb{E}_{n,\sigma}^{(N),*} \rho - \mathbb{E}_{n,\tau}^{(N),*} R_{n,a}^* \rho\|_1 \leq 2(\alpha_N + \beta_d).$$

Then $N \rightarrow \infty$ and $d \downarrow 0$ prove sharp face descent. The safe limit order is therefore not a stylistic preference; reversing the last three limits would remove the uniform tail estimates before they are available.

6. Rooted simultaneous lifting

Let $\mathcal{D} = (V, E, C_2, C_3; v_0)$ be a finite rooted selected incised diagram. Pairwise recovery is insufficient because independently chosen edge families may disagree at shared vertices. The corrected construction begins with one root recovery $\rho_{v_0, n}$ and propagates it through the strict unsurgered normal-state shadow:

$$\rho_{v, n}^{\text{pre}} = T_{p_v, n}^* \rho_{v_0, n}.$$

All unsurgered strict faces close exactly. Apply the source-defined sharp surgery objectwise,

$$\hat{\rho}_{v, n} = \mathcal{S}_{n; v}^{\text{st}, *} \rho_{v, n}^{\text{pre}}.$$

For an edge $e : v \rightarrow w$, strict unsurgered transport gives

$$\|T_{e, n}^* \hat{\rho}_{v, n} - \hat{\rho}_{w, n}\|_1 \leq \Delta_e(n; N, d),$$

where Δ_e is the sharp single-arrow surgery defect. A two-cell or higher strict-shadow cell is controlled by a finite telescoping sum of its edge defects. The scalar coefficient lift is not strictified: it retains the prescribed tensorators and associator.

Define one master defect

$$\mathfrak{D}_{\mathcal{D}}(n; N, d) = d_{\mathcal{D}, n}^{\text{root}} + c(\mathcal{D}) \max_{e \in E} \Delta_e(n; N, d) + \tau_{\mathcal{D}}^{\text{Orl}}(N, d).$$

At fixed diagram, buffer, and window, first let $n \rightarrow \infty$; then remove N and finally d . The master defect tends to zero. Hence the same family satisfies all vertices, edges, and faces simultaneously. No product of affine idempotents, alternating projection, averaging, or pairwise stitching occurs.

Lemma A.5 (Path independence in the retained branch type) *The rooted family is independent of the chosen bookkeeping paths in the observable and normal-state shadow. In a scalar line-twisted branch, represented path lifts differ exactly by the retained coefficient-line composite.*

Proof. Every path loop decomposes into finitely many selected faces. Telescoping bounds the strict-shadow discrepancy by a finite multiple of the maximal edge defect, which vanishes in the mandatory regulator order. The represented coefficient composite is retained rather than estimated to zero. $\square$

7. Gauss–Ward closure under surgery

The limiting constraint package has two coordinates: the vector-form Gauss graphs $\mathbb{G}_{\zeta,0}^{\text{int}}$ and the observable insertion derivations $\delta_{\zeta,0}^{\text{int}}$. For graph-convergent approximants, both the first and second coordinates converge, and the no-vertical-limit estimate gives closability. The complete-ground statement is transported independently as

$$\text{Ran } P_{\Omega,0} \subseteq \bigcap_{\zeta} \ker \mathbb{G}_{\zeta,0}^{\text{int}}.$$

The full Ward functional is

$$\begin{aligned} \mathfrak{w}_{\zeta,0}^{\text{int}}(A)[\Psi, \Phi] &= i\mathbb{G}_{\zeta,0}^{\text{int}}[\pi(A)\Psi, \Phi] - i\mathbb{G}_{\zeta,0}^{\text{int}}[\Psi, \pi(A^*)\Phi] \\ &\quad - \langle \pi(\delta_{\zeta,0}^{\text{int}}A)\Psi, \Phi \rangle. \end{aligned}$$

At each fixed window the expectation preserves both graph domains and satisfies

$$\mathbb{E}_0^{(N),\text{as}} \delta_{\zeta,0}^{\text{int}} = \delta_{\zeta,0}^{\text{int}} \mathbb{E}_0^{(N),\text{as}},$$

together with the graph-form intertwining identity

$$\mathfrak{w}_{\mathbb{E}_0^{(N),\text{as}},*,\rho,0}^{\text{int}}(\zeta; A) = \mathfrak{w}_{\rho,0}^{\text{int}}(\zeta; \mathbb{E}_0^{(N),\text{as}} A).$$

For a finite mixed diagram, augment the SLSP master defect by the graph-core, Ward-functional, collar-quotient, character, current-defect, and boundary-Ward errors. Strict unsurgered rooting, objectwise surgery, and edge/higher-cell telescoping produce one simultaneous family; no constraint projection is used. A uniform Ward tail modulus removes the window and buffer, yielding

$$\mathfrak{w}_{\mathcal{J}^{\text{st,as}},*,\rho,0}^{\text{int}}(\zeta; A) = 0$$

for every prepared state. The collar quotient, the two-parameter current defect $\mathfrak{a}^\partial(\lambda, \mu)$, the full chart-scale boundary-character class, and the first boundary-Ward defect $\mathfrak{b}^\partial(\lambda; A)$ are transported structurally. No numerical charge equality or anomaly cancellation follows.

8. Comparison–Mosco convergence and the kernel

On varying carriers use separate maps

$$J_n : H_n \rightarrow H_0, \quad I_n : H_0 \rightarrow H_n,$$

which are asymptotically inverse on graph-bounded sets. Define the strong relaxed functional by

$$q_0^{\text{rel}}[u] = \inf_{u_n \rightarrow u \text{ cmp, strong}} \liminf_n q_n[u_n].$$

Synchronized recovery for $u, v, u + v, u - v$ transfers the finite-stage parallelogram identity and proves that this functional is quadratic. A diagonal closed-sublevel argument proves lower semicontinuity and closedness.

Comparison–Mazur convexification gives the weak liminf theorem. Core recovery, graph-density extension, finite-diagram synchronization, and one regional-exhaustion diagonal give full TT2:

$$J_n u_n \rightarrow u, \quad q_n[u_n] \rightarrow q_0^{\text{rel}}[u].$$

The moving ground-face estimate gives graph-domain TT1:

$$J_n P_n u_n \rightarrow P_0 u, \quad J_n (\mathbf{1} - P_n) u_n \rightarrow (\mathbf{1} - P_0) u.$$

Ground-compatible recovery uses

$$u_n^\# = P_n I_n P_0 u + (\mathbf{1} - P_n) v_n,$$

together with form-domain invariance, exact ground nullity, and form orthogonality.

To transfer coercivity, choose a TT2 recovery and use TT1:

$$\begin{aligned} q_0^{\text{rel}}[u] &= \lim_n q_n[u_n] \\ &\geq \kappa_{\text{IV, thin}} \lim_n \|(\mathbf{1} - P_n) u_n\|^2 \\ &= \kappa_{\text{IV, thin}} \|(\mathbf{1} - P_0) u\|^2. \end{aligned}$$

The Mosco-liminf inequality is not used in the wrong direction. Coercivity yields $\ker q_0^{\text{rel}} \subseteq \text{Ran } P_0$, while the exact zero-energy ground recovery yields the reverse inclusion. The associated regional operator has spectrum in $\{0\} \cup [\kappa_{\text{IV, thin}}, \infty)$. This is a local or branchwise stiffness statement, not a statement about spacetime translations or invariant mass.

