

Supplementary Material for “Positive-Thickness Reconstruction for Yang–Mills Wilson-Tube Systems: Local-Normal Nets and Controlled Center-Banded Descent”

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This supplement records the extended technical material supporting the positive-thickness reconstruction theorem. It collects conventions and independence ledgers, obstruction witnesses, quadratic-form and leakage details, normal-descent domain arguments, finite-propagation and geometric estimates, sectorial Gauss–Ward calculations, thin-readiness records, the strict fixed-stratum control model, and the Friedrichs and linked-flux repair proofs. No additional headline theorem is asserted beyond the claims stated in the main article.

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Appendix A: Conventions and branch architecture

This appendix records the conventions needed to read the branch-sensitive statements without importing stronger conclusions than the hypotheses justify.

1. Operator-algebraic maps

All observable algebras are von Neumann algebras. A morphism in $\mathbf{W}^*\mathbf{Alg}_n$ is a normal unital $*$ -homomorphism. Normality means ultraweak continuity and is equivalent to the existence of a bounded predual map. It does not imply injectivity. When $T : \mathcal{M}_0 \rightarrow \mathcal{M}_1$ is normal, the state transport is contravariant:

$$T_* : \mathcal{M}_{1,*} \rightarrow \mathcal{M}_{0,*}, \quad T_*(\omega) = \omega \circ T. \quad (\text{A.1})$$

For composable maps T_a and T_b ,

$$(T_b T_a)_* = T_{a,*} T_{b,*}. \quad (\text{A.2})$$

This order is used in every scale-bordism cocycle formula.

A normal spatial isomorphism is stronger than a normal homomorphism. A coefficient-line unitary

$$U : \mathcal{H}_0 \hat{\otimes} L \rightarrow \mathcal{H}_1 \quad (\text{A.3})$$

implements a faithful algebra map because unitary conjugation is injective. In the weak branch, U is replaced by a normal W^* -correspondence or an ultraweakly closed intertwiner space; no equality of spectra, multiplicities, or kernels follows.

2. Graph triples and conjugate duals

A graph triple is

$$\mathcal{V} \xrightarrow{j} \mathcal{H} \xrightarrow{j^*} \mathcal{V}^*, \quad (\text{A.4})$$

where $\mathcal{V}$ is a Hilbert space continuously and densely embedded in the pivot Hilbert space $\mathcal{H}$.

A bounded Hermitian graph form is represented by

$$\mathbb{G} : \mathcal{V} \rightarrow \mathcal{V}^*. \quad (\text{A.5})$$

If $J : \mathcal{V}_0 \rightarrow \mathcal{V}_1$ is a graph unitary, its conjugate-dual inverse is

$$J^{-*} = (J^{-1})^* : \mathcal{V}_0^* \rightarrow \mathcal{V}_1^*. \quad (\text{A.6})$$

Graph covariance is

$$\mathbb{G}_1 J = J^{-*} \mathbb{G}_0, \quad (\text{A.7})$$

equivalently

$$g_1[J\Psi, J\Phi] = g_0[\Psi, \Phi]. \quad (\text{A.8})$$

A bounded map $\mathbb{G} : \mathcal{V} \rightarrow \mathcal{V}^*$ need not take values in $j^*\mathcal{H}$, so it need not define a Hilbert-space operator. A densely defined symmetric pivot realization is closable. A closed symmetric operator need not be self-adjoint. A self-adjoint operator has a spectral measure and an exponentiated unitary group, but the existence of that group does not by itself prove that it implements the intended gauge action on the observable algebra.

3. Complete ground sectors

The invariant ground datum is a projection P_Ω , not a vector. Its normal-state face is

$$\mathfrak{S}_\Omega = \{\rho \geq 0 : \text{Tr } \rho = 1, \rho = P_\Omega \rho P_\Omega\}. \quad (\text{A.9})$$

Statements such as Gauss annihilation, Ward invariance, and entropy surgery are formulated for the entire range of P_Ω or for all states in $\mathfrak{S}_\Omega$. No cyclicity, separating property, Reeh–Schlieder property, or one-dimensionality is assumed.

4. Banded classes and effective descent

A coefficient-line descent class is an equivalence class of line objects, line multiplications, and scalar tensorators. Its vanishing means that coherent units can be chosen in the lines. This is called coefficient strictification. It is distinct from effective descent, which asks for an actual global object in the target category. Thus

$$[\widehat{\Omega}]_{\text{cs,ctrl}} = 0 \not\Rightarrow \text{a global Hilbert representation.} \quad (\text{A.10})$$

Similarly, the boundary-character class $[\chi^\partial]_{\text{cs}}$ is independent of both $[\hat{\Omega}]_{\text{cs,ord}}$ and $[\hat{\Omega}]_{\text{cs,ctrl}}$. Vacuum coefficient strictification does not automatically provide global collar-charge zero-points.

The notation $[\Omega]_{\text{band}}$ is intrinsic and degree-free. A Čech class in degree two is written only after a constant abelian band and a good cover are selected. When central reduction fails, the correct object is correspondence-valued; a scalar phase is not imposed.

5. Exact, controlled, and weak branches

The article uses the following branch table.

Exact branch.: The algebraic transport is a normal homomorphism, injective only when separately proved; the represented transport is coefficient-line unitary; the analytic conclusion is equality of the stated forms, states, seams, or operators on their specified domains.

Controlled branch.: The algebraic or represented transport carries a finite-stage defect; the analytic conclusion is an explicit graph or predual estimate whose modulus decays along the selected cofinal family.

Weak branch.: The transport is correspondence-valued; the conclusion is a bimodule or relation-valued intertwining statement and not unitary equivalence.

An exact algebraic face does not force an exact analytic face. A controlled defect tending to zero does not change a topological obstruction class. A weak correspondence does not imply unitary equivalence.

6. Five independent axes

For bookkeeping, a completed thick object has branch signature

$$\mathfrak{b} = (b_{\text{alg}}, b_{\text{rep}}, b_{\text{an}}, b_{\text{op}}, b_{\text{sec}}). \quad (\text{A.11})$$

The entries record:

- whether algebra transport is strict, pseudofunctorial, or correspondence-valued;
- whether representation transport is unitary-line or weak correspondence transport;
- whether analytic transport is exact or controlled;
- whether only graph forms, closed operators, self-adjoint operators, or exponentiated groups are available;
- whether the chart-scale system lies in Sector I, controlled Sector II, or Sector III.

No component determines another. In particular,

$$\begin{aligned}
&\text{normal algebra map} \not\Rightarrow \text{injective map}, \\
&[\widehat{\Omega}]_{\text{cs,ctrl}} = 0 \not\Rightarrow \text{effective Hilbert descent}, \\
&\text{state surgery} \not\Rightarrow \text{graph-space surgery}, \\
&\bar{q}_r \geq \kappa_*(I - P_{\Omega,r}) \not\Rightarrow \text{a thin positive threshold}.
\end{aligned} \tag{A.12}$$

These non-implications are part of the theorem architecture, not merely cautionary remarks.

Appendix B: Controlled scale cohomology and abstract sector witnesses

This appendix separates three statements that must not be conflated: the ordinary chart-scale descent class, the controlled class relevant to uniformly local refinement, and the physical fixed-thickness $SU(3)$ class derived in Appendix I. The rooted scale nerve is contractible in unrestricted ordinary cohomology; controlled Sector II records the absence of a strictification selected from uniformly local, scale-natural shell data.

1. Cyclic carry cocycles

Let $N \geq 2$, $\zeta_N = e^{2\pi i/N}$, and let $\tilde{a} \in \{0, \dots, N-1\}$ denote the preferred representative of $a \in \mathbb{Z}_N$. Define

$$\epsilon_N(a, b) = \frac{\tilde{a} + \tilde{b} - \widetilde{a+b}}{N} \in \{0, 1\}. \tag{B.1}$$

The carry identity is

$$\epsilon_N(a, b) + \epsilon_N(a+b, c) = \epsilon_N(b, c) + \epsilon_N(a, b+c). \tag{B.2}$$

For $m \in \mathbb{Z}_N$, put

$$\Omega_m(a, b, c) = \zeta_N^{m \tilde{a} \epsilon_N(b, c)}. \quad (\text{B.3})$$

Proposition B.1 (Cyclic pentagon and detector). *The cochain Ω_m is normalized and satisfies $\delta\Omega_m = 1$. Moreover,*

$$I_N(\Omega) = \prod_{b=0}^{N-1} \Omega(1, b, 1) \quad (\text{B.4})$$

is invariant under normalized two-coboundaries, and

$$I_N(\Omega_m) = \zeta_N^m. \quad (\text{B.5})$$

Thus Ω_m is nontrivial in $H^3(B\mathbb{Z}_N; U(1))$ for $m \neq 0$.

