

# Positive-Thickness Reconstruction for Yang–Mills Wilson-Tube Systems: Local-Normal Nets and Controlled Center-Banded Descent

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We develop a conditional positive-thickness spacetime reconstruction for gauge-invariant Yang–Mills Wilson-tube systems. The front end constructs regional time-zero physical data, selected Cauchy anchoring, typed realization defects, effective anchored descent, fixed-scale lifting two-stacks, and rooted correspondence-valued cofinal sections. Under explicit regular-support, finite-atlas, positive-margin, category-recognition, realization-identification, and coercive-provenance hypotheses, the incised regional basis and retained positive-thickness realization are recovered. On the selected source-free physical branch, relaxation of the compressed meridional defect is incompatible with relaxation of the retained Sector-III detector to the flat point channel. This identifies the persistent obstruction carrier with the prescribed codimension-two worldsheet; only then is the incised spacetime defined by removing that carrier, so the incision is derived rather than imposed as a regulator or boundary condition. For each selected finite admissible protected cover, weakly contractible certificate fibers recover the corresponding Čech nerve, on which the physical lifting stack is identified with the retained positive-thickness  $SU(3)$  realization. The complete-ground decoration is preserved through reconstruction, with a certified coercive bound greater than  $337/1000$ . The terminal output is a coherent directed system of finite positive-thickness realizations; zero-thickness reconstruction and spectral closure are deferred to Papers IV and V.

Keywords: algebraic quantum field theory; Yang–Mills theory; von Neumann algebras; Wilson observables; gerbes; higher descent; Gauss law; relative entropy; quadratic forms

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## I. INTRODUCTION

The local formulation of quantum field theory separates two questions that are often conflated. The first asks whether bounded observables can be assigned to spacetime regions in an isotone and local manner. The second asks whether a preferred family of representations, ground sectors, charges, and unbounded generators can be chosen globally and coherently. In algebraic quantum field theory the first question is encoded by a net of operator algebras, while the second belongs to representation theory, superselection structure, covariance, and spectral analysis; see Refs.<sup>1–5</sup>. For gauge theories the separation is especially important. Gauge-invariant Wilson observables can be localizable even when charged or vacuum data require nontrivial descent, boundary degrees of freedom, or projective implementation. The purpose of this article is to make that separation precise for a positive-thickness Yang–Mills reconstruction built from gauge-invariant Wilson-tube systems.

The construction belongs to a sequence of papers. Paper I, *Coarse Transport, Central Twist, and Finite-Scale Coercivity for Yang–Mills Wilson-Tube Hamiltonians: Physical Wilson-Channel Reduction and Exact Ultraviolet–Infrared Leakage*, establishes the objectwise finite-scale physicalization and coercive mechanism after removal of the complete ground sector<sup>6</sup>. Paper II, *Scale-Bordism Criteria for Uniform Coercivity and Center-Banded Vacuum Descent in Yang–Mills Wilson-Tube Systems*, selects or characterizes branches carrying one common admissible profile and organizes their comparison maps, coefficient lines, entropy selection, and form bounds<sup>7</sup>. Those results are deliberately time-zero. The present paper supplies the positive-thickness spacetime layer; the zero-thickness limit is not taken here.

A central logical repair is that the incision is not assumed at the beginning of that spacetime layer. The selected time-zero branch is first extended regionally on ordinary Minkowski testing regions and compared with local positive-thickness spacetime candidates through selected Cauchy data. The failure of simultaneous anchored lifting determines the common carrier  $\Sigma_{\text{obs}}^{\Gamma_{\text{cof}}}$ . Only after a faithful regular-support presentation has supplied stratified tubular control, finite adapted atlases, and positive finite-stage margins do we pass to the incised regional category. The regional realization-identification theorem then proves CP4 for the ordinary observable shadow. The Paper-I/Paper-II coercive certificate

is reintroduced only after these steps and is not used to create or regularize the habitat. The incision is therefore not a disposable preliminary device: it is the recovered spacetime realization on which the coherent directed family of finite positive-thickness Yang–Mills packets exists and on which all later regional, Cauchy, Čech, and selected-scale comparisons are represented.

The conditional theorem is accompanied by two concrete tests of its architecture. First, a selected linked-flux  $SU(3)$  branch is assembled over an incised region category from the local operator  $A_m = \eta \nabla_m^* \nabla_m + V_m$ . A uniform Dirichlet-collar energy estimate and its Mosco limit prove finite propagation, causal locality, and time-slice. At fixed thickness the meridional deck carry acts on triality-sensitive complete-ground coefficient lines and produces a physical Sector-III class. Along the exact dyadic thickness orbit, a center-root collar corridor produces an ordinarily exact but controlled-nontrivial scale class on the neutral fiber branch. Second, an independent four-character  $SU(2)$  Wilson model repeats the full finite physicalization, including a nonzero mixed block, and extends it through arbitrary tensor depth with globally strict coefficient descent. The  $SU(3)$  system tests both fixed-thickness and controlled scale twisting; the  $SU(2)$  system is a strict control showing that center sensitivity alone does not force higher twist.

The observable setting is von Neumann algebraic. At each selected thickness

$$\varepsilon \in E_{\text{cof}} \subset (0, \infty), \quad \inf E_{\text{cof}} = 0, \quad (\text{I.1})$$

we construct a regional assignment

$$\mathfrak{A}_\varepsilon^W : \mathbf{Reg}_{\text{inc}}^{\varepsilon, \text{cp}}(M, \Sigma) \longrightarrow \mathbf{W}^* \mathbf{Alg}_n, \quad (\text{I.2})$$

where the recovered background is

$$M_{\text{inc}} = \mathbb{R}^{3,1} \setminus \Sigma_{\text{obs}}^{\Gamma_{\text{cof}}} \quad (\text{I.3})$$

after the front-end recognition and realization-identification theorems. The carrier is first obtained as the complement of the maximal simultaneous lifting habitat; its controlled stratified and collared geometry is a separate H1 conclusion under the Regular-Support Presentation package. On each chosen selected admissible finite cover, the testing diagram is

its rooted or marked Čech nerve. CP1–CP3 are geometric on that diagram; observable CP4 is the presentation independence of the ordinary regional algebra after descent and follows, for the selected physical branch, from Theorem II.260. Arbitrary-union additivity and a zero-thickness Haag–Kastler net require additional arguments and are not consequences of compatible-family descent alone.

The endpoint of the inhabited front end is consequently not one isolated thick packet. For each selected finite admissible demand  $\mathfrak{d}$ , it is an intrinsic datum

$$\mathfrak{D}_{\text{YM},\mathfrak{d}}^{\text{thick,fin}},$$

and enlargement of finite demand gives coherent comparison morphisms after admissible common refinement. The resulting directed finite-thick system is the object prepared for Paper IV. Paper III does not take its zero-thickness limit.

### Guide to the load-bearing construction

The argument should be read through the following dependency chain. This is not merely a section synopsis: it identifies the results and witnesses whose removal would break the construction.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |       |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|
| <p>selected time-zero branch <math>\implies</math> persistent frustum/linked-flux obstruction</p> <p style="padding-left: 40px;"><math>\implies</math> derived obstruction carrier</p> <p style="padding-left: 40px;"><math>\implies</math> incised spacetime realization</p> <p style="padding-left: 40px;"><math>\implies</math> admissible site and local thick packets</p> <p style="padding-left: 40px;"><math>\implies</math> protected Cauchy, Čech, and scale coherence</p> <p style="padding-left: 40px;"><math>\implies</math> coherent finite-thick Yang–Mills system.</p> | (I.4) |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|

*a. Selected input.* Paper II supplies the rooted physical branch, sector selection, admissibility certificates, comparison data, and finite-scale coercive provenance. Paper III does not repeat branch selection and does not use coercivity to manufacture the spacetime habitat.

*b. Persistent obstruction and carrier.* The frustum geometry, source-free nonrelaxation argument, concentration exclusion, and linked-flux detector prove persistence of the selected

obstruction. The calculation is load bearing because it establishes that the obstruction occurs on the selected Yang–Mills branch and cannot be removed by an admissible point relaxation. The resulting failure carrier is then derived as

$$\Sigma_{\text{obs}}^{\text{YM,vac}} = \Sigma_{\text{pres}}.$$

The retained gerbe class and the carrier are related but not identified: the former records higher coherence, whereas the latter is the geometric locus on which the required ordinary lifting realization fails.

*c. Derived incision and habitat.* Only after persistence and carrier equality have been proved do we define

$$M_{\text{inc}}^{\text{YM,vac}} := M \setminus \Sigma_{\text{obs}}^{\text{YM,vac}}.$$

The incision is therefore derived rather than imposed. It is not a regulator, a cutoff, an auxiliary boundary condition, or a device chosen to force coercivity. The spacetime-realization theorem proves that this complement, with its induced controlled geometry, is the selected ambient realization on which the finite positive-thickness system can exist.

*d. Local realization and coherence.* On the incised habitat, admissible regions and covers define the selected Grothendieck site. Protected Cauchy data, regional restriction, Green operators, and finite-thickness packets supply local existence. Rooted or marked Čech nerves record overlap data, while glued scale bordisms, comparison cells, and the associator supply regional, Cauchy, and selected-scale coherence. These results show that the packets form one refinement-compatible system rather than an unrelated family of local models.

*e. Intrinsic endpoint and handoff.* For each finite admissible demand  $\mathfrak{d}$ , the intrinsic output is  $\mathfrak{D}_{\text{YM},\mathfrak{d}}^{\text{thick,fin}}$ . Presentation choices are identified under admissible common refinement, while control data certify that the comparison remains inside the selected admissible class. The terminal output of this paper is the coherent directed system of these finite positive-thickness equivalence classes. Effective thin descent and construction of the thin observable system belong to Paper IV; no-leakage and invariant-mass spectral closure belong to Paper V.

### **Selected branch.:**

Principal conclusion: a rooted physical finite-scale input with sector and coercive provenance.

Essential certificate: the Paper-II branch-selection and admissibility package.

**Persistent obstruction.:**

Principal conclusion: the selected vacuum lifting problem cannot relax through the frustum to the flat point channel. Essential witness: source-free nonrelaxation, concentration exclusion, and  $I_3(\Omega_m) = \omega^m \neq 1$ .

**Carrier derivation.:**

Principal conclusion:  $\Sigma_{\text{obs}}^{\text{YM,vac}} = \Sigma_{\text{pres}}$ . Essential witness: the failure-locus argument together with the retained tubular/frustum presentation.

**Incised habitat.:**

Principal conclusion:  $M_{\text{inc}}^{\text{YM,vac}}$  supports the selected positive-thickness realization. Essential certificate: H1–H3 support regularity, collars, endpoint margins, and causal control.

**Local thick realization.:**

Principal conclusion: regional packets, Green data, and protected Cauchy comparison exist on the selected site. Essential certificate: CIR and the finite-propagation/Friedrichs witness package.

**Čech and scale coherence.:**

Principal conclusion: local packets assemble coherently under overlap and selected finite-demand refinement. Essential certificate: P4–P5 effectivity, RID/CP4, glued-bordism comparison cells, and associator coherence.

**Finite-thick endpoint.:**

Principal conclusion: the classes  $\{\mathfrak{D}_{\text{YM},\mathfrak{d}}^{\text{thick,fin}}\}$  form the Paper-IV handoff system. Essential certificate: common-refinement invariance and the inherited complete-ground coercive packet.

A calculation is therefore load bearing precisely when it proves nontriviality, persistence, nonemptiness, sector realization, or a hypothesis used in this chain. Longer derivations and duplicate checks may be relocated without changing the architecture only when the theorem and its irreducible witness remain in the manuscript.

## How to read the paper and its verification surfaces

The manuscript is not intended to be read as one undifferentiated linear sequence. A reader seeking the logical architecture should begin with this introduction, the input and main-theorem ledger in Section III, the pre-incision front end, and the final finite-thick handoff. A reader auditing the obstruction should follow the frustum, nonrelaxation, carrier, and incised-habitat chain. A reader auditing local spacetime realization should follow the regional net, protected Cauchy, Green-operator, and finite-propagation results. A reader auditing coherence should follow the rooted or marked Čech nerves, realization-identification cells, selected scale bordisms, and associator. The explicit  $SU(3)$  and  $SU(2)$  realizations may then be read as branch-specific inhabitation and control tests of that architecture.

Four evidentiary surfaces are kept distinct.

### Primary proof surface.:

The body of the manuscript contains the statements, hypotheses, definitions, and proofs required for the Paper-III finite positive-thickness endpoint. No numerical output is used as a premise in a theorem.

### Irreducible witness surface.:

Selected calculations remain in the body when they establish nontriviality, persistence, nonemptiness, sector realization, or satisfaction of a theorem hypothesis. These witnesses instantiate the analytic or cohomological mechanism but do not replace the proof in which that mechanism is used.

### Technical supplement surface.:

Expanded energy estimates, universal-cover calculations, scale-cohomology derivations, global-local assembly details, and strict-control computations are indexed in the accompanying technical supplement map. They provide traceability and expanded derivation without altering the logical dependency chain of the manuscript.

### Numerical-audit and reproducibility surface.:

Independent certificate programs and their recorded outputs check representative finite computations, including the strict  $SU(2)$  seed and finite  $Z_3$  cocycle identities. These artifacts

verify the implemented calculations and package integrity; they are not evidence substituted for mathematical proof.

The accompanying supplement therefore contains a theorem–witness–artifact ledger. For each load-bearing stage it records the principal conclusion, the irreducible witness retained in the manuscript, the expanded derivation or audit artifact, and whether that artifact is logically required, explanatory, or independently reproducible. Readers interested only in the theorem chain may ignore the audit layer; readers wishing to reproduce representative calculations can follow the ledger directly to the corresponding files.

*f. Model-verification status.* The architecture below is not identified with an inhabited Yang–Mills branch merely because its prestacks and defects are well typed. The closure audit in Theorem II.299 establishes the selected positive-thickness front end, while the full manuscript remains Outcome B because the model-specific downstream H4–H19 synthesis has not yet been integrated into this lineage. The retained  $SU(3)$  package supplies P1–P5 on the selected dyadic system: regional extension, protected stationary Cauchy anchoring, common-core defect naturality, effective anchored descent, fixed-scale lifting stacks, and a rooted correspondence-valued normal lax section. This stationary branch is realizable on every selected region and has empty carrier.

The intended nonstationary Sector-III problem is constructed separately as a rooted physical lifting stack. The selected vacuum Yang–Mills substack is source free and carries one cofinal transverse concentration-exclusion certificate. A hypothetical lift through the straight worldsheet would produce an admissible point relaxation; the diagonal Orlicz estimate would remove its complete-ground meridional defect, while the retained linked detector  $I_3(\Omega_m) = \omega^m \neq 1$  forbids relaxation to the flat point channel. Hence

$$M_{\text{reg}}^{\text{YM,vac}} = M \setminus \Sigma_{\text{pres}}, \quad \Sigma_{\text{obs}}^{\text{YM,vac}} = \Sigma_{\text{pres}}.$$

This is a branch-specific dynamical carrier theorem, not an identification of the stationary and nonstationary dynamics and not an inference from gerbe twisting alone. The explicit detector and tubular data therefore inhabit H1 and H2 on the selected vacuum branch. The endpoint-normalized physical H3 ledger records the actual support endpoint of every regional, chart, Cauchy, scale, background, and mixed comparison. Positivity of all finitely

many actual endpoint margins gives one safety ratio below one at the selected positive thicknesses, closing H3.1–H3.10. The selected protected physical Cauchy diamonds satisfy CIR0–CIR5. For a chosen finite admissible protected cover, finite certificate choices on all nonempty intersections form weakly contractible fibers, and vertical localization recovers the corresponding rooted or marked Čech nerve. This is not a theorem about every open cover or an unrestricted ambient topology. The source-free lifting stack and the retained positive-thickness realization are pseudonaturally biequivalent on that same Čech nerve, and observable CP4 is proved for its ordinary descended shadow. The inherited complete-ground packet is a cartesian local decoration of this simplicial diagram: P4–P5 descent and every RID coherence cell preserve its projection, core, preform, and source-certified constant. Thus the selected front end is closed without inverting a scale correspondence or deriving a target bound from a one-sided defect.

The use of Wilson observables is motivated by the gauge-invariant formulation introduced by Wilson<sup>8,9</sup> and by the rigorous lattice and constructive gauge-theory literature<sup>10,11</sup>. The present framework is not a Euclidean lattice construction and does not derive its finite-propagation estimates from a lattice transfer matrix. Instead, a selected positive-thickness time-zero package supplies bounded Wilson-tube generators, physical quotient relations, and primitive Cauchy propagators. Their supports are organized by upward-closed filters rather than by a presumed unique minimal support. This is necessary because retracing, subdivision, cancellation, and physical compression can produce many equivalent tube presentations.

The first main conclusion is ordinary operator-algebraic descent. Chartwise represented Wilson-tube cores are completed to von Neumann algebras and glued through normal spatial isomorphisms. The resulting observable algebra does not carry a gerbe label. It is an ordinary regional algebra, independent of the chosen chart, local Cauchy slice, selected representative within a fixed-thickness fiber, and admissible presentation. The descent mechanism is a normal equalizer of a presentation groupoid. A point that matters for correctness is that evaluation identifies the compatible-family algebra with the isotropy fixed-point algebra unless observable isotropy acts trivially. The construction hypotheses include precisely that triviality on the ultraweakly generating physical Wilson core.

Locality has two positive-thickness forms. If the controlled thickenings of two regions

are spacelike separated, then their descended algebras commute exactly in every common ambient regional algebra:

$$\mathcal{O}_1^{+\varepsilon} \perp \mathcal{O}_2^{+\varepsilon} \implies [\mathcal{A}_\varepsilon^W(\mathcal{O}_1), \mathcal{A}_\varepsilon^W(\mathcal{O}_2)] = 0. \quad (\text{I.5})$$

If only the underlying thin regions are spacelike separated, collar overlap and quasilocal propagation are measured by an intrinsic defect

$$\|[A, B]\| \leq \eta_\varepsilon^{\text{desc}}(\mathcal{O}_1, \mathcal{O}_2) \|A\| \|B\|, \quad \eta_\varepsilon^{\text{desc}} \longrightarrow 0. \quad (\text{I.6})$$

The convergence is pointwise for every thin-spacelike pair and uniform on finite families with a fixed positive spacelike margin. This is analogous in spirit to finite-velocity and Lieb–Robinson estimates<sup>12,13</sup>, but the hypotheses and carriers here are adapted to Wilson-tube transport on incised spacetime.

The second main conclusion concerns the selected complete-ground representation system. For each adapted chart  $\alpha$  and thickness  $\varepsilon$  the local datum is

$$(\mathcal{H}_{\alpha,\varepsilon}, \pi_{\alpha,\varepsilon}, P_{\Omega,\alpha,\varepsilon}), \quad (\text{I.7})$$

where  $P_{\Omega,\alpha,\varepsilon}$  is the complete ground-sector projection. No one-dimensionality or preferred vacuum vector is assumed. On a chart overlap the comparison is allowed to be twisted by a Hermitian coefficient line:

$$\mathbb{U}_{\alpha\beta,\varepsilon} : \mathcal{H}_{\beta,\varepsilon} \hat{\otimes} L_{\alpha\beta,\varepsilon} \longrightarrow \mathcal{H}_{\alpha,\varepsilon}. \quad (\text{I.8})$$

The coefficient-line reduction is not automatic from spatial equivalence. Before reduction, the ambiguity is a torsor under the ground-normalizing commutant. Scalar gerbe language is used only after the selected physical comparison system reduces that ambiguity to a central one-dimensional band.

The overlap lines have multiplication maps on triple intersections, and the direct and composite representation routes differ by scalar tensorators. This gives a fixed-thickness banded class

$$[\Omega_\varepsilon]_{\text{band}}. \quad (\text{I.9})$$

Across scale, the selected comparison arrows retain more information than their numerical endpoints. Their algebra maps, representation lines, scale tensorators, and mixed chart–scale cells assemble over a Grothendieck-type chart–scale bicategory. We retain both the

unrestricted ordinary class

$$[\widehat{\Omega}]_{\text{cs,ord}} \tag{I.10}$$

and the class in the uniformly local, band-preserving, scale-natural controlled complex,

$$[\widehat{\Omega}]_{\text{cs,ctrl}}. \tag{I.11}$$

This distinction is forced by the rooted refinement category: its unrestricted positive-degree cohomology with constant coefficients is contractible, while the canonical root primitive remembers absolute depth and is not an admissible uniformly controlled strictification.