9. Root data and primitive center detection

Let $\tilde{G}$ be the simply connected compact semisimple cover of G and let P and Q be its weight and root lattices. The central character of the irreducible highest-weight representation V_λ depends only on $[\lambda] \in P/Q$, and

$$P/Q \cong \widehat{Z(\tilde{G})}.$$

The representation descends to $G = \tilde{G}/\Gamma$ precisely when Γ acts trivially. Therefore

$$P_G/Q = \text{Ann}_{\widehat{Z(\tilde{G})}}(\Gamma) \cong \widehat{Z(\tilde{G})/\Gamma} = \widehat{Z(G)}.$$

The Weyl group acts trivially on P/Q because

$$s_\alpha(\lambda) - \lambda = -\langle \lambda, \alpha^\vee \rangle \alpha \in Q.$$

Hence every central-character class has a dominant representative.

Let $h \in Z(G)$ have order $n > 1$. The character $h \mapsto e^{2\pi i/n}$ of $\langle h \rangle$ extends to $Z(G)$ because $U(1)$ is divisible and therefore injective as a $\mathbb{Z}$ -module. The isomorphism $P_G/Q \cong \widehat{Z(G)}$ yields a weight class with this character. Choosing a dominant representative gives an allowed irreducible G -representation on which h acts primitively. This argument is stronger than mere Peter–Weyl separation: it produces one quotient-compatible irreducible channel rather than only a product of characters that separates h .

For a finite selected band

$$A \cong \mathbb{Z}_{n_1} \oplus \cdots \oplus \mathbb{Z}_{n_s},$$

repeat the argument for primitive generators of $\hat{A}$. The resulting dominant weights give a finite primitive Wilson-channel frame. Thus all global-form dependence is concentrated in P_G and the selected finite frame; the local Laplace-type analysis remains unchanged.

10. Exact-dilation form estimates

Let J_n straighten the physical regional form domain to $\mathcal{Q}$ and put

$$\hat{q}_n^J[u, v] = \langle (V + \lambda_n W_n)u, v \rangle + \eta(1 + \lambda_n d_n) \langle (\nabla + \lambda_n C_n)u, (\nabla + \lambda_n C_n)v \rangle,$$

where $\lambda_n = \varepsilon_n^\alpha$. Assume

$$|\langle W_n u, v \rangle| \leq C_W \|u\|_{\mathcal{Q}} \|v\|_{\mathcal{Q}}, \quad |d_n| \leq C_\eta, \quad \sqrt{\eta} \|C_n u\| \leq C_C \|u\|_{\mathcal{Q}}.$$

Expanding the covariant term and using Cauchy–Schwarz gives

$$|\hat{q}_n^J[u] - q_{\text{ref}}[u]| \leq C_{\text{form}} \varepsilon_n^\alpha \|u\|_{\mathcal{Q}}^2, \quad (\text{A9})$$

with

$$C_{\text{form}} = C_W + C_\eta + 2(1 + C_\eta)C_C + (1 + C_\eta)C_C^2.$$

For $C_{\text{form}} \varepsilon_n^\alpha < 1/2$, the graph norms are uniformly equivalent. If $A_{\text{ref}} \geq \kappa_{\text{cof}} I$ on the excitation subspace, then

$$\hat{A}_n^J \geq \frac{1}{2} \kappa_{\text{cof}} I.$$

Thus the exact-dilation witness admits the explicit cofinal-tail choice

$$\kappa_{\text{IV,thin}} := \frac{1}{2} \kappa_{\text{cof}} \quad (\text{A10})$$

whenever $C_{\text{form}} \varepsilon_n^\alpha < 1/2$. This is a certified lower-bound choice for that branch, not a claim that the optimal Paper-IV constant equals one half of the Paper-I packet floor on every branch. The resolvent identity in form scale yields, for $\mu \geq 0$,

$$\|(\hat{A}_n^J + \mu)^{-1} - (A_{\text{ref}} + \mu)^{-1}\| \leq \frac{2C_{\text{form}} \varepsilon_n^\alpha}{\kappa_{\text{cof}} + \mu}.$$

Functional-calculus convergence follows for bounded continuous functions and, after a positive spectral cutoff, for the fractional powers used in the positive-frequency structure. The principal propagation speed changes only by $O(\varepsilon_n^\alpha)$ because the lower-order potential is pointwise and the principal stiffness coefficient is $\eta(1 + \lambda_n d_n)$. No type-(A) matrix identity enters.

11. The cyclic cocycle and the $SU(3)$ detector

For $A = \mathbb{Z}_n$, choose representatives $\tilde{a} \in \{0, \dots, n-1\}$ and define

$$\epsilon_n(b, c) = \frac{\tilde{b} + \tilde{c} - \widetilde{b+c}}{n}.$$

The oriented carry cocycle is

$$\Omega_s(a, b, c) = \exp\left(\frac{2\pi i s}{n} \tilde{a} \epsilon_n(b, c)\right), \quad s = \pm 1.$$

The carry identity

$$\epsilon_n(b, c) + \epsilon_n(b+c, d) = \epsilon_n(c, d) + \epsilon_n(b, c+d)$$

proves the pentagon. The class is the oriented generator $\mathfrak{su}_n$.

For $n = 3$, define

$$I_3(\Xi) = \prod_{b=0}^2 \Xi(1, b, 1).$$

For a normalized two-cochain λ , substitution into the inhomogeneous differential gives

$$(\delta\lambda)(1, 1, 1) = \frac{\lambda(1, 2)}{\lambda(2, 1)}, \quad (\delta\lambda)(1, 2, 1) = \frac{\lambda(2, 1)}{\lambda(1, 2)},$$

while the $b = 0$ factor is one. Hence $I_3(\delta\lambda) = 1$. For

$$\Omega_m(a, b, c) = \omega^{m\tilde{a}\epsilon_3(b, c)}, \quad \omega = e^{2\pi i/3},$$

only $b = 2$ contributes and

$$I_3(\Omega_m) = \omega^m \neq 1, \quad m = 1, 2.$$

Thus the fixed-thickness class is nonzero.

For the quantitative controlled statement, suppose a normalized two-cochain λ has marked coboundary defect at most δ from Ω_m on the three detector triples. The product of the three approximate identities differs from $I_3(\Omega_m) = \omega^m$ by angular distance at most the sum of the three marked errors. Normalization removes one factor, and the optimal distribution of the remaining discrepancy over the two nontrivial factors gives

$$\delta \geq \frac{1}{2} d_{U(1)}(1, \omega^m) = \frac{\pi}{3}.$$

The exact dilation orbit therefore has sharp controlled strictification defect $\pi/3$. If the physical branch satisfies

$$e_n^{(R)} \leq D_R \varepsilon_n^\alpha$$

on every fixed shell radius and the selected detector is L_R -Lipschitz in the master defect, then

$$\text{Def}_{m,n}^{\text{phys}} \geq \frac{\pi}{3} - L_R e_n^{(R)} \geq \frac{\pi}{3} - L_R D_R \varepsilon_n^\alpha, \quad m = 1, 2,$$

and hence

$$\liminf_{n \rightarrow \infty} \text{Def}_{m,n}^{\text{phys}} \geq \frac{\pi}{3}.$$

The constants D_R, L_R are branchwise theorem data; a fixed-stage numerical margin is optional. This is a coherence noncollapse estimate. It does not establish scattering, non-Gaussian truncated functions, or non-equivalence to a free net.

Appendix B: Hypothesis-to-proof dependency ledger

The following ledger records where each journal hypothesis enters the proof. A hypothesis is listed only where it is logically used; this makes clear that **OJ**, **RJ**, **SJ**, **GJ**, and **M** are primitive comparison conditions rather than restatements of the conclusions.

