

Supplementary Material

Scale-Bordism Criteria for Uniform Coercivity and Center-Banded Vacuum Descent in Yang–Mills Wilson–Tube Systems

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This supplementary document reproduces the frozen technical-supplement boundary from the citation-authoritative Paper II long-form release. It contains the complete $SU(2)$ certificate, constructive-scope records, source registry, reproducibility certificate, typed downstream handoff, and data and acknowledgment material. No mathematical claim is strengthened or altered by this production edition.

Chapter 13: Complete $\mathrm{SU}(2)$ Certificate and Constructive Scope

S.1 The complete physicalized $\mathrm{SU}(2)$ Galerkin seed

S.1.1 Purpose and scope

This appendix verifies the complete finite physical package used in Section 7 for one exact four-character Peter–Weyl compression; see [19, 20] for compact-group and character background. The calculation includes the interacting ground projection, removal of the complete ground sector, the Gram-normalized physical Wilson channel, magnetic recompression, the exact ultraviolet–infrared mixed block, and a positive resolved coercive constant.

All matrices use the ordered character basis displayed below, and every quoted interval is outward-rounded in the standard sense of verified interval computation [21–23]. The certificate is specific to this $\mathrm{SU}(2)$ fixed-stratum realization.

The example has two distinct numerical lower bounds. The exact one-channel block reduction gives

$$3.113554839353\dots,$$

whereas the robust threshold profile used in the main text deliberately retains the smaller certificate

$$3.093219423836\dots$$

The second value has explicit decimal slack in every input constant and is the quantity denoted by $\hat{\kappa}_{\mathrm{SU}(2)}$ in (22). Neither number is identified with a continuum spectral gap.

S.1.2 The four-character class-function compression

Work in the conjugation-invariant Hilbert space

$$L^2(\mathrm{SU}(2))^{\mathrm{Ad} \mathrm{SU}(2)}.$$

For $j \in \frac{1}{2}\mathbb{N}_0$, let χ_j be the irreducible character of spin j . Schur orthogonality gives

$$\langle \chi_j, \chi_k \rangle = \delta_{jk}.$$

Retain

$$\mathcal{K}_2 = \mathrm{span} \left\{ \chi_0, \chi_{\frac{1}{2}}, \chi_1, \chi_{\frac{3}{2}} \right\} \tag{190}$$

and use this ordered basis throughout the appendix.

Choose the electric normalization

$$H_{\mathrm{el}}^{(2)} \chi_j = 4j(j+1) \chi_j.$$

Then

$$H_{\text{el}}^{(2)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 15 \end{pmatrix}. \quad (191)$$

The fundamental character satisfies

$$\chi_{\frac{1}{2}} \chi_j = \chi_{j+\frac{1}{2}} + \chi_{j-\frac{1}{2}}, \quad \chi_{-\frac{1}{2}} := 0.$$

The nonnegative Wilson multiplier

$$1 - \frac{1}{2} \chi_{\frac{1}{2}}$$

has compression

$$H_{\text{mag}}^{(2)} = \begin{pmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 1 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & 1 \end{pmatrix}. \quad (192)$$

Its eigenvalues are

$$1 - \cos \frac{k\pi}{5}, \quad k = 1, 2, 3, 4,$$

so it is strictly positive on the retained space.

The complete finite Hamiltonian is

$$H^{(2)} = H_{\text{el}}^{(2)} + H_{\text{mag}}^{(2)} = \begin{pmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 4 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 9 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & 16 \end{pmatrix}. \quad (193)$$

All entries in (191)–(193) are exact rational numbers.