The present construction is adjacent to Roberts net cohomology, superselection sectors over region posets, connections and bundles over posets, and presheaves of tensor categories<sup>14–19</sup>. Those theories encode charges, holonomy, and bundle or gerbe structure over a spacetime base. Here the primary observable assignment has already descended ordinarily; the classified object is the selected complete-ground implementation over the product of chart and refinement data, with a separate controlled regularity imposed on admissible scale strictifications. The construction also uses nonabelian descent and gerbe theory<sup>20–23</sup> and bicategorical Grothendieck constructions<sup>24,25</sup>.

The controlled sector trichotomy is

$$\begin{aligned} \text{I : } & [\Omega_\varepsilon]_{\text{band}} = 0 \text{ for every } \varepsilon, \quad [\widehat{\Omega}]_{\text{cs,ctrl}} = 0, \\ \text{II}_{\text{ctrl}} : & [\Omega_\varepsilon]_{\text{band}} = 0 \text{ for every } \varepsilon, \quad [\widehat{\Omega}]_{\text{cs,ord}} = 0, \quad [\widehat{\Omega}]_{\text{cs,ctrl}} \neq 0, \\ \text{III : } & [\Omega_\varepsilon]_{\text{band}} \neq 0 \text{ for at least one } \varepsilon. \end{aligned} \tag{I.12}$$

Controlled Sector II is ordinary strictifiability without uniformly local, scale-natural strictifiability. Appendix B proves the root contraction, defines the controlled complex, and gives a strictly refining cyclic witness. Appendix I derives the physical fixed-thickness  $SU(3)$  class directly from meridional geometry and triality.

The third contribution is an analytic package compatible with this sectorial descent. Compactly supported interior gauge parameters define Gauss graph forms on common energy graph spaces. Under stronger hypotheses these forms admit closed, self-adjoint, and exponentiated realizations. Physical regional observables lie in the spectral strong commutant whenever self-adjointness is available. The complete ground sector is Gauss-null chartwise;

in a twisted sector these kernels glue as a coefficient-line-banded subobject rather than as one global Hilbert subspace. Collar-nontrivial parameters are retained as possible charge classes modulo interior constraints. This is consonant with the role of Gauss law and edge data in gauge theory<sup>26–29</sup>.

The Ward identities are formulated first at graph-form level. Boundary charge zero-points can transform affinely across charts or scales, but scalar shifts cancel from the adjoint Ward action. Thus global charge operators require more structure than global Ward derivations. Relative entropy is defined intrinsically on the regional von Neumann algebras using faithful normal semifinite reference weights, following the operator-algebraic relative-entropy framework of Araki and its monotonicity refinements<sup>30–32</sup>. The state-selection map called entropy surgery is primary on normal states. It is not assumed to act linearly on Hilbert or form domains. Along selected scale-bordism arrows, surgery defects satisfy a one-cocycle law and entropy production is additive under gluing.

Finally, the inherited positive-thickness quadratic forms close and retain the branchwise uniform lower bound selected in Paper II from the objectwise Paper-I estimates

$$\bar{q}_{\alpha,r;\mathcal{O}}[\Psi] \geq \kappa_* \|(I - P_{\Omega,\alpha,r})\Psi\|^2. \quad (\text{I.13})$$

The representation theorem for closed forms<sup>33,34</sup> then gives a positive self-adjoint form operator with zero spectral projection equal to the complete ground projection. This is an objectwise thick statement. It is not a Poincaré mass operator and does not imply a limiting spectral threshold. A simple  $\ell^2$ -example shows that complete ground sectors can grow so that every limiting direction becomes null even though the same positive constant  $\kappa_*$  holds at every positive thickness. A later thin reconstruction therefore needs projection tightness and a recovery or upper-identification theorem in addition to a Mosco-type lower-bound argument<sup>35</sup>.

The article is organized as follows. Section II constructs P1–P5, derives the common obstruction carrier, proves the conditional H1–H3 controlled geometry, recovers the incised regional category, establishes RID/CP4, and reintroduces the inherited complete-ground coercive provenance. Section III records the unchanged H4–H19 downstream hypotheses, separates the observable/descent and analytic theorems, and gives the full dependency ledger.

Section IV constructs the thick observable net and locality estimates. Section V develops fixed-thickness and total chart-scale descent. Section VI establishes the gauge, Ward, entropy, and closed-form package. Section VII treats background covariance and identifies the exact conditional endpoint. Section VIII gives the linked-flux  $SU(3)$  global local-normal assembly and the independent strict  $SU(2)$  control. The appendices collect the branch conventions, controlled root-contraction and cyclic-band arguments, closed-form leakage, domain and propagation arguments, thin-readiness conditions, the concise  $SU(2)$  certificate, and the Friedrichs/linked-flux repairs. Full realization calculations and reproducibility ledgers are supplied in the accompanying technical supplement.

## II. PRE-INCISION HABITAT FORMATION AND REGIONAL REALIZATION RECOVERY

### A. Inherited marked time-zero branch and selected scale notation

Let

$$M = \mathbb{R}^{3,1}, \quad \Gamma_{\text{cof}} \xrightarrow{e} E_{\text{cof}} \subset (0, \infty), \quad e(r) = \varepsilon_r, \quad \inf E_{\text{cof}} = 0. \quad (\text{II.1})$$

An object of the selected Paper-II branch carries the marked time-zero package

$$\mathcal{P}_r = \left( \mathcal{H}_r^0, \pi_r^0, P_{\Omega,r}^0, \mathfrak{W}_r^{\text{phys}}, q_r^0, \mathcal{C}_r^0 \right), \quad (\text{II.2})$$

where the invariant ground datum is the complete projection  $P_{\Omega,r}^0$ , not a preferred vector. A selected scale arrow

$$a : r_0 \longrightarrow r_1 \quad (\text{II.3})$$

is not identified with its numerical endpoint pair. Its regional and represented correspondences, coefficient line or normal correspondence, analytic comparison class, multiplication maps, reassociation cells, and rooted branch marking remain part of the datum.

The purpose of this section is to construct the logically prior regional and spacetime lifting problem on ordinary Minkowski testing regions, derive its common obstruction carrier, recover the controlled incised testing category, and identify the resulting lifting gerbe with the retained positive-thickness Paper-III realization. No coercive constant is used until the final provenance subsection.

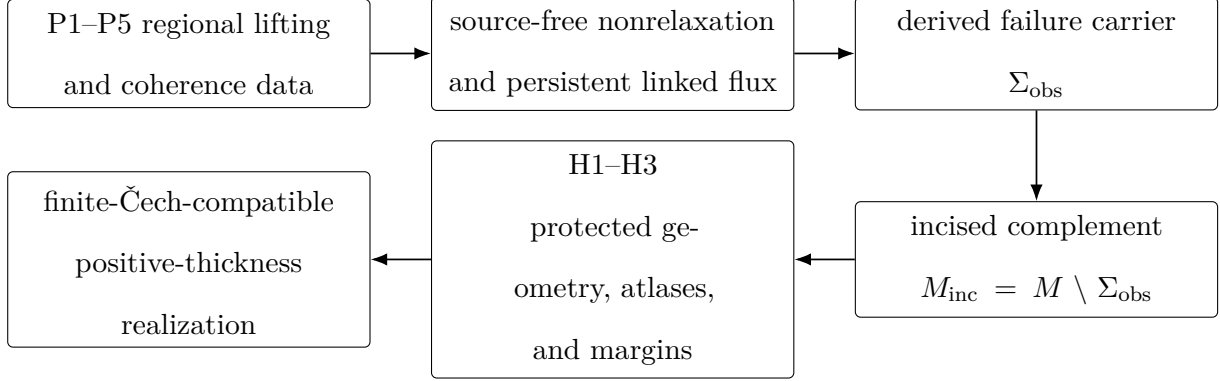


FIG. 1. Logical order of the inhabited Paper-III front end. The incision is formed only after the persistent obstruction carrier has been identified.

**Reader’s guide.** The incision is not chosen in advance. The argument first poses the regional lifting problem on ordinary Minkowski testing regions, separates the stationary architecture witness from the nonstationary physical branch, and uses source-free nonrelaxation to identify the persistent failure carrier. Only then is the admissible complement formed and equipped with finite obstruction-adapted atlases, positive margins, and a coherent regional realization. The output is a controlled positive-thickness habitat, not a thin theory.

## B. P1: regional time-zero physical extension

The finite physical packages of Paper I and the marked scale-bordism branch of Paper II determine two faces of the required front-end diagram: the objectwise physical face and the selected scale face. They do not determine the spatial regional face or the mixed regional–scale face. We therefore state the weakest regional extension datum from which the required prestack can be constructed.

**Definition II.1** (Selected compactly presented spatial site). An object of  $\mathbf{SpReg}^{0,\text{cp}}$  is a tuple

$$\mathbb{B} = (B, \mathbf{\Lambda}_B, \mathfrak{F}_B) \quad (\text{II.4})$$

with the following properties.

1.  $B \Subset \mathbb{R}^3$  is a bounded regular open set.

2. For every  $r \in \Gamma_{\text{cof}}$ ,  $\Lambda_{B,r} \in B$  is a selected finite spatial cellular presentation supporting the Wilson-tube carriers used at scale  $r$ .
3. The geometric refinement maps attached to the selected scale arrows carry  $\Lambda_{B,r}$  into the selected presentation at the target scale.
4.  $\mathfrak{F}_{B,r}$  is a nonempty upward-directed family of compact support presentations cofinal under enlargement inside  $B$ .
5. Every finite family of selected subregions and scale arrows used in a demand diagram admits a finite common cellular refinement.

An admissible arrow  $\mathbf{i} : \mathbb{B}' \hookrightarrow \mathbb{B}$  consists of the open inclusion  $i : B' \hookrightarrow B$ , compatible cellular monomorphisms  $\Lambda_{B',r} \hookrightarrow \Lambda_{B,r}$ , and monotone maps of the selected support families. Composition is induced by composition of these data. We replace the resulting category by one fixed essentially small skeleton. Finite compatible covers define the topology  $\mathcal{J}_{\text{Sp}}^0$ , although only the underlying category is used in P1.

For  $\mathbb{B} \in \mathbf{SpReg}^{0,\text{cp}}$ , write  $\downarrow \mathbb{B}$  for the category of selected admissible subregions of  $\mathbb{B}$ . The contravariant prestack restriction constructed below is restriction of a regional packet from  $\downarrow \mathbb{B}$  to a smaller downset. This must not be confused with the covariant inclusion of local observable algebras appearing inside the packet.

**Definition II.2** (Physical-package correspondence bicategory). Let  $\mathbf{PhysPkgCorr}^0$  denote the selected  $(2,1)$ -category whose objects are tuples

$$\begin{aligned} \mathcal{P}_{B,r}^0 = & \left( \mathcal{H}_{B,r}^0, \pi_{B,r}^0, P_{\Omega,B,r}^0, \mathcal{W}_{B,r}^{0,\text{phys}}, q_{B,r}^0, \mathcal{C}_{B,r}^0; \right. \\ & \left. \mathbf{J}_{B,r}, G_{B,r}, \mathbf{F}_{B,r}^{\text{phys}}, \mathbf{M}_{B,r}, \mathbf{D}_{B,r}, \text{Supp}_{B,r} \right). \end{aligned} \quad (\text{II.5})$$

Here  $\mathbf{J}$ ,  $G$ ,  $\mathbf{F}^{\text{phys}}$ ,  $\mathbf{M}$ , and  $\mathbf{D}$  collectively denote the surviving channel map, Gram operator, normalized physical channel, magnetic recompressions, and channel–residual–ultraviolet decomposition inherited from Paper I. The support datum is an upward-closed filter of admissible compact carriers.

A one-morphism is an admissible physical package correspondence carrying:

1. a quotient-level observable correspondence;

2. a represented Hilbert correspondence or spatial implementer;
3. a complete-ground intertwining relation;
4. maps on the physical-channel coefficient spaces;
5. exact or explicitly controlled transport of Gram operators, magnetic recompressions, decompositions, and inherited forms;
6. transport of the declared algebraic, form, and graph cores; and
7. monotone transport of support filters.

A two-morphism is an invertible unitary intertwiner preserving all displayed decorations and all marked data. Composition is the selected balanced composition of correspondences, with its declared associator and unitors.

**Hypothesis II.3** (P1: Regional Physical Extension package). The following data and estimates are available.

(RPE0) Regional physical quotients. For every selected  $(B, r)$ , there is a gauge-invariant Wilson presentation algebra  $\mathcal{W}_{B,r}^{0,\text{gi}}$  and a regional physical ideal  $\mathcal{I}_{B,r}^{0,\text{phys}}$  such that

$$\mathcal{W}_{B,r}^{0,\text{phys}} = \mathcal{W}_{B,r}^{0,\text{gi}} / \mathcal{I}_{B,r}^{0,\text{phys}}. \quad (\text{II.6})$$

Thus the regional observable algebra is obtained by regional physicalization, not by naively restricting an unphysical ambient algebra.

(RPE1) Regional finite packages. For every selected  $(B, r)$ , an object  $\mathcal{P}_{B,r}^0$  of **PhysPkgCorr**<sup>0</sup> is given. The projection  $P_{\Omega,B,r}^0$  is the complete-ground projection. No preferred vector in its range is selected.

(RPE2) Regional physical bordisms. For every admissible inclusion  $i : B' \hookrightarrow B$  and every  $r \in \Gamma_{\text{cof}}$ , there is an admissible regional physical bordism

$$\mathbb{R}_i^r : \mathcal{P}_{B,r}^0 \rightsquigarrow \mathcal{P}_{B',r}^0. \quad (\text{II.7})$$

For composable inclusions  $B'' \xrightarrow{j} B' \xrightarrow{i} B$ , there is an invertible compositor

$$\rho_{j,i}^r : \mathbb{R}_j^r \circ \mathbb{R}_i^r \Longrightarrow \mathbb{R}_{i \circ j}^r, \quad (\text{II.8})$$

together with invertible unit cells. The regional pentagon and triangle identities hold.

(RPE3) Selected scale transport. For every selected scale arrow  $a : r \rightarrow s$  and every selected region  $B$ , the Paper-II bordism data restrict to an admissible correspondence

$$\mathbb{T}_a^B : \mathcal{P}_{B,r}^0 \rightsquigarrow \mathcal{P}_{B,s}^0. \quad (\text{II.9})$$

It retains its coefficient line or normal correspondence, complete-ground restriction, analytic comparison class, multiplication cell

$$\mu_{b,a}^B : \mathbb{T}_b^B \circ \mathbb{T}_a^B \Longrightarrow \mathbb{T}_{ba}^B, \quad (\text{II.10})$$

and coefficient reassociator. The selected scale pentagon and unit identities hold in the same admissible correspondence bicategory.

(RPE4) Mixed regional–scale cells. For every admissible inclusion  $i : B' \hookrightarrow B$  and every selected scale arrow  $a : r \rightarrow s$ , there is an invertible marking-preserving Beck–Chevalley cell

$$\beta_{i,a} : \mathbb{R}_i^s \circ \mathbb{T}_a^B \Longrightarrow \mathbb{T}_a^{B'} \circ \mathbb{R}_i^r. \quad (\text{II.11})$$

The two independent mixed coherence identities hold. For composable regional arrows  $B'' \xrightarrow{j} B' \xrightarrow{i} B$ , the two pasted cells from  $\mathbb{R}_{ioj}^s \mathbb{T}_a^B$  to  $\mathbb{T}_a^{B''} \mathbb{R}_{ioj}^r$  agree. Suppressing the ambient associators and unitors, this identity is

$$\beta_{ioj,a} \rho_{j,i}^s = \rho_{j,i}^r (\beta_{j,a} * \mathbb{R}_i^r) (\mathbb{R}_j^s * \beta_{i,a}). \quad (\text{II.12})$$

For composable scale arrows  $r \xrightarrow{a} s \xrightarrow{b} t$ , the two pasted cells from  $\mathbb{R}_i^t \mathbb{T}_b^B \mathbb{T}_a^B$  to  $\mathbb{T}_{boa}^{B'} \mathbb{R}_i^r$  agree. Suppressing the ambient associators and unitors, this identity is

$$\beta_{i,boa} (\mathbb{R}_i^t * \mu_{b,a}^B) = (\mu_{b,a}^{B'} * \mathbb{R}_i^r) (\mathbb{T}_b^{B'} * \beta_{i,a}) (\beta_{i,b} * \mathbb{T}_a^B). \quad (\text{II.13})$$

Equations (II.12) and (II.13) are equalities of fully parenthesized pasted invertible two-cells after restoring the indicated associators.

(RPE5) Quotient, ground, channel, and recompression compatibility. The regional bordisms are induced at the physical quotient level. The chosen representatives preserve the physical ideals in the sense required to descend to (II.6). Their represented correspondences intertwine the physical actions and satisfy

$$U_i^r (P_{\Omega,B,r}^0 \hat{\otimes} I) = P_{\Omega,B',r}^0 U_i^r \quad (\text{II.14})$$

whenever the correspondence is written through a spatial implementer  $U_i^r$ . The induced coefficient-space maps intertwine the physical channels. Consequently the Gram and magnetic recompression data obey the declared exact covariance identities or belong to the declared admissible controlled classes. A controlled relation is never replaced by literal equality.

(RPE6) Core and analytic bounds. Every regional and scale correspondence carries the declared algebraic, form, and graph cores into the corresponding target cores. On every finite demand shape there are finite constants satisfying, for the regional core map  $R_{i,r}$ ,

$$\|R_{i,r}\xi\|_{\text{graph},B',r} \leq C_{i,r}\|\xi\|_{\text{graph},B,r}, \quad (\text{II.15})$$

and

$$\left|q_{B',r}^0(R_{i,r}\xi, R_{i,r}\eta) - q_{B,r}^0(\xi, \eta)\right| \leq \delta_{i,r}^q \|\xi\|_{q,B,r} \|\eta\|_{q,B,r}. \quad (\text{II.16})$$

Analogous finite-shape bounds hold for selected scale and mixed comparisons. The constants may depend on the finite demand shape. No cofinal-uniform bound is asserted here unless it is separately included in the selected branch profile.

(RPE7) Support-filter transport. Support presentations descend through the physical quotient. Regional bordisms act monotonically on the upward-closed support filters, selected scale arrows preserve the declared support class, and the mixed cell  $\beta_{i,a}$  identifies the two transported support filters.

(RPE8) Marked type and roots. Every regional scale diagram carries the inherited marked reconstruction branch

$$\tau = \left(t_{\text{sc}}^{\text{vac}}, \{[\Omega_{e(r)}]_{\text{band}}\}_{r \in \Gamma_{\text{cof}}}, [\hat{\Omega}]_{\text{cs,ord}}, [\hat{\Omega}]_{\text{cs,ctrl}}\right). \quad (\text{II.17})$$

Regional bordisms, mixed cells, and vertical equivalences are root preserving and act by band-local admissible equivalences on the surviving coefficient diagrams. Hence they preserve  $\tau$ .

*Remark II.4* (Exact status of Hypothesis II.3). Hypothesis II.3 is not a formal consequence of the objectwise Paper-I packages and the Paper-II scale arrows. A model-specific proof must still construct regional physical presentations, regional correspondences, and the mixed cells  $\beta_{i,a}$ . The selected retained  $SU(3)$  branch supplies such a proof on the ordinary pre-incision

dyadic time-zero basis in Theorem II.13. For arbitrary branches or unrestricted regional sites, P1 remains a hypothesis.

**Theorem II.5** (Regional time-zero physical extension). *Under Hypothesis II.3, there exists a marked contravariant pseudofunctor*

$$\boxed{\mathbf{H}_{\Gamma, \tau}^{0, \text{phys}} : (\mathbf{SpReg}^{0, \text{cp}})^{\text{op}} \longrightarrow 2\mathbf{Gpd}.} \quad (\text{II.18})$$

For  $\mathbb{B} \in \mathbf{SpReg}^{0, \text{cp}}$ , an object of  $\mathbf{H}_{\Gamma, \tau}^{0, \text{phys}}(\mathbb{B})$  is the decorated pseudofunctor

$$\mathcal{P}_{\mathbb{B}} : (\downarrow \mathbb{B})^{\text{op}} \times \Gamma_{\text{cof}} \longrightarrow \mathbf{PhysPkgCorr}^0 \quad (\text{II.19})$$

whose object, regional-arrow, scale-arrow, and mixed-arrow data are those of Hypothesis II.3, and whose marked type is  $\tau$ . Its one-morphisms are marking-preserving pseudonatural equivalences and its two-morphisms are invertible marking-preserving modifications.