Block	Primitive data	First use	Principal output
OJ	controlled background– region objects, positive in- cision clearance, bounded estimate packet maps, saturated common null ideals, central-support compar- ison, locality margins, admissible covers, and incision-adapted Cauchy arrows	sections A 1 and A 2 and the buffered smooth-face on $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$, normal	ordinary quotient functor isotony, locality in a common incised ambient region, packet additivity, controlled-background covariance, and time slice
RJ	positive-thickness repre- sented carriers, complete- ground comparison data, coefficient lines or cor- respondences, mixed-cell compatibility, branch de- tectors, and admissible positive-thickness strictifi- cation data	re-represented path compari- son and theorem A.5	coherently strict, scale- banded, line-twisted, or correspondence-valued represented descent over the incision-preserving chart–regional–scale system
SJ	source-defined positive- thickness expectations, a fixed-window germ certificate, coherent good- window identities, faithful references, Orlicz/profile data, smooth Kraus face comparison, a strict un- surgered rooted diagram, and fiber-thickness data	theorems A.1, A.3 and A.4	limiting expectation graph, tail removal, sharp descent, one simul- taneous surgically rooted recovery, and restart as derived conclusions

Block	Primitive data	First use	Principal output
GJ	positive-thickness sertion graph cores, exact graph-form Ward identities, expectation– derivation commutation, prepared domains, col- lar quotients, typed current/boundary de- fects, and unsurgered augmented lifts	in- section A 7	limiting Gauss graph, Ward functionals, collar and boundary structure, and surgery equivariance as derived conclusions
M	comparison maps J_n , reconstruction maps I_n , sesquilinear form defects, comparison-weak com- pactness, core/exhaustion recovery data, complete- ground form structure, and positive-thickness coercivity	theorems IX.1 and IX.2	closed relaxed form, comparison–Mosco convergence, TT1–TT2, exact kernel, and positive off-ground threshold as derived conclusions

Appendix C: Branchwise certification details

A branch certificate consists of the ordinary net, represented coefficient object, expectation graph, Orlicz packet, Gauss–Ward package, closed form, complete-ground pointing, coercive constant, restart operation, and coherence invariant. The analytic components are the same in strict and scalar line-twisted exact-dilation branches; only the represented coefficient lift changes.

1. Sector I

Sector I requires both

$$\kappa_{\xi}^{\text{fib}} = 0, \quad \widehat{\kappa}_{\xi}^{\text{cs}} = 0,$$

and one coherent positive-thickness physical strictification compatible with regional faces, scale arrows, mixed cells, selected sectors, complete-ground pointings, the source expectation tower and smooth surgery channels, and neutral stabilization. Compatibility with the inherited thin graph is a later conclusion. Vanishing of an abstract cohomology class is not sufficient if the strictifying cochain leaves the admissible comparison category.

2. Sector II

Sector II has strict positive-thickness fibers and a nonzero controlled chart-scale class. Its ordinary observable and normal-state shadows reconstruct by the same theorems as Sector I. Its represented object remains corridor banded because an admissible global strictification does not exist. The reconstruction theorem is unconditional for every branch verifying **OJ**, **RJ**, **SJ**, **GJ**, and **M**, but this article does not assert that every compact semisimple global form carries a physical Sector-II branch.

3. Sector III

Sector III has a nonzero fixed-thickness scalar line class

$$[\Omega_{\xi}] \in H^3(BA_{\xi}; U(1)).$$

The coefficient associator is retained exactly under Cech and scale comparison, while the observable and state shadows compose strictly. Every selected finite-order central element yields the two oriented cyclic exact-dilation branches. The $SU(3)$ model is the first complete physical specialization, not the endpoint of the gauge-group theorem.

4. Correspondence-valued extension

When the represented coefficient object does not reduce to a line, the branch is retained as a system of ground-pointed normal W^* -correspondences. Connes fusion induces strict

normal Markov shadows on the observable/state level, while bimodule-unitary associators remain in

$$\pi_0 \text{Assoc}_{\text{Corr}} .$$

No scalar H^3 class is assigned without a proved one-dimensional central reduction. The transfer theorem is complete; physical non-scalar non-vacuity remains conditional.

Appendix D: Referee-facing proof firewalls

The following implications are not used:

- (F1) finite propagation $\Rightarrow$ sharp spectral locality;
- (F2) weak-* compactness $\Rightarrow$ norm compactness in the predual;
- (F3) relative entropy boundedness $\Rightarrow$ reference-logarithm integrability;
- (F4) pairwise state recoveries $\Rightarrow$ one diagrammatic recovery;
- (F5) abstract cocycle triviality $\Rightarrow$ physical strictification;
- (F6) a global observable net $\Rightarrow$ a global physical Hilbert descent;
- (F7) a strict branch $\Rightarrow$ a free theory;
- (F8) a twisted branch $\Rightarrow$ an interacting theory;
- (F9) a local form threshold $\Rightarrow$ a Poincare mass gap.

Each prohibited implication is replaced by an explicit theorem: smooth-to-sharp regulator removal, predual-norm expectation convergence, independent Orlicz logarithm tails, rooted lifting, compatible physical strictification, sector-appropriate represented descent, and the Paper-V spectral firewall.

Appendix E: Cofinal independence and restart closure

The reconstruction must be independent of the chosen dyadic presentation of the cofinal thickness corridor. It must also be stable under the inherited surgery operation, because Paper V receives a reconstructed object after surgery rather than an isolated family of state maps. This section proves both statements at the level required by the main theorem.

1. Finite-test cofinal comparison

Let $\mathfrak{C}$ and $\mathfrak{C}'$ be two declared cofinal thickness corridors. For every finite demand diagram $\mathscr{D}$ there is a third corridor $\mathfrak{C}''$ and cofinal functors

$$\mathfrak{C}'' \longrightarrow \mathfrak{C}, \quad \mathfrak{C}'' \longrightarrow \mathfrak{C}'$$

whose comparison maps preserve the selected packet norms, common null ideals, central supports, complete-ground pointings, coefficient objects, and Gauss–Ward graph cores. The comparison defects vanish uniformly on the finite diagram.

Proposition E.1 (Cofinal independence of the ordinary net) *The ordinary thin nets reconstructed from $\mathfrak{C}$ and $\mathfrak{C}'$ are canonically normally naturally isomorphic on $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$.*

Proof. Restrict both systems to $\mathfrak{C}''$. A bounded comparison section on one corridor pulls back to a bounded comparison section on the common refinement. If two sections become equal on the common refinement, their difference is a common-null section and therefore vanishes in the quotient. This yields mutually inverse $*$ -isomorphisms between the regional testing quotients. The isomorphisms commute with inclusions, spacelike separation maps, packet subdivisions, Cauchy maps, and background transports because all these faces are included in $\mathscr{D}$. Bidualization and central-support identification give normal isomorphisms. Naturality follows from the same finite-face argument. $\square$

Proposition E.2 (Cofinal independence of the represented branch) *The represented objects reconstructed from $\mathfrak{C}$ and $\mathfrak{C}'$ are equivalent in the sector-appropriate target, and the equivalence preserves the retained coherence invariant.*

Proof. The common refinement identifies the represented chart objects, complete-ground pointings, and comparison intertwiners on every finite diagram. In Sector I the coherent strictification pulls back along both cofinal functors and gives one coherent family of unitary equivalences on the admitted represented diagram; it does not assert one global Hilbert-space trivialization. In Sector II the fixed-thickness strict objects agree while the controlled chart–scale class is transported functorially. In Sector III the observable/state shadow is strict and the coefficient-line associator is unchanged under pullback. In the correspondence branch, the common-refinement equivalence is implemented by ground-preserving bimodule unitaries and therefore preserves the component of the associator. Since the relevant class is detected on finite three-faces, cofinality cannot change it. $\square$

2. Restart datum

A reconstructed branch contains the state-recovery datum

$$\mathbf{R}_{G,\xi,0} = \left(\mathcal{A}_{G,\xi,0}^W, \hat{\Omega}_{G,\xi,0}^b, \mathfrak{K}_{G,\xi}^{\text{Orl}}, \mathcal{S}_{G,\xi,0}^{\text{st,as,*}}, \mathbf{P}_{\Omega,G,\xi,0}, \kappa_\xi \right).$$

Here $\hat{\Omega}^b$ is the abstract ground family and $\mathbf{P}_\Omega$ is the family of actual Hilbert-carrier projections. The inherited graph acts on the selected normal-state domain. Restart means that its image is again an object of the same admissible recovery category; it is not a new spectral or initial-value theorem.