Proof. Equation (B.2) cancels the exponent in $\delta\Omega_m$. The factors in $I_N(\delta\lambda)$ telescope for every normalized two-cochain λ . In (B.3), $\epsilon_N(b, 1)$ is nonzero only for $b = N - 1$. $\square$

2. Ordinary root contraction

Let $\mathcal{R}$ have objects r_n , $n \geq 0$, and a unique arrow $a_{n;k} : r_n \rightarrow r_{n+k}$ for every $k \geq 0$. The object r_0 is initial. An ordinary normalized two-cochain is a family $\lambda_n(k, \ell)$, and

$$(\delta_{\mathcal{R}}\lambda)_n(k, \ell, p) = \frac{\lambda_{n+k}(\ell, p)\lambda_n(k, \ell + p)}{\lambda_n(k + \ell, p)\lambda_n(k, \ell)}. \quad (\text{B.6})$$

Theorem B.2 (Ordinary cone contraction). *For constant $U(1)$ coefficients, $H^q(B\mathcal{R}; U(1)) = 0$ for $q > 0$. For the depth-independent cyclic scale cocycle*

$$\Omega_r^{\text{sc}}(k, \ell, p) = \zeta_N^{r \tilde{k} \epsilon_N(\ell, p)}, \quad (\text{B.7})$$

the explicit primitive

$$\lambda_{r;n}^{\text{root}}(k, \ell) = \Omega_r^{\text{sc}}(n, k, \ell) \quad (\text{B.8})$$

satisfies $\delta_{\mathcal{R}}\lambda_r^{\text{root}} = \Omega_r^{\text{sc}}$.

Proof. The initial object supplies an extra degeneracy of the nerve. Applying the three-cocycle identity to the cone arrow $r_0 \rightarrow r_n$ gives the explicit formula. Direct substitution of (B.8) into (B.6) yields (B.7). $\square$

The primitive is not stationary. For $r \neq 0$,

$$\frac{\lambda_{r;n+1}^{\text{root}}(N-1, 1)}{\lambda_{r;n}^{\text{root}}(N-1, 1)} = \zeta_N^r. \quad (\text{B.9})$$

It remembers the absolute depth even when adjacent renormalized shell packages are isomorphic.

3. Uniformly scale-controlled cochains

A decorated scale simplex records a finite shell neighborhood of each vertex, all comparisons among its vertices, the complete-ground projections, coefficient lines, multiplication maps, and the declared analytic controls. A normalized p -cochain is *uniformly scale controlled* when it is determined by a single finite-radius functional of this decorated simplex, depends only on reduced band grades, is natural under control-preserving isomorphisms, and is uniformly Lipschitz in the shell metric. Face restriction is nonexpansive, so the ordinary coboundary preserves these conditions.

Proposition B.3 (Controlled cochain complex). *The uniformly scale-controlled normalized cochains form a subcomplex*

$$(C_{\text{sc,ctrl}}^\bullet(\mathfrak{D}; U(1)), \delta) \subseteq (C_{\text{ord}}^\bullet(N, \mathcal{R}; U(1)), \delta). \quad (\text{B.10})$$

On an exact renormalized dilation orbit, naturality forces depth stationarity and band preservation reduces the complex canonically to the normalized cochain complex of $\mathbb{Z}_N$.

Proof. Every term in the coboundary is the value of the original cochain on a face of one decorated simplex. Finite radius, naturality, normalization, and band preservation are inherited by face restriction, and the Lipschitz constants add. Exact shell translation identifies all source depths; reduced-grade dependence then gives the normalized $\mathbb{Z}_N$ complex. $\square$

Theorem B.4 (Controlled cyclic nonvanishing and sharp defect). *On the exact dilation orbit,*

$$[\Omega_r^{\text{sc}}]_{\text{sc,ctrl}} = ru_N \in H^3(B\mathbb{Z}_N; U(1)). \quad (\text{B.11})$$

For $N = 3$ and $r = 1, 2$, every normalized two-cochain satisfies

$$\max_{b=1,2} d_{U(1)}(\Omega_r^{\text{sc}}(1, b, 1), (\delta\lambda)(1, b, 1)) \geq \frac{\pi}{3}, \quad (\text{B.12})$$

and the bound is sharp.

Proof. Proposition B.3 reduces the equation to the normalized group complex. Proposition B.1 excludes a primitive. For the quantitative bound, the product of the two marked residuals is ω^r ; the geodesic triangle inequality gives $2 \max d \geq 2\pi/3$. Choosing equal residual phases gives equality. $\square$

4. Abstract strictly refining controlled Sector II

Assign thickness $e(r_n) = 2^{-n}$ and grade $|a_{n;k}| = k \pmod{N}$. Every nonidentity arrow strictly decreases thickness; every fixed-thickness chart category is strict. Give the scale coefficient lines reassociator

$$\alpha_{c,b,a} = \Omega_m(|c|, |b|, |a|)I. \quad (\text{B.13})$$

Theorem B.5 (Abstract strictly refining controlled Sector-II witness). *For $m \neq 0$, every fixed-thickness class is zero, the unrestricted ordinary scale class is zero by Theorem B.2, and the controlled total class is nonzero by Theorem B.4. The system is therefore controlled Sector II. It uses no nonidentity equal-thickness scale endomorphism.*

This theorem establishes categorical nonemptiness only. It does not derive the coefficient system from a Yang–Mills Hamiltonian. The physical $SU(3)$ scale corridor and the fixed-thickness linked-flux class are separate constructions in Section VIII and Appendix I.

Appendix C: Closed forms, complete kernels, and leakage

1. Closure of the coercive preform

Let $\mathcal{H}$ be a Hilbert space, P an orthogonal projection, and q^{pre} a densely defined nonnegative closable form on a core $\mathcal{C}$. Assume

$$P\mathcal{C} \subseteq \mathcal{C}, \quad q^{\text{pre}}[P\psi, \phi] = 0, \quad (\text{C.1})$$

and

$$q^{\text{pre}}[\psi] \geq \kappa \|(I - P)\psi\|^2, \quad \kappa > 0. \quad (\text{C.2})$$

Let $\bar{q}$ be its closure, with graph norm

$$\|\psi\|_q^2 = \|\psi\|^2 + \bar{q}[\psi]. \quad (\text{C.3})$$

Lemma C.1 (Graph continuity of the ground projection). *The projection P extends to a contraction on $D(\bar{q})$ in the graph norm, and*

$$\bar{q}[P\psi] = 0, \quad \bar{q}[\psi] = \bar{q}[(I - P)\psi]. \quad (\text{C.4})$$

Proof. Choose $\psi_n \in \mathcal{C}$ converging to ψ in the graph norm. By (C.1),

$$q^{\text{pre}}[P(\psi_n - \psi_m)] = 0, \quad (\text{C.5})$$

so $P\psi_n$ is Cauchy in the graph norm exactly when it is Cauchy in $\mathcal{H}$. It converges to $P\psi$. Closure gives $P\psi \in D(\bar{q})$ and $\bar{q}[P\psi] = 0$. Polarization of (C.1) and passage to the closure eliminate all cross terms, giving (C.4). $\square$

Theorem C.2 (Closed coercive decomposition). *Under (C.1)–(C.2),*

$$D(\bar{q}) = P\mathcal{H} \hat{\oplus} (I - P)D(\bar{q}), \quad (\text{C.6})$$

$$\bar{q}[\psi] \geq \kappa \|(I - P)\psi\|^2, \quad (\text{C.7})$$

and

$$\ker \bar{q} = P\mathcal{H}. \quad (\text{C.8})$$

If K is the positive self-adjoint operator associated with $\bar{q}$, then

$$K = 0_{P\mathcal{H}} \oplus K^\perp, \quad K^\perp \geq \kappa I, \quad (\text{C.9})$$

so

$$E_K(\{0\}) = P, \quad \text{spec}(K) \subseteq \{0\} \cup [\kappa, \infty). \quad (\text{C.10})$$

Proof. The domain decomposition follows from Lemma C.1. Let $\psi_n \in \mathcal{C}$ converge to ψ in the graph norm. Then $(I - P)\psi_n \rightarrow (I - P)\psi$ in $\mathcal{H}$, and lower semicontinuity gives

$$\bar{q}[\psi] = \lim_n q^{\text{pre}}[\psi_n] \geq \kappa \lim_n \|(I - P)\psi_n\|^2, \quad (\text{C.11})$$

which is (C.7). If $\bar{q}[\psi] = 0$, the bound gives $(I - P)\psi = 0$, so $\psi \in P\mathcal{H}$; the reverse inclusion follows from Lemma C.1. The representation theorem for closed nonnegative forms gives K . The form vanishes on $P\mathcal{H}$ and is bounded below by κ on the orthogonal complement, giving (C.9) and (C.10). $\square$

If $U : \mathcal{H}_0 \rightarrow \mathcal{H}_1$ is unitary, carries P_0 to P_1 , maps a core for q_0^{pre} to a core for q_1^{pre} , and preserves the preform, then it extends to a graph unitary and

$$K_1 = UK_0U^*. \quad (\text{C.12})$$

This applies to exact chart and scale transports. For a controlled graph comparison V_a^q , the defect (VI.42) is bounded on the product graph space and composes by (VI.43). A small such defect is weaker than norm-resolvent convergence and does not imply convergence of spectral projections.