For an admissible inclusion  $j : \mathbb{B}' \hookrightarrow \mathbb{B}$ , the restriction two-functor is precomposition with the canonical inclusion

$$(\downarrow \mathbb{B}')^{\text{op}} \times \Gamma_{\text{cof}} \longrightarrow (\downarrow \mathbb{B})^{\text{op}} \times \Gamma_{\text{cof}}. \quad (\text{II.20})$$

The Grothendieck construction of (II.18) distinguishes:

1. cartesian arrows over spatial regional restrictions;
2. internal selected scale-transport correspondences; and
3. vertical marking-preserving equivalences.

The construction preserves physical Wilson quotients, complete-ground projections, physical channels, Gram and magnetic recompression data, inherited forms and their declared cores, coefficient correspondences, analytic comparison classes, support filters, roots, and the marked type  $\tau$ .

*Proof.* Fix  $\mathbb{B} \in \mathbf{SpReg}^{0, \text{cp}}$ .

*Step 1: object assignment.* For each pair  $(\mathbb{C}, r)$ , with  $\mathbb{C} \in \downarrow \mathbb{B}$  and  $r \in \Gamma_{\text{cof}}$ , RPE0 and RPE1 assign the object  $\mathcal{P}_{\mathbb{C}, r}^0$  of  $\mathbf{PhysPkgCorr}^0$ . Because the regional Wilson algebra is the quotient (II.6), this assignment is already made in the physical category; no later quotient is silently applied to an unphysical regional restriction.

*Step 2: pure regional arrows.* An arrow of  $(\downarrow \mathbb{B})^{\text{op}}$  is represented by an admissible inclusion  $i : C' \hookrightarrow C$ . Assign to it the regional bordism  $\mathbb{R}_i^r$  from RPE2. The compositor  $\rho_{j,i}^r$ , its regional pentagon, and the regional unit triangles are exactly the compositor, pentagon, and unit axioms for pseudofunctoriality in the regional variable.

*Step 3: pure scale arrows.* For a selected scale arrow  $a : r \rightarrow s$ , assign  $\mathbb{T}_a^C$ . The cells  $\mu_{b,a}^C$ , the retained coefficient reassociator, the scale pentagon, and the scale unit identities are exactly the pseudofunctor coherence data in the scale variable. A nonzero banded reassociator is retained; no strictification is imposed.

*Step 4: mixed arrows and independence of factorization.* Every arrow in  $(\downarrow \mathbb{B})^{\text{op}} \times \Gamma_{\text{cof}}$  can be represented by a regional arrow and a scale arrow in either order. The cell  $\beta_{i,a}$  of RPE4 compares the two images. For two regional arrows and one scale arrow, equation (II.12) identifies the two boundary pastings of the mixed regional cube. For one regional arrow and two scale arrows, equation (II.13) identifies the two boundary pastings of the mixed scale cube. Together with the pure regional and pure scale pentagons, these are precisely the product-pseudofunctor coherence conditions. Hence the value assigned to a composite mixed arrow is independent, up to the specified invertible two-cell, of the chosen regional–scale factorization. This constructs  $\mathcal{P}_{\mathbb{B}}$  in (II.19).

*Step 5: retention of the physical package.* RPE5 ensures that every regional arrow is defined after passage to the physical quotient, intertwines the represented physical action and the complete-ground corner, and transports the physical channel. Gram operators and magnetic recompressions are functorial consequences of the channel comparison only in the exact branch; in the controlled branch the specified admissible defect is retained as part of the one-morphism. Thus this step does not replace controlled covariance by equality.

RPE6 transports the distinct algebraic, form, and graph cores with the bounds (II.15)–(II.16). These bounds show that the inherited forms and analytic comparison classes are well-defined under the regional and mixed compositions used above. The constants are only finite-shape constants, exactly as stated. RPE7 gives the corresponding support-filter functoriality.

*Step 6: fixed marked type.* By RPE8, every regional, scale, mixed, and vertical comparison is

root-preserving and band-local. Such a comparison changes the relative analytic cocycle and the coefficient reassociator only by their declared admissible coboundaries. Therefore the marked classes in (II.17) are unchanged. This uses the representative invariance mechanism proved in Paper II, together with the new spatial band-locality assertion in RPE8. In particular, restriction cannot change a strict branch into a twisted branch or conversely.

*Step 7: the fiber is a two-groupoid.* Take as objects all selected decorated pseudofunctors  $\mathcal{P}_{\mathbb{B}}$  of marked type  $\tau$ . Take as one-morphisms marking-preserving pseudonatural equivalences and as two-morphisms invertible marking-preserving modifications. Every one- and two-morphism is invertible at the declared categorical level, so the resulting bicategory is a 2-groupoid.

*Step 8: contravariant regional restriction.* For  $j : \mathbb{B}' \hookrightarrow \mathbb{B}$ , the selected subregion category  $\downarrow \mathbb{B}'$  is a full selected subcategory of  $\downarrow \mathbb{B}$ . Precomposition with (II.20) defines

$$j^* : \mathbf{H}_{\Gamma, \tau}^{0, \text{phys}}(\mathbb{B}) \longrightarrow \mathbf{H}_{\Gamma, \tau}^{0, \text{phys}}(\mathbb{B}'). \quad (\text{II.21})$$

It acts on pseudonatural equivalences and modifications by the same precomposition. For  $\mathbb{B}'' \xrightarrow{k} \mathbb{B}' \xrightarrow{j} \mathbb{B}$ , precomposition gives  $(j \circ k)^* = k^* \circ j^*$  strictly after the chosen skeletal identifications. If the selected downset inclusions are retained only up to equivalence, the canonical associator of precomposition supplies the invertible compositor; the corresponding pentagon follows from associativity of functor composition. The unit comparison is analogous. Hence (II.18) is a contravariant pseudofunctor.

The three arrow classes in the Grothendieck construction are now distinct: cartesian arrows arise from the functors  $j^*$ , selected scale arrows are internal one-morphisms  $\mathbb{T}_a^C$  of an object, and vertical arrows are pseudonatural equivalences in a fixed fiber. This proves every assertion of the theorem.  $\square$

*Remark II.6 (Circularity firewall).* The inherited forms  $q_{B,r}^0$  and their core-comparison defects are transported as decorations of the regional package. No lower coercive bound, no spectral threshold, no invariant momentum operator, and no vacuum spectral projection beyond the already inherited complete-ground projection is used to define the site, the regional quotient, the regional bordisms, or the mixed cells. In particular, none of  $\hat{\kappa}_r$ ,  $\kappa_*$ ,  $m_0$ ,  $P_\mu$ ,  $P^2$ , or  $\Pi_0$  enters P1.

**Proposition II.7** (Finite-cellular factorizing witness for P1). *Suppose that, at every selected scale, each finite spatial cell  $e$  carries a finite physical package  $\mathcal{P}_{e,r}^0$ , and that for a finite selected cell set  $E(B,r)$  the regional package factorizes as*

$$\mathcal{H}_{B,r}^0 = \widehat{\bigotimes_{e \in E(B,r)} \mathcal{H}_{e,r}^0}, \quad P_{\Omega,B,r}^0 = \bigotimes_{e \in E(B,r)} P_{\Omega,e,r}^0, \quad (\text{II.22})$$

*with regional physical algebra obtained by quotienting the tensor presentation by the sum of the local physical ideals. Assume that the regional form is the finite sum of the local forms on the algebraic tensor core, that the scale transport factorizes cellwise with strict coefficient line  $\mathbb{C}$ , and that support is finite cell support. Then Hypothesis II.3 holds on the finite-cellular subsite, with marked Sector-I type.*

*Proof.* For a cell inclusion  $E(B',r) \subseteq E(B,r)$ , the Hilbert factorization is

$$\mathcal{H}_{B,r}^0 \cong \mathcal{H}_{B',r}^0 \widehat{\otimes} \mathcal{H}_{B \setminus B',r}^0. \quad (\text{II.23})$$

The corresponding tensor-factor bimodule gives the regional physical bordism. Equation (II.22) gives

$$P_{\Omega,B,r}^0 = P_{\Omega,B',r}^0 \widehat{\otimes} P_{\Omega,B \setminus B',r}^0, \quad (\text{II.24})$$

so complete-ground transport uses only the full ground projection and does not choose a vector. The quotient by the sum of local physical ideals is compatible with removal of tensor factors, while the channel, Gram, and magnetic recompression data factorize by hypothesis. Finite sums of local forms satisfy RPE6 with finite constants, and cell supports satisfy RPE7.

The coefficient lines are  $\mathbb{C}$ , so the scale multiplication and reassociation are strict. Regional restriction and cellwise scale transport commute through the canonical Hilbert tensor associator. The two mixed coherence laws reduce to the pentagon and naturality identities for the tensor product. The marked type is therefore the strict Sector-I type and is preserved under every regional restriction. Thus RPE0–RPE8 hold.  $\square$

*Remark II.8* (Status of the explicit Yang–Mills witnesses). The finite  $SU(2)$  Galerkin package of Paper I seeds the strict finite-cellular witness of Proposition II.7, but it does not prove the selected  $SU(3)$  branch. Conversely, the already incised linked-flux  $SU(3)$  realization does not by itself derive P1. Theorem II.13 closes the gap by returning to the tensor-local retained

$SU(3)$  seed and package-adapted dyadic scale maps and rebuilding their regional restriction system on ordinary pre-incision time-zero space. The physical linked-flux interpretation is kept outside that proof.

### C. The selected $SU(3)$ pre-incision regional branch

The model-specific P1 verification is carried by the retained finite Peter–Weyl package before any incision is selected. The construction below uses only the ordinary time-zero space  $\mathbb{R}^3$ , the nested dyadic carrier system, the finite retained  $SU(3)$  character windows, and the package-adapted scale correspondences. In particular, no obstruction carrier, tubular neighborhood of an obstruction carrier, linked collar, post-incision selector, entropy surgery, coercive lower bound, or limiting operator is used.

**Definition II.9** (Selected dyadic pre-incision time-zero site). Let  $\mathbf{SpReg}^{0,\text{dyad}}$  be the full selected subcategory of  $\mathbf{SpReg}^{0,\text{cp}}$  whose objects are relatively compact open sets  $B \subseteq \mathbb{R}^3$  equipped with the restrictions of one fixed nested dyadic cubulation. An arrow  $i : B' \hookrightarrow B$  is an ordinary open inclusion together with the induced inclusion of every retained dyadic cell set.

Let  $\Gamma_{SU(3)}^{\text{pre}}$  be the rooted dyadic analytic spine with objects

$$r_j = (\varepsilon_j, h_j, L_j), \quad \varepsilon_j = 2^{-j-5}, \quad h_j = 2^{-j-1}, \quad L_j = j + 2, \quad j \geq 2,$$

and its selected refinement arrows  $a_{kj} : r_j \rightarrow r_k, j \leq k$ . No carrier-clearance or linking datum is included in  $\Gamma_{SU(3)}^{\text{pre}}$ .

Fix a countable locally finite dyadic carrier forest

$$\Lambda_2 \subset \Lambda_3 \subset \cdots, \quad \Lambda_j \setminus \Lambda_{j-1} \text{ finite on compact sets.}$$

Each persistent label  $\lambda \in \Lambda_j$  has a fixed closed dyadic control cube  $\widehat{Q}_\lambda$  and, at every later scale  $k \geq j$ , a spacelike Wilson loop with closed tube

$$T_{\lambda,k} \subseteq \text{int}(\widehat{Q}_\lambda), \quad \text{rad}(T_{\lambda,k}) \leq \varepsilon_k.$$

At each finite scale the tubes are pairwise disjoint. Under dyadic refinement, the source construction's distinguished descendant of an old carrier is relabelled by the same persistent

label  $\lambda$ ; genuinely added descendants receive new labels. Thus old labels persist and new labels are only added.

For each  $(B, r_j)$ , define the finite regional label set

$$E_j(B) = \{\lambda \in \Lambda_j : \hat{Q}_\lambda \subset B\}.$$

The fixed control cube, rather than the possibly smaller descendant tube, is used in this selection rule. This is the boundary-stability convention that makes regional restriction commute with scale refinement.

**Lemma II.10** (Basis and finite-refinement properties). *The category  $\mathbf{SpReg}^{0,\text{dyad}}$  is an ordinary pre-incision time-zero basis in the following sense.*

1. *The locally finite persistent carrier forest and nested pairwise-disjoint tube family in Definition II.9 exist.*
2. *For every compact  $K \subset U \subseteq \mathbb{R}^3$ , there is an object  $B \in \mathbf{SpReg}^{0,\text{dyad}}$  with  $K \subset B \subseteq U$ .*
3. *Every finite family of objects and selected scale arrows admits a finite common dyadic refinement.*
4. *For every inclusion  $B' \subset B$  and every  $j$ ,  $E_j(B') \subset E_j(B)$ ; for every  $j \leq k$ , persistence of labels gives a canonical inclusion*

$$\iota_{kj}^B : E_j(B) \hookrightarrow E_k(B)$$

*natural in  $B$ .*

*Proof.* Enumerate the dyadic control cubes by increasing birth level and assign a summable volume budget to the labels born inside each fixed control cube. At a finite stage, only finitely many previously chosen tubes meet a compact control cube. Choose their transverse radii so that their total volume there is less than one half of the cube volume. The complement then contains an open ball. Place the next translated model loop and tube in such a ball, with radius small enough to preserve every finitely many relevant ancestor budget, and at later stages shrink the tube inside the same control cube. Induction gives the persistent locally finite pairwise-disjoint family.

Compactness gives a positive distance from  $K$  to  $\mathbb{R}^3 \setminus U$ . A sufficiently fine dyadic mesh has a finite union of cubes whose interiors contain  $K$  and whose closures lie in  $U$ . Taking a common finer dyadic level proves the third assertion. The fourth follows from the fixed control cube selection rule and persistence of labels.  $\square$

For the retained cutoff  $L_j$ , write

$$\mathcal{K}_j = \text{span}\{\chi_{(p,q)} : p + q \leq L_j\},$$

let  $\mathcal{A}_j^{\text{ret}} \subset B(\mathcal{K}_j)$  be the finite-dimensional gauge-invariant algebra generated by the retained electric and compressed Wilson operators, let  $H_j$  be the retained finite Hamiltonian, and let

$$P_{\Omega,j} = \mathbf{1}_{\{E_{0,j}\}}(H_j)$$

be its complete ground projection. Let  $F_j^{\text{phys}}$ ,  $G_j$ ,  $B_{\text{mag},j}^{\text{phys}}$ , and the channel-residual-ultraviolet flags denote the finite physical package already constructed at cutoff  $L_j$ . Only their finite-dimensionality and their exact flag transport are used below.

**Definition II.11** (Selected regional  $SU(3)$  packet). For  $(B, r_j)$ , define

$$\mathcal{H}_{B,j}^0 := \widehat{\bigotimes_{\lambda \in E_j(B)} \mathcal{K}_{j,\lambda}}, \quad (\text{II.25})$$

$$\mathcal{W}_{B,j}^{0,\text{gi}} := \widehat{\bigotimes_{\lambda \in E_j(B)} \mathcal{A}_{j,\lambda}^{\text{ret}}}, \quad (\text{II.26})$$

$$\mathcal{I}_{B,j}^{0,\text{phys}} := 0, \quad \mathcal{W}_{B,j}^{0,\text{phys}} := \mathcal{W}_{B,j}^{0,\text{gi}} / \mathcal{I}_{B,j}^{0,\text{phys}}, \quad (\text{II.27})$$

$$P_{\Omega,B,j}^0 := \widehat{\bigotimes_{\lambda \in E_j(B)} P_{\Omega,j,\lambda}}, \quad (\text{II.28})$$

$$q_{B,j}^0 := \sum_{\lambda \in E_j(B)} (H_{j,\lambda} - E_{0,j}I). \quad (\text{II.29})$$

The representation is the defining tensor-product representation. The algebraic, form, and Gauss-graph cores are all declared to be the full finite-dimensional space  $\mathcal{H}_{B,j}^0$ , while the Ward derivation domain is the full finite-dimensional algebra  $\mathcal{W}_{B,j}^{0,\text{phys}}$ .

The interior Gauss generators are the infinitesimal conjugation generators on the retained character space. They vanish on  $\mathcal{K}_j$ , hence the regional Gauss graph form is the zero form. The retained adjoint Ward derivations are bounded commutator derivations on the finite matrix algebra and therefore have full domain. No collar charge or boundary module is part of this pre-incision packet.

The regional physical channel, Gram operator, magnetic recompression, and channel–residual–ultraviolet flags are the orthogonal sums of one-cell physical channels, with every other tensor factor anchored by its complete ground range. The support filter is the upward closure of finite unions of the tube supports  $T_{\lambda,j}$ .

**Definition II.12** (Regional and scale correspondences). For  $i : B' \hookrightarrow B$ , put  $D_j(B, B') = E_j(B) \setminus E_j(B')$ . The regional correspondence  $\mathbb{R}_i^j$  is the tensor-factor correspondence determined by

$$\mathcal{H}_{B',j}^0 \widehat{\otimes} \bigotimes_{\lambda \in D_j(B, B')} \text{Ran } P_{\Omega,j,\lambda} \longrightarrow \mathcal{H}_{B,j}^0,$$

with the canonical ordered tensor reassociation. This notation denotes the complete ground ranges themselves; it does not select unit vectors in them. The corresponding observable map inside the packet is the covariant inclusion  $A \mapsto A \otimes I$ .

For  $a_{kj} : r_j \rightarrow r_k$ , use the package-adapted one-cell isometry class  $U_{kj} : \mathcal{K}_j \rightarrow \mathcal{K}_k$  carrying the complete-ground and physical-channel flags exactly. On a region  $B$ , tensor  $U_{kj}$  over the persistent old labels  $\iota_{kj}^B(E_j(B))$  and anchor every genuinely added label in  $E_k(B) \setminus \iota_{kj}^B(E_j(B))$  by its complete ground range. This gives a regional scale correspondence  $\mathbb{T}_{kj}^B$ .

For a marked center band  $m \in \mathbb{Z}_3$ , decorate  $\mathbb{T}_{kj}^B$  by the formal one-dimensional coefficient line  $\mathcal{L}_{kj}^{(m)}$  of grade  $k - j \pmod{3}$ . Its multiplication cells carry the normalized cyclic reassociator

$$\Omega_m(a, b, c) = \omega^{\tilde{m}\tilde{a}\epsilon_3(b,c)}, \quad \epsilon_3(b, c) = \left\lfloor \frac{\tilde{b} + \tilde{c}}{3} \right\rfloor,$$

where tildes denote the standard lifts in  $\{0, 1, 2\}$ . This is coefficient data only. At P1 no geometric linking interpretation and no physical nontriviality statement is made.

**Theorem II.13** (Selected  $SU(3)$  regional time-zero physical extension). *For every  $m \in \mathbb{Z}_3$ , the data of Definitions II.9 and II.11, together with Construction II.12, verify RPE0–RPE8 of Hypothesis II.3 on*

$$\mathbf{SpReg}^{0,\text{dyad}} \times \Gamma_{SU(3)}^{\text{pre}}.$$

*Consequently there is a marked contravariant pseudofunctor*

$$\boxed{\mathbf{H}_{SU(3),m}^{0,\text{phys}} : \left(\mathbf{SpReg}^{0,\text{dyad}}\right)^{\text{op}} \longrightarrow 2\mathbf{Gpd}}$$

whose regional packets are explicitly inhabited by the selected retained positive-thickness  $SU(3)$  finite package.

More precisely:

1. regional observable packets, support restriction, and time-zero isotony are exact;
2. the physical Wilson quotient is the identity quotient of the already gauge-invariant retained presentation and is natural under every regional and selected scale correspondence;
3. complete-ground projections, one common analytic core, interior Gauss graph forms, and Ward domains restrict exactly;
4. coefficient lines, the branch marking  $m$ , and the selected root are preserved under regional restriction;
5. regional physical bordisms compose by canonical tensor reassociation;
6. the selected scale correspondences are natural under inclusions, and the Beck–Chevalley cells are canonical tensor-permutation unitaries;
7. regional form transport is exact, while scale-form transport has a finite declared defect on every finite demand shape.

No incision, obstruction clearance, linked collar, or coercive lower bound is used in this construction.

*Proof.* We verify the clauses in the order in which they enter P1.

*RPE0–RPE1: regional physical objects.* The retained character windows consist of conjugation-invariant functions, so  $\mathcal{A}_j^{\text{ret}}$  is already a gauge-invariant physical presentation. Thus the zero ideal in Definition II.11 is a genuine physical quotient, not a quotient omitted by notation. Finite tensor products give the regional Hilbert space, represented observable algebra, full finite physical channel package, complete-ground projection, bounded preform, and support filter. The ground datum is the entire projection  $P_{\Omega, B, j}^0$ .