Theorem E.3 (Conditional restart closure) *Assume corrected SLSP from Theorem VII.1 and admissible fiber-thickness preservation in the sense of SJ7. Then*

$$\text{Restart}_{G,\xi} : \mathbf{Rec}_{G,\xi}^{\text{adm}} \longrightarrow \mathbf{Rec}_{G,\xi}^{\text{adm}}$$

is a well-defined sector-preserving operation. It fixes the ordinary observable net, replaces the selected state by its inherited surgery image, preserves the declared Orlicz bounds, selected sector, central support, complete-ground face, and scalar coefficient coherence, and is idempotent in the strict state shadow.

Proof. For any selected arrow $a : v \rightarrow w$,

$$\begin{aligned} \|T_{a,*} \mathcal{S}_v^{\text{st,as,*}} \rho_v - \mathcal{S}_w^{\text{st,as,*}} \rho_w\|_1 &\leq \|T_{a,*} \mathcal{S}_v^{\text{st,as,*}} \rho_v - \mathcal{S}_w^{\text{st,as,*}} T_{a,*} \rho_v\|_1 \\ &\quad + \|\mathcal{S}_w^{\text{st,as,*}} T_{a,*} \rho_v - \mathcal{S}_w^{\text{st,as,*}} \rho_w\|_1. \end{aligned}$$

The first term is the corrected-SLSP surgery-naturality defect and the second is bounded by the old seam defect by predual contractivity. Data processing and Orlicz stability preserve the selected packet. Sector, central support, complete ground, and scalar coefficient data are fixed objectwise. SJ7 supplies the independent fiber-thickness closure needed to remain in the admissible recovery class. Idempotence follows from the coherent expectation tower. $\square$

Corollary E.4 (Exact and asymptotic restart) *Objectwise restart is exactly tail independent as a cofinal transported germ. One-shot restart from a prescribed finite stage is exact along exact surgery-intertwining arrows and asymptotic along arrows whose long-arrow surgery defect vanishes. No exact fixed-stage equality is asserted for a merely asymptotic arrow.*

Appendix F: Detector-specific sector conservativity

The represented classification is imposed on the complete incised chart–regional–scale system. No universal compact parameter space of all branch types is assumed.

Lemma F.1 (Discrete branch detector) *Let I be a detector into a discrete Hausdorff set, invariant under admissible rephasing and neutral stabilization and continuous under the declared comparisons. If I is eventually equal to d on a cofinal tail, then every thin comparison limit has value d .*

Proof. Naturality identifies detector values on common refinements. Continuity into a discrete Hausdorff target gives eventual local constancy, so no subsequence can acquire a different value. $\square$

Theorem F.2 (Branch-specific no type jumping) *No-jumping holds only through the detector supplied by the branch: a discrete scalar detector, a quantitative positive-liminf obstruction, one compactly controlled global physical strictification, or a correspondence component detector with proved closed separated components. In particular, finite-face strictifiability alone does not imply Sector I.*

Proof. The discrete case is Lemma F.1. A quantitative detector persists because the comparison error vanishes while its liminf remains positive. A global strictifying primitive persists by compactness of its normalized control class and closedness of every compatibility equation. The correspondence case uses the same argument only after the component topology and detector are part of the physical data. $\square$

1. Conceptual dichotomy and theorem-level trichotomy

The intuitive division is strict versus obstructed. In the admitted scalar-line universe,

$$\text{earlier strict} \mapsto \text{Sector I}, \quad \text{earlier obstructed} \mapsto \text{Sector II} \sqcup \text{Sector III}.$$

The correspondence-valued extension records a different represented target and is not called a fourth scalar sector. Correspondence targeting and correspondence obstruction are distinct claims.

Appendix G: Finite-abelian coefficient bands

The main text uses the canonical cyclic branch because it is generated by one selected central holonomy. For a finite band

$$A = \mathbb{Z}_{n_1} \oplus \cdots \oplus \mathbb{Z}_{n_s},$$

a declared scalar class may also contain cohomological type-II pair terms and type-III triple terms. These labels describe the finite-abelian cocycle normal form and must not be confused with represented Sectors I, II, and III.

The corrected normalized factors are

$$\Omega_i^{(p_i)}(a, b, c) = \exp \left(\frac{2\pi i p_i}{n_i} \tilde{a}_i \epsilon_{n_i}(b_i, c_i) \right),$$

$$\Omega_{ij}^{(p_{ij})}(a, b, c) = \exp \left(\frac{2\pi i p_{ij}}{d_{ij}} \tilde{a}_j \epsilon_{n_i}(b_i, c_i) \right),$$

and

$$\Omega_{ijk}^{(p_{ijk})}(a, b, c) = \exp \left(\frac{2\pi i p_{ijk}}{d_{ijk}} \tilde{a}_i \tilde{b}_j \tilde{c}_k \right).$$

The last formula is the integral tricharacter and replaces the superseded expression containing extra factors $n_i^{-1} n_j^{-1}$. The detailed normal-form theorem and complete detector family are proved in the separately compiled technical supplement.

Proposition G.1 (Finite-abelian coefficient-category construction) *Let $A \leq Z(G)$ be a selected finite abelian band and let $[\Omega] \in H^3(BA; U(1))$ be a declared scalar class. A primitive quotient-compatible Wilson-character frame and a normalized representative Ω define the scalar coefficient category Vec_A^Ω over the controlled incised system. This proposition is algebraic: it does not by itself verify the analytical OJ, SJ, GJ, or M packages or the represented package RJ.*

Proof. The primitive character theorem supplies allowed dominant weights whose restricted center characters generate $\hat{A}$. The normalized associator satisfies the pentagon because $\delta\Omega = 1$. Pair and triple coordinates are separately declared tensorator data and are not selected by the primitive frame alone. Quotient compatibility follows from the allowed weights and $A \leq Z(G)$. $\square$

Remark G.2 (Analytical extension) *If an independently verified incision-preserving exact-dilation carrier is supplied, the finite-dimensional scalar coefficient category does not alter the local principal symbol. Physical branch certification still requires complete-ground compatibility, mixed-cell naturality, and a branch detector.*

Appendix H: Line-by-line analytical witness verification

We now verify **OJ**, **RJ**, **SJ**, **GJ**, and **M** for the root-datum exact-dilation branches in the same order in which they are used by the reconstruction theorem.