2. A leakage counterexample

Let

$$\mathcal{H} = \ell^2(\mathbb{N}) \quad (\text{C.13})$$

and let P_n be the projection onto

$$\text{span}\{e_1, \dots, e_n\}. \quad (\text{C.14})$$

Define

$$q_n[\psi] = \|(I - P_n)\psi\|^2. \quad (\text{C.15})$$

Each form is closed and satisfies

$$q_n[\psi] \geq \|(I - P_n)\psi\|^2, \quad \ker q_n = P_n\mathcal{H}, \quad (\text{C.16})$$

with the uniform constant $\kappa_* = 1$.

However,

$$P_n \longrightarrow I \quad \text{strongly}, \quad (\text{C.17})$$

and therefore

$$q_n[\psi] \longrightarrow 0 \quad (\text{C.18})$$

for every $\psi \in \mathcal{H}$. The pointwise and Mosco limit is the zero form, whose kernel is all of $\mathcal{H}$. Thus a uniform endpoint estimate does not prevent the complete ground sectors from absorbing every limiting direction.

Proposition C.3 (No automatic threshold transfer). *The family (C.15) proves*

$$\left(q_n \geq \kappa_*(I - P_n) \text{ for all } n\right) \not\Rightarrow \left(q_\infty \geq \kappa_*(I - P_\infty)\right) \quad (\text{C.19})$$

for any nontrivial limiting ground complement. Uniform positive-thickness coercivity alone is therefore insufficient for a thin spectral threshold.

3. A sufficient transfer pattern

Suppose the forms q_n are realized in a common comparison Hilbert space, the projections satisfy

$$P_n \rightarrow P_\infty \quad \text{strongly}, \quad (\text{C.20})$$

and a candidate closed form q_∞ has a strong recovery property: for every $\psi \in D(q_\infty)$ there are $\psi_n \rightarrow \psi$ with

$$q_n[\psi_n] \longrightarrow q_\infty[\psi]. \quad (\text{C.21})$$

Then the endpoint bounds imply

$$\begin{aligned} q_\infty[\psi] &= \lim_n q_n[\psi_n] \\ &\geq \kappa_* \lim_n \|(I - P_n)\psi_n\|^2 \\ &= \kappa_* \|(I - P_\infty)\psi\|^2. \end{aligned} \quad (\text{C.22})$$

The recovery or upper-identification statement is essential. The usual Mosco lower-bound condition alone has the opposite logical direction: it bounds q_∞ above by a liminf of approximating energies and cannot by itself produce (C.22).

In the Yang–Mills program, (C.20) is the ground-sector non-leakage input and (C.21) is the limiting energy-identification input. Neither is proved in the present article.

Appendix D: Normal compatible-family descent and unbounded invariance

This appendix collects two technical mechanisms used repeatedly in the main text: normal equalizer descent for bounded observables and the passage from graph-form invariance to strong commutation for self-adjoint constraints.

1. Compatible families as a normal equalizer

Let $\mathcal{G}$ be a small groupoid and let

$$\mathfrak{M} : \mathcal{G} \longrightarrow \mathbf{W}^* \mathbf{Alg}_{\mathbf{n}} \quad (\text{D.1})$$

be a functor such that every arrow is represented by a normal $*$ -isomorphism. Write $\mathcal{M}_p = \mathfrak{M}(p)$ and $\Phi_\nu = \mathfrak{M}(\nu)$. The bounded product

$$\prod_{p \in \text{Ob } \mathcal{G}}^{\ell^\infty} \mathcal{M}_p \quad (\text{D.2})$$

is a von Neumann algebra with coordinatewise operations and predual ℓ^1 -sum; see Refs.^{41,42} for the standard operator-algebraic background.

For every arrow $\nu : p \rightarrow q$, define the normal linear difference map

$$D_\nu((X_r)_r) = X_q - \Phi_\nu(X_p). \quad (\text{D.3})$$

The compatible-family algebra is

$$\text{Desc}(\mathfrak{M}) = \bigcap_{\nu \in \text{Mor } \mathcal{G}} \ker D_\nu. \quad (\text{D.4})$$

Proposition D.1 (Normal equalizer). *The algebra $\text{Desc}(\mathfrak{M})$ is a von Neumann subalgebra of (D.2).*

Proof. Each D_ν is ultraweakly continuous because coordinate evaluation and Φ_ν are normal. Hence $\ker D_\nu$ is ultraweakly closed. The intersection (D.4) is therefore ultraweakly closed. Compatibility is preserved by adjoints and products because the Φ_ν are $*$ -homomorphisms. The compatible families contain the unit, so they form a von Neumann subalgebra. $\square$

Fix an object p . Its isotropy group acts normally on $\mathcal{M}_p$. Let

$$\mathcal{M}_p^{\text{Iso}(p)} = \{X \in \mathcal{M}_p : \Phi_\gamma(X) = X \text{ for all } \gamma : p \rightarrow p\}. \quad (\text{D.5})$$

Theorem D.2 (Evaluation and isotropy). *If $\mathcal{G}$ is connected, coordinate evaluation is a normal $*$ -isomorphism*

$$\text{ev}_p : \text{Desc}(\mathfrak{M}) \xrightarrow{\cong} \mathcal{M}_p^{\text{Iso}(p)}. \quad (\text{D.6})$$

In particular, evaluation is onto $\mathcal{M}_p$ if and only if the observable isotropy acts trivially.

Proof. A compatible family has X_p fixed by every loop at p , proving that evaluation lands in (D.5). It is injective because connectedness lets one transport X_p to every object. Conversely, choose for each q an arrow $\nu_q : p \rightarrow q$ and set $X_q = \Phi_{\nu_q}(X)$. If ν'_q is another choice, then $(\nu'_q)^{-1}\nu_q$ is an isotropy loop at p , so the fixed-point condition gives the same X_q . The resulting family is compatible and evaluates to X . Normality of evaluation and its inverse follows from normality of the component maps. $\square$

A one-object groupoid with isotropy group H illustrates the necessity of the condition: the descended algebra is $\mathcal{M}^H$, not $\mathcal{M}$. In the Wilson-tube construction, observable isotropy loops agree with the identity on the ultraweakly generating physical core; normality then forces them to be the identity on the bicommutant.

2. Descent of natural transformations

Let $R : \mathcal{G}_1 \rightarrow \mathcal{G}_0$ be a functor and let

$$J_p : \mathfrak{M}_0(Rp) \longrightarrow \mathfrak{M}_1(p) \quad (\text{D.7})$$

be normal unital $*$ -homomorphisms satisfying

$$\Phi_\nu^{(1)} J_p = J_q \Phi_{R\nu}^{(0)} \quad (\text{D.8})$$

for every $\nu : p \rightarrow q$. Define

$$\text{Desc}(J, R)((X_r)_r) = (J_p(X_{Rp}))_p. \quad (\text{D.9})$$

Proposition D.3 (Normal descent of a natural transformation). *Equation (D.9) defines a normal unital $*$ -homomorphism between the compatible-family algebras. It is injective or invertible only when the component family has the corresponding property and the comparison functor sees the relevant connected components.*

Proof. Naturality (D.8) proves that the image family is compatible. Coordinatewise normality gives normality on the product and hence on the equalizer. The final statement follows by evaluating in chosen presentations and applying Theorem D.2; normality alone contains no injectivity information. $\square$

Regional inclusions, strict background maps, and homomorphic scale maps are all instances of Proposition D.3. It also proves composition: two natural transformations descend to the composite transformation because their component maps compose pointwise.

3. Normal extension of commutator estimates

Let $\mathfrak{A}_0 \subseteq \mathcal{M}_0$ and $\mathfrak{A}_1 \subseteq \mathcal{M}_1$ be unital $*$ -subalgebras that are strongly dense in their bicommutants inside a common representation. Suppose

$$\|[A, B]\| \leq \eta \|A\| \|B\| \quad (\text{D.10})$$

for all $A \in \mathfrak{A}_0$, $B \in \mathfrak{A}_1$.