*RPE2: regional bordisms and composition.* For  $B'' \subset B' \subset B$ , the complement cell set decomposes as the disjoint ordered union

$$D_j(B, B'') = D_j(B', B'') \sqcup D_j(B, B').$$

Hence the two iterated complete-ground tensor correspondences and the direct correspondence differ only by the canonical tensor associator and permutation. These give the compositor  $\rho_{B'',B',B}^j$ . Mac Lane coherence gives the regional pentagon and triangle. No ground vector is needed because tensoring is by the ground ranges as Hilbert correspondences.

*RPE3: selected scale transport.* The package-adapted maps satisfy  $U_{\ell k} U_{kj} = U_{\ell j}$  at the level of their selected isometry classes and carry the ground and physical-channel flags exactly. Persistence of labels gives  $\iota_{\ell k}^B \iota_{kj}^B = \iota_{\ell j}^B$ . Added cells are always anchored by complete-ground ranges. Therefore the regional scale correspondences compose through the canonical tensor compositor. The coefficient-line multiplication and its reassociator supply the remaining scale compositor and pentagon. For  $m = 0$  this coefficient face is strict; for  $m = 1, 2$  it is retained as marked center-banded data.

*RPE4: mixed regional-scale cells.* Naturality of  $\iota_{kj}^B$  gives a commutative square of persistent-label inclusions for every  $B' \subset B$ . Both composites  $\mathbb{R}_i^k \mathbb{T}_{kj}^B$  and  $\mathbb{T}_{kj}^{B'} \mathbb{R}_i^j$  apply the same one-cell flag maps to the same retained cells and anchor the same remaining cells in complete ground ranges. Their only difference is tensor ordering. The canonical tensor-permutation unitary is  $\beta_{i,akj}$ . The mixed regional and mixed scale identities reduce respectively to naturality and the pentagon for the tensor associator, tensored with the coefficient-line compositor. Thus the Beck–Chevalley cells are invertible, marking preserving, and coherent.

*RPE5: quotient, ground, channel, and recompression data.* The regional algebra inclusions preserve the zero physical ideal exactly. On the regional correspondence one has, as an equality of range projections,

$$P_{\Omega,B,j}^0 = P_{\Omega,B',j}^0 \widehat{\otimes} \widehat{\bigotimes_{\lambda \in D_j(B,B')}} P_{\Omega,j,\lambda}.$$

The same tensor-factor calculation transports the one-excitation physical channel, its Gram operator, magnetic recompression, and the declared channel–residual–ultraviolet flags on the retained summands. The selected one-cell scale maps carry the ground and channel flags exactly; the remaining finite analytic quantities stay in their declared controlled comparison classes. No controlled statement is replaced by equality.

*RPE6: common cores and form bounds.* All spaces are finite dimensional, so the full Hilbert space is simultaneously an algebraic, form, Gauss-graph, and Ward core. Regional ground

anchoring is exact for the shifted tensor-sum preform:

$$(\mathbb{R}_i^j)^* q_{B,j}^0 \mathbb{R}_i^j = q_{B',j}^0,$$

because every added shifted one-cell Hamiltonian annihilates its complete ground range. For a scale arrow put

$$\delta_{kj}^{\text{cell}} := \left\| U_{kj}^* (H_k - E_{0,k} I) U_{kj} - (H_j - E_{0,j} I) \right\| < \infty.$$

Then on a region  $B$ ,

$$\left\| (\mathbb{T}_{kj}^B)^* q_{B,k}^0 \mathbb{T}_{kj}^B - q_{B,j}^0 \right\| \leq |E_j(B)| \delta_{kj}^{\text{cell}}.$$

This is the required finite-demand form-defect bound. The corresponding graph bound follows from equivalence of norms in finite dimension. Neither estimate uses a positive lower bound for  $q_{B,j}^0$ .

The interior Gauss generators vanish on each retained character window, so their graph forms restrict exactly. The Ward derivations are bounded and have full domain; tensor restriction and the package-adapted scale maps preserve those domains. This is the entire P1 Gauss–Ward content. Boundary charges and post-incision Ward characters are not invoked. *RPE7: support filters.* Regional restriction deletes precisely the tube supports attached to cells outside  $B'$ . A selected scale arrow retains every old carrier label, replaces its tube by the nested later-scale tube inside the same control cube, and adds only finitely many selected carrier labels on each compact region. Hence finite carrier support and its upward closure are preserved. The two mixed transports produce the same support family, so  $\beta_{i,a}$  identifies them.

*RPE8: marking and roots.* Regional correspondences act trivially on the formal coefficient line and do not change its grade. Scale composition uses the fixed coefficient multiplication and  $\Omega_m$ , while the distinguished least dyadic level is preserved by every admitted restriction. Therefore the selected root, the vacuum scale type, and the marked center band  $m$  are invariant. This asserts preservation of the pre-incision marked coefficient system only; it does not identify  $m = 1, 2$  with a physical linked-flux class on ordinary Minkowski space.

The preceding verifications give RPE0–RPE8. Applying Theorem II.5 yields the displayed contravariant pseudofunctor. □

**Corollary II.14** (Model-specific P1 closure for the selected retained branch). *On the ordinary pre-incision dyadic time-zero basis and the selected dyadic analytic scale spine, obligations P1.1–P1.13 are discharged for the retained  $SU(3)$  branch. The following statements remain outside this corollary:*

1. *identification of the marked  $m = 1, 2$  coefficient system with the physical linked-flux class;*
2. *derivation and regularity of the obstruction carrier;*
3. *effective zero-defect lifting, stackification, CIR, RID, and the comparison with the later incised realization;*
4. *every thin-limit, Haag–Kastler, spectral, confinement, or scattering conclusion.*

*Thus P1 is no longer the first unresolved model-specific edge. The next edge is the P2 anchoring theorem identifying selected Cauchy extraction with this explicit pre-incision regional packet.*

**Remark II.15** (Physical linked-flux firewall). The algebraic coefficient cocycle in P1 is a retained marking. Its later clock–shift realization on a linked Wilson– $t$  Hooft cell is evidence that the marked branch is physically inhabited after incision; it is not used to construct the ordinary pre-incision regional prestack. Accordingly this P1 theorem neither presupposes the incision nor proves that the dynamically derived obstruction carrier equals the straight worldsheet used by the explicit post-incision witness.

**Corollary II.16** (P1 closure status). *At the abstract front-end level, Hypothesis II.3 and Theorem II.5 close the general P1 bridge. At the model-specific level, Theorem II.13 verifies RPE0–RPE8 for the selected retained  $SU(3)$  branch on the ordinary pre-incision dyadic time-zero basis and selected dyadic analytic scale spine. No claim is made for unrestricted regional sites or for physical linked-flux realization before the obstruction carrier is derived.*

#### **D. P2: selected admissible Cauchy-data extraction and anchoring**

For fixed  $r$ , write  $\mathbf{H}_r^{0,\text{phys}}(\mathbb{B})$  for the fixed-scale evaluation of the P1 prestack at  $(\mathbb{B}, r)$ , retaining the complete regional package and its marking-preserving equivalences.

The regional time-zero prestack constructed in Theorem II.5 does not by itself identify a spacetime realization candidate with its data on a Cauchy hypersurface. A second bridge is required: a selected hyperbolic trace–solution equivalence on the declared candidate cores, compatible with the regional and scale directions already fixed in P1. This subsection states the weakest such bridge used below and derives the time-zero extraction pseudofunctor and its anchor.

**Definition II.17** (Selected admissible Cauchy presentation). Let  $O \in \mathbf{Reg}_{\text{pre}}^{r,\text{cp}}(M)$ . A selected admissible Cauchy presentation of  $O$  is a tuple

$$\mathfrak{c} = (N, S, \iota, B) \tag{II.30}$$

with the following data and properties.

1.  $N \Subset M$  is a selected causally regular, relatively compact, globally hyperbolic testing region belonging to the pre-incision site.
2.  $S$  is a selected smooth spacelike Cauchy hypersurface or Cauchy patch for the relevant development in  $N$ , and  $\iota : S \hookrightarrow N$  is the declared embedding.
3.  $B \Subset S$  is a selected spatial footprint. It is equipped with a fixed spatial presentation identifying it with the underlying region of an object  $\mathbb{B}_{\mathfrak{c}} \in \mathbf{SpReg}^{0,\text{cp}}$ . The cellular and support-filter data of  $\mathbb{B}_{\mathfrak{c}}$  are part of the selection.
4. The selected causal development satisfies

$$O \Subset D_N(B) \Subset N, \tag{II.31}$$

after the chosen harmless shrinkings allowed by the testing site.

5. The chart, support, scale, and finite-demand refinements required to compare the spacetime packet with the P1 packet on  $B$  are included in the selected presentation.

The letter  $S$ , rather than  $\Sigma$ , is used for the Cauchy hypersurface because  $\Sigma$  is reserved for the obstruction carrier.

**Definition II.18** (Selected Cauchy comparison groupoid). For a fixed testing region  $O$ , let  $\mathbf{Cauc}_{\text{adm}}(O)$  be one essentially small groupoid with objects the selected admissible Cauchy presentations of  $O$ . An arrow  $u : \mathfrak{c} \rightarrow \mathfrak{c}'$  consists of the following selected comparison data:

1. a causally regular comparison region  $N_u$  and admissible restriction maps from the relevant parts of  $N$  and  $N'$  into  $N_u$ ;
2. a selected common spatial refinement  $\mathbb{B}_u \in \mathbf{SpReg}^{0,\text{cp}}$  with admissible inclusions

$$k_u : \mathbb{B}_u \hookrightarrow \mathbb{B}_{\mathfrak{c}}, \quad k'_u : \mathbb{B}_u \hookrightarrow \mathbb{B}_{\mathfrak{c}'}; \quad (\text{II.32})$$

3. a selected finite-energy Cauchy slab in  $N_u$  joining the two restricted Cauchy patches;
4. common chart, support, and scale refinements on every finite demand shape used in the comparison.

The reverse slab and the same common refinement represent an inverse arrow. Composition is performed after a selected finite common refinement. The chosen comparison compositors and unitors make this an admissible comparison groupoid. No arrow is inserted merely because two smooth Cauchy surfaces exist in the same globally hyperbolic region.

**Definition II.19** (Cauchy-pointed candidate two-groupoid). Fix  $r \in \Gamma_{\text{cof}}$  and a selected presentation  $\mathfrak{c} = (N, S, \iota, B)$ . The two-groupoid  $\mathbf{CohReal}_{r,\mathfrak{c}}^{\text{cand}}(N)$  has as objects raw positive-thickness realization candidates  $\mathfrak{P}_N$  carrying the following data before any zero-defect or coherence condition is imposed:

1. chartwise physical Wilson presentation quotients and represented observable packets on the selected local cores;
2. algebraic, form, propagation, and graph cores, kept as distinct domains;
3. primitive finite-energy trace and solution maps for the selected Cauchy presentation;
4. typed interior Gauss graph-form and interior Ward maps on their declared candidate domains, without imposing their transport or descent identities;
5. support filters and their primitive causal propagation maps;
6. complete-ground projections or complete-ground corners, but no chosen ground vector;
7. physical channel maps, Gram operators, magnetic recompressions, and channel–residual–ultraviolet decompositions;

8. inherited quadratic forms on the declared form cores;
9. coefficient lines or normal correspondences, with their multiplication cells and reassociators;
10. relative refinement-to-admissible classes and the selected scale and mixed comparison cells;
11. roots and marked reconstruction-branch data.

The word “candidate” means that the displayed maps and cells have the correct types; the Gauss, Ward, analytic, Cech, mixed, branch, and anchor identities are not imposed here. One-morphisms are marking-preserving candidate equivalences intertwining the selected trace and solution maps. Two-morphisms are invertible modifications preserving the same data.

The Grothendieck construction over  $\mathbf{Cauc}_{\text{adm}}(O)$  is denoted

$$\mathbf{CohReal}_r^{\text{cand}, \text{Cau}}(O) = \int_{\mathfrak{c} \in \mathbf{Cauc}_{\text{adm}}(O)} \mathbf{CohReal}_{r, \mathfrak{c}}^{\text{cand}}(N). \quad (\text{II.33})$$

For compatibility with the shorter notation in the earlier front patch, set

$$\mathbf{Cand}_r^{\text{Cau}}(O) := \mathbf{CohReal}_r^{\text{cand}, \text{Cau}}(O). \quad (\text{II.34})$$

**Definition II.20** (Independently extracted Cauchy-data package). Let  $\mathfrak{P}_N$  be a candidate and let  $\gamma_{\mathfrak{c}, r}$  and  $E_{\mathfrak{c}, r}$  denote its selected trace and solution maps on the declared cores. The package

$$\text{Data}_{\mathfrak{c}, r}(\mathfrak{P}_N) \quad (\text{II.35})$$

is constructed from the spacetime candidate as follows.

1. Its algebraic data core is the trace image  $\gamma_{\mathfrak{c}, r} \mathcal{C}_{N, r}^{\text{prop}}$ , modulo only the physical relations that survive the trace.
2. Its Hilbert space is the completion of that image in the selected Cauchy energy norm; it is not defined to be  $\mathcal{H}_{B, r}^0$ .
3. If  $A$  is a traced physical Wilson observable and  $u$  is trace data, its represented action is

$$\pi_{\mathfrak{c}, r}^{\text{Data}}(A)u = \gamma_{\mathfrak{c}, r} \pi_{N, r}(\tilde{A}) E_{\mathfrak{c}, r} u, \quad (\text{II.36})$$

where  $\tilde{A}$  is any admissible spacetime representative. Quotient stability below makes the right-hand side independent of the representative.

4. The complete-ground projection is the transported projection

$$P_{\Omega, \mathfrak{c}, r}^{\text{Data}} = \gamma_{\mathfrak{c}, r} P_{\Omega, N, r} E_{\mathfrak{c}, r} \quad (\text{II.37})$$

on the completed data space.

5. The data form is

$$q_{\mathfrak{c}, r}^{\text{Data}}[u, v] = q_{N, r}^{\text{prop}}[E_{\mathfrak{c}, r}u, E_{\mathfrak{c}, r}v] \quad (\text{II.38})$$

on the transported form core, with the declared admissible correction retained if the branch is controlled rather than exact.

6. The channel, Gram, recompression, decomposition, support, coefficient, multiplication, reassociation, relative-class, root, and marked data are obtained by the corresponding trace-solution transport maps, not by forgetting them.

No normal-state domain is introduced at this stage. Normal-state structures belong to the later regional local-normal construction.

**Hypothesis II.21** (P2: Selected Cauchy Time-Slice Anchor package). Assume Hypothesis II.3. For every selected  $r \in \Gamma_{\text{cof}}$ , the following data and estimates are available.

(CTA0) Selected presentation system. The groupoids  $\mathbf{Cauc}_{\text{adm}}(O)$ , their regional restriction functors, their selected common refinements, and their comparison compositors are fixed. They are stable under the selected scale arrows on every finite demand shape.

(CTA1) Trace-solution equivalence on declared cores. For every  $\mathfrak{c} = (N, S, \iota, B)$ , there are maps

$$\gamma_{\mathfrak{c}, r} : \mathcal{C}_{N, r}^{\text{prop}} \longrightarrow \mathcal{C}_{\mathfrak{c}, r}^{\text{Data}}, \quad E_{\mathfrak{c}, r} : \mathcal{C}_{\mathfrak{c}, r}^{\text{Data}} \longrightarrow \mathcal{C}_{N, r}^{\text{prop}}, \quad (\text{II.39})$$

with invertible candidate-level unit and counit cells

$$v_{\mathfrak{c}, r} : \mathbf{1} \Longrightarrow \gamma_{\mathfrak{c}, r} E_{\mathfrak{c}, r}, \quad \epsilon_{\mathfrak{c}, r} : E_{\mathfrak{c}, r} \gamma_{\mathfrak{c}, r} \Longrightarrow \mathbf{1} \quad (\text{II.40})$$

on the selected solution subcategory. The triangle identities hold. The counit is asserted only on candidate solutions in the selected development, not on arbitrary spacetime test sections.

(CTA2) Finite-energy and graph estimates. The trace and solution maps are closable on their declared domains and extend to the selected completions. On every finite demand shape there are finite constants such that

$$\|\gamma_{\mathbf{c},r}\xi\|_{E,B,r} \leq C_{\mathbf{c},r}^{\text{tr}}\|\xi\|_{\text{graph},N,r}, \quad (\text{II.41})$$

$$\|E_{\mathbf{c},r}u\|_{\text{graph},N,r} \leq C_{\mathbf{c},r}^{\text{sol}}\|u\|_{E,B,r}. \quad (\text{II.42})$$

On form cores, the transported form obeys either the exact identity or an explicit admissible estimate

$$\left|q_{\mathbf{c},r}^{\text{Data}}[u, v] - q_{B,r}^0[x, y]\right| \leq \delta_{\mathbf{c},r}^q \|x\|_{q,B,r} \|y\|_{q,B,r} \quad (\text{II.43})$$

whenever  $u$  and  $v$  are the anchor images of  $x$  and  $y$ . The constants may depend on the finite demand shape. No cofinal-uniform estimate is asserted unless it belongs to the inherited common profile.

(CTA3) Physical quotient and support stability. The trace and solution maps preserve the regional physical ideals in both directions in the sense needed for quotient descent. If  $\tilde{A}_1 - \tilde{A}_2$  lies in the spacetime physical ideal, then

$$\gamma_{\mathbf{c},r}\pi_{N,r}(\tilde{A}_1 - \tilde{A}_2)E_{\mathbf{c},r} = 0 \quad (\text{II.44})$$

on the data core. Support carriers are mapped to their selected Cauchy shadows and back to the corresponding causal developments. The resulting maps of upward-closed support filters are monotone and compatible with the P1 regional physical bordisms.

(CTA4) Decoration transport. Trace–solution transport intertwines the represented physical observable action and preserves, exactly or in the declared controlled class:

1. the complete-ground projection or complete-ground corner;
2. the physical Wilson channel;
3. Gram operators and magnetic recompressions;
4. channel, residual, and ultraviolet decompositions;
5. inherited forms and their algebraic, form, and graph cores;
6. coefficient lines or normal correspondences;

7. multiplication maps and reassociators;
8. ordinary Čech representatives in their admissible classes;
9. controlled chart–scale representatives in their admissible classes;
10. relative refinement-to-admissible classes;
11. roots and the marked reconstruction branch  $\tau$ .

No part of CTA4 selects a vector in the complete-ground range.

(CTA5) Regional pseudonaturality. For an admissible regional restriction  $i : \mathbb{B}' \hookrightarrow \mathbb{B}$ , let  $\mathfrak{c}|_i$  be the selected restricted Cauchy presentation and let  $\text{Res}_{i,r}$  denote candidate restriction. There is an invertible marking-preserving cell

$$\vartheta_{i,\mathfrak{c},r}^{\text{reg}} : \mathbb{R}_i^r \circ \text{Data}_{\mathfrak{c},r} \Longrightarrow \text{Data}_{\mathfrak{c}|_i,r} \circ \text{Res}_{i,r}. \quad (\text{II.45})$$

These cells satisfy the regional identity and composition laws with the P1 compositors  $\rho_{j,i}^r$ .

(CTA6) Scale pseudonaturality and the mixed cube. For every selected scale arrow  $a : r \rightarrow s$ , candidate scale transport  $\mathbb{T}_a^N$  and time-zero transport  $\mathbb{T}_a^B$  are related by an invertible marking-preserving cell

$$\vartheta_{\mathfrak{c},a}^{\text{sc}} : \mathbb{T}_a^B \circ \text{Data}_{\mathfrak{c},r} \Longrightarrow \text{Data}_{\mathfrak{c},s} \circ \mathbb{T}_a^N. \quad (\text{II.46})$$

The cells satisfy the scale identity and composition laws with the multiplication cells  $\mu_{b,a}$  and the coefficient reassociators. Moreover, for each pair  $(i, a)$ , the following two fully parenthesized pasted cells have the same source and target and are equal:

$$\mathbb{R}_i^s \mathbb{T}_a^B \text{Data}_{\mathfrak{c},r} \Longrightarrow \text{Data}_{\mathfrak{c}|_i,s} \mathbb{T}_a^{N|i} \text{Res}_{i,r}. \quad (\text{II.47})$$

The first route uses the P1 Beck–Chevalley cell  $\beta_{i,a}$ , then  $\vartheta_{i,\mathfrak{c},r}^{\text{reg}}$ , then  $\vartheta_{\mathfrak{c}|_i,a}^{\text{sc}}$ . The second uses  $\vartheta_{\mathfrak{c},a}^{\text{sc}}$ , then  $\vartheta_{i,\mathfrak{c},s}^{\text{reg}}$ , then the candidate regional–scale comparison cell. This is the P2 mixed regional–scale coherence cube.