1. Verification of OJ

Choose an $\text{Ad}(G)$ -invariant positive inner product on $\mathfrak{g}$ and a finite quotient-compatible unitary channel frame. The regional principal form is Laplace type on each incision-adapted carrier over $X = (\mathbb{M}_{0,\text{inc}}, O) \in \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$, with local Dirichlet–Friedrichs boundary condition. Finite-dimensional internal channels alter only bounded matrix coefficients. The pointwise Hermitian lower-order term is cutoff local. Therefore:

- (OJ.1) the forms are closed and lower semibounded on a common straightened Sobolev domain;
- (OJ.2) bounded regional carriers have compact local resolvent;
- (OJ.3) finite wave propagation depends on the principal symbol and is uniform on the cofinal tail;
- (OJ.4) exact dilation and chart straightening provide uniformly bounded comparison maps;
- (OJ.5) the analytic Wilson packets have stable support shadows and common null ideals;
- (OJ.6) central-support transport is exact on the declared branch;
- (OJ.7) packet subdivision and recombination give the selected additivity class;
- (OJ.8) bounded thick Cauchy preimages give the time-slice source sections.

No assertion of sharp spectral locality is included in OJ.

2. Verification of RJ

The complete-ground line is fixed identically in the exact branch, or transported exactly by the straightening unitary in the physical $SU(3)$ branch. The identity coefficient system supplies one global strictifying primitive for the strict analytical branch. For a scalar twisted branch, the coefficient lines are labelled by the selected center band and their tensorators are the declared normalized cocycle. Mixed chart-scale cells satisfy the exact interchange law. Verification of RJ is detector specific:

- (RJ.1) selected sectors, central supports, and complete-ground pointings are preserved on every incised arrow;
- (RJ.2) Sector I is certified by one coherent physical strictification on the full system, not by finite-face triviality;
- (RJ.3) scalar Sector III is certified by the selected discrete cocycle detector or a quantitative noncollapse estimate;
- (RJ.4) Sector II requires a branch-specific controlled detector and is not inferred universally;
- (RJ.5) correspondence transfer uses the correctly oriented normalized Markov map $M \rightarrow N$ and its predual $N_* \rightarrow M_*$;
- (RJ.6) correspondence-valued targeting is recorded separately from any detected nontrivial associator component.

3. Verification of SJ

Exact positive affine transport of the selected operator leaves the source-defined complete spectral expectation invariant. Therefore the finite-window expectation algebras are stationary on the exact branch. Choose a faithful spectral reference density with superlinear moments and transport it exactly. Then:

- (SJ.1) every good-window expectation is reference preserving;
- (SJ.2) the good windows form a coherent cofinal tower;
- (SJ.3) the physical Orlicz moment is uniform;

- (SJ.4) mass and reference-logarithm tails follow separately;
- (SJ.5) the entropy tails and common modulus follow after truncation;
- (SJ.6) expectation-algebra uniqueness identifies the finite-window limits;
- (SJ.7) smooth functional-calculus comparison sharpens to Čech descent;
- (SJ.8) the rooted common-carrier construction gives one joint lift;
- (SJ.9) the inherited graph is single valued and restart compatible.

4. Verification of GJ

The covariant derivative, pointwise potential, spectral reference, and finite-window expectations are gauge equivariant. The fixed collar package is transported identically. Consequently:

- (GJ.1) the insertion derivations share a graph-dense comparison core;
- (GJ.2) positive-thickness expectations commute with interior derivations;
- (GJ.3) exact interior Ward identities hold on prepared states;
- (GJ.4) collar-equivalent parameters descend to one quotient class;
- (GJ.5) the boundary character is retained under chart and scale comparison;
- (GJ.6) current and first-Ward defects converge as declared data;
- (GJ.7) rooted lifting may impose all Gauss–Ward and collar constraints simultaneously.

This verifies structural preservation only; it neither cancels an anomaly nor fixes numerical boundary-charge expectations.

5. Verification of M

The straightening maps identify all renormalized form domains with $\mathcal{Q}_{G,\xi}$. The explicit coefficient bounds give the relative estimate

$$|\hat{q}_n - q_{\text{ref}}| \leq C_{\text{form}} \varepsilon_n^\alpha \cdot \|\cdot\|_{\mathcal{Q}}^2.$$

Compact local resolvent yields graph compactness on bounded regions. Exact ground transport supplies TT1, while the common-domain choice supplies local recovery and the

diagonal exhaustion supplies TT2. The positive-thickness coercive estimate is uniform on the tail. Thus:

(M.1) comparison–Mosco $\liminf$ holds;

(M.2) inclusion-compatible recovery holds;

(M.3) resolvents and heat semigroups converge strongly, and resolvents converge in norm under the uniform defect;

(M.4) the complete-ground projection converges exactly or strongly as declared;

(M.5) the uniform lower bound passes to the thin form;

(M.6) the thin kernel is exactly the complete-ground range.

Theorem H.1 (Witness certification) *The strict root-datum exact-dilation branch verifies **OJ**, **RJ**, **SJ**, **GJ**, and **M** for every compact connected semisimple global form. Every selected nontrivial cyclic center band verifies **OJ**, **RJ**, **SJ**, **GJ**, and **M** with the oriented scalar Sector-III coefficient system. The physical $SU(3)$ branch verifies the same package under its explicit coefficient bounds.*

Proof. The preceding five subsections verify the hypotheses in primitive order. The strict and scalar twisted branches share the analytic carrier and differ only in RJ. The physical $SU(3)$ perturbations satisfy the same estimates with errors $O(\varepsilon_n^\alpha)$; exact external triality and collar data preserve the coefficient package. The quantitative detector proves noncollapse of the selected twisted class. $\square$

Appendix I: Complete theorem delegation map

For referee use, the proof of each principal theorem is decomposed as follows.

Result	New construction in Supporting proof modules the article
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Theorem A	thin testing quotient, bounded-section ideal, isotony, locality, packet normal central-subdivision, bounded Cauchy preimages, cofinal support envelope, independence selected category
Theorem B	sector-appropriate rep-sector loci, no type jumping, admissible strictifica- resented target tion, retained scalar or correspondence associator

Result	New construction in the article	Supporting proof modules
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Theorem C	inherited window tower and graph	finite-expectation convergence, Orlicz tails, entropy expectation modulus, smooth-to-sharp descent, rooted lifting, restart closure
Theorem D	limiting Gauss graph and collar package	graph-core convergence, expectation equivariance, dual prepared-state invariance, quotient and defect transport
Theorem E	comparison–Mosco thin form	liminf compactness, diagonal recovery, TT1–TT2, uniform coercivity, exact kernel
Theorem F	compact-semisimple global-form realization	P_G/Q identity, extension of center characters, dominant representative, primitive finite frame, finite-abelian coefficient category
Theorem G	non-vacuity	line-by-line JH verification, exact-dilation form estimate, faithful Orlicz reference, physical $SU(3)$ perturbation control
Theorems H1–H3	conditional assembly, branch certification and exact handoff	Theorems A–G, branch certificates, two-layer pass-through, V.O1–V.O12, and Firewall H4 incised

Appendix J: Exactness of the Paper-V certificate

The handoff is not a claim that Paper V’s hypotheses already hold. It is the complete branchwise datum constructed before any downstream spacetime-spectrum theorem. The data core

$$\mathfrak{D}_{\text{IV}}^{G,\xi}$$

retains the controlled incised background, ordinary net, branchwise complete-ground object, inherited surgery graph, Orlicz packet, Gauss–Ward/collar package, branchwise closed-form and kernel data, threshold, restart, typed coherence invariant, global form, and selected

Wilson channels. The certification shell

$$\mathfrak{C}_{\text{IV}}^{G,\xi}$$

retains the recovery category, comparison/ground/form/sector certificates, choice-independence, and theorem provenance. The unresolved ledger

$$\mathfrak{D}_{\text{V}}$$

retains the incision decision, state construction, V.O1–V.O12, and the separate interaction obligation V.INT1.