S.1.3 Exact ground projection

The characteristic polynomial of $H^{(2)}$ is

$$\det(\lambda I - H^{(2)}) = \frac{1}{16} (16\lambda^4 - 480\lambda^3 + 4356\lambda^2 - 12932\lambda + 8561). \quad (194)$$

Let $E_0^{(2)}$ be the smallest real root. Numerically,

$$0.918058177772 < E_0^{(2)} < 0.918058177773. \quad (195)$$

The four eigenvalues are

$$\begin{aligned} E_0^{(2)} &= 0.9180581777722170525\dots, \\ E_1^{(2)} &= 4.0319221933103229455\dots, \\ E_2^{(2)} &= 9.0143808889852464909\dots, \\ E_3^{(2)} &= 16.0356387399322135109\dots \end{aligned} \quad (196)$$

An exact algebraic ground vector can be written as follows. Set

$$\begin{aligned} a_0(E) &= 1, \\ a_1(E) &= 2(1 - E), \\ a_2(E) &= 4(4 - E)(1 - E) - 1, \\ a_3(E) &= 2(9 - E)a_2(E) - a_1(E), \end{aligned} \tag{197}$$

and

$$w(E) = \begin{pmatrix} a_0(E) \\ a_1(E) \\ a_2(E) \\ a_3(E) \end{pmatrix}.$$

The recurrence obtained from the tridiagonal eigenvalue equation gives

$$H^{(2)}w(E_0^{(2)}) = E_0^{(2)}w(E_0^{(2)}).$$

Define

$$\Omega_2 = \frac{w(E_0^{(2)})}{\|w(E_0^{(2)})\|}. \tag{198}$$

With positive first coordinate,

$$\Omega_2 = \begin{pmatrix} 0.9867860150249531 \\ 0.1617180884400741 \\ 0.0100254653234385 \\ 0.0003323665295096 \end{pmatrix} + O(10^{-16}). \tag{199}$$

The vacuum datum is the complete spectral projection

$$P_{\text{vac}}^{(2)} = \mathbf{1}_{\{E_0^{(2)}\}}(H^{(2)}) = |\Omega_2\rangle\langle\Omega_2|. \tag{200}$$

The vector in (198) is used only to display the rank-one projection. Its phase is not part of the finite package.

Numerically,

$$P_{\text{vac}}^{(2)} = \begin{pmatrix} 0.973746639449 & 0.159581148049 & 0.009892988975 & 0.000327974643 \\ 0.159581148049 & 0.026152740129 & 0.001621299088 & 0.000053749680 \\ 0.009892988975 & 0.001621299088 & 0.000100509955 & 0.000003332129 \\ 0.000327974643 & 0.000053749680 & 0.000003332129 & 0.000000110468 \end{pmatrix} + O(10^{-13}). \tag{201}$$

S.1.4 Raw Wilson excitation and exact vacuum removal

Let the coefficient graph be the strict two-vertex graph with normalized excitation vector

$$g_+ = \frac{1}{\sqrt{2}}(1, -1).$$

Only the one-dimensional graph-excitation complement is retained. Define the raw Wilson channel by

$$F_2^W g_+ = \phi_W := \frac{-16\chi_0 + 97\chi_{\frac{1}{2}} + 10\chi_1}{\sqrt{9765}}. \tag{202}$$

The coefficient identity

$$16^2 + 97^2 + 10^2 = 9765$$

shows that $\|\phi_W\| = 1$. The half-integral character component is nonzero, so the retained vector is not center blind.

Vacuum removal gives

$$T_2 g_+ = (I - P_{\text{vac}}^{(2)}) \phi_W. \quad (203)$$

The raw vacuum overlap is

$$\langle \Omega_2, \phi_W \rangle = -1.6869479570427 \dots \times 10^{-5}. \quad (204)$$

Consequently, the scalar Gram operator is

$$G_2 = T_2^* T_2 = G_2 I, \quad G_2 = 1 - |\langle \Omega_2, \phi_W \rangle|^2, \quad (205)$$

and

$$0.999999999715 < G_2 < 0.999999999716. \quad (206)$$

In particular, the physical channel survives complete ground-sector removal with a very large margin.