(CTA7) Cauchy comparison and higher coherence. For every arrow  $u : \mathfrak{c} \rightarrow \mathfrak{c}'$ , restriction to the common footprint  $\mathbb{B}_u$  yields an invertible comparison

$$\chi_{u,r}(\mathfrak{P}) : \mathbb{R}_{k_u}^r \text{Data}_{\mathfrak{c},r}(\mathfrak{P}) \Longrightarrow \mathbb{R}_{k'_u}^r \text{Data}_{\mathfrak{c}',r}(\mathfrak{P}). \quad (\text{II.48})$$

It is induced by the selected evolution operator  $\gamma_{\mathfrak{c}',r} E_{\mathfrak{c},r}$  on the common comparison slab. For composable comparison arrows  $\mathfrak{c}_0 \xrightarrow{u} \mathfrak{c}_1 \xrightarrow{v} \mathfrak{c}_2$ , after restriction to the selected triple common refinement there is an invertible modification

$$\omega_{v,u,r} : \chi_{v,r} \circ \chi_{u,r} \Longrightarrow \chi_{v \circ u, r}. \quad (\text{II.49})$$

For three composable arrows, the two pastings of the  $\omega$ 's agree. Thus the comparison system satisfies the bicategorical cocycle, or quadruple-overlap pentagon, required for presentation independence.

(CTA8) Selected branch essential image. A candidate  $\mathfrak{P}_N$  is declared to realize the inherited selected branch on  $\mathfrak{c}$  only if there is a specified marking-preserving candidate equivalence

$$\lambda_{\mathfrak{c},r} : \text{Evol}_{\mathfrak{c},r} \left( \mathcal{P}_{B,r}^0 \right) \xrightarrow{\cong} \mathfrak{P}_N \Big|_{D_N(B)} \quad (\text{II.50})$$

compatible with regional restriction, selected scale transport, coefficient data, roots, and the comparison cells of CTA7. More precisely, for every  $u : \mathfrak{c} \rightarrow \mathfrak{c}'$ , after restriction to the selected common footprint there is an invertible modification

$$\ell_{u,r} : \chi_{u,r}^{\text{cand}} \circ \mathbb{R}_{k_u}^r (\lambda_{\mathfrak{c},r}) \Longrightarrow \mathbb{R}_{k'_u}^r (\lambda_{\mathfrak{c}',r}) \circ \chi_{u,r}^0, \quad (\text{II.51})$$

where  $\chi_{u,r}^0$  is the selected P1 identification of the two restricted inherited packages and  $\chi_{u,r}^{\text{cand}}$  is candidate Cauchy evolution. The  $\ell$ 's are coherent with the  $\omega$ 's on triple comparisons. Here  $\text{Evol}_{\mathfrak{c},r}$  is constructed from  $E_{\mathfrak{c},r}$ ; it is not assumed to exist for arbitrary time-zero objects or arbitrary Cauchy presentations.

*Remark II.22* (Exact status of Hypothesis II.21). P1 supplies the regional time-zero packages and their regional-scale pseudofunctoriality. It does not supply a spacetime evolution equation, a finite-energy solution theory, trace maps, Cauchy comparison operators, or a time-slice essential-image statement. Papers I–II therefore do not imply CTA0–CTA8. Hypothesis II.21 is the minimal selected hyperbolic input used to construct the anchor; it is weaker than an unrestricted Cauchy-slice axiom because it is imposed only on the declared presentation groupoids and candidate cores.

**Theorem II.23** (Selected admissible Cauchy-data extraction and anchor). *Assume Hypotheses II.3 and II.21. Then the following hold.*

1. For each fixed presentation  $\mathbf{c} = (N, S, \iota, B)$  and scale  $r$ , the construction

$$\mathrm{TZ}_{\mathbf{c},r} := \mathrm{Data}_{\mathbf{c},r} : \mathbf{CohReal}_{r,\mathbf{c}}^{\mathrm{cand}}(N) \longrightarrow \mathbf{H}_r^{0,\mathrm{phys}}(\mathbb{B}) \quad (\mathrm{II}.52)$$

is a pseudofunctor.

2. These fiberwise pseudofunctors assemble, using CTA5 and CTA7, into a pseudofunctor

$$\mathrm{TZ}_r : \mathbf{CohReal}_r^{\mathrm{cand},\mathrm{Cau}} \longrightarrow \int_{\mathbb{B} \in \mathbf{SpReg}^{0,\mathrm{cp}}} \mathbf{H}_r^{0,\mathrm{phys}}(\mathbb{B}). \quad (\mathrm{II}.53)$$

When the footprint  $\mathbb{B}$  is fixed, the abbreviated notation used below is

$$\mathrm{TZ}_r : \mathbf{CohReal}_r^{\mathrm{cand}}(N) \longrightarrow \mathbf{H}_r^{0,\mathrm{phys}}(\mathbb{B}).$$

3. If  $\mathfrak{P}_N$  realizes the selected branch through  $\lambda_{\mathbf{c},r}$ , there is a marking-preserving anchor equivalence

$$\boxed{\eta_{\mathbf{c},r} : \mathcal{P}_{B,r}^0 \xrightarrow{\cong} \mathrm{Data}_{\mathbf{c},r}(\mathfrak{P}_N).} \quad (\mathrm{II}.54)$$

It is the composite

$$\mathcal{P}_{B,r}^0 \xrightarrow{v_{\mathbf{c},r}} \mathrm{Data}_{\mathbf{c},r} \mathrm{Evol}_{\mathbf{c},r}(\mathcal{P}_{B,r}^0) \xrightarrow{\mathrm{Data}(\lambda_{\mathbf{c},r})} \mathrm{Data}_{\mathbf{c},r}(\mathfrak{P}_N). \quad (\mathrm{II}.55)$$

4. The anchor preserves every decoration listed in CTA4. In particular, it intertwines complete-ground projections,

$$\eta_{\mathbf{c},r} P_{\Omega,B,r}^0 = P_{\Omega,\mathbf{c},r}^{\mathrm{Data}} \eta_{\mathbf{c},r}, \quad (\mathrm{II}.56)$$

and it does not choose a vector in either ground range.

5. The anchors are pseudonatural under the regional physical bordisms of P1 and under selected scale arrows. They obey the mixed regional-scale cube of CTA6.

6. If  $\mathbf{c}$  and  $\mathbf{c}'$  are connected by an arrow of  $\mathbf{Cauc}_{\mathrm{adm}}(O)$ , their extracted packages become equivalent after the selected common regional refinement, through  $\chi_{u,r}$ . The anchor routes are related by the derived invertible modification

$$\zeta_{u,r} : \chi_{u,r} \circ \mathbb{R}_{k_u}^r(\eta_{\mathbf{c},r}) \Longrightarrow \mathbb{R}_{k'_u}^r(\eta_{\mathbf{c}',r}) \circ \chi_{u,r}^0. \quad (\mathrm{II}.57)$$

These equivalences satisfy the triple-comparison modifications  $\omega_{v,u,r}$ , the compatibility of  $\zeta$  with  $\omega$ , and the corresponding pentagon. Hence Cauchy independence holds up to specified admissible equivalence, not literal equality.

*Proof.* The proof is divided into seven steps corresponding to the six assertions.

*Step 1: quotient-level well-definedness.* The algebraic data core is constructed from the trace image of the independently specified spacetime candidate core. Suppose two spacetime representatives  $\tilde{A}_1$  and  $\tilde{A}_2$  determine the same physical Wilson class. By CTA3 their difference acts trivially after trace and solution transport, as stated in (II.44). Therefore (II.36) is independent of the representative and defines an action of the physical quotient. The same clause shows that a support presentation changed by a physical relation yields the same upward-closed support filter. Thus the extracted observable and support data are quotient-level objects, not restrictions of an unphysical ambient packet.

*Step 2: completion, represented action, and forms.* By (II.41), the trace map is bounded from the declared graph core to the Cauchy energy completion. By (II.42), the solution map is bounded in the reverse direction on the selected data core. The unit and counit of CTA1 imply that the two maps are mutually inverse up to the specified invertible cells on the selected solution subcategory. Hence the trace norm is nondegenerate modulo exactly the selected physical null relations.

For a physical observable  $A$ , the operator in (II.36) is closable on the data core because the spacetime action preserves the candidate graph core and both comparison maps are graph-bounded. Intertwining of products and adjoints follows by inserting the unit and counit cells and using the corresponding properties of the spacetime representation. The data form (II.38) is well defined on the transported form core. CTA2 retains an exact form identity only in the exact branch; in a controlled branch the defect remains the declared admissible defect and is bounded by (II.43). No lower bound for that form is used.

*Step 3: complete-ground and channel decorations.* CTA4 gives  $P_{\Omega,N,r}E_{\mathfrak{c},r} = E_{\mathfrak{c},r}P_{\Omega,\mathfrak{c},r}^{\text{Data}}$  on the data core and the corresponding trace relation. The triangle identities then show that (II.37) is an idempotent self-adjoint projection on the completed data space. The same transport identities intertwine the physical channel maps. Gram operators, being the declared channel composites, and magnetic recompressions, being the declared recompressions of the same transported channel data, are therefore carried by the exact or controlled relations specified in CTA4. The argument applies componentwise to the channel, residual, and ultraviolet

decompositions. It uses only the complete-ground projection and never selects a ground vector.

*Step 4: pseudofunctoriality for a fixed presentation.* Let  $F : \mathfrak{P}_N \rightarrow \mathfrak{Q}_N$  be a candidate equivalence. Define  $\text{Data}_{\mathfrak{c},r}(F)$  by conjugating the core map of  $F$  with trace and solution:

$$\text{Data}_{\mathfrak{c},r}(F) = \gamma_{\mathfrak{c},r}^{\mathfrak{Q}} F E_{\mathfrak{c},r}^{\mathfrak{P}}. \quad (\text{II.58})$$

CTA4 implies that this correspondence preserves all decorations. For composable equivalences  $F$  and  $G$ , insert the counit  $E^{\mathfrak{Q}} \gamma^{\mathfrak{Q}} \Rightarrow \mathbf{1}$  between them. The resulting invertible cell is the compositor

$$\text{Data}(G) \circ \text{Data}(F) \Longrightarrow \text{Data}(G \circ F). \quad (\text{II.59})$$

The triangle identities for CTA1 imply the pseudofunctor pentagon and unit laws. On two-morphisms, trace–solution conjugation gives an invertible modification. This proves (II.52).

*Step 5: construction and preservation of the anchor.* The first arrow in (II.55) is the unit of the trace–solution equivalence evaluated on the independently supplied P1 package. The second is the image under the data pseudofunctor of the branch-realization comparison  $\lambda_{\mathfrak{c},r}$ . Both arrows are marking-preserving equivalences; therefore their composite is an equivalence. This construction is not tautological: the target was defined from the spacetime candidate in Definition II.20, while  $\lambda_{\mathfrak{c},r}$  is the substantive essential-image input.

Each decoration is preserved by the unit in CTA1, by CTA4, and by the corresponding component of  $\lambda_{\mathfrak{c},r}$ . Equation (II.56) follows by pasting the two complete-ground intertwining squares. The channel, Gram, recompression, decomposition, form, coefficient, multiplication, reassociation, relative-class, root, and marked-type assertions follow from the analogous pasted squares or invertible two-cells. Controlled identities remain controlled; no strictification is introduced by the anchor.

*Step 6: regional and scale pseudonaturality.* For a regional inclusion  $i$ , paste  $\mathbb{R}_i^r \eta_{\mathfrak{c},r}$  with  $\vartheta_{i,\mathfrak{c},r}^{\text{reg}}$  and the regional compatibility cell of  $\lambda_{\mathfrak{c},r}$ . The result is an invertible modification from the restrict-then-anchor route to the anchor-then-extract route. The regional composition law follows from the composition law in CTA5 and the P1 compositor  $\rho_{j,i}^r$ .

For a selected scale arrow  $a : r \rightarrow s$ , the same construction with  $\vartheta_{c,a}^{\text{sc}}$ , the scale compatibility of  $\lambda$ , and the multiplication cells  $\mu_{b,a}$  gives the scale pseudonaturality modification. The equality of the two pastings around (II.47) is exactly CTA6. Hence regional and scale pseudonaturality are mutually compatible rather than separately asserted.

*Step 7: comparison of two selected presentations.* Let  $u : \mathfrak{c} \rightarrow \mathfrak{c}'$ . Restrict both data packages to the selected common footprint  $\mathbb{B}_u$ . On the common data core define the comparison by the selected Cauchy evolution  $\gamma_{c',r} E_{c,r}$ . Its inverse is  $\gamma_{c,r} E_{c',r}$ . The unit and counit cells of CTA1 show that the two composites are isomorphic to the identity. CTA2 supplies boundedness on the completed data spaces, and CTA3–CTA4 show that the comparison preserves every quotient, support, represented, ground, channel, form, coefficient, and marking decoration. Pasting the unit cells of CTA1 with the modification  $\ell_{u,r}$  from CTA8 gives the anchor-comparison modification  $\zeta_{u,r}$  in (II.57). This proves (II.48) together with anchor independence.

For three presentations, the two-step evolution and the direct evolution are related by  $\omega_{v,u,r}$ . For four presentations, the two ways of parenthesizing a three-step evolution agree by the last clause of CTA7. Thus the extracted packages form a pseudofunctor over the selected Cauchy comparison groupoid. Together with CTA5 this assembles the fiberwise pseudofunctors into (II.53). Presentation independence is therefore equivalence after the declared common refinement. No statement has been made about an unselected Cauchy surface.  $\square$

*Remark II.24* (No arbitrary Cauchy-slice principle). Theorem II.23 compares only selected admissible Cauchy presentations connected by arrows of  $\mathbf{Cauc}_{\text{adm}}(O)$ . It does not assert a time-slice theorem for every smooth Cauchy hypersurface, every globally hyperbolic neighborhood, or every comparison map. The dependence on a selected presentation is removed only up to the explicitly constructed equivalences  $\chi$ ,  $\omega$ , and  $\zeta$ .

*Remark II.25* (Circularity firewall for P2). The trace–solution equivalence may transport the inherited forms as part of the data package, but neither its existence nor the definition of the anchor uses a coercive lower bound, a spectral threshold, an invariant momentum operator, a collar charge, entropy surgery, or a post-incision normal-state construction. In particular,

none of  $\hat{\kappa}_r$ ,  $\kappa_*$ ,  $m_0$ ,  $P_\mu$ ,  $P^2$ , or  $\Pi_0$  enters CTA0–CTA8 or the definition of  $\text{TZ}_r$ .

**Proposition II.26** (Finite-cylinder witness for P2). *Let  $\mathcal{P}_{B,r}^0$  be any finite-dimensional P1 package. Suppose  $K_{B,r} = K_{B,r}^*$  is a bounded generator whose unitary group  $U_{B,r}(t) = e^{-itK_{B,r}}$  preserves the physical quotient representation, the complete-ground projection, the physical channel, Gram and recompression data, the inherited form, support labels, coefficient data, roots, and markings. Assume also that the P1 regional and selected scale intertwiners commute with this unitary evolution and that the resulting comparison cells preserve the marked type. On the cylinder  $N = (-T, T) \times B$ , define the candidate solution with Cauchy trace at time  $t$  by*

$$\gamma_{t,r}\Psi = \Psi(t), \quad E_{t,r}u(s) = U_{B,r}(s-t)u. \quad (\text{II.60})$$

*Then CTA0–CTA8 hold on the finite-cylinder presentation groupoid, with strict comparison*

$$\chi_{t',t,r} = U_{B,r}(t' - t), \quad \chi_{t'',t',r}\chi_{t',t,r} = \chi_{t'',t,r}. \quad (\text{II.61})$$

*Proof.* Finite dimensionality makes every core equal as a vector space and every operator bounded, while the domain labels may still be retained. The maps in (II.60) are inverse and unitary, so CTA1–CTA2 hold with constants one in the Hilbert norm and finite constants in every chosen equivalent graph norm. Preservation of the displayed package data is the hypothesis on  $U_{B,r}(t)$ , giving CTA3–CTA4. Regional and scale compatibility follows from the stated commutation of the P1 intertwiners with the chosen evolutions; this is an explicit finite-dimensional equation and supplies CTA5–CTA6. The group law gives (II.61), hence CTA7. Taking  $\lambda_{t,r}$  to be the selected evolution identification gives CTA8. No spectral lower bound for  $K_{B,r}$  is used.  $\square$

## E. The selected $SU(3)$ stationary Cauchy suspension

The selected retained  $SU(3)$  packet of Theorem II.13 is finite dimensional at every regional and scale stage. It therefore admits an explicit spacetime suspension before any incision is chosen. The suspension used here is the zero-speed symmetric-hyperbolic system

$$\partial_t \Psi = 0. \quad (\text{II.62})$$

Its purpose is to prove the Cauchy extraction and anchoring interface, not to replace the later nontrivial thick Hamiltonian dynamics. All geometric and physical nonclaims from the P1 theorem remain in force.

**Definition II.27** (Protected dyadic stationary Cauchy presentation). Let  $B \in \mathbf{SpReg}^{0,\text{dyad}}$ . A protected stationary presentation over  $B$  is a tuple

$$\mathbf{c} = (N, I_t, t, B^+; S_t, \iota_t, B_t)$$

with the following data.

1.  $B^+ \Subset \mathbb{R}^3$  is a selected dyadic protective envelope with  $\overline{B} \Subset B^+$ .
2.  $I_t = (t - \delta, t + \delta) \Subset \mathbb{R}$  is a selected finite interval centered at the presentation time  $t$ .
3.  $N \Subset M$  is a selected causally convex Minkowski double cone centered at time  $t$ , with central Cauchy hypersurface  $S_t$ , and whose interior contains the compact protected slab  $\overline{I_t} \times \overline{B^+}$ .
4. The selected slice and footprint are  $S_t = N \cap (\{t\} \times \mathbb{R}^3)$ ,  $\iota_t : S_t \hookrightarrow N$ , and  $B_t = \{t\} \times B$ , equipped with the vertical-translation identification  $B_t \cong B$  and the persistent dyadic carrier labels of Definition II.9.
5. A testing region  $O \Subset N$  is admitted only when  $O \Subset D_N(B_t)$ .

The selected groupoid  $\mathbf{Cauc}_{SU(3)}^{\text{stat}}(O; B)$  has these presentations as objects. Its arrows are generated by changes of the selected time slice, protective-envelope restriction, and finite common slab refinement, always with the same persistent footprint labels. Regional changes of footprint are kept separate and are represented by the P1 bordisms  $\mathbb{R}_i^j$ .

**Lemma II.28** (Coverage and finite comparison refinement). *The protected stationary presentations have the following properties.*

1. *Every relatively compact Minkowski testing region is contained in the development of a selected dyadic Cauchy patch of the displayed form.*
2. *Every finite family of selected times, protective envelopes, regional inclusions, and arrows in  $\Gamma_{SU(3)}^{\text{pre}}$  admits a protected finite common refinement.*

3. *Vertical transport of every persistent carrier in  $E_j(B)$  remains inside the protective slab, and regional restriction and selected scale refinement preserve its label.*

*Proof.* For a compact set  $K \Subset M$  and a time  $t$ , define its causal shadow on the slice  $\{t\} \times \mathbb{R}^3$  by

$$\text{Sh}_t(K) = \{x \in \mathbb{R}^3 : \text{there is } (s, y) \in K \text{ with } |x - y| \leq |s - t|\}.$$

This set is compact. If  $\text{Sh}_t(\overline{O}) \Subset B$ , every inextendible causal curve through a point of  $O$  meets the slice  $t$  at a point of  $B_t$ ; hence  $O \Subset D_M(B_t)$ . Lemma II.10 supplies a selected dyadic  $B$  containing the compact shadow and a protective dyadic  $B^+$  containing  $\overline{B}$ . A sufficiently large relatively compact Minkowski double cone contains both  $\overline{O}$  and the compact slab  $\overline{I}_t \times \overline{B}^+$ ; its constant-time section is Cauchy, so the same causal-shadow argument gives  $O \Subset D_N(B_t)$ .