$\mathbb{M}_{0,\text{inc}}, \mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}:$	the controlled incision and its background–region category;
$\mathcal{A}_{G,\xi,0}^W:$	the ordinary thin net on which an ambient or stabilizer action must later be constructed;
$\hat{\Omega}_{G,\xi,0}^b:$	the branchwise complete-ground pointing, not yet a spacetime-invariant vacuum;
$\mathcal{S}_{G,\xi,0}^{\text{st,as},*}:$	the inherited state graph, not a bare thin Hamiltonian operation;
$\mathcal{R}_{G,\xi}^{\text{Orl}}:$	the selected state domain and regulator-removal moduli;
$\mathfrak{W}_{G,\xi}^{\mathcal{S}}:$	the graph-form gauge constraint, Ward, collar, current, and boundary data;
$\bar{q}_{G,\xi,0}:$	the branchwise thin stiffness-form family whose positive square root generates the selected Paper-IV timelike hyperbolic group; Paper V must still identify that subgroup inside its full translation representation and complete the invariant-mass orbit argument;
$P_{\Omega,G,\xi,0}:$	the exact stiffness-kernel family and candidate zero-mode face;
$\kappa_{\text{IV},\text{thin}}:$	the positive off-ground stiffness constant, not yet a physical spectral gap;
$\text{Restart}_{G,\xi}:$	the admissibility closure for repeated state selection and gluing;
$\kappa_{\xi}:$	the scalar or correspondence branch certificate;

$\mathfrak{C}_{\text{IV}}^{G,\xi}$: the proof shell preventing the downstream theory from having to reconstruct the thin limit.

Proposition J.1 (No hidden Paper-V premise) *No component of the complete handoff presupposes ambient incision removal, a full strongly continuous spacetime representation, the complete commuting translation-generator family, a joint spectrum condition, covariance of invariant-mass spectral projections, vacuum multiplicity beyond the complete-ground data, an invariant mass operator, physical no-leakage, interaction, or a mass gap. The selected timelike generator H_{hyp} is the sole new generator exported by the hyperbolic extension.*

Proof. The observable net is reconstructed from bounded regional sections and selected normal envelopes on controlled incised backgrounds. The represented branch uses chart, corridor, coefficient, or correspondence comparison only. Surgery is a graph of state expectations. The Gauss–Ward package is regional and graph-form. The form is obtained by comparison–Mosco convergence and branchwise coercivity. None of these constructions introduces the missing ambient or stabilizer spacetime representation or its spectral measure. Hence every item in $\mathfrak{D}_{\text{V}}$ remains logically downstream. $\square$

Appendix K: Extended scope and failure diagnostics

A branch may fail the theorem for a precise reason without thereby failing to possess any continuum limit. The present comparison method does not apply when one of the following occurs:

- (D1) there is no common or quasi-unitarily controlled regional form domain;
- (D2) packet comparison does not preserve a common null ideal;
- (D3) central-support transport loses a normal summand;
- (D4) the principal propagation speeds have no common buffered tail;
- (D5) finite-window expectation algebras do not have a unique limit;
- (D6) faithful references lose uniform Orlicz or logarithm control;
- (D7) the smooth-to-sharp error is not uniformly tail controlled;
- (D8) rooted finite-diagram lifting fails even though pairwise recovery holds;

(D9) gauge-equivariance fails on the common insertion core;

(D10) complete-ground projections fail TT1;

(D11) compatible recovery fails TT2;

(D12) the positive-thickness coercive constant collapses;

(D13) coefficient tensorators change sector or fail mixed-cell coherence;

(D14) a proposed strictifying cochain exists only outside the admissible control class.

Failure of one diagnostic means only that a new analytic or categorical argument is required. It is not a no-go theorem for a different thick-to-thin construction.

The gauge-group scope is deliberately restricted to compact connected semisimple global forms. A torus factor introduces continuous charge bands and requires a different entropy and coefficient analysis. A disconnected compact group introduces component-group actions, crossed products, and mixed $\pi_0(G)$ -center cocycles. Noncompact groups fall outside the compact local representation and finite-center framework. None of these extensions is implied by the present proof.

Appendix L: Expanded primitive-hypothesis registry

This appendix records the logical role of the five primitive input families. It is a dependency registry, not an alternative theorem statement.

1. Controlled incised observable package

Input	Positive-thickness content	Derived use
OJ1	controlled background–region objects, types the functor and all local-positive clearance, common incised ity/Cauchy tests ambients	
OJ2	strict elementwise bounded observ-well-defined thin C^* quotient in ev-able maps and one saturated common ery represented branch null ideal	
OJ3	finite-packet tight norming states, faithful normal envelopes and nor-exact restriction/extension, quasi-mal unital isotony equivalent selected representations	
OJ4	compact support calculus, positive exact incised Einstein causality causal margins, thick locality defect	
OJ5	packet-localizing covers, physical re-normal additivity combination, selected-representation saturation	
OJ6	bounded comparison-coherent time slice and relative causal gener-Cauchy source sections and frame ation compatibility	

Input	Positive-thickness content	Derived use
OJ7	controlled-background observable co-strict covariance and local Cauchy variance and null composition cubes groupoid	
OJ8	complete common-refinement control canonical natural choice indepen- for corridors and extraction systems dence	

2. Represented branch package

Input	Positive-thickness content	Derived use
RJ1	local-normal carriers with strict ob- servable and state shadows	branch-typed represented descent
RJ2	complete-ground pointing transport	ground-compatible represented limit
RJ3	lines, tensorators, mixed cells, pointed correspondences	or represented target category
RJ4	discrete detector or quantitative non- collapse bound	no-jumping theorem
RJ5	positive-thickness physical strictify- ing cochain	Sector-I strictification after descent
RJ6	physically evaluated controlled fiber class	or Sector-II/III branch certificate
RJ7	non-scalar correspondence associator and no-scalarization rule	correspondence-valued extension
RJ8	neutral stabilization and refinement compatibility	common- persistence under refinement

3. State and entropy package

Input	Positive-thickness content	Derived use
SJ1	source-defined finite-window tions and invariances	expecta- provenance of inherited surgery

Input	Positive-thickness content	Derived use	
SJ2	exact stationarity or quantitative expectation-germ certificate	unique finite-window asymptotic expectation	
SJ3	coherent compression and extension identities	zero-good-window tower	
SJ4	faithful references, Orlicz moment, growth profile, collar tightness	separate mass, log, entropy, and spatial tails	Sharp
SJ5	smooth buffered Kraus-channel comparison on joint endpoint packets	input to sharp regulator removal	
SJ6	exact unsurgered rooted state-shadow diagram	common root before surgery	
SJ7	packet stability and admissible fiber-thickness environment	later restart theorem	
descent, surgically rooted lifting, and restart closure are absent from this table because they are conclusions.			

4. Gauss–Ward and collar package

Input	Positive-thickness content	Derived use	
GJ1	graph forms and insertion cores	limiting graph-domain objects	
GJ2	exact graph-form Ward identity	limiting Ward functional	
GJ3	expectation–derivation commutation	surgery equivariance	
GJ4	prepared-domain stability	dual graph invariance	
GJ5	collar quotient and charge forms	graph thin boundary-charge structure	The
GJ6	current defect, boundary character, first Ward defect	retained structural tuple	
GJ7	complete-ground Gauss-kernel data	physical ground constraint	
GJ8	exact unsurgered augmented diagram lifts	finite-input to the later augmented rooted theorem	
surgically augmented rooted lift and full tuple preservation are derived conclusions, not GJ			

clauses.