Proposition D.4 (Kaplansky extension). *The same estimate holds for all $X \in \mathcal{M}_0$ and $Y \in \mathcal{M}_1$:*

$$\|[X, Y]\| \leq \eta \|X\| \|Y\|. \quad (\text{D.11})$$

Proof. By Kaplansky density, choose nets $A_i \rightarrow X$ and $B_j \rightarrow Y$ strongly- $*$ with $\|A_i\| \leq \|X\|$ and $\|B_j\| \leq \|Y\|$. For fixed j , bounded strong- $*$ convergence gives $[A_i, B_j] \rightarrow [X, B_j]$ strongly. The operator norm is lower semicontinuous under strong convergence on bounded sets, so

$$\|[X, B_j]\| \leq \eta \|X\| \|B_j\|. \quad (\text{D.12})$$

Taking the second bounded strong- $*$ limit gives (D.11). $\square$

4. Graph invariance and spectral strong commutation

Let a be a densely defined closed semibounded form on $\mathcal{H}$ with associated self-adjoint operator A . Let $B \in B(\mathcal{H})$ satisfy

$$BD(a) \subseteq D(a), \quad a[B\psi, \phi] = a[\psi, B^*\phi] \quad (\text{D.13})$$

for all $\psi, \phi \in D(a)$.

Lemma D.5 (Resolvent commutation). *For every λ below the lower bound of A ,*

$$B(A - \lambda)^{-1} = (A - \lambda)^{-1}B. \quad (\text{D.14})$$

Proof. For $f \in \mathcal{H}$, let $u = (A - \lambda)^{-1}f$. The vector $u \in D(a)$ is characterized by

$$a[u, v] - \lambda \langle u, v \rangle = \langle f, v \rangle \quad (\text{D.15})$$

for every $v \in D(a)$. Apply (D.13) to obtain

$$a[Bu, v] - \lambda \langle Bu, v \rangle = \langle Bf, v \rangle. \quad (\text{D.16})$$

Uniqueness of the variational solution gives $Bu = (A - \lambda)^{-1}Bf$. $\square$

The functional calculus then gives commutation with every bounded Borel function of A , in particular with all spectral projections. Hence

$$\{A\}'_{\text{str}} = \{B \in B(\mathcal{H}) : BE_A(\Delta) = E_A(\Delta)B \text{ for all Borel } \Delta\} \quad (\text{D.17})$$

is a von Neumann algebra.

Theorem D.6 (Strong commutation from a core). *Let $\mathfrak{A}$ be a represented unital $*$ -algebra that preserves $D(a)$ and satisfies (D.13). Then*

$$\mathfrak{A}'' \subseteq \{A\}'_{\text{str}}. \quad (\text{D.18})$$

Proof. Lemma D.5 places every $B \in \mathfrak{A}$ in the spectral strong commutant. Since (D.17) is a von Neumann algebra, it contains the bicommutant generated by $\mathfrak{A}$. $\square$

This is the argument used for the Gauss generators. It is stronger and safer than trying to extend a formal commutator equality by ultraweak density.

5. Commutant isotropy does not preserve graph domains

Let $\mathcal{H} = \ell^2(\mathbb{Z})$, let $\mathcal{M} = \mathbb{C}I$, and let

$$\mathcal{V} = \left\{ x = (x_n) : \sum_{n \in \mathbb{Z}} 2^{2|n|} |x_n|^2 < \infty \right\}. \quad (\text{D.19})$$

Every unitary lies in $\mathcal{M}'$. Choose a permutation σ of $\mathbb{Z}$ that sends an infinite sequence $n_k \rightarrow \infty$ to $\sigma(n_k) = k$. The induced permutation unitary Z can map a vector concentrated on the n_k with weighted-square summability into one whose coefficients at k fail the weighted condition, or conversely. Hence

$$Z \in \mathcal{M}' \quad \text{but} \quad Z\mathcal{V} \neq \mathcal{V}. \quad (\text{D.20})$$

This is why the selected Cauchy and overlap commutant defects must satisfy explicit domain and graph-form isotropy conditions before unbounded path independence is asserted.

Appendix E: Finite controlled geometry and causal propagation

This appendix records the finite-stage geometric facts used in the support and covariance arguments. The statements are local to compact test diagrams and do not assert a global resolution of every possible incision.

1. Finite adapted atlases

Let $\Sigma \subset M$ be a closed controlled stratified subset. Fix a finite family consisting of relatively compact admissible regions, compact support pairs, collar footprints, and local Cauchy slabs. Local finiteness of the stratification and compactness imply that only finitely many strata meet the selected family. Choose tubular neighborhoods for those strata and shrink the regions so that all relevant closures remain inside the control charts.

Proposition E.1 (Finite adapted refinement). *Every finite geometric test family admits a finite precompact atlas $\{U_\alpha\}$ with the following properties:*

1. *every selected closure and causal shadow is covered by finitely many U_α ;*

2. *chart transitions preserve the selected tubular and collar controls;*
3. *every selected region, support carrier, and Cauchy slice has positive distance from the unwanted chart and incision boundaries;*
4. *any two such finite atlases possess a common adapted refinement;*
5. *a strict controlled background embedding transports the atlas to an adapted atlas of its image.*

Proof. Cover each compact set by finitely many adapted control charts and take the finite union. Precompact shrinking gives positive buffer distances. Intersections of two finite adapted atlases, followed by precompact shrinking, give a common refinement. Strict background embeddings preserve the control structures by definition, so transported charts remain adapted. Distances are understood intrinsically in the transported image; an embedding need not preserve all ambient-target geodesic distances. $\square$

The control charts can be resolved along the finitely many incident strata to manifolds with corners

$$\beta_\alpha : \widehat{U}_\alpha \longrightarrow \widetilde{U}_\alpha \tag{E.1}$$

with incision front faces $\mathfrak{F}_{S,\alpha}$. Adapted restrictions and chart transitions lift and preserve the incidence structure. This geometric resolution does not imply that the principal gauge bundle extends across the front face; that is a separate bundle datum.

2. Margins and thickening thresholds

Let h be a control Riemannian metric on a compact stage. If $\mathcal{O}^{+\delta}$ is produced by a controlled thickening profile with Lipschitz constant $C_{\mathcal{O}}$, then

$$d_h(\overline{\mathcal{O}^{+\delta}}, \Sigma) \geq d_h(\overline{\mathcal{O}}, \Sigma) - C_{\mathcal{O}}\delta. \tag{E.2}$$

For two compact regions define a continuous spacelike-margin functional $\mu_U(\mathcal{O}_1, \mathcal{O}_2) > 0$ on a common globally hyperbolic neighborhood. Controlled thickening gives

$$\mu_U(\mathcal{O}_1^{+\delta_1}, \mathcal{O}_2^{+\delta_2}) \geq \mu_U(\mathcal{O}_1, \mathcal{O}_2) - C_1\delta_1 - C_2\delta_2. \tag{E.3}$$

Thus a finite family has a single positive threshold $\delta_{\mathfrak{F}}$ preserving chart containment, incision clearance, and all selected causal-separation relations. This proves the finite-stage exact-locality threshold (IV.22).

3. Causal shadows

Let N be globally hyperbolic with Cauchy slices Σ_s . For compact $K \subseteq \Sigma_{s_1}$, the shadow (IV.5) is compact because the causal diamond between compact sets and Cauchy slices is compact. It is monotone and preserves finite unions. If $s_1 < s_2 < s_3$, every causal curve from K to Σ_{s_3} meets Σ_{s_2} , while concatenation gives the reverse inclusion. Hence (IV.6) holds.

Assume the finite-stage shadow maps are Hausdorff-Lipschitz:

$$d_H(\text{Sh}_{s_2, s_1}(K), \text{Sh}_{s_2, s_1}(K')) \leq \Lambda d_H(K, K'). \quad (\text{E.4})$$

Then

$$\text{Sh}_{s_2, s_1}(K^{+\varepsilon}) \subseteq \text{Sh}_{s_2, s_1}(K)^{+\Lambda\varepsilon}, \quad (\text{E.5})$$

which is (IV.7). In a standard constant-time Minkowski frame, $\Lambda = 1$.

A strict controlled background map $\psi : N \rightarrow N'$ is causally compatible on its image. Therefore

$$\psi(\text{Sh}_{s_2, s_1}^N(K)) = \text{Sh}_{\psi s_2, \psi s_1}^{\psi N}(\psi K), \quad (\text{E.6})$$

and similarly for domains of dependence and causal hulls. These identities establish the geometric faces of the background–scale–Cauchy coherence diagram.