The vacuum-removed vector and its normalized physicalization are

$$T_2 g_+ = \begin{pmatrix} -0.1618971509585235 \\ 0.9816051255955704 \\ 0.1011962925775352 \\ 0.0000000056068504 \end{pmatrix} + O(10^{-16}), \quad (207)$$

$$\begin{aligned} F_2^{\text{phys}} g_+ &= G_2^{-1/2} T_2 g_+ =: s_2 \\ &= \begin{pmatrix} -0.1618971509815598 \\ 0.9816051257352427 \\ 0.1011962925919344 \\ 0.0000000056068504 \end{pmatrix} + O(10^{-16}). \end{aligned} \quad (208)$$

By construction,

$$\|s_2\| = 1, \quad P_{\text{vac}}^{(2)} s_2 = 0. \quad (209)$$

S.1.5 Physical magnetic recompression

The recompressed magnetic operator on the one-dimensional physical coefficient space is

$$B_{\text{mag},2}^{\text{phys}} = (F_2^{\text{phys}})^* H_{\text{mag}}^{(2)} F_2^{\text{phys}} = b_{\text{mag},2}^{\text{phys}} I, \quad (210)$$

where

$$1.059584273164 < b_{\text{mag},2}^{\text{phys}} < 1.059584273165. \quad (211)$$

This is the recompression of the actual magnetic Hamiltonian after exact vacuum removal and Gram normalization. No comparison form has been inserted independently.

Let

$$\hat{H}^{(2)} = H^{(2)} - E_0^{(2)} I.$$

The exact channel diagonal is

$$3.114097481076 < \langle s_2, \hat{H}^{(2)} s_2 \rangle < 3.114097481077. \quad (212)$$

The electric and magnetic contributions are both present in this shifted channel constant.

S.1.6 Infrared–ultraviolet decomposition

Set

$$P_{\text{ch}}^{(2)} = |s_2\rangle\langle s_2|, \quad P_{\text{res}}^{(2)} = 0, \quad (213)$$

and define

$$P_{\text{UV}}^{(2)} = I - P_{\text{vac}}^{(2)} - P_{\text{ch}}^{(2)}. \quad (214)$$

Thus

$$\text{Ran}(I - P_{\text{vac}}^{(2)}) = \text{Ran } P_{\text{ch}}^{(2)} \oplus \text{Ran } P_{\text{UV}}^{(2)}.$$

There is no residual infrared sector and hence no channel–residual block.

The ultraviolet compression has eigenvalues

$$8.096308674897\dots, \quad 15.117361132938\dots \quad (215)$$

Therefore

$$P_{\text{UV}}^{(2)} \widehat{H}^{(2)} P_{\text{UV}}^{(2)} \geq c_{\text{UV},2}^{\text{act}} P_{\text{UV}}^{(2)}, \quad c_{\text{UV},2}^{\text{act}} > 8.096308674897. \quad (216)$$

The complete ultraviolet–infrared mixed operator is

$$K_2 = P_{\text{UV}}^{(2)} \widehat{H}^{(2)} P_{\text{ch}}^{(2)}. \quad (217)$$

Since the electric part is diagonal in the character basis and the channel is not an exact eigenvector, this block is nonzero. Its norm satisfies

$$0.051998558862 < \|K_2\| < 0.051998558863. \quad (218)$$

Thus the witness genuinely exercises exact mixed-block control rather than relying on an infrared reducing-subspace assumption.

For reference, choose an orthonormal ultraviolet basis that diagonalizes the ultraviolet compression. Then the mixed column is

$$\begin{pmatrix} -0.008362855475\dots \\ -0.051321659873\dots \end{pmatrix}, \quad (219)$$

and its Euclidean norm is the value in (218).