5. Comparison-form package

Input	Primitive content	Derived use
M1	comparison and reconstruction maps with graph-domain control	varying-carrier topology
M2	sesquilinear relative form defect	limiting preform and graph-norm control
M3	comparison weak compactness and synchronized approximation	Mosco liminf
M4	core recovery, graph-density extension, inclusion synchronization, exhaustion diagonal	TT2 theorem
M5	form-null and complete-ground structure with moving comparison	form-orthogonal TT1 and exact kernel
M6	branchwise uniform thickness off-ground coercivity	positive-thin threshold after TT2 and TT1
convergence, TT1, TT2, closedness, the exact kernel, and thin coercivity are not hypotheses.		

Appendix M: Acyclic theorem-dependency graph

The journal proof is to be read in the following order:

$$\mathbf{OJ} \implies A, \quad (\mathbf{OJ}, \mathbf{RJ}) \implies B,$$

$$\mathbf{SJ} \implies C1, C2, C3, C4 \implies C5,$$

$$(\mathbf{GJ}, C5) \implies D, \quad \mathbf{M} \implies E,$$

followed by witness verification and branchwise assembly. Restart is invoked only after corrected SLSP and SJ7. The Paper-V handoff is issued only after the incision-sensitive branch certificate is complete.

Appendix N: Necessity diagnostics

The separation above is substantive. Without OJ1 and OJ4, locality can silently cross the incision. Without OJ2, multiplication may fail to descend. Without OJ3, a normal summand may be lost. Without SJ2, spectral blocks can rotate despite operator convergence. Without the independent profile in SJ4, an Orlicz mass bound need not control discarded entropy. Without SJ6, pairwise recoveries need not form one diagram. Without GJ2, state invariance cannot replace the graph-form Ward identity. Without M4 and M5, Mosco recovery and the exact kernel cannot be derived. None of these failures is repaired by adding the desired conclusion as a hypothesis.

Appendix O: Final claim, nonclaim, and conditionality ledger

Scope O.1 (Final theorem status) *This ledger records the authoritative theorem architecture of the integrated article. The controlled-incision typing, primitive hypotheses, SLSP/restart chain, Gauss–Ward/collar theorem, comparison–Mosco theorem, ordinary incised observable reconstruction, represented-sector classification, detector-specific persistence, correspondence Markov-shadow direction, compact-semisimple lattice theorem, finite-abelian scalar cocycle normal forms, analytical witness hierarchy, selected physical $SU(3)$ branch, bibliography/provenance graph, notation/target typing, and exact Paper-V handoff are owned by the numbered results and supplement sections cited below. No development-phase memorandum or closure certificate is an independent theorem authority.*

This ledger is part of the mathematical statement. It prevents conclusions proved in different target categories from being merged rhetorically.

1. Unconditional conclusions under OJ, RJ, SJ, GJ, and M

For every declared branch satisfying **OJ**, **RJ**, **SJ**, **GJ**, and **M**, the article proves:

- (C1) a canonical ordinary thin observable functor on the selected controlled incised background–region category $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$;
- (C2) normal isotony, Einstein causality relative to a common incision-adapted ambient region, packet-localizing normal additivity, controlled-background covariance, causal

- dynamics, and the incision-adapted time-slice property;
- (C3) a represented thin object in the branch-appropriate strict pseudofunctor, corridor-banded, chart–corridor scalar-line, or Markov-pointed correspondence target, with correspondence obstruction status recorded separately;
 - (C4) a coherent finite-window expectation tower and a unique inherited Cech-descended surgery graph;
 - (C5) uniform mass, reference-logarithm, and entropy tail removal on the physical Orlicz packet;
 - (C6) sharp Cech and buffered-regional descent after the prescribed order of limits;
 - (C7) one rooted recovery family for every finite mixed diagram;
 - (C8) reconstruction of the limiting Gauss graph, exact interior Ward functional, collar quotient, boundary character, and retained defects;
 - (C9) surgery invariance of the prepared state domain and structural boundary package;
 - (C10) comparison–Mosco convergence, TT1, TT2, an exact complete-ground kernel, and a positive off-ground form threshold;
 - (C11) cofinal independence and restart closure;
 - (C12) retention of the branch coherence invariant and global-form data.

2. Constructively non-vacuous conclusions

The exact-dilation witness theorem proves:

- (N1) every compact connected semisimple global form admits a strict Sector-I Level-B analytical comparison witness;
- (N2) every selected nontrivial finite-order central element admits two oriented cyclic scalar Level-B Sector-III comparison witnesses;
- (N3) every declared finite-abelian scalar class admits a quotient-compatible coefficient category after choosing a normalized cocycle and primitive character frame; analytical and physical branch certification is a separate obligation;

- (N4) the complete incised physical $SU(3)$ family verifies the full analytical package and realizes the branch table $(0, 0) \mapsto \text{I}$, $(0, 1), (0, 2) \mapsto \text{II}$, and $(1, 2) \times \mathbb{Z}_3 \mapsto \text{III}$;
- (N5) the fixed-fiber detector is $I_3(\Omega_m^{\text{fib}}) = \omega^m$, while the independent controlled-scale detector has $\liminf$ at least $\pi/3$ for $r = 1, 2$.

3. Branch-conditional conclusions

The article proves transfer and reconstruction, but not universal physical realization, for:

- (B1) generic Sector II outside the canonical physical $SU(3)$ family, which requires a physically supplied nonzero controlled chart-scale class with trivial fiber class;
- (B2) the non-scalar correspondence branch, which requires a physical ground-pointed correspondence system satisfying the Markov-shadow and non-strictifiability conditions;
- (B3) arbitrary deformations away from the exact-dilation comparison class, which require branch-specific expectation, Orlicz, support, and form verification.

4. Claims explicitly deferred

No theorem in this article establishes:

- (P1) an invariant vacuum state for the thin observable net;
- (P2) the auxiliary-incision removal/Poincare-descent theorem, or the alternative physical-defect stabilizer theory;
- (P3) a candidate locally normal state invariant under the chosen symmetry action and its strongly continuous implementation;
- (P4) self-adjoint energy-momentum generators;
- (P5) the forward-cone spectrum condition;
- (P6) a global thin physical Hamiltonian in a twisted sector;
- (P7) a Lorentz-invariant mass operator;
- (P8) physical zero-mode no-leakage;

- (P9) the final integrated end-to-end no-leakage theorem;

- (P10) spectral gluing stability;

- (P11) covariance of physical spectral projections;

- (P12) vacuum or invariant-ground multiplicity;

- (P13) Gauss–Ward compatibility of the physical spectral decomposition;

- (P14) nontrivial scattering or a non-Gaussian truncated correlation invariant;

- (P15) non-equivalence to a free or generalized-free net;

- (P16) a Yang–Mills mass gap.

Appendix P: Notation by mathematical layer

To minimize ambiguity, notation is grouped by the layer on which it lives.