4. From generator propagation to regional locality

Suppose each elementary Wilson label w on Σ_{s_1} is assigned a propagated label $\mathcal{P}_{s_2, s_1} w$ satisfying:

1. the propagated carrier lies in the causal shadow of the original thick carrier;
2. physical relations, adjoints, and products are preserved;
3. reverse propagation is inverse on the physical quotient;

4. propagation composes under subdivision of the Cauchy path.

Then the formal assignment descends to the algebraic isomorphism (IV.8), and support exactness (IV.9) follows by induction on Wilson words and linearity. If two propagated thick carriers are causally disjoint, each pair of elementary factors commutes; repeated use of the Leibniz identity for commutators proves commutation of the full words and polynomials. Proposition D.4 then extends the result to the regional von Neumann closures.

In the quasilocal branch, choose localized approximants satisfying (IV.20). Expanding

$$[A, B] = [A - A_0, B - B_0] + [A - A_0, B_0] + [A_0, B - B_0] \quad (\text{E.7})$$

and using $\|[X, Y]\| \leq 2\|X\|\|Y\|$ gives (IV.21). The exact and quasilocal branches are therefore logically distinct: asymptotic leakage control does not imply exact positive-thickness commutation unless the leakage vanishes.

Appendix F: Sectorial gluing of Gauss kernels and collar Ward data

This appendix records the gluing statements that distinguish the strict and twisted gauge sectors. The central point is that complete-ground annihilation is first a chartwise kernel condition; a global operator equation is available only after the represented and unbounded systems descend effectively.

1. Chart covariance of graph constraints

Let ξ_β be a compactly supported interior parameter on a chart overlap and let

$$\xi_\alpha = \mathbb{C}_{\alpha\beta}^g \xi_\beta \quad (\text{F.1})$$

be its transport by the chart diffeomorphism and gauge transition function. The overlap unitary restricts to a graph unitary

$$\mathbb{U}_{\alpha\beta,r}^{V,G} : \mathcal{V}_{\beta,r}^G \hat{\otimes} L_{\alpha\beta,r} \longrightarrow \mathcal{V}_{\alpha,r}^G. \quad (\text{F.2})$$

Graph covariance is

$$\mathbb{G}_{\alpha,r}(\xi_\alpha) \mathbb{U}_{\alpha\beta,r}^{V,G} = (\mathbb{U}_{\alpha\beta,r}^{V,G})^{-*} (\mathbb{G}_{\beta,r}(\xi_\beta) \hat{\otimes} I). \quad (\text{F.3})$$

In a closed operator branch, (F.3) implies equality of the transported domains and conjugacy of the closures. In a self-adjoint branch it implies conjugacy of the spectral measures.

Suppose the complete-ground graph projection satisfies

$$\mathbb{G}_{\alpha,r}(\xi_\alpha)P_{\Omega,\alpha,r}^{V,G} = 0. \quad (\text{F.4})$$

Then (F.3) and (V.6) give

$$\mathbb{U}_{\alpha\beta,r}^{V,G}(\ker \mathbb{G}_{\beta,r}(\xi_\beta) \hat{\otimes} L_{\alpha\beta,r}) = \ker \mathbb{G}_{\alpha,r}(\xi_\alpha). \quad (\text{F.5})$$

In particular, the complete-ground subspaces form a coefficient-line-banded kernel object.

Proposition F.1 (Sectorial ground annihilation). *At a fixed thickness, the family*

$$\mathcal{H}_{\Omega,\alpha,r} = P_{\Omega,\alpha,r} \mathcal{H}_{\alpha,r} \quad (\text{F.6})$$

is a Gauss-null descent subobject. If $[\Omega_{e(r)}]_{\text{band}} = 0$ and effective Hilbert and Gauss-domain descent hold, it gives a global projection $P_{\Omega,r}$ and global constraints satisfying

$$G_r(\xi)P_{\Omega,r} = 0. \quad (\text{F.7})$$

If the fixed-thickness class is nonzero, the chartwise family (F.6) is the invariant conclusion and (F.7) is not asserted.

Proof. Equation (F.5) proves the descent-subobject statement. A coefficient trivialization converts the overlap correspondences to an ordinary unitary cocycle, but the global operator equation still requires that the Hilbert objects and unbounded domains descend effectively. Under those additional hypotheses the local kernel equations glue to (F.7). $\square$

Across a scale arrow $a : r_0 \rightarrow r_1$, exact graph covariance is

$$\mathbb{G}_{\alpha,r_1}(\mathbb{R}_a^g \xi) \mathbb{V}_{\alpha;a}^{V,G} = (\mathbb{V}_{\alpha;a}^{V,G})^{-*}(\mathbb{G}_{\alpha,r_0}(\xi) \hat{\otimes} I). \quad (\text{F.8})$$

It transports the complete Gauss-null ground subspaces exactly. In the controlled branch define the graph defect by the difference of the two sides of (F.8). For composable arrows it satisfies the same transported-sum law as (VI.43); therefore finite glued chains inherit the edgewise decay modulus.

2. Collar quotient and splitting independence

Let the exact sequence (VI.2) be fixed. For $\lambda \in \mathfrak{g}_\partial$, choose any admissible lift ξ . If ξ' is another lift, then $\xi' - \xi \in \mathfrak{g}_{\text{int}}$. Hence the quotient class (VI.19) is independent of the lift.

A splitting s^∂ gives the representative

$$\mathbb{Q}_s^\partial(\lambda) = \mathbb{G}^{\text{adm}}(s^\partial \lambda). \quad (\text{F.9})$$

If $\tilde{s}^\partial = s^\partial + \tau$, where $\tau : \mathfrak{g}_\partial \rightarrow \mathfrak{g}_{\text{int}}$, then

$$\mathbb{Q}_{\tilde{s}}^\partial(\lambda) = \mathbb{Q}_s^\partial(\lambda) + \mathbb{G}^{\text{int}}(\tau \lambda). \quad (\text{F.10})$$

On $\mathcal{K}^{G, \text{int}}$, the second term vanishes, proving that the reduced charge form is intrinsic.

The charge may transform across a chart overlap by an affine scalar term as in (VI.23).

On a triple overlap, consistency gives

$$\chi_{\alpha\gamma}^{\text{ch}}(\lambda) = \chi_{\alpha\beta}^{\text{ch}}(\lambda) + \chi_{\beta\gamma}^{\text{ch}}(\lambda) \quad (\text{F.11})$$

with the parameter transported to the appropriate chart. Scale characters satisfy an analogous composition law, and chart and scale characters obey a mixed square. Changing local charge zero-points by scalar linear functionals changes these characters by a total coboundary. Their total class $[\chi^\partial]_{\text{cs}}$ is therefore intrinsic.

A continuous infinitesimal character $\chi : \mathfrak{g}_\partial \rightarrow i\mathbb{R}$ satisfies

$$\chi([\lambda, \mu]) = 0. \quad (\text{F.12})$$

It factors through the continuous abelianization. In particular, if the boundary algebra is topologically perfect, every such infinitesimal character vanishes. Nontrivial affine data must then arise from a global or discrete quotient, a germ-level structure, or a central extension rather than from an arbitrary infinitesimal semisimple gauge algebra.

3. Ward action versus charge zero-points

Let Q_α^∂ and Q_β^∂ be self-adjoint representatives related by (VI.23). Define the adjoint derivation

$$\mathcal{D}_\lambda^\alpha(A) = i[Q_\alpha^\partial(\lambda), A]. \quad (\text{F.13})$$

The scalar shift commutes with A , so

$$\mathcal{D}_\lambda^\alpha \circ \text{Ad}_{\mathbb{U}_{\alpha\beta}} = \text{Ad}_{\mathbb{U}_{\alpha\beta}} \circ \mathcal{D}_\lambda^\beta. \quad (\text{F.14})$$

Thus the Ward derivation descends even when the local charge zero-points do not.

If the current defect (VI.24) is scalar,

$$\mathfrak{a}^\partial(\lambda, \mu) = c^\partial(\lambda, \mu)I, \quad (\text{F.15})$$

then

$$[\mathcal{D}_\lambda, \mathcal{D}_\mu] + \mathcal{D}_{[\lambda, \mu]} = 0 \quad (\text{F.16})$$

although the charge representation is projective. A scalar current cocycle therefore changes the implementation but not the observable Ward algebra.

Theorem F.2 (Charge descent is stronger than Ward descent). *A global family of collar-charge operators requires triviality of the boundary-character class together with effective unbounded charge descent. The adjoint Ward derivations require neither a choice of global scalar zero-point nor the vanishing of a scalar current cocycle. They still require effective descent of the observable, Hilbert, and insertion-domain systems.*

4. Ground-state Ward identities

Let ρ be a normal state supported in $P_{\Omega, \alpha, r}$. From (VI.15) and the interior Ward identity,

$$\rho(\pi(\delta_\xi A)) = 0. \quad (\text{F.17})$$

This holds for every state in the complete ground face, not only for a vector state. On chart overlaps the pushforward of ρ by $\mathbb{U}_{\alpha\beta, r}$ satisfies the corresponding identity in the target chart. Therefore the complete-ground Ward functionals form a sectorial descent family. In Sector I, after effective descent, they become ordinary global Ward identities; in Sector III they remain coefficient-line banded.