S.1.7 The sharp one-channel block bound

Let

$$\begin{aligned} c_{\text{ch},2}^{\text{act}} &= \langle s_2, \widehat{H}^{(2)} s_2 \rangle, \\ c_{\text{UV},2}^{\text{act}} &= \lambda_{\min} \left(P_{\text{UV}}^{(2)} \widehat{H}^{(2)} P_{\text{UV}}^{(2)} \big|_{\text{Ran } P_{\text{UV}}^{(2)}} \right), \\ \varepsilon_{\text{mix},2}^{\text{act}} &= \|K_2\|. \end{aligned} \quad (220)$$

The scalar comparison matrix is

$$M_2^{\text{act}} = \begin{pmatrix} c_{\text{ch},2}^{\text{act}} & -\varepsilon_{\text{mix},2}^{\text{act}} \\ -\varepsilon_{\text{mix},2}^{\text{act}} & c_{\text{UV},2}^{\text{act}} \end{pmatrix}. \quad (221)$$

Its smaller eigenvalue satisfies

$$3.113554839353 < \kappa_2^{\text{sharp}} < 3.113554839354. \quad (222)$$

Consequently,

$$\hat{H}^{(2)} \geq \kappa_2^{\text{sharp}}(I - P_{\text{vac}}^{(2)}). \quad (223)$$

The exact spectral gap is

$$3.113864015538 < E_1^{(2)} - E_0^{(2)} < 3.113864015539. \quad (224)$$

Thus the sharp block comparison captures more than 99.99% of the exact finite Galerkin gap without inserting the excited eigenvalue into the estimate.

S.1.8 A robust threshold-profile certificate

The cofinal theory does not require maximal finite-scale constants. It requires constants with enough slack to persist inside a compact good stratum. We therefore declare the rational decimal thresholds

$$\boxed{\begin{aligned} c_{\text{ch},2}^{\text{cert}} &= \frac{1546885249}{500000000} = 3.093770498, \\ c_{\text{UV},2}^{\text{cert}} &= 8, \\ \varepsilon_{\text{mix},2}^{\text{cert}} &= \frac{13}{250} = 0.052. \end{aligned}} \quad (225)$$

The intervals (212), (216), and (218) imply

$$c_{\text{ch},2}^{\text{act}} > c_{\text{ch},2}^{\text{cert}}, \quad c_{\text{UV},2}^{\text{act}} > c_{\text{UV},2}^{\text{cert}}, \quad \varepsilon_{\text{mix},2}^{\text{act}} < \varepsilon_{\text{mix},2}^{\text{cert}}. \quad (226)$$

The determinant margin is exactly

$$\begin{aligned} c_{\text{ch},2}^{\text{cert}} c_{\text{UV},2}^{\text{cert}} - (\varepsilon_{\text{mix},2}^{\text{cert}})^2 \\ = 24.747459984 > 0. \end{aligned} \quad (227)$$

The corresponding subcriticality ratio is

$$\rho_2^{\text{cert}} = \frac{(\varepsilon_{\text{mix},2}^{\text{cert}})^2}{c_{\text{ch},2}^{\text{cert}} c_{\text{UV},2}^{\text{cert}}} = 0.000109251801392 \dots < 1. \quad (228)$$

The smaller eigenvalue of the certified scalar matrix is

$$\boxed{\begin{aligned} \hat{\kappa}_{\text{SU}(2)} &= \frac{c_{\text{ch},2}^{\text{cert}} + c_{\text{UV},2}^{\text{cert}} - \sqrt{(c_{\text{ch},2}^{\text{cert}} - c_{\text{UV},2}^{\text{cert}})^2 + 4(\varepsilon_{\text{mix},2}^{\text{cert}})^2}}{2} \\ &= 3.0932194238368030870 \dots \end{aligned}} \quad (229)$$

Therefore

$$\boxed{H^{(2)} \geq E_0^{(2)} I + 3.093219423836 \dots (I - P_{\text{vac}}^{(2)})}. \quad (230)$$

This is the robust value quoted in the main text.