For finitely many demands, take a common dyadic refinement of all spatial presentations, a common protective envelope, and one larger causally regular comparison region containing the finite union of the protected compact slabs and the selected compact causal slabs joining their central Cauchy patches. The fixed control-cube rule and persistence of labels then make every regional and scale square commute. Finally, each tube lies in its fixed control cube contained in  $B$ , and the entire vertical tube lies in the protected slab contained in  $N$ .  $\square$

**Definition II.29** (Stationary suspension of the selected regional packet). Fix  $m \in \mathbb{Z}_3$ , a scale  $r_j$ , a region  $B$ , and a protected presentation  $\mathbf{c}$ . For  $u \in \mathcal{H}_{B,j}^0$ , let

$$\Psi_u(s) = u, \quad s \in I_t.$$

The propagation core and its selected completion are

$$\mathcal{C}_{N,B,j}^{\text{stat}} = \{\Psi_u : u \in \mathcal{H}_{B,j}^0\}, \quad \|\Psi_u\|_{\text{stat}} = \|u\|.$$

The candidate  $\text{Susp}_{\mathbf{c},j}^{(m)}(\mathcal{P}_{B,j}^0)$  is defined by the following rules.

1. The physical Wilson algebra acts pointwise:  $(\pi_N(A)\Psi_u)(s) = \Psi_{Au}(s)$ .
2. The complete-ground projection, preform, channel, Gram, recompression, decomposition, Gauss graph form, Ward derivations, coefficient line, multiplication cells, reassociator, root, and branch marking are the constant pullbacks of their P1 values.

3. The support of the label  $\lambda$  is the protected vertical world tube

$$\mathbb{T}_{\lambda,j}^c = (I_t \times T_{\lambda,j}) \cap N.$$

Its shadow on every selected slice is the same labelled tube  $T_{\lambda,j} \subset B_t$ .

4. For a regional arrow  $i : B' \hookrightarrow B$ , define  $\text{Susp}(\mathbb{R}_i^j)$  to be the constant correspondence whose underlying decorated Hilbert correspondence is  $\mathbb{R}_i^j$  on every selected slice. For a selected scale arrow  $a_{kj} : r_j \rightarrow r_k$ , define  $\text{Susp}(\mathbb{T}_{kj}^B)$  analogously from the P1 scale correspondence. Their compositors, coefficient cells, and mixed cells are the constant suspensions of  $\rho$ ,  $\mu$ ,  $\Omega_m$ , and  $\beta$ .

For a selected slice  $S_t$ , define

$$\gamma_{c,j}(\Psi_u) = u, \quad E_{c,j}(u) = \Psi_u. \quad (\text{II.63})$$

Thus  $\gamma$  is evaluation using the persistent-label identification  $B_t \cong B$ , and  $E$  is constant solution extension.

**Theorem II.30** (Selected  $SU(3)$  Cauchy extraction and anchoring). *For every  $m \in \mathbb{Z}_3$ , the stationary suspension of Definition II.29 verifies CTA0–CTA8 of Hypothesis II.21 on the protected dyadic presentation system and the scale spine  $\Gamma_{SU(3)}^{\text{pre}}$ . Consequently:*

1. *the selected Cauchy extraction pseudofunctor is explicitly inhabited;*
2. *for every protected presentation there is an exact marking-preserving anchor*

$$\eta_{c,j}^{(m)} : \mathcal{P}_{B,j}^0 \xrightarrow{\cong} \text{Data}_{c,j}(\text{Susp}_{c,j}^{(m)}(\mathcal{P}_{B,j}^0));$$

3. *the anchor is pseudonatural under regional restriction and selected scale transport, and its regional–scale mixed cube is the P1 Beck–Chevalley cube;*
4. *every selected Cauchy comparison is independent of the finite zigzag in the protected comparison groupoid;*
5. *on the stationary essential image, suspension and extraction are pseudonaturally inverse:*

$$\text{TZ}_{SU(3),m}^{\text{stat}} \circ \text{Susp}_{SU(3),m} \cong \mathbf{1}, \quad \text{Susp}_{SU(3),m} \circ \text{TZ}_{SU(3),m}^{\text{stat}} \cong \mathbf{1}. \quad (\text{II.64})$$

*The equivalences preserve the physical Wilson quotient, complete-ground projection, full common core, interior Gauss graph form, Ward domain, physical channel package, preform, support filter, coefficient line, reassociator, root, and marked band  $m$ . All trace, solution, anchor, and comparison norms are one, and the anchor form defect is zero.*

*Proof.* We verify CTA0–CTA8 directly.

*CTA0: presentation system.* Lemma II.28 gives coverage, regional restriction, finite common refinement, and finite-demand scale stability. The selected Cauchy comparison arrows are generated by time-slice changes and protective-slab refinements with fixed persistent labels. Reversal of a time change or slab refinement supplies the inverse arrow. Composition is performed in the protected common refinement, so the selected comparisons form the required essentially small groupoid.

*CTA1–CTA2: trace, solution, and estimates.* Equation (II.63) gives

$$\gamma_{c,j} E_{c,j} = \mathbf{1}, \quad E_{c,j} \gamma_{c,j} = \mathbf{1}$$

on the declared solution core. The unit, counit, and triangle cells are therefore identities. The solution space is finite dimensional and its chosen graph and energy norms are the P1 Hilbert norm. Hence the trace and solution constants are one. The extracted form is

$$q_{c,j}^{\text{Data}}[u, v] = q_{B,j}^0[u, v],$$

so  $\delta_{c,j}^q = 0$ . No lower bound for this form is used. The system (II.62) is symmetric hyperbolic with positive time matrix  $I$ , vanishing spatial matrices, and propagation speed zero on the selected carrier labels.

*CTA3: quotient and support.* The P1 physical ideal is zero on this retained branch, and the pointwise candidate action is defined on the already physical quotient. Thus quotient stability is exact. Constant extension sends every labelled tube to its protected vertical world tube and trace returns the same labelled slice tube. Regional restriction deletes exactly the labels outside the smaller region, while selected scale arrows retain old labels and add only the declared new labels. The support-filter maps are therefore monotone and commute with the P1 regional bordisms.

*CTA4: decoration transport.* Every candidate decoration was defined as the constant pullback of the P1 decoration. Trace and solution consequently intertwine all represented observables and preserve the complete-ground projection, channel, Gram, recompression, decomposition, preform, common core, Gauss graph form, Ward domain, coefficient line, multiplication cell, reassociator, relative class, root, and marking exactly. The construction uses the complete projection and never chooses a vector in its range.

*CTA5: regional pseudonaturality.* For  $i : B' \hookrightarrow B$ , both routes in (II.45) have the same constant underlying correspondence module  $\mathbb{R}_i^j$ , with the same quotient, ground, core, form, support, and marking decorations. Hence  $\vartheta_{i,c,j}^{\text{reg}}$  is the identity two-cell after the canonical tensor reassociation. The composition law is exactly the P1 law for  $\rho_{j,i}^r$ .

*CTA6: scale pseudonaturality and the mixed cube.* For  $a_{kj}$ , both routes in (II.46) have the same constant underlying scale correspondence  $\mathbb{T}_{kj}^B$ . Thus  $\vartheta_{c,a_{kj}}^{\text{sc}}$  is the identity marking-preserving cell, with the coefficient multiplication and reassociator retained from P1. Around the mixed regional–scale cube, the two suspended routes differ precisely by the P1 cell  $\beta_{i,a_{kj}}$ . The equality of the two boundary pastings is therefore (II.12) and (II.13), not a new strictification.

*CTA7: selected Cauchy comparison.* For a time-slice or protected-slab comparison  $u : c \rightarrow c'$ , one has on the common footprint

$$\chi_{u,j} = \gamma_{c',j} E_{c,j} = \mathbf{1}.$$

The inverse, unit, and compositor are identities. Hence  $\omega_{v,u,j} = \mathbf{1}$ , and every triple and quadruple comparison coherence holds strictly. This proves independence under the entire selected protected comparison system, but says nothing about an unselected Cauchy hypersurface.

*CTA8: essential image and anchor.* The candidate is the explicitly constructed evolution  $\text{Susp}_{c,j}^{(m)}(\mathcal{P}_{B,j}^0)$ . Therefore the branch-realization comparison  $\lambda_{c,j}$  is the identity candidate equivalence. Its regional, scale, coefficient, root, and Cauchy compatibility modifications  $\ell_{u,j}$  are identities or the inherited P1 tensor coherence cells. Applying the general construction (II.55) gives  $\eta_{c,j}^{(m)} = \mathbf{1}$  after the persistent-label identification. The same identities give both pseudoinverse relations in (II.64).

Thus CTA0–CTA8 hold and every assertion follows from Theorem II.23.  $\square$

**Corollary II.31** (Model-specific P2 closure for the selected retained branch). *On the protected pre-incision dyadic Cauchy basis, obligations P2.1–P2.9 are discharged for the selected retained  $SU(3)$  branch. The closure is a stationary Cauchy-suspension theorem. It does not identify this suspension with the later prescribed-incision linked-flux dynamics, and it does not prove interaction, a nonzero propagation speed, arbitrary-slice independence, P3 defect naturality, P4 effectivity, P5 stackification, the dynamically derived obstruction carrier, CIR, RID, thinning, or any spectral conclusion.*

*Remark II.32* (Stationary-suspension firewall). The equation  $\partial_t \Psi = 0$  is used because P2 asks whether the selected time-zero packet admits an explicitly inhabited Cauchy extraction and return map. It proves that interface without importing the post-incision Hamiltonian, collar current, entropy surgery, coercive constant, or linked flux. The nontrivial support-exact Hamiltonian dynamics constructed later for the prescribed-incision  $SU(3)$  witness remains a distinct theorem and is not inferred from the present stationary suspension.

*Remark II.33* (Status of the retained  $SU(2)$  and  $SU(3)$  witnesses). A finite  $SU(2)$  package can enter Proposition II.26 after a package-preserving bounded evolution has been supplied. This witnesses the logical inhabitation of the selected-anchor architecture, not a hyperbolic spacetime Yang–Mills theorem. For the retained  $SU(3)$  branch, Theorem II.30 now supplies the missing pre-incision anchor by an explicit protected stationary suspension. The linked-flux  $SU(3)$  construction later in Paper III remains a distinct post-incision nontrivial dynamical witness. Neither theorem is used to derive the obstruction carrier, and no collar or spectral estimate enters the P2 stationary anchor.

**Corollary II.34** (P2 closure and remaining model obligation). *At the abstract front-end level, Hypothesis II.21 and Theorem II.23 close the general P2 bridge. At the model-specific level, Theorem II.30 verifies CTA0–CTA8 for the selected retained  $SU(3)$  branch on the protected pre-incision dyadic Cauchy basis. The result is a stationary suspension and does not identify that suspension with the later prescribed-incision linked-flux dynamics. No unrestricted Cauchy independence theorem is claimed.*

## F. The five-component pre-incision defect

The word “common” in the common-core requirement does not mean that the observable algebraic core, the quadratic-form core, the Gauss graph core, and the Ward insertion domain are literally the same completed object. It means that they belong to one transport-stable core packet with one dense algebraic nucleus on which every comparison appearing in the defect calculus can be evaluated before any closure is taken.

**Definition II.35** (Finite pre-incision demand datum). A finite pre-incision demand datum is a tuple

$$\mathfrak{d} = (O, \mathcal{U}, \mathfrak{c}, \mathbb{B}, \Gamma_{\mathfrak{d}}, \mathcal{F}_{\mathfrak{d}}), \quad (\text{II.65})$$

where:

1.  $O \in M = \mathbb{R}^{1,3}$  is a selected ordinary Minkowski testing region;
2.  $\mathcal{U} = \{U_{\alpha} \rightarrow O\}_{\alpha \in A}$  is a finite selected chart cover with a fixed finite common refinement system;
3.  $\mathfrak{c} = (N, S, \iota, B)$  is a selected admissible Cauchy presentation and  $\mathbb{B}$  is the corresponding object of  $\mathbf{SpReg}^{0, \text{cp}}$ ;
4.  $\Gamma_{\mathfrak{d}} \subset \Gamma_{\text{cof}}$  is a finite full demand subcategory containing every scale object and arrow used in the calculation; and
5.  $\mathcal{F}_{\mathfrak{d}}$  is the finite list of regional, chart, scale, regional-chart, chart-scale, regional-scale, and anchor faces whose boundary composites must be compared.

Passing to a refinement of  $\mathfrak{d}$  means refining the chart cover, shrinking or restricting the region through an admissible arrow, enlarging the finite scale demand category, or replacing  $\mathfrak{c}$  by a selected admissibly comparable presentation. No arbitrary Cauchy presentation and no unrestricted scale diagram is introduced.

**Definition II.36** (Common-core packet). Let  $\mathfrak{P}$  be an object of  $\mathbf{CohReal}_r^{\text{cand}, \text{Cau}}(O)$  and let  $U_{\alpha}$  be a chart in  $\mathfrak{d}$ . A pre-incision common-core packet is

$$\mathfrak{c}_{\alpha, r}(O) = \left( \mathcal{A}_{\alpha, r}^{\text{phys}, \text{alg}}(O), \mathcal{A}_{\alpha, r}^{W, \text{int}}(O), \mathcal{C}_{\alpha, r}^{\cap}(O), \mathcal{C}_{\alpha, r}^{\text{prop}}(O), \mathcal{C}_{\alpha, r}^q(O), \mathcal{C}_{\alpha, r}^G(O), \mathcal{C}_{\alpha, r}^W(O), V_{\alpha, r}^G(O) \right), \quad (\text{II.66})$$

with the following meaning.

1.  $\mathcal{A}_{\alpha,r}^{\text{phys,alg}}(O)$  is the quotient-level physical Wilson  $*$ -algebraic core.
2.  $\mathcal{A}_{\alpha,r}^{W,\text{int}}(O)$  is a support-controlled bulk Ward insertion  $*$ -algebra containing  $\mathcal{A}_{\alpha,r}^{\text{phys,alg}}(O)$ . It contains no collar charge, boundary current, or entropy-surgery datum.
3.  $\mathcal{C}_{\alpha,r}^\cap(O)$  is a dense invariant algebraic nucleus satisfying

$$\mathcal{C}_{\alpha,r}^\cap(O) \subseteq \mathcal{C}_{\alpha,r}^{\text{prop}}(O) \cap \mathcal{C}_{\alpha,r}^q(O) \cap \mathcal{C}_{\alpha,r}^G(O) \cap \mathcal{C}_{\alpha,r}^W(O). \quad (\text{II.67})$$

4.  $\mathcal{C}^{\text{prop}}$  is the finite-energy Cauchy propagation core from P2;  $\mathcal{C}^q$  is a core for the inherited closable quadratic form;  $\mathcal{C}^G$  is a core for the Gauss graph norm; and  $\mathcal{C}^W$  is invariant under the represented interior Ward insertion algebra and the Gauss graph maps used in the Ward formula.
5.  $V_{\alpha,r}^G(O)$  is the completion of  $\mathcal{C}_{\alpha,r}^G(O)$  in the selected Gauss graph norm. The primary Gauss datum is a bounded graph-dual map on  $V^G$ , not an automatically available self-adjoint Hilbert-space operator.

There is no normal-state domain in (II.66). Normal-state spaces, preduals, boundary characters, and entropy surgery enter only after the obstruction carrier has acquired support regularity and the regional local-normal construction has been completed.

For every elementary comparison  $f$  occurring in  $\mathfrak{d}$ , write  $s(f)$  and  $t(f)$  for its source and target packet. The comparison data include a coefficient line or normal correspondence  $\mathbb{L}_f$ , core maps

$$J_f^\cap, \quad J_f^{\text{prop}}, \quad J_f^q, \quad J_f^G, \quad J_f^W, \quad (\text{II.68})$$

a parameter map  $c_f^\mathfrak{g}$  on compactly supported interior gauge parameters, and a map  $c_f^A$  on the Ward insertion algebra. For a regional inclusion,  $c_f^\mathfrak{g}$  is the selected extension-by-zero map in the covariant parameter direction. For chart and scale arrows it is the declared bundle or scale comparison. The maps in (II.68) are not silently identified with one operator; they act on the indicated domains and are required to agree on the common nucleus.

**Definition II.37** (Interior Gauss graph-form defect). Let

$$G_{\alpha,r}^{\text{int}}(\xi) : V_{\alpha,r}^G(O) \longrightarrow \left(V_{\alpha,r}^G(O)\right)^* \quad (\text{II.69})$$

be a bounded Hermitian graph-dual map, continuously linear in  $\xi \in \mathfrak{g}_{\text{int},\alpha,r}(O)$ . For an elementary comparison  $f$ , define

$$\Gamma_{f,r}^{G,\text{int}}(\xi) = \left(J_f^G\right)^* G_{t(f)}^{\text{int}} \left(c_f^{\mathfrak{g}} \xi\right) J_f^G - \left(G_{s(f)}^{\text{int}}(\xi) \hat{\otimes} I_{\mathbb{L}_f}\right). \quad (\text{II.70})$$

Its target is the Banach space

$$\mathfrak{X}_{f,r}^{G,\text{int}} = \mathcal{L}_b \left( \mathfrak{g}_{\text{int},s(f)}, \mathcal{B}_{\text{herm}} \left( V_{s(f)}^G \hat{\otimes} \mathbb{L}_f, \left( V_{s(f)}^G \hat{\otimes} \mathbb{L}_f \right)^* \right) \right). \quad (\text{II.71})$$

The equation  $\Gamma_{f,r}^{G,\text{int}} = 0$  is graph-form transport. It does not assert closability of a pivot-Hilbert preoperator, self-adjointness, exponentiation, or existence of one global Gauss operator. For the finite demand datum, set

$$\Gamma_r^{G,\text{int}} = \left( \Gamma_{f,r}^{G,\text{int}} \right)_{f \in \mathcal{F}_{\mathfrak{d}}^G}, \quad \mathfrak{X}_r^{G,\text{int}}(\mathfrak{d}) = \prod_{f \in \mathcal{F}_{\mathfrak{d}}^G} \mathfrak{X}_{f,r}^{G,\text{int}}. \quad (\text{II.72})$$

**Definition II.38** (Interior adjoint Ward defect). For  $A \in \mathcal{A}_{\alpha,r}^{W,\text{int}}(O)$ ,  $\xi \in \mathfrak{g}_{\text{int},\alpha,r}(O)$ , and  $\Psi, \Phi \in \mathcal{C}_{\alpha,r}^W(O)$ , set

$$\begin{aligned} w_{\alpha,r}^{\text{int}}(\xi; A)[\Psi, \Phi] &= i \left\langle G_{\alpha,r}^{\text{int}}(\xi) \pi_{\alpha,r}(A) \Psi, \Phi \right\rangle_G - i \left\langle G_{\alpha,r}^{\text{int}}(\xi) \Psi, \pi_{\alpha,r}(A^*) \Phi \right\rangle_G \\ &\quad - \left\langle \pi_{\alpha,r}(\delta_{\xi} A) \Psi, \Phi \right\rangle. \end{aligned} \quad (\text{II.73})$$

Here  $\langle \cdot, \cdot \rangle_G$  denotes the graph-dual pairing. The interior Ward transport defect is

$$\begin{aligned} \Gamma_{f,r}^{W,\text{int}}(\xi; A)[\Psi, \Phi] &= w_{t(f)}^{\text{int}} \left( c_f^{\mathfrak{g}} \xi; c_f^A A \right) \left[ J_f^W \Psi, J_f^W \Phi \right] \\ &\quad - w_{s(f)}^{\text{int}}(\xi; A)[\Psi, \Phi]. \end{aligned} \quad (\text{II.74})$$

The target is explicitly

$$\mathfrak{X}_{f,r}^{W,\text{int}} = \mathcal{L}_b \left( \mathfrak{g}_{\text{int},s(f)} \hat{\otimes} \mathcal{A}_{s(f)}^{W,\text{int}}, \text{Sesq}_b \left( \mathcal{C}_{s(f)}^W \hat{\otimes} \mathbb{L}_f \right) \right), \quad (\text{II.75})$$

with the projective tensor topology on the two input factors and the selected Ward graph topology on the vector domain. Set

$$\Gamma_r^{W,\text{int}} = \left( \Gamma_{f,r}^{W,\text{int}} \right)_{f \in \mathcal{F}_{\mathfrak{d}}^W}, \quad \mathfrak{X}_r^{W,\text{int}}(\mathfrak{d}) = \prod_{f \in \mathcal{F}_{\mathfrak{d}}^W} \mathfrak{X}_{f,r}^{W,\text{int}}. \quad (\text{II.76})$$

No quotient by boundary characters occurs pre-incision. Thus  $\Gamma^{W,\text{int}} = 0$  means exact interior adjoint Ward descent only.