Appendix G: Thin-reconstruction criteria

The present theorem ends with a complete positive-thickness diagram. This appendix records the additional criteria that a subsequent reconstruction must verify before any

zero-thickness object can be claimed. The criteria are stated on each fixed finite background–scale–chart–Cauchy test stage. They are not consequences of Theorems [III.17](#) and [III.18](#) unless explicitly indicated.

TR1: Cofinal corridor control.:

The selected scale-bordism arrows must admit a cofinal subsystem on which all chart, support, representation, and analytic corridor choices are simultaneously comparable. The arrow labels must be retained; endpoint thickness pairs are insufficient in controlled Sector II.

TR2: Observable compactness.:

Regional unit-ball elements on the selected Wilson cores must have convergent representatives in a common normal comparison carrier. The limit must be independent of the chosen presentation and compatible with regional inclusions.

TR3: Normal-state tightness.:

Finite-entropy and complete-ground normal states must be tight under the contravariant predual scale maps. Limit points must remain normal on the candidate regional algebras.

TR4: Sectorially typed representation compactness.:

In Sector I one may seek ordinary Hilbert comparison carriers. In controlled Sector II the carrier must retain the controlled scale twisting, and in Sector III it must remain coefficient-line or correspondence valued. Compactness is not allowed to strictify the sector by fiat.

TR5: Graph-domain recovery.:

The Gauss and closed-form graph spaces must possess a common limiting core or an equivalent recovery mechanism. Convergence of bounded observables does not control unbounded domains.

TR6: Entropy lower semicontinuity and continuity control.:

Relative entropy must be lower semicontinuous on the chosen tight state sets. To pass the surgery-bordism balance law through a controlled defect, one also needs a continuity modulus converting predual distance into entropy control on those sets.

TR7: Seam convergence.:

State seams, represented seams, and vector-form seams must converge and remain compatible with glued scale composition. The limiting seam must be independent of parenthesization and of the selected finite corridor decomposition.

TR8: Defect convergence.:

The locality, Gauss, Ward, surgery, and closed-form defects must vanish uniformly on each fixed finite test stage. The cocycle formulas then guarantee vanishing on every fixed finite glued chain.

TR9: Gauge and Ward stability.:

Gauge-parameter transport, insertion domains, complete-ground kernels, collar quotient classes, and Ward derivations must remain stable under the comparison maps. A limiting Ward action does not automatically supply global limiting charge zero-points.

TR10: Causal and background stability.:

The limiting regional assignment must preserve the causal-development and background-covariance squares. Exact thin locality and a time-slice theorem require separate proofs after the regional algebras are constructed.

TR11: Coercive liminf readiness.:

The objectwise estimate already gives

$$\liminf_r \bar{q}_r[\Psi_r] \geq \kappa_* \liminf_r \|(I - P_{\Omega,r})\Psi_r\|^2. \quad (\text{G.1})$$

This is a readiness inequality, not a bound for a limiting form.

TR12: Cofinal independence.:

Two cofinal extraction systems and two choices of comparison carriers must possess a common refinement on which the resulting candidate thin objects are canonically equivalent in the appropriate sector.

Two additional threshold-transfer conditions are indispensable:

TT1 : $P_{\Omega,r}$ is tight and converges to a candidate $P_{\Omega,\infty}$,

TT2 : the limiting form admits recovery sequences or an equivalent upper identification. (G.2)

Under TR11, TT1, and TT2, the argument in Appendix C yields

$$q_\infty[\Psi] \geq \kappa_* \|(I - P_{\Omega,\infty})\Psi\|^2. \quad (\text{G.3})$$

Without TT1 and TT2, the leakage example (C.15) applies.

Theorem G.1 (Typed thin-readiness statement). *Assume TR1–TR12 on every finite test stage. Then the positive-thickness diagram has compatible observable, state, graph-domain, entropy, seam, defect, and cofinal-comparison witnesses for formulating a sector-preserving zero-thickness reconstruction. The statement does not construct the thin algebra, representation, Gauss/Ward system, entropy surgery, or closed form. A positive limiting threshold additionally requires TT1 and TT2.*

The division of labor is therefore precise. The present article constructs the thick diagram and proves its internal coherence. A later reconstruction must build the thin objects from the readiness witnesses. A subsequent non-leakage theorem must establish TT1 and TT2 strongly enough to preserve the positive threshold. None of those later conclusions is encoded into the definition of readiness.

Appendix H: Certified $SU(2)$ strict-control realization

This appendix records the proof data behind Theorem VIII.4. Its purpose is not to supply a second continuum construction. It verifies nonemptiness of the strict finite/depth interface after the complete interacting-ground physicalization and the scale-bordism requirements are imposed simultaneously.

1. Fusion reconstruction and complete ground projection

The $SU(2)$ character fusion rule

$$\chi_{1/2}\chi_j = \chi_{j-1/2} + \chi_{j+1/2}, \quad \chi_{-1/2} = 0, \quad (\text{H.1})$$

shows that compression of multiplication by $\chi_{1/2}$ to (VIII.25) is

$$X = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (\text{H.2})$$

With electric normalization $4j(j+1)$,

$$H_{\text{el}} = \text{diag}(0, 3, 8, 15), \quad H_{\text{mag}} = I - \frac{1}{2}X, \quad (\text{H.3})$$

which gives (VIII.26). Its scaled characteristic polynomial is

$$p(\lambda) = 16\lambda^4 - 480\lambda^3 + 4356\lambda^2 - 12932\lambda + 8561. \quad (\text{H.4})$$

The polynomial is square free, and exact Sturm counting isolates one simple root in each of four disjoint intervals. In particular, the lowest root is

$$E_0^{(2)} = 0.9180581777722170525783499839449488 \dots \quad (\text{H.5})$$

and the complete projection

$$P_{\text{vac}}^{(2)} = 1_{\{E_0^{(2)}\}}(H^{(2)}) \quad (\text{H.6})$$

has rank one. A numerical eigenvector may be used to evaluate intervals, but it is not part of the invariant datum.

2. Physicalization and the nonzero mixed block

Let $F^W(1) = \phi^W$ with (VIII.27). Since F^W is normalized,

$$G = 1 - |\langle g_+, \phi^W \rangle|^2 = 0.9999999997154206590229225986252481 \dots, \quad (\text{H.7})$$

where g_+ is any normalized representative of the complete ground line. Therefore F^{phys} in (VIII.28) is well defined and satisfies

$$(F^{\text{phys}})^* F^{\text{phys}} = 1, \quad P_{\text{vac}}^{(2)} F^{\text{phys}} = 0. \quad (\text{H.8})$$

The magnetic scalar must be recomputed after this reduction:

$$B_{\text{mag}}^{\text{phys}} = (F^{\text{phys}})^* H_{\text{mag}} F^{\text{phys}} = 1.059584273164392957736871096703039 \dots \quad (\text{H.9})$$

Set

$$P_{\text{ch}} = F^{\text{phys}} (F^{\text{phys}})^*, \quad P_{\text{res}} = 0, \quad P_{\text{UV}} = I - P_{\text{vac}}^{(2)} - P_{\text{ch}}, \quad (\text{H.10})$$

and $H^\flat = H^{(2)} - E_0^{(2)} I$. The interval certificate gives

$$\begin{aligned} P_{\text{ch}} H^\flat P_{\text{ch}} &= 3.114097481076100331337464231810777 \dots P_{\text{ch}}, \\ P_{\text{UV}} H^\flat P_{\text{UV}} &\geq 8.096308674897366858801704823980880 \dots P_{\text{UV}}. \end{aligned} \quad (\text{H.11})$$

The exact mixed block

$$K^{(2)} = P_{\text{UV}} H^\flat P_{\text{ch}} \quad (\text{H.12})$$

is nonzero and satisfies the norm in (VIII.29). Taking

$$c_{\text{ch}} = 3.093770498, \quad c_{\text{UV}} = 8, \quad \varepsilon_{\text{mix}} = 0.052, \quad (\text{H.13})$$

gives the exact determinant margin

$$c_{\text{ch}} c_{\text{UV}} - \varepsilon_{\text{mix}}^2 = 24.747459984 > 0. \quad (\text{H.14})$$

The smaller eigenvalue of the scalar comparison matrix yields (VIII.30). This proof uses the channel, ultraviolet, and leakage decomposition; it is not replaced by direct diagonalization of $H^{(2)}$.