S.1.9 Package ledger

Package component	Verification in the four-character model
Exact Hamiltonian	The rational matrices (191), (192), and (193).
Complete vacuum datum	The spectral projection (200); the displayed vector is only a computational representative.
Raw Wilson channel	The normalized center-sensitive vector (202).
Vacuum removal	The exact compression (203), with nonzero Gram scalar (206).
Physicalization	The Gram-normalized vector (208), satisfying (209).
Magnetic recompression	The actual recompressed magnetic scalar (211).
Infrared block	The one-dimensional channel projection (213); the residual block is zero.
Ultraviolet block	The exact projection (214) and positive lower bound (216).
Exact mixed block	The nonzero operator (217) with norm (218).
Sharp resolved bound	The one-channel comparison (222).
Robust certified bound	The slack threshold profile (225) and final estimate (230).

Theorem S.1.1 (Complete physicalized SU(2) seed). *The Hamiltonian (193), complete ground projection (200), and raw Wilson vector (202) determine a finite physicalized good package with positive Gram operator, positive physical magnetic recompression, positive channel and ultraviolet diagonal constants, zero residual sector, and a nonzero exact ultraviolet–infrared mixed block. The robust threshold profile (225) yields*

$$H^{(2)} \geq E_0^{(2)} I + \hat{\kappa}_{\text{SU}(2)}(I - P_{\text{vac}}^{(2)}), \quad \hat{\kappa}_{\text{SU}(2)} > 3.0932.$$

Proof. The exact ground projection and physicalized channel are established in Sections S.1.3 and S.1.4. The recompressed magnetic scalar is positive by (211). The orthogonal decomposition (213)–(214) has no residual component. The diagonal estimates and exact mixed-block norm are given by (212), (216), and (218). The certified constants are dominated by the exact values as in (226), and their determinant is positive by (227). The sharp scalar two-block estimate therefore gives (229) and (230). $\square$

S.1.10 Scope of the witness

The appendix proves finite good-locus nonemptiness for the displayed four-character compression. It does not prove:

- (i) that the same constants persist after adding further Peter–Weyl sectors;
- (ii) that the raw channel remains uniformly transverse to the complete ground sector under arbitrary refinement;
- (iii) that the finite ground projections converge;
- (iv) that the robust finite bound survives a continuum or thin-observable limit.

Those are separate extension, compactness, and no-leakage questions. The role of this witness is narrower: every analytic operation appearing in the Paper-I finite package and used by the Paper-II cofinal-selection theorem is realized end to end in one exact SU(2) finite system.

S.2 Perturbative stability and constructive scope

S.2.1 Vacuum-compatible interstage interactions

Let $P_n^{(0)}$ be the product ground projection of the recursive family and let

$$B_n = \sum_{m=2}^n W_m^{(n)}, \quad P_n^{(0)} W_m^{(n)} = W_m^{(n)} P_n^{(0)} = 0.$$

If

$$\delta_\infty = \sum_{m=2}^{\infty} \|W_m\| < \kappa_{\text{rec}},$$

then

$$\hat{H}_n^{\text{int}} \geq (\kappa_{\text{rec}} - \delta_\infty)(I - P_n^{(0)}). \quad (231)$$

The exact ground projection and vacuum overlap are unchanged. For geometrically decaying interactions, graph-map perturbations and analytic curvature corrections are summable.

S.2.2 Moving vacuum sectors

For a general perturbation B_n , using the standard isolated-spectral-subspace perturbation estimates [14, 15], with

$$\|B_n\| \leq \delta_* < \kappa_{\text{rec}}/4,$$

the low-energy Riesz projection satisfies

$$\|P_n^{\text{isl}} - P_n^{(0)}\| \leq \frac{2\delta_*}{\kappa_{\text{rec}} - 2\delta_*} < 1.$$

For rank one this island is the complete exact ground sector. In higher rank, a separate no-splitting mechanism is required; irreducible symmetry is one sufficient condition. Under rank one or exact no splitting, positive moved overlap, and preservation of the band marking,

$$H_n - E_{0,n}I \geq (\kappa_{\text{rec}} - 2\delta_*)(I - P_{\text{vac},n}), \quad (232)$$

and the strict or twisted banded type is unchanged.