**Definition II.39** (Admissible analytic-defect target). For each analytic comparison face  $F \in \mathcal{F}_\mathfrak{d}^T$  and each analytic type

$$\nu \in \{\text{prop}, q, G, W, \pi, \text{ch}, \text{res}, \text{uv}\}, \quad (\text{II.77})$$

let  $\mathfrak{D}_F^{(\nu)}$  be the declared Banach or complete locally convex space of typed comparison defects and let  $\mathfrak{A}_F^{T,(\nu)} \subseteq \mathfrak{D}_F^{(\nu)}$  be the closed class of admissible defects. If  $\mathfrak{A}_F^{T,(\nu)}$  is a closed linear subspace, use the ordinary quotient. In general, write

$$\mathfrak{D}_F^{(\nu)} // \mathfrak{A}_F^{T,(\nu)} \quad (\text{II.78})$$

for the pointed collapse quotient whose basepoint is the whole admissible class. The optional excess seminorm is

$$\|[D]\|_{\text{exc}} = \inf_{A \in \mathfrak{A}_F^{T,(\nu)}} \|D - A\|. \quad (\text{II.79})$$

For the actual typed defect  $D_F^{(\nu)}$ , define

$$\Gamma_{F,r}^{T,(\nu)} = [D_F^{(\nu)}] \in \mathfrak{D}_F^{(\nu)} // \mathfrak{A}_F^{T,(\nu)}. \quad (\text{II.80})$$

The aggregate analytic target and defect are

$$\mathfrak{X}_r^T(\mathfrak{d}) = \prod_{F \in \mathcal{F}_\mathfrak{d}^T} \prod_{\nu} \left( \mathfrak{D}_F^{(\nu)} // \mathfrak{A}_F^{T,(\nu)} \right), \quad \Gamma_r^T = \left( \Gamma_{F,r}^{T,(\nu)} \right)_{F,\nu}. \quad (\text{II.81})$$

The condition  $[\Gamma_r^T]_{\text{adm}} = 0$  means membership in the declared admissible class, not strict flattening of every representative.

**Definition II.40** (Complete pre-incision coherence residual). Let  $\mathcal{F}_\mathfrak{d}^{\check{C}}$  contain precisely the following faces used by the candidate model:

1. fixed-thickness Cech tetrahedra and their unit faces;
2. regional associativity and unit faces;
3. selected scale associativity and unit faces;
4. regional-chart comparison squares;
5. chart-scale coherence prisms;

6. regional-scale Beck–Chevalley cubes from P1; and
7. the mixed regional-scale–Cauchy cube from P2 whenever the anchor is transported around that cube.

For  $F \in \mathcal{F}_{\mathfrak{d}}^{\check{C}}$ , choose the common parenthesis and coefficient correspondence prescribed by the candidate bicategory and let  $C_F^+$  and  $C_F^-$  denote the two boundary composites. Define

$$R_F^{\check{C}} = C_F^+ (C_F^-)^{-1} \in \text{Aut}(C_F^-), \quad R_r^{\check{C}} = (R_F^{\check{C}})_{F \in \mathcal{F}_{\mathfrak{d}}^{\check{C}}} . \quad (\text{II.82})$$

The coherence target is the finite product groupoid

$$\mathfrak{X}_r^{\check{C}}(\mathfrak{d}) = \prod_{F \in \mathcal{F}_{\mathfrak{d}}^{\check{C}}} \text{Aut}(C_F^-), \quad (\text{II.83})$$

with arrows induced by simultaneous whiskering and two-cells induced by the chosen common-refinement modifications. The equation  $R_r^{\check{C}} = \mathbf{1}$  requires equality of all boundary composites after the declared associators and unitors are inserted. It does not require the fixed-thickness multiplier, the banded Cech class, or the controlled chart-scale class to be trivial.

**Definition II.41** (Pre-incision branch and anchor defect). For every elementary comparison  $f$ , let  $\tau_{s(f)}$  and  $\tau_{t(f)}$  be the marked reconstruction-branch objects inherited from Paper II. The type residual is

$$\Gamma_{f,r}^{\text{type}} = (f^* \tau_{t(f)}, \tau_{s(f)}; m_f), \quad (\text{II.84})$$

where  $m_f$  is the declared marking comparison. Its zero object means that  $m_f$  is an admissible root-preserving equivalence in the marked branch groupoid; it does not mean that the banded class is zero.

Let  $\Gamma_r^{\text{TZ}}$  be the tuple of residual modifications for:

1. regional pseudonaturality of the anchors  $\eta_{c,r}$ ;
2. selected scale pseudonaturality of the anchors;
3. the P2 mixed regional-scale anchor cube;
4. the selected Cauchy comparison modification  $\zeta_{u,r}$ ; and
5. the triple and quadruple selected-Cauchy comparison coherences.

Each residual is the ratio of the two specified boundary modifications after transport to one common target. Set

$$\Gamma_r^{B,\text{pre}} = \left( \Gamma_r^{\text{type}}, \Gamma_r^{\text{TZ}} \right). \quad (\text{II.85})$$

Let  $\mathfrak{X}_r^{\text{type}}(\mathfrak{d})$  be the product of the marked-comparison groupoids over the elementary comparisons, and let  $\mathfrak{X}_r^{\text{TZ}}(\mathfrak{d})$  be the product of the automorphism groupoids of the common target anchors over the listed anchor faces. Then

$$\mathfrak{X}_r^{B,\text{pre}}(\mathfrak{d}) = \mathfrak{X}_r^{\text{type}}(\mathfrak{d}) \times \mathfrak{X}_r^{\text{TZ}}(\mathfrak{d}). \quad (\text{II.86})$$

The zero condition preserves roots, marked branch data, and the selected time-zero anchor up to the specified invertible modifications. Literal equality is not required.

Combining the preceding definitions gives

$$\boxed{\mathbf{O}_{r,\text{pre}} = \left( \Gamma_r^{G,\text{int}}, \Gamma_r^{W,\text{int}}, \Gamma_r^T, R_r^{\check{C}}, \Gamma_r^{B,\text{pre}} \right)}. \quad (\text{II.87})$$

The pre-incision zero condition is

$$\Gamma_r^{G,\text{int}} = 0, \quad \Gamma_r^{W,\text{int}} = 0, \quad [\Gamma_r^T]_{\text{adm}} = 0, \quad R_r^{\check{C}} = \mathbf{1}, \quad \Gamma_r^{B,\text{pre}} = 0. \quad (\text{II.88})$$

In particular, (II.88) does not require

$$[\Omega]_{\text{band}} = 0, \quad D_{b,a}^{(\nu)} = 0, \quad [\hat{\Omega}]_{\text{cs,ctrl}} = 0. \quad (\text{II.89})$$

A coherent nonneutral gerbe class and controlled nonzero analytic comparison defect are compatible with zero pre-incision obstruction.

**Definition II.42** (Pointed defect two-prestack). For a finite demand datum  $\mathfrak{d}$ , let  $X_{r,\text{pre}}(\mathfrak{d})$  be the product of the five component targets defined in (II.72), (II.76), (II.81), (II.83), and (II.86). Each factor is pointed by its zero or identity object.

Let  $\mathbf{Cmp}(\mathfrak{d})$  be the selected comparison two-groupoid generated by regional restrictions, chart refinements, scale arrows, selected Cauchy comparisons, mixed comparison cells, and marking-preserving candidate equivalences. Its objects are the refined finite demand data, its one-morphisms are the induced invertible comparison maps on all five target factors, and its two-morphisms are the induced conjugacy or whiskering modifications. Define

$$\mathfrak{X}_{r,\text{pre}}(\mathfrak{d}) = \mathbf{Cmp}(\mathfrak{d}) \ltimes X_{r,\text{pre}}(\mathfrak{d}) \quad (\text{II.90})$$

by the Grothendieck action construction. Thus an object is a five-component defect tuple, a one-morphism is a selected invertible transport of defect tuples, and a two-morphism is the corresponding invertible modification. Restriction of finite demand data gives the pointed two-prestack  $\mathfrak{X}_{r,\text{pre}}$ .

## G. Pre-incision and post-incision defects

Before support regularity, only interior Gauss–Ward data and the selected time-zero anchor are available. Collar charges, boundary Ward characters, normal-state surgery, and entropy cocycles therefore do not enter  $\mathbf{O}_{r,\text{pre}}$ .

Only after Hypotheses II.140–II.178, the controlled-incision regional category, and the regional normal-state calculus have been constructed may one define

$$\Gamma_r^{W,\text{full}} = \left( \Gamma_r^{W,\text{int}}, \Gamma_r^{W,\partial} / \text{Char}_\partial \right), \quad (\text{II.91})$$

and

$$\Gamma_r^{B,\text{full}} = \left( \Gamma_r^{B,\text{pre}}, R_r^{\text{Surg}} \right). \quad (\text{II.92})$$

These full defects audit the completed incised realization. They do not retroactively redefine the obstruction carrier.

## H. P3: common-core defect naturality

**Hypothesis II.43** (P3: Common-Core Defect-Naturality package). Assume P1 and P2. For every finite demand datum  $\mathfrak{d}$ , the following conditions hold.

(DN0) Core packet and density. The packet (II.66) exists in every selected chart and scale. The nucleus  $\mathcal{C}^\cap$  is invariant under the quotient-level physical observable core, the interior Ward insertion core, the selected trace and solution maps, and every elementary comparison used in  $\mathfrak{d}$ . It is dense in the Hilbert completion, a form core for  $q$ , and a graph core for  $V^G$ .

(DN1) Physical quotient stability. The regional physical ideals are stable under regional, chart, Cauchy, and scale transport and under the interior gauge derivations. If  $I$  belongs to a physical ideal, then

$$\pi(I)|_{\mathcal{C}^\cap} = 0, \quad \pi(\delta_\xi I)|_{\mathcal{C}^\cap} = 0. \quad (\text{II.93})$$

Hence the Gauss and Ward expressions are independent of unphysical representatives.

(DN2) Graph, form, and Ward bounds. On every compact support stage  $K$  and finite demand shape, there are finite constants

$$C_{\mathfrak{d},K}^G, \quad C_{\mathfrak{d},K}^W, \quad C_{\mathfrak{d},K}^q, \quad C_{\mathfrak{d},K}^{\text{tr}} < \infty \quad (\text{II.94})$$

such that all  $J_f^G$  are graph bounded with graph-bounded inverses on comparison equivalences, all  $J_f^W$  preserve the Ward multiplier domain, all  $J_f^q$  are bounded for the form graph norm, and the P2 trace and solution maps obey the corresponding graph estimates. The constants may depend on  $\mathfrak{d}$  and  $K$ ; no cofinal-uniform bound is inferred here.

(DN3) Gauss and Ward intertwining data. The parameter maps and insertion maps satisfy

$$c_f^A(\delta_\xi A) = \delta_{c_f^{\mathfrak{g}} \xi}(c_f^A A) \quad (\text{II.95})$$

on the interior insertion algebra. The graph-dual pullback and the represented observable action are jointly continuous on the selected Ward stage. The common nucleus is invariant under  $A$ ,  $A^*$ , and every graph map that appears in (II.73).

(DN4) Admissible analytic classes. Every  $\mathfrak{A}_F^{T,(\nu)}$  is closed in its declared topology, contains the base admissible defect, and is preserved by regional restriction, chart refinement, selected scale transport, Cauchy comparison, coefficient reassociation, and marking-preserving equivalence. Composition of admissible defects is admissible after the exact correction cell prescribed by the candidate bicategory.

(DN5) Complete coherence-face ledger. The face set  $\mathcal{F}_{\mathfrak{d}}^{\check{C}}$  contains every associativity, unit, Cech, regional-chart, chart-scale, regional-scale, and anchor-mixed face used by the chosen candidate model. Whiskering and common refinement identify the two boundary composites in one fixed automorphism group before  $R_F^{\check{C}}$  is formed. No required mixed face is omitted.

(DN6) Branch and anchor target stability. The marked branch groupoids, root data, anchor comparison groupoids, and their zero objects are preserved by every elementary comparison. The P2 cells  $\vartheta^{\text{reg}}$ ,  $\vartheta^{\text{sc}}$ , the mixed cube,  $\chi$ ,  $\omega$ , and  $\zeta$  act on the same retained coefficient and root data used in (II.84).

(DN7) Mixed naturality. For every regional-scale, chart-scale, regional-chart, or regional-scale-Cauchy square or cube, the two transports of each core packet are connected by the

declared invertible comparison cell. On the common nucleus the induced maps on Gauss parameters, Ward insertions, graph-dual forms, analytic-defect targets, and anchor targets agree after that cell.

(DN8) Closure branch, when used. The primary P3 defect is defined before operator closure. If a candidate also retains a pivot-Hilbert Gauss preoperator or a closed form, that preoperator is closable on  $\mathcal{C}^\cap$ , each comparison maps core to core, and graph conjugacy carries its closure domain onto the declared target domain. No self-adjointness or spectral strong commutation is inferred from DN8.

*Remark II.44* (Exact status of Hypothesis II.43). P1 supplies the regional and mixed physical-package maps, and P2 supplies the selected trace, solution, and anchor comparisons. Neither theorem supplies the transport-stable Gauss graph completions, the Ward multiplier domain, the physical-ideal derivation law, or the closed admissible analytic-defect classes. DN0–DN8 are therefore a new common-core analytic input, not a renaming of P1 or P2 and not a consequence of Papers I–II alone.

**Theorem II.45** (Common-core defect naturality). *Under Hypotheses II.3, II.21, and II.43, the assignment*

$$\mathbf{O}_{r,\text{pre}} : \mathbf{CohReal}_r^{\text{cand,Cau}} \longrightarrow \mathfrak{X}_{r,\text{pre}} \quad (\text{II.96})$$

*is a well-defined pseudonatural morphism of pre-incision two-prestacks on the selected Minkowski testing site. Here*

$$\mathfrak{X}_{r,\text{pre}} = \mathfrak{X}_r^{G,\text{int}} \times \mathfrak{X}_r^{W,\text{int}} \times \mathfrak{X}_r^T \times \mathfrak{X}_r^{\tilde{C}} \times \mathfrak{X}_r^{B,\text{pre}} \quad (\text{II.97})$$

*is the pointed defect target defined above. Componentwise:*

1. *all five defects are defined from one common-core packet while retaining the distinction among algebraic, form, graph, propagation, and Ward domains;*
2. *they descend through the physical quotient;*
3. *they are natural under admissible regional restriction, chart refinement, selected scale transport, and selected Cauchy comparison;*
4. *they are invariant under marking-preserving candidate equivalence;*

5. the two routes around every declared mixed square or cube give canonically identified defect values; and
6. the homotopy full zero locus

$$\ker \mathbf{O}_{r,\text{pre}} \subseteq \mathbf{CohReal}_r^{\text{cand,Cau}} \quad (\text{II.98})$$

is a two-subprestack.

No implication from zero defect to physical local realizability is asserted in this theorem.

*Proof.* We prove the six assertions in the order stated.

*Step 1: one packet, distinct domains.* By DN0, every expression in (II.70)–(II.84) is first defined on  $\mathcal{C}^\cap$ . The Gauss expression extends uniquely to the graph completion because  $G^{\text{int}}(\xi)$  is bounded from  $V^G$  to  $(V^G)^*$  and  $J_f^G$  is graph bounded. The form comparisons are evaluated on  $\mathcal{C}^q$ , the Ward expressions on  $\mathcal{C}^W$ , and Cauchy transport on  $\mathcal{C}^{\text{prop}}$ . Their restrictions agree on  $\mathcal{C}^\cap$  by the last sentence preceding (II.68). Thus the defects use one declared core packet without identifying the completions or their norms. The finite constants in DN2 guarantee that every target element is continuous in the topology declared in its definition.

*Step 2: passage to the physical quotient.* Let  $A$  and  $A + I$  be two Ward representatives with  $I$  in the regional physical ideal. By DN1,  $\pi(I)|_{\mathcal{C}^\cap} = 0$  and  $\pi(\delta_\xi I)|_{\mathcal{C}^\cap} = 0$ . Subtracting the two versions of (II.73) therefore gives zero term by term. The same ideal stability makes the represented observable, channel, Gram, recompression, residual, ultraviolet, and support comparisons independent of representative. The Gauss graph map is already part of the quotient-level candidate packet and its parameter transport does not use an unphysical observable representative. Hence every component of (II.96) survives the physical quotient.

*Step 3: Gauss naturality.* Let  $f : s \rightarrow t$  and  $g : t \rightarrow u$  be composable elementary comparisons. Insert the compositor  $J_{g \circ f}^G \cong J_g^G J_f^G$  and the coefficient associator. On the common nucleus one obtains

$$\Gamma_{g \circ f}^{G,\text{int}}(\xi) = (J_f^G)^* \Gamma_g^{G,\text{int}}(c_f^{\mathfrak{g}} \xi) J_f^G + \Gamma_f^{G,\text{int}}(\xi). \quad (\text{II.99})$$

The equation follows by adding and subtracting  $(J_f^G)^* G_t(c_f^{\mathfrak{g}} \xi) J_f^G$  after all coefficient factors have been transported to the source. DN2 extends (II.99) continuously to the graph com-

pletion. Regional restriction, chart refinement, scale transport, and Cauchy comparison are the corresponding choices of  $f$ . For a vertical candidate equivalence  $E$ , substituting  $J_f^G E^G$  into (II.70) shows that the new defect is the conjugate of the old one; hence its zero object is invariant. DN7 identifies the conjugations arising from the two routes around every mixed face.

*Step 4: Ward naturality.* Apply  $c_f^A$  and  $c_f^g$  to (II.73). The first two terms transform by graph conjugacy from Step 3 and invariance of  $\mathcal{C}^W$ . The third transforms by (II.95). Therefore

$$\Gamma_{g \circ f}^{W, \text{int}} = f^* \Gamma_g^{W, \text{int}} + \Gamma_f^{W, \text{int}} \quad (\text{II.100})$$

as jointly continuous sesquilinear forms on the transported Ward stage, with the coefficient associator understood. This proves regional, chart, scale, Cauchy, and vertical-equivalence naturality. DN7 again identifies the two mixed routes. Since no boundary character space occurs in the target, the conclusion is exactly interior adjoint Ward descent.

*Step 5: analytic, coherence, branch, and anchor components.* For an analytic face  $F$ , transport induces a continuous map  $\mathfrak{D}_F^{(\nu)} \rightarrow \mathfrak{D}_{F'}^{(\nu)}$ . DN4 maps  $\mathfrak{A}_F^{T, (\nu)}$  into  $\mathfrak{A}_{F'}^{T, (\nu)}$ , so it descends to the pointed collapse quotient (II.78). Consequently the basepoint condition  $[\Gamma^T]_{\text{adm}} = 0$  is preserved even when the chosen defect representative is nonzero.

For a coherence face  $F$ , whiskering both boundary composites by an elementary comparison gives

$$R_{F'}^{\check{C}} = W_f R_F^{\check{C}} W_f^{-1} \quad (\text{II.101})$$

inside the common target automorphism group selected in DN5. Thus  $R_F^{\check{C}} = \mathbf{1}$  is natural. Equation (II.101) does not set either boundary composite or their coherent multiplier equal to the identity separately.

DN6 carries the marking comparison  $m_f$ , roots, and every P2 anchor modification to the corresponding target comparison. The residual of a transported anchor square is therefore the whiskered residual of the source square; the same boundary calculation applies to the mixed anchor cube and the triple and quadruple Cauchy comparisons. This proves naturality of  $\Gamma^{B, \text{pre}}$  and preserves nontrivial marked branch type.

*Step 6: pseudonaturality and the zero subprestack.* Steps 3–5 supply the component transformations for every elementary arrow. Equations (II.99), (II.100), and (II.101), together with

the exact correction cells of DN4–DN7, verify the identity and composition axioms for the transformation into (II.97). The higher compatibility for mixed faces is precisely DN7 and the complete face ledger DN5. Hence (II.96) is pseudonatural.

Every elementary transport preserves the five basepoint conditions in (II.88); vertical equivalences carry zero objects to isomorphic zero objects. Therefore the homotopy full zero locus is closed under restriction, comparison, one-morphisms, and two-morphisms. It is the two-subprestack (II.98). The reverse assertion that this zero locus is the prestack of physically coherent realizations is not used and remains the separate P4 local-completeness theorem.  $\square$

*Remark II.46 (Closures and operator branches).* The proof uses graph-dual boundedness and form-graph boundedness. If DN8 is invoked, a core identity passes to closures because graph convergence is preserved in both directions and the comparison maps carry the closure graph onto the target closure graph. This yields conjugacy of the declared closed operators or forms. It does not prove essential self-adjointness, equality of spectral measures without self-adjointness, or commutation with a later local von Neumann algebra.