3. Tensor depth and exact bordism transport

For $n \geq 1$, let

$$\mathcal{H}_n^{(2)} = (\mathcal{K}^{(2)})^{\otimes n}, \quad H_n^{(2)} = \sum_{r=1}^n I^{\otimes(r-1)} \otimes H^{(2)} \otimes I^{\otimes(n-r)}. \quad (\text{H.15})$$

Strong commutation of the tensor summands gives

$$E_{0,n}^{(2)} = n E_0^{(2)}, \quad P_{\text{vac},n}^{(2)} = (P_{\text{vac}}^{(2)})^{\otimes n}. \quad (\text{H.16})$$

Let $V = \text{Ran } P_{\text{vac}}^{(2)}$ and define the phase-free anchored coefficient space

$$\mathcal{E}_n = \bigoplus_{r=1}^n V^{\otimes(r-1)} \otimes \mathbb{C} \otimes V^{\otimes(n-r)}. \quad (\text{H.17})$$

After removal of the joint ground line, distinct shell channels are orthogonal, hence $G_n = GI_{\mathcal{E}_n}$. The global mixed operator is an orthogonal direct sum of copies of the seed block:

$$K_n^{(2)} = \bigoplus_{r=1}^n K^{(2,r)}, \quad \|K_n^{(2)}\| = \|K^{(2)}\|. \quad (\text{H.18})$$

No channel vector is sent to a multiple-excitation sector. The common Paper-II profile gives

$$H_n^{(2)} - E_{0,n}^{(2)} I \geq 0.24975(I - P_{\text{vac},n}^{(2)}), \quad (\text{H.19})$$

while the tensor-sum argument gives the stronger bound with the seed constant in (VIII.30).

For a batch insertion of k ground shells, canonical tensor reassociation defines

$$J_{n;k}^{(2)} : \mathcal{H}_n^{(2)} \widehat{\otimes} V^{\widehat{\otimes} k} \longrightarrow \mathcal{H}_{n+k}^{(2)}. \quad (\text{H.20})$$

Its restriction to complete ground lines is unitary and has overlap one. The old coefficient embedding transports the shifted Hamiltonian, ground projection, raw and physicalized channels, Gram operator, centered magnetic compression, and mixed block in the exact pullback directions. For all $k, \ell \geq 0$,

$$J_{n+k;\ell}^{(2)} \left(J_{n;k}^{(2)} \widehat{\otimes} I \right) = J_{n;k+\ell}^{(2)} \quad (\text{H.21})$$

after canonical tensor identification. The coefficient reassociator is therefore the identity.

4. Center marking and neutral stabilization

The center unitary

$$Z\chi_j = (-1)^{2j}\chi_j \quad (\text{H.22})$$

satisfies $ZXZ^{-1} = -X$. The two background fibers

$$H^{[\epsilon]} = H_{\text{el}} + I - \frac{(-1)^\epsilon}{2} X, \quad \epsilon \in \mathbb{Z}_2, \quad (\text{H.23})$$

obey $ZH^{[\epsilon]}Z^{-1} = H^{[\epsilon+1]}$. The physicalization and the depth bordisms are covariant across these fibers. A k -shell fundamental marking has parity $k \bmod 2$, but (H.21) still gives identity reassociation. This proves that a physical center grade does not itself force a higher obstruction.

If the system is tensored with a genuinely neutral factor having trivial gauge and center action, grade zero, strict transport, and identity reassociator, the stabilized reassociator is the product of two identity cocycles. Thus strictness is preserved. If the neutral factor has excitations, its gap may lower the ultraviolet constant, but it cannot alter the zero cohomology class.

- **Complete ground datum:** rank-one spectral projection at $E_0^{(2)} = 0.918058177772\dots$
- **Gram scalar:** $G = 0.999999999715\dots$
- **Physical magnetic recompression:** $1.059584273164\dots$
- **Mixed norm:** $\|K^{(2)}\| = 0.0519985588624\dots \neq 0$.
- **Resolved seed constant:** $3.093219423837\dots$
- **Depth propagation:** $G_n = GI$ and $\|K_n^{(2)}\| = \|K^{(2)}\|$.
- **Strict classes:** $[\Omega^{(2)}]_{\text{band}} = [\hat{\Omega}^{(2)}]_{\text{cs,ctrl}} = 0$.
- **Scope:** fixed-stratum depth exhaustion only.

Appendix I: Local Friedrichs propagation and fixed-thickness linked flux

This appendix records the two realization theorems used in Section VIII. The first is restricted to a local zero-order internal term; a spectral lower bound alone is not a propagation theorem. The second derives the $SU(3)$ fixed-thickness cocycle from meridional geometry and the center action on a selected complete-ground Wilson/collar line.

1. The local incised-space operator

Let

$$X_{\text{inc}} = \mathbb{R}^3 \setminus \ell, \quad \ell = \{(0, 0, z) : z \in \mathbb{R}\}, \quad (\text{I.1})$$

and let $E_m \rightarrow X_{\text{inc}}$ be a finite-rank Hermitian bundle with smooth flat unitary connection ∇_m and positive meridional holonomy $\omega^m I$. For the causal canonical branch, take

$$A_m = \eta \nabla_m^* \nabla_m + M_{V_m}, \quad \eta > 0, \quad (\text{I.2})$$

where

$$V_m \in C^\infty(X_{\text{inc}}; \text{End}_{\text{Herm}} E_m), \quad V_m(x) \geq -C_V I. \quad (\text{I.3})$$

Thus M_{V_m} is pointwise and commutes with scalar cutoffs. An abstract nonlocal h_{int} is excluded from the causal statement.

For $\rho > 0$, put

$$X_\rho = \{x : \text{dist}(x, \ell) > \rho\} \quad (\text{I.4})$$

and define the Dirichlet form

$$\mathfrak{a}_{m,\rho}[u, v] = \eta \langle \nabla_m u, \nabla_m v \rangle + \langle V_m u, v \rangle \quad (\text{I.5})$$

on

$$Q_{m,\rho} = H_0^1(X_\rho; E_m) \cap D(M_{V_m, +}^{1/2}). \quad (\text{I.6})$$

The shifted form $\mathfrak{a}_{m,\rho} + (C_V + 1)\|\cdot\|^2$ is closed. Its Friedrichs operator is denoted $A_{m,\rho}$. The global minimal form $\mathfrak{a}_m$ is the closure of $C_c^\infty(X_{\text{inc}}; E_m)$ in the analogous shifted form norm.

Proposition I.1 (Uniform collar positivity). *Assume*

$$\mathfrak{a}_m[v] \geq \kappa_{\text{cof}} \|v\|^2, \quad v \in Q_m, \quad (\text{I.7})$$

with $\kappa_{\text{cof}} > 0$. Extension by zero $J_\rho : L^2(X_\rho; E_m) \rightarrow L^2(X_{\text{inc}}; E_m)$ satisfies

$$J_\rho Q_{m,\rho} \subseteq Q_m, \quad \mathfrak{a}_m[J_\rho u, J_\rho v] = \mathfrak{a}_{m,\rho}[u, v]. \quad (\text{I.8})$$

Consequently,

$$A_{m,\rho} \geq \kappa_{\text{cof}} I \quad (\text{I.9})$$

uniformly in ρ .

Proof. Approximate $u \in H_0^1(X_\rho; E_m) \cap D(V_{m,+}^{1/2})$ in the collar form norm by compactly supported smooth sections. Their zero extensions are Cauchy in the global form norm and the integrals agree. Apply (I.7) to the zero extension. $\square$

In a local unitary frame, $\nabla_m^* \nabla_m = -\Delta + \text{lower-order terms}$. Hence

$$\sigma_{\text{prin}}(\partial_t^2 + A_{m,\rho}) = (-\tau^2 + \eta|\xi|^2)I, \quad (\text{I.10})$$

and the characteristic speed is at most $\sqrt{\eta}$.

2. Fixed-collar and collar-limit propagation

Let d_ρ be the intrinsic distance on X_ρ and $K \subset X_\rho$ closed. Choose $\vartheta \in W^{1,\infty}(\mathbb{R})$ with $\vartheta = 0$ on $(-\infty, 0]$, $\vartheta = 1$ on $[1, \infty)$, and $\vartheta' \geq 0$, and set

$$\chi_{\delta,t}(x) = \vartheta\left(\frac{d_\rho(x, K) - \sqrt{\eta}t}{\delta}\right). \quad (\text{I.11})$$

For a smooth solution $u_{tt} + A_{m,\rho}u = f$, use the shifted energy density

$$e[u] = \frac{1}{2}\left(|u_t|^2 + \eta|\nabla_m u|^2 + \langle (V_m + C_V + 1)u, u \rangle\right) \quad (\text{I.12})$$

and flux $j[u] = -\eta \operatorname{Re}\langle u_t, \nabla_m u \rangle$. Since $|j| \leq \sqrt{\eta}e$, the weighted exterior energy

$$E_{\delta,\rho}(t) = \int_{X_\rho} \chi_{\delta,t}^2 e[u(t)] \quad (\text{I.13})$$

satisfies

$$\frac{d}{dt}E_{\delta,\rho}(t) \leq (C_V + 2)E_{\delta,\rho}(t) + \frac{1}{2} \int_{X_\rho} \chi_{\delta,t}^2 |f(t)|^2. \quad (\text{I.14})$$

The cutoff flux is nonpositive, and the Dirichlet trace gives no collar-boundary source. The constants are independent of ρ and δ .