S.2.3 Exact scope of the active supplement

The active supplement supplies two verification layers: the complete one-scale physicalized SU(2) certificate, and the perturbative and convention-dependent consequences stated in this and the following section. The selected triality-sensitive SU(3) packet enters through Paper I and is not presented as a second Galerkin calculation here. These results verify nonemptiness and perturbative stability of the Paper-II finite mechanism in the displayed SU(2) habitat; they do not identify it with the full unmodified constant-coupling $3 + 1$ -dimensional lattice Yang–Mills Hamiltonian.

S.3 Conditional comparison of banded globalizations

The preceding section proves existence and inequivalence of the strict and twisted fixed-stratum depth globalizations without ordering them. An optional numerical comparison requires additional conventions

$$\mathbf{w} = (\{w_\nu\}, w_{\text{rel}}, w_{\text{band}}, \ell_Z, \text{Vol}_{\text{WT}}, \mathcal{G}_{\text{rep}}).$$

After reducing over the representative groupoid, let

$$\overline{\mathcal{S}}_{D,\mathbf{w}}(\mathfrak{D}_D) = \inf_{g \in \mathcal{G}_{\text{rep}}} \mathcal{S}_{D,\mathbf{w}}^{\text{raw}}(g \cdot \mathfrak{D}_D),$$

and for a compatible exhaustion, either full-scale cofinal or explicitly fixed-stratum depth-exhaustive, set

$$\mathcal{J}_{\text{cmp},\mathbf{w}}^\infty(\mathfrak{S}) = \limsup_{n \rightarrow \infty} \overline{\mathcal{S}}_{D_n,\mathbf{w}}(\mathfrak{S}|_{D_n}). \quad (233)$$

Numerical comparison requires the same channels, norms, weights, band length, volume normalization, and representative groupoid. It is evaluated on the coefficient-line reassociator class, not on scalar compositors or a selected local tensorator. Existence of minimizers and invariance under common refinement require additional compactness, semicontinuity, and asymptotic-negligibility hypotheses.

By Theorem 7.4, the finite analytic contribution is identical on the paired fixed-stratum depth globalizations. Hence

$$\Delta_{\text{pair},\mathbf{w}}^\infty = w_{\text{band}} \Delta \mathcal{J}_{\text{band}}^\infty. \quad (234)$$

In the calibrated extensive $\mathbb{Z}_2$ convention with one nontrivial coefficient reassociator evaluation per shell of volume v_{sh} ,

$$\boxed{\Delta_{\text{pair},\mathbf{w}}^\infty = \frac{4w_{\text{band}}}{v_{\text{sh}}}}. \quad (235)$$

Thus $w_{\text{band}} = 0$ gives a tie, while $w_{\text{band}} > 0$ makes the strict globalization lower cost under that declared convention. This is not a dynamical vacuum-selection theorem. The formula is conditional on the declared normalization and does not supply a physical ordering without those additional conventions.

Part VII

Audit, Provenance, and Back Matter

Chapter 14: Source Registry and Reproducibility Certificate

Frozen source packages

The long-form baseline is generated from the following mounted archives:

Archive	SHA-256
Paper_II_Journal_v0.6.1_Acknowledgment_Update_2026-07-18.zip	b1f9de2c74a5b3caa47632fa848e2f951775b685dcd099143815bf1425e4e364
Paper_II_Technical_Supplement_v0.6.0.zip	af7d632586489d289bebb663fa2502473c419acda5783385664219b970bd7587
Original article source archive	8dfe72d7df564395b565e6eb8f0b161edceae373a6017959ff202438f2a9ff3c
Original supplement source archive	e9dc01d971a49f44f108882a309a9b1d7f849a0fd70d8013ca95a3428b8dd801

Source inventory

The certified article tree contains 21 TeX files, 371 unique labels, and 104 theorem-like environments. The certified supplement tree contains four TeX files, 60 unique labels, and one theorem environment. No duplicate label occurs within either authoritative tree. The long-form integration retains all article modules and all three active supplement modules. External-document prefixes in the supplement are removed only in the integration copies so that the same labels resolve internally.