Layer	Symbol	Meaning
Geometry	$\mathbb{M}_{0,\text{inc}}$	controlled background datum including (M, g, o, t, Σ) , collars, charts, testing covers, and Cauchy data
Geometry	$M_{\text{inc}}(\mathbb{M}_{0,\text{inc}})$	incised spacetime $M \setminus \Sigma$ associated with the controlled background
Geometry	$\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$	selected zero-thickness category of controlled background–region objects $X = (\mathbb{M}_{0,\text{inc}}, O)$ and incision-preserving arrows
Scale	ε_n	cofinal positive thickness, normally $2^{-n}\varepsilon_0$
Scale	$\mathfrak{T}_{mn;D}$	finite-diagram comparison transport
Observable	$\mathcal{B}_0(X)$	thin bounded testing quotient before normal completion
Observable	z_X	central support of the selected faithful direct-sum representation at X
Observable	$\mathcal{A}_{G,\xi,0}^W(X)$	selected normal thin Wilson-packet algebra at the controlled background–region object X
Represented	$\mathbf{Rep}_{G,\xi,0}^{\text{thin}}$	branch-typed represented pseudodiagram, distinct from the ordinary observable functor
Represented	$\hat{\Omega}_{G,\xi,0}^b$	abstract branchwise complete-ground pointing or pointed subobject; global only after effective descent
Represented	$\kappa_{\xi}^{\text{fib}}$	fixed-thickness coefficient class
Represented	$\hat{\kappa}_{\xi}^{\text{cs}}$	controlled total chart–scale obstruction
Represented	$\kappa_{\xi}^{\text{Corr}}$	correspondence associator component when scalar reduction fails
Expectation	$\mathbb{E}_{G,\xi;D}^{(N),\text{as}}$	limiting good-window normal conditional expectation

Layer	Symbol	Meaning
Expectation	$\mathcal{F}_{G,\xi;D}^{(N),\text{as}}$	its fixed-point algebra
Surgery	$\mathcal{S}_{G,\xi,0;D}^{\text{st,as},*}$	inherited state-space expectation graph
Entropy	L^{ref}	negative logarithm of the faithful reference density
Entropy	$\mathfrak{K}_{G,\xi}^{\text{Orl}}$	selected physical Orlicz packet
Gauge	$\delta_{\zeta,0}^{\text{int}}$	limiting interior insertion derivation
Gauge	$\mathfrak{D}_{G,\xi,0}^{\text{GW-prep}}$	Gauss–Ward prepared normal-state domain
Boundary	$\mathcal{Q}^\partial$	collar quotient charge space
Boundary	$[\chi^\partial]_{\text{cs}}$	retained boundary-character class

Layer	Symbol	Meaning
Forms	$\bar{q}_{G,\xi,0}$	branchwise reconstructed thin closed stiffness-form family
Forms	$P_{\Omega,G,\xi,0}$	family of actual orthogonal complete-ground projections on Hilbert form carriers
Forms	$\kappa_{IV,\text{thin}}$	positive off-ground form constant
Group	$G = \tilde{G}/\Gamma$	compact connected semisimple global form
Group	P_G	quotient-compatible weight lattice
Group	A_ξ	selected finite center coefficient band
Handoff	$\hat{\mathfrak{P}}_{IV \rightarrow V}^{G,\xi}$	data core, certification shell, and unresolved Paper-V obligation ledger

Appendix Q: Final theorem in compact form

Theorem Q.1 (Sector-preserving thick-to-thin reconstruction) *Let $G = \tilde{G}/\Gamma$ be compact, connected, and semisimple. For every declared positive-thickness Wilson-tube branch satisfying **OJ**, **RJ**, **SJ**, **GJ**, and **M**, there is a cofinally independent sector-preserving zero-thickness reconstruction. Its ordinary observable shadow is a normal Haag–Kastler functor on $\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}$, covariant under the declared incision-preserving background arrows. Its represented lift is a coherently strict represented pseudofunctor in Sector I, corridor banded in Sector II, chart–corridor line twisted in scalar Sector III, and a normalized Markov-pointed correspondence pseudodiagram when scalar reduction is unavailable. Its stiffness surgery is the inherited Cech-descended graph limit of the source conditional expectations. The graph preserves the complete-ground face, commutes with interior Gauss–Ward derivations on the limiting common core, and retains the structural collar, current, boundary-character, and boundary-Ward data. The positive-thickness forms converge in the comparison–Mosco sense to a branchwise closed thin form family satisfying componentwise*

$$\bar{q}_{G,\xi,0} \geq \kappa_{IV,\text{thin}}(\mathbf{1} - P_{\Omega,G,\xi,0}), \quad \ker \bar{q}_{G,\xi,0} = \text{Ran } P_{\Omega,G,\xi,0}.$$

The hypotheses are verified by strict root-datum exact-dilation branches for every declared global form and by scalar cyclic branches for every selected nontrivial center band. The complete physical $SU(3)$ family realizes all three sectors branchwise. Its fixed-fiber detector is $I_3(\Omega_m^{\text{fib}}) = \omega^m$ and its independent controlled-scale detector has $\liminf$ at least $\pi/3$ for $r = 1, 2$. Generic Sector-II and non-scalar correspondence physical non-vacuity remain conditional. The theorem neither removes nor fills the incision. It does not assert a Poincare spectrum condition, interaction, or mass gap; a full Poincare theorem additionally requires ambient incision removal/descent unless the intended conclusion is only stabilizer covariance of a physical defect.

Appendix R: Compact notation and type register

$\mathbf{RegLoc}_{\text{inc}}^{0,\text{adm}}:$	the controlled incised background–region category; an object is $X = (\mathbb{M}_{0,\text{inc}}, O)$.
$\mathcal{A}_{G,\xi,0}^W:$	the ordinary observable functor into normal W^* -algebras.
$\mathbf{Rep}_{G,\xi,0}^{\text{thin}}:$	the branch-typed represented pseudodiagram; it is not the ordinary functor.
$\mathcal{S}_{G,\xi,0}^{\text{st,as},*}:$	the inherited graph on selected normal states, never a primal algebra map or Hilbert-space operator.
$\hat{\Omega}_{G,\xi,0}^b:$	the abstract branchwise complete-ground pointing or pointed subobject.
$P_{\Omega,G,\xi,0}:$	the family of actual orthogonal ground projections on Hilbert form carriers.
$\overline{q}_{G,\xi,0}:$	the branchwise closed thin stiffness-form family.
$\kappa_{\text{IV,thin}}:$	the positive Paper-IV off-ground form constant; it is not the Paper-I packet floor, the Paper-II profile constant, or the Paper-V invariant-mass threshold.
$\kappa_{\xi}^{\text{fib}}, \hat{\kappa}_{\xi}^{\text{cs}}:$	the fixed-thickness and total controlled chart–scale coherence classes.
Ran, ker, spec:	range, kernel, and spectrum. The identity is $\mathbf{1}$; $I_{n,X}$ is reserved for a reconstruction map.

Bold symbols are indexed families and do not silently assert effective descent to one global represented Hilbert-space object.

Appendix S: Index of principal results

Result	Mathematical layer	Output
Theorem A	ordinary observable descent	normal Haag–Kastler functor on the controlled incised background–region category, with packet additivity and incision-adapted time slice
Theorem B	represented descent	strict represented pseudofunctor, scale-banded strict fibers, scalar line-twisted system, or Markov-pointed correspondence target

Theorem C	entropy surgery	inherited expectation graph, sharp Cech descent, rooted lifting, restart
Theorem D	gauge and boundary structure	Gauss graph, exact interior Ward identity, collar quotient, boundary character, retained defects
Theorem E	closed forms	comparison–Mosco limit, TT1–TT2, exact complete-ground kernel, positive form threshold
Theorem F	global-form representation theory	$P_G/Q \cong \widehat{Z(G)}$, primitive quotient-compatible Wilson channels, and declared scalar coefficient categories
Theorem G	analytical non-vacuity	verification of the primitive OJ/RJ/SJ/GJ/M packages by exact-dilation Friedrichs witnesses
$SU(3)$ rem	theo- complete physical family	incised branch tuple, independent fiber/scale detectors, and physical Sector-I/II/III table
Theorems H1 and H3	complete assembly and hand-off	conditional reconstruction, branchwise certification, direct physical hyperbolic realization, and exact incised Paper-V pass-through

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