Theorem I.2 (Fixed-collar finite propagation). *If the initial data are supported in K and the forcing is absent outside their forward cone, then*

$$\operatorname{supp} u(t) \subseteq \{x : d_\rho(x, K) \leq \sqrt{\eta}|t - t_0|\}. \quad (\text{I.15})$$

Equivalently, $d_\rho(E, F) > \sqrt{\eta}|t|$ implies

$$1_F \cos(tA_{m,\rho}^{1/2})1_E = 0, \quad 1_F A_{m,\rho}^{-1/2} \sin(tA_{m,\rho}^{1/2})1_E = 0. \quad (\text{I.16})$$

Proof. The weighted exterior energy is initially zero. Gronwall's inequality and (I.14) keep it zero. Let $\delta \downarrow 0$ and use Fatou's lemma. General finite-energy data follow by approximation. The operator statements are the position- and velocity-data cases. $\square$

For $\rho_n \downarrow 0$, the zero-extended form domains increase. Every compactly supported smooth section of X_{inc} belongs to all sufficiently small collar domains, and the global domain is the closure of their union. The liminf and recovery conditions therefore give Mosco convergence of the shifted forms^{35,43,44}. The associated operators converge in embedded strong resolvent sense. Proposition I.1 removes a zero-frequency singularity; the first-order wave generators then converge by Trotter–Kato, uniformly on compact time intervals.

Theorem I.3 (Incised Friedrichs/collar finite propagation). *The global finite-energy Cauchy problem for $\partial_t^2 + A_m$ is well posed, and its retarded, advanced, and causal propagators satisfy*

$$\text{supp}(E_m f) \subseteq J_{\sqrt{\eta}}(\text{supp } f). \quad (\text{I.17})$$

The result is independent of the collar sequence. On every relatively compact collar-Cauchy admissible region N , the Green sequence

$$0 \longrightarrow C_c^\infty(N; E_m) \xrightarrow{P_m} C_c^\infty(N; E_m) \xrightarrow{E_m} \text{Sol}_{\text{sc}}(N) \longrightarrow 0 \quad (\text{I.18})$$

is exact. Consequently the quotient symplectic functor is causal, extension by zero along causally compatible inclusions is injective, and a neighborhood of a Cauchy section generates the same Weyl algebra.

Proof. Approximate data and sources by collar data with supports in arbitrarily small intrinsic neighborhoods. The collar solutions vanish outside the enlarged cones by Theorem I.2; compact-time strong convergence and bounded multiplication by characteristic functions preserve vanishing, and the neighborhood enlargement is then removed. Exactness follows from Cauchy uniqueness, the retarded/advanced identities, and the standard Cauchy cutoff $P_m(\chi u) = [P_m, \chi]u$, whose support remains local because V_m is pointwise. This is the direct local mechanism underlying the comparison results; the related Laplace-type and Green-hyperbolic frameworks are discussed in Refs.^{45–50}. $\square$

3. Fixed-thickness linked-flux geometry

Fix one positive thickness. The universal angular cover is

$$\widetilde{X}_{\text{inc}} = (0, \infty) \times \mathbb{R}_{\tilde{\theta}} \times \mathbb{R}, \quad T(r, \tilde{\theta}, z) = (r, \tilde{\theta} + 2\pi, z). \quad (\text{I.19})$$

For $m \in \mathbb{Z}_3$, define

$$E_m = (\widetilde{X}_{\text{inc}} \times \mathbb{C}^3) / ((T\tilde{x}, v) \sim (\tilde{x}, \omega^m v)). \quad (\text{I.20})$$

The induced flat connection has positive meridional holonomy $\omega^m I_3$.

Choose three angular chart objects and strict lifted corridors Γ_n advancing the lifted angle by $2\pi n/3$. Then

$$\Gamma_n \Gamma_k = \Gamma_{n+k}, \quad \Gamma_3 = T. \quad (\text{I.21})$$

Transporting the three chart objects to one selected base chart by complete-ground comparisons gives a coefficient-preserving skeleton with underlying one-groupoid $B\mathbb{Z}_3$; the deck transformation remains the coefficient carry C_m . For the preferred lifts $\tilde{\alpha} \in \{0, 1, 2\}$,

$$\tilde{U}_{m,\alpha}\tilde{U}_{m,\beta} = C_m^{\epsilon_3(\alpha,\beta)}\tilde{U}_{m,\alpha+\beta}. \quad (\text{I.22})$$

The canonical endpoint package contains a declared rank-one, center-natural line

$$L_{m,\alpha} \subset \text{Hom}_{\text{Gauss},\Omega,\text{cen}}(V_{j,m}^\Omega \hat{\otimes} \mathcal{C}_\alpha, V_{j+\alpha,m}^\Omega), \quad \dim L_{m,\alpha} = 1, \quad (\text{I.23})$$

where $\mathcal{C}_\alpha$ has triality α :

$$\rho_\alpha(\omega I_3) = \omega^{\tilde{\alpha}} I. \quad (\text{I.24})$$

Rank one is selected endpoint data, not a multiplicity-one theorem for the full intertwiner space.

Proposition I.4 (Meridional carry–triality commutation). *For $\ell_\alpha \in L_{m,\alpha}$, center-background naturality gives*

$$C_{m,j+\alpha}^\Omega \ell_\alpha = \ell_\alpha (C_{m,j}^\Omega \hat{\otimes} \rho_\alpha(\omega^m I_3)). \quad (\text{I.25})$$

Hence the induced action on the line is

$$C_m \cdot \ell_\alpha = \omega^{m\tilde{\alpha}} \ell_\alpha. \quad (\text{I.26})$$

Moving one carry line K_m through $L_{m,\alpha}$ is therefore the ordinary tensor flip followed by this conjugation action:

$$\ell \hat{\otimes} k \longmapsto \omega^{m\tilde{\alpha}} k \hat{\otimes} \ell. \quad (\text{I.27})$$

The pairwise products, with all carry lines placed to the left, are

$$\mu_{m;\alpha,\beta}^{\text{fib}} : L_{m,\alpha} \hat{\otimes} L_{m,\beta} \longrightarrow K_m^{\epsilon_3(\alpha,\beta)} \hat{\otimes} L_{m,\alpha+\beta}. \quad (\text{I.28})$$

Theorem I.5 (Physical fixed-thickness linked-flux class). *The two parenthesized products of three fixed-thickness coefficient lines differ by*

$$\Omega_m^{\text{fib}}(\alpha, \beta, \gamma) = \omega^{m\tilde{\alpha}\epsilon_3(\beta,\gamma)}. \quad (\text{I.29})$$

The marked triple and detector are

$$\Omega_m^{\text{fib}}(1, 2, 1) = \omega^m, \quad I_3(\Omega_m^{\text{fib}}) = \prod_{b=0}^2 \Omega_m^{\text{fib}}(1, b, 1) = \omega^m. \quad (\text{I.30})$$

Thus

$$[\Omega_m^{\text{fib}}] = mu_3 \in H^3(B\mathbb{Z}_3; U(1)), \quad (\text{I.31})$$

and the $m = 1, 2$ branches are Sector III at every selected positive thickness. The class survives complete-ground compression, center-natural rephasing, and genuinely neutral stabilization; the induced observable adjoint maps compose strictly.

Proof. The left-associated product produces $K_m^{\epsilon_3(\alpha, \beta) + \epsilon_3(\alpha + \beta, \gamma)}$. The right-associated product first produces $K_m^{\epsilon_3(\beta, \gamma)}$ to the right of $L_{m, \alpha}$. Proposition I.4 moves it left and supplies the scalar in (I.29); the carry identity makes the remaining powers equal. The detector is invariant under normalized coboundaries by Proposition B.1. Scalars on external coefficient lines cancel under adjoint action, and neutral factors contribute the identity cocycle. $\square$

The fixed-thickness class is generated by meridional geometry, not by scale transport. It is logically independent of the controlled scale class of Appendix B.

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