Computational execution record

The included program `verify_su2_certificate.py` was executed afresh in the extracted certified package. All ten displayed arithmetic and interval checks passed. The rerun reproduces the lowest eigenvalue, Gram scalar, magnetic recompression, ultraviolet floor, mixed-block norm, robust and sharpened two-block values, and the common-profile value 0.24975.

Upstream SU(3) provenance. This Paper-II package does not contain the Paper-I program that derives κ_{cof} . The long form therefore records an upstream source pointer rather than inventing or duplicating that certificate. The Paper-III $\mathbb{Z}_3$ linked-flux program certifies a different cohomological witness and is not the source of the Paper-I coercive constant.

Hostile-referee disposition

The integrated source preserves the certified firewalls: fixed-stratum depth exhaustion is not advertised as full cofinality; the complete-ground projection is not replaced by a preferred vector; $\kappa_*(q)$ is not identified with κ_{cof} ; the explicit $\text{SU}(2)$ realization is not renamed as the selected $\text{SU}(3)$ branch; physical scale arrows are not promoted to equivalences; coefficient-line equivalence is not physical scale equivalence; the two cohomology classes remain distinct; Paper-II strict/twisted alternatives are not Paper-III sectors; and no finite-scale estimate is advertised as a mass gap.

Chapter 15: Typed Handoff and Downstream Obligation Ledger

Paper-I intake

Paper II consumes the Paper-I typed finite packet [1]

$$\left(H_\gamma, E_{0,\gamma}, P_{\text{cg},\gamma}, F_\gamma^W, T_\gamma, G_\gamma, F_\gamma^{\text{phys}}, P_{\text{ch},\gamma}, P_{\text{res},\gamma}, P_{\text{UV},\gamma}, K_\gamma \right),$$

with the active semantic alias $P_{\text{cg},\gamma} = P_{\text{vac},\gamma}$. Here G_γ is the Gram operator and not the gauge group G .

Paper-II export

The downstream datum is the typed finite-depth obstruction-and-coercivity packet displayed in the conclusion. It retains the geometric scale category, the complete physicalized packages, line-assisted correspondences, the coefficient-line Hilbert descent object, the separate relative and band classes, the threshold profile, and the Paper-II profilewise constant.

Claims reserved for Papers III–V

Paper II does not construct an incision geometry, a regional W^* -system, a positive-thickness spacetime realization, a Friedrichs or Mosco thin operator, a limiting Hilbert object, a Poincaré representation, a joint spectral measure, an invariant-mass operator, a no-light-cone-leakage theorem, interaction, confinement, or a relativistic mass gap. Those obligations remain downstream exactly as typed by the five-paper dependency graph [2–4].

Chapter 16: Data, Code, and Acknowledgments

Data and code availability

All Paper-II LaTeX sources, the integrated supplement modules, the visible $SU(2)$ verification program, fresh rerun output, audit ledgers, and checksums are included in the source release accompanying this long-form manuscript. The Paper-I $SU(3)$ finite-packet program [1] remains an upstream certificate and is identified by source pointer rather than copied without its authoritative archive.

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Use of Generative Artificial Intelligence

The author used OpenAI’s ChatGPT, including GPT-4.5 through GPT-5.6, as an interactive research and adversarial editorial tool during the development of this manuscript and the broader five-paper series. Its use included assistance with organizing mathematical material, testing formulations and proof strategies, improving exposition, checking notation and cross-references, preparing and reviewing $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$ source, and supporting computational and document-audit workflows.

The conceptual framework, ontologies, definitions, conjectures, identification of the principal obstructions, load-bearing components of the proofs, and overall proof strategies originated with and remain solely those of the author.

All mathematical claims, definitions, theorems, proofs, computations, interpretations, citations, and final wording were reviewed and approved by the author, who assumes full responsibility for the content of the manuscript. No generative artificial-intelligence system is credited as an author.

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