*Remark II.47 (Circularity firewall).* The inherited forms may occur as decorations and their graph norms may be used to state continuity. No coercive lower bound, mass parameter, momentum operator, vacuum spectral projection, collar charge, boundary Ward character, entropy surgery map, or post-incision normal state is used to define  $\mathfrak{C}_{\alpha,r}(O)$  or any component of  $\mathbf{O}_{r,\text{pre}}$ . In particular, none of  $\hat{\kappa}_r$ ,  $\kappa_*$ ,  $m_0$ ,  $P_\mu$ ,  $P^2$ , or  $\Pi_0$  enters P3.

**Proposition II.48** (Finite-dimensional strict witness for P3). *Assume the finite-cellular P1 witness and the finite-cylinder P2 witness. Suppose further that on each finite cell packet:*

1. *every core in (II.66) is the same finite-dimensional vector space before completion, while the labels are retained;*
2. *a finite-dimensional real interior parameter space acts through bounded Hermitian Gauss maps  $G_r^{\text{int}}(\xi)$ ;*
3. *the Ward insertion derivation is implemented on the finite insertion algebra and is intertwined by the regional and scale maps;*

4. all chart, regional, scale, and anchor cells are strict or the canonical tensor associators;  
and

5. the admissible analytic class contains the declared finite comparison defects and is stable under the tensor-factor transports.

Then DN0–DN8 hold and the P3 defect map is natural on the finite-cylinder subsite. The witness may carry nontrivial marked coefficient-line data provided its Cech and mixed residuals satisfy  $R^{\tilde{C}} = \mathbf{1}$ .

*Proof.* Finite dimensionality gives density, closability, and equivalence of all selected graph norms. Gauge-stability of the finite physical ideals gives DN1. Boundedness of every linear map gives DN2–DN3 with finite-demand constants equal to the maxima of finitely many operator norms. Stability of the declared analytic classes gives DN4. Strict cells and canonical tensor associators satisfy the complete face ledger DN5 and mixed naturality DN7. The P2 anchor cells preserve roots and markings by hypothesis, giving DN6. Every preoperator is bounded and hence closed, so DN8 holds. Theorem II.45 applies.  $\square$

*Remark II.49* (Status of the retained Yang–Mills witnesses). The finite SU(2) control can seed the strict finite-dimensional witness after the regional and Cauchy structures required by P1–P2 are supplied. The linked-flux SU(3) construction in the later post-incision manuscript verifies substantial chart, scale, and interior Gauss–Ward covariance in one selected realization. Neither witness, by itself, proves DN0–DN8 for the general pre-incision candidate prestack or for an unrestricted cofinal diagram.

**Corollary II.50** (Abstract P3 bridge and model obligation). *At the abstract front-end level, Hypothesis II.43 and Theorem II.45 close the P3 bridge. For a model-specific Yang–Mills realization, verification of DN0–DN8 remains an explicit common-core and quotient-stability proof obligation unless a separate regional–chart–scale Gauss–Ward theorem is cited without using the derived obstruction carrier or any post-incision structure.*

## I. Selected $SU(3)$ common-core defect naturality

The P1–P2 construction is finite dimensional at every selected regional, chart, Cauchy, and scale stage. This removes the unbounded-domain ambiguity that is present in the general candidate prestack, but it does not remove the need to type every defect through its declared correspondence. We now make that typing explicit and verify DN0–DN8 without using the derived obstruction carrier or any post-incision datum.

**Definition II.51** (Selected dyadic P3 demand system). Fix a region  $B \in \mathbf{SpReg}^{0,\text{dyad}}$ , a finite dyadic cover  $\mathcal{U} = \{U_\alpha \rightarrow B\}_{\alpha \in A}$  closed under the finite intersections used in the Čech nerve, a protected P2 Cauchy presentation  $\mathfrak{c}$ , and a finite full subcategory  $\Gamma_{\mathfrak{d}} \subset \Gamma_{SU(3)}^{\text{pre}}$ . Every chart and every nonempty finite chart intersection carries the P1 regional packet obtained by replacing  $B$  with that dyadic region. Chart restriction and chart refinement are therefore the corresponding P1 regional correspondences. The selected Cauchy arrows are those of Definition II.27, and the scale arrows are the package-adapted arrows  $a_{kj}$  of Definition II.9.

The face ledger contains all regional, chart, Cauchy, scale, and mixed faces formed from this finite data, including all Čech tetrahedra, regional–chart squares, chart–scale prisms, P1 regional–scale Beck–Chevalley cubes, and P2 regional–scale–Cauchy cubes. Refinement means passage to a finite dyadic common refinement, enlargement of the finite scale demand, or replacement by a protected P2-comparable Cauchy presentation. No arbitrary chart, slice, or scale arrow is admitted.

Let  $\mathbf{CohReal}_{SU(3),m}^{\text{dyad,stat}}$  denote the full selected two-subprestack generated by the explicitly constructed P1–P2 packet and its marking-preserving transports and equivalences over these demand data. Let  $\mathfrak{X}_{SU(3),m,\text{pre}}$  be the restriction of the pointed P3 defect target to the same selected comparison system.

**Definition II.52** (Selected retained common-core packet). For a selected dyadic chart  $U$  and scale  $r_j$ , set

$$\mathcal{A}_{U,j}^{\text{phys,alg}} = \mathcal{A}_{U,j}^{W,\text{int}} = \mathcal{W}_{U,j}^{0,\text{phys}}, \quad (\text{II.102})$$

$$\mathcal{C}_{U,j}^\cap = \mathcal{C}_{U,j}^{\text{prop}} = \mathcal{C}_{U,j}^q = \mathcal{C}_{U,j}^G = \mathcal{C}_{U,j}^W = \mathcal{H}_{U,j}^0, \quad (\text{II.103})$$

$$V_{U,j}^G = \mathcal{H}_{U,j}^0. \quad (\text{II.104})$$

Give  $V_{U,j}^G$  its Hilbert norm, which is also the selected Gauss graph norm because the retained conjugation generator vanishes.

Let

$$\mathfrak{g}_{U,j}^{\text{int}} = \bigoplus_{\lambda \in E_j(U)} \mathfrak{su}(3)_\lambda. \quad (\text{II.105})$$

The infinitesimal conjugation action on the retained character space is zero, so define

$$G_{U,j}^{\text{int}}(\xi) = 0, \quad \delta_\xi A = 0 \quad \left( \xi \in \mathfrak{g}_{U,j}^{\text{int}}, A \in \mathcal{W}_{U,j}^{0,\text{phys}} \right). \quad (\text{II.106})$$

For a regional or chart inclusion,  $c_f^{\mathfrak{g}}$  extends the parameter by zero on the added persistent labels. For a selected scale arrow, it sends old-label components through the persistent-label inclusion and assigns zero to genuinely new labels. The map  $c_f^A$  is the represented tensor-factor inclusion or package-adapted scale map. The maps  $J_f^\cap, J_f^{\text{prop}}, J_f^q, J_f^G, J_f^W$  are the corresponding P1 or suspended P2 decorated correspondences, restricted to the indicated full finite-dimensional domains.

The complete-ground comparison residual is retained in the analytic  $\pi$ -slot as

$$\Gamma_{f,j}^\Omega = (J_f^\cap)^* P_{\Omega,t(f)} J_f^\cap - \left( P_{\Omega,s(f)} \hat{\otimes} I_{L_f} \right), \quad (\text{II.107})$$

a typed bounded operator on the transported source packet. Thus no projection living in a different Hilbert space is subtracted before the comparison map is inserted.

**Definition II.53** (Selected finite-stage admissible analytic classes). For regional, chart, Cauchy, represented-action, complete-ground, channel, Gram, recompression, residual, ultraviolet, Gauss, and Ward comparison slots, let the admissible class be the zero subspace because P1–P2 transport these data exactly. For a selected scale-form face  $F$ , let  $\mathfrak{A}_F^{T,(q)}$  be the linear span of:

1. the actual finite scale-form residual of the P1 packet;
2. all of its pullbacks and whiskerings inside the finite demand system; and
3. the exact compositor and Beck–Chevalley correction terms required to compare two factorizations of the same selected scale arrow.

All these spaces are finite dimensional. The class is fixed by the P1 package and the finite demand shape before the P3 defect map is evaluated; it is not enlarged candidate by candidate.

On refinement, the corresponding class is generated by the refined finite orbit, so every selected transport maps the source admissible class into the target admissible class.

**Theorem II.54** (Selected  $SU(3)$  common-core five-defect naturality). *For every marked band  $m \in \mathbb{Z}_3$ , the P1 regional prestack of Theorem II.13, the stationary P2 Cauchy suspension of Theorem II.30, and Definitions II.51–II.53 verify DN0–DN8 of Hypothesis II.43 on the selected finite dyadic regional–chart–Cauchy–scale demand system.*

*Consequently the five-component defect map*

$$\mathbf{O}_{SU(3),m,\text{pre}} : \mathbf{CohReal}_{SU(3),m}^{\text{dyad,stat}} \longrightarrow \mathfrak{X}_{SU(3),m,\text{pre}} \quad (\text{II.108})$$

*is pseudonatural. On the explicitly selected P1–P2 object one has*

$$\Gamma^{G,\text{int}} = 0, \quad \Gamma^{W,\text{int}} = 0, \quad [\Gamma^T]_{\text{adm}} = 0, \quad R^{\check{C}} = \mathbf{1}, \quad \Gamma^{B,\text{pre}} = 0. \quad (\text{II.109})$$

*The equality in the analytic slot is membership in the selected admissible class; it does not assert that every scale-form residual vanishes.*

*Proof.* We verify the eight DN clauses and then identify the five defect values.

*DN0: one transport-stable common-core packet.* By Definition II.11, every selected regional and chart Hilbert space is finite dimensional, and the algebraic, form, Gauss, Ward, and propagation domains are the full space. The P2 suspension is constant in time and uses the same finite-dimensional domain on every protected slice. Hence the equalities in (II.104) define one invariant nucleus that is dense, a form core, and a graph core. P1 regional and scale correspondences and P2 trace, solution, and Cauchy maps preserve it exactly.

*DN1: physical quotient and complete-ground stability.* The selected physical ideal is zero. It is therefore stable under every regional, chart, Cauchy, and scale map and under every interior derivation, and

$$\pi(I)|_{\mathcal{C}^\cap} = 0, \quad \pi(\delta_\xi I)|_{\mathcal{C}^\cap} = 0$$

holds identically. The P1 regional correspondences tensor by complete-ground ranges, the package-adapted scale maps carry complete-ground flags exactly, and the P2 trace and solution maps preserve the same projection. Therefore (II.107) is zero for every admitted comparison. This is equality of complete projections, not a choice of a ground vector.

*DN2: finite-demand bounds.* Every map acts between finite-dimensional spaces. For one finite demand shape, take  $C_{\mathfrak{d},K}^G$ ,  $C_{\mathfrak{d},K}^W$ ,  $C_{\mathfrak{d},K}^q$ , and  $C_{\mathfrak{d},K}^{\text{tr}}$  to be the maxima of the finitely many relevant operator and inverse-operator norms. They are finite. The P2 trace and solution constants are exactly one. These are finite-demand constants only; no cofinal-uniform estimate is inferred.

*DN3: Gauss and Ward intertwining.* Equation (II.106) gives

$$c_f^A(\delta_\xi A) = 0 = \delta_{c_f^g \xi}(c_f^A A).$$

The graph-dual Gauss maps are zero and all represented actions are bounded on the full domain. Hence the Gauss and Ward expressions are jointly continuous and invariant under every admitted comparison. Direct substitution into Definitions II.37 and II.38 gives  $\Gamma_f^{G,\text{int}} = 0$  and  $\Gamma_f^{W,\text{int}} = 0$  for every regional, chart, Cauchy, scale, and mixed edge.

*DN4: admissible analytic classes.* All non-form analytic residuals vanish because P1 transports the represented algebra, complete-ground projection, channel, Gram, recompression, residual, and ultraviolet flags exactly, and P2 suspends them unchanged. The selected scale-form residual need not vanish. Definition II.53 places it in a finite-dimensional linear space containing all of its finite pullbacks, whiskerings, and exact composition corrections. Such a subspace is closed, and the generating rule makes it stable under regional restriction, chart refinement, selected scale composition, P2 Cauchy comparison, coefficient reassociation, and marking-preserving equivalence. Thus  $[\Gamma^T]_{\text{adm}} = 0$  without converting a one-sided or nonzero form discrepancy into an equality of forms or a transported lower bound.

*DN5: complete face ledger and coherence residual.* Definition II.51 includes every face used by the selected model. Regional and chart compositors are canonical tensor associators and permutations, so their pentagons, triangles, and Čech refinement faces commute by monoidal coherence. The coefficient reassociator

$$\Omega_m(a, b, c) = \omega^{\widetilde{ma}\epsilon_3(b,c)}$$

is normalized and satisfies the cyclic three-cocycle identity; hence its scale pentagon commutes although its cohomology class need not be trivial. The P1 cells  $\beta_{i,a}$  satisfy the regional-scale Beck–Chevalley laws, and the P2 stationary anchor cells are identities or those inherited P1

cells. Therefore the two boundary composites of every listed face agree after the declared associators and unitors are inserted, and  $R^{\check{C}} = \mathbf{1}$ .

*DN6: branch and anchor stability.* P1 transports the coefficient line, normalized reassociator, selected root, and marked band  $m$  without changing the branch object. P2 trace, solution, anchor, and protected Cauchy comparison maps preserve the same data exactly. Thus every type comparison  $m_f$  is a root-preserving equivalence and every regional, scale, mixed, and Cauchy anchor residual is the identity modification. Hence  $\Gamma^{B,\text{pre}} = 0$ . This preserves the marked center-banded branch and does not neutralize it.

*DN7: mixed naturality.* For regional-chart squares, both routes are iterated P1 regional correspondences. For regional-scale and chart-scale faces, naturality of the persistent-label inclusion  $\iota_{kj}^B$  identifies the old-label factors and both routes add the same complete-ground factors. The resulting comparison is the P1 tensor-permutation cell. The P2 regional-scale-Cauchy cube is the stationary suspension of that same cell. Since the Gauss and Ward maps are zero, their two transports agree identically; the analytic classes are stable by DN4; and the root, marking, and anchor targets agree by DN6. Thus every mixed face satisfies DN7.

*DN8: closure branch.* Every selected form and preoperator is bounded on a finite-dimensional space. It is therefore closed, its full domain is carried onto the target full domain, and graph conjugacy is ordinary bounded conjugacy. No self-adjointness, spectral measure, or strong-commutation statement is needed.

The eight clauses now hold. The general naturality theorem II.45 gives the pseudonaturality of (II.108). The component calculations above give (II.109). The zero condition is only the P3 transport-stable obstruction condition; its sufficiency and effectivity are not used here. □

**Corollary II.55** (Model-specific P3 closure for the selected retained branch). *On the selected finite dyadic regional-chart-Cauchy-scale system, obligations P3.1–P3.9 are discharged for the retained pre-incision  $SU(3)$  branch. The homotopy-full zero locus contains the explicitly constructed P1–P2 object and is stable under every admitted regional, chart, protected Cauchy, selected scale, mixed, and marking-preserving comparison.*

*Remark II.56* (P3 firewall and remaining edge). The proof uses finite-dimensionality, the

zero physical ideal, conjugation invariance of retained characters, persistent-label tensor transport, the P2 stationary anchor, and the normalized cyclic reassociator. It uses no incision, obstruction carrier, collar charge, boundary Ward character, entropy surgery, coercive constant, closed Hamiltonian from the later prescribed-incision witness, thin limit, or spectral statement. It does not by itself prove that every zero-defect candidate admits an effective anchored lift. That inverse-to-extraction and descent statement is established for the same selected branch in Subsection II K.

**Corollary II.57** (P3 closure status). *Hypothesis II.43 and Theorem II.45 remain the general conditional bridge. Theorem II.54 verifies its hypotheses for one explicitly inhabited retained pre-incision  $SU(3)$  branch on the selected finite dyadic demand system. No unrestricted regional, chart, Cauchy, or cofinal-scale naturality is asserted.*

## J. P4: defect exactness

P3 proves that the five-component zero locus is a transport-stable homotopy-full two-subprestack of the candidate prestack. That statement does not yet identify the zero locus with the physically coherent realization prestack. To formulate that identification without circularity, we first define coherent realization independently of the defect map.

**Definition II.58** (Pre-incision local realization bicategory). For a selected testing region  $U \in M$  and a fixed selected scale object  $r$ , let

$$\mathbf{LocPkg}_r^{+, \text{pre}}(U) \tag{II.110}$$

be the selected  $(2, 1)$ -category whose objects are actual chart-local positive-thickness realization packets

$$\mathfrak{R}_{\alpha, r}(U) = \left( \mathcal{W}_{\alpha, r}^{\text{phys}}(U), \mathcal{H}_{\alpha, r}(U), \pi_{\alpha, r}, P_{\Omega, \alpha, r}, \text{Supp}_{\alpha, r}; \mathfrak{C}_{\alpha, r}, G_{\alpha, r}^{\text{int}}, \delta_{\alpha, r}^{\text{int}}, \text{An}_{\alpha, r}, \tau_{\alpha, r} \right). \tag{II.111}$$

Here:

1.  $\mathcal{W}_{\alpha, r}^{\text{phys}}(U)$  is an actual regional physical quotient, not an unphysical ambient algebra with notation restricted to  $U$ ;

2.  $(\mathcal{H}_{\alpha,r}, \pi_{\alpha,r}, P_{\Omega,\alpha,r})$  is the represented packet with complete-ground projection and no selected vector;
3.  $\text{Supp}_{\alpha,r}$  is the support-filter datum;
4.  $\mathfrak{C}_{\alpha,r}$  is the P3 common-core packet, retaining the distinct propagation, form, Gauss, and Ward domains;
5.  $G^{\text{int}}$  and  $\delta^{\text{int}}$  are the interior Gauss graph-form and Ward insertion actions on the declared domains;
6.  $\text{An}_{\alpha,r}$  denotes the typed admissible analytic comparison structure; and
7.  $\tau_{\alpha,r}$  is the inherited rooted marked reconstruction branch.

A one-morphism is a quotient-level, support-controlled, marking-preserving physical correspondence carrying all displayed decorations on their declared domains. A two-morphism is an invertible decorated intertwiner. No collar, boundary-character, normal-state, surgery, spectral, or mass datum is part of (II.111).

**Definition II.59** (Effective anchored descent object). Fix a finite demand datum

$$\mathfrak{d} = (O, \mathcal{U}, \mathfrak{c}, \mathbb{B}, \Gamma_{\mathfrak{d}}, \mathcal{F}_{\mathfrak{d}}) \quad (\text{II.112})$$

from Definition II.35. An effective anchored descent object at  $r$  consists of the following data.

1. For every chart  $U_{\alpha}$  in  $\mathcal{U}$ , an object  $\mathfrak{R}_{\alpha,r}(U_{\alpha})$  of  $\mathbf{LocPkg}_r^{+, \text{pre}}(U_{\alpha})$ .
2. For every selected chart overlap, regional inclusion, and selected scale comparison appearing in  $\mathfrak{d}$ , a decorated physical correspondence with its coefficient line or normal correspondence.
3. Multiplication cells, reassociators, regional compositors, scale compositors, and every mixed cell listed in Definition II.40, satisfying the corresponding pentagon, triangle, prism, and cube equations.
4. For every typed analytic face  $F$  and type  $\nu$ , a chosen admissible correction certificate

$$\mathfrak{a}_F^{(\nu)} = (D_F^{(\nu)}, A_F^{(\nu)}, \text{heta}_F^{(\nu)}), \quad A_F^{(\nu)} \in \mathfrak{A}_F^{T,(\nu)}, \quad (\text{II.113})$$

where  $\theta_F^{(\nu)}$  identifies the actual comparison defect with the selected admissible representative in the typed correction groupoid. The representative need not be zero.

5. A selected admissible Cauchy presentation and an anchor  $\eta_{c,r}$  satisfying the P2 regional, scale, mixed, and selected-Cauchy comparison modifications.
6. Root and marking data representing the inherited branch  $\tau$ , with no neutralization requirement on its banded or controlled classes.
7. An effectivity certificate exhibiting the preceding coherent descent presentation as an object of the selected stack of local positive-thickness realization packets on  $O$ .

One-morphisms are marking- and anchor-preserving pseudonatural equivalences of these data, including maps between the chosen analytic correction certificates. Two-morphisms are invertible modifications. The resulting  $(2, 1)$ -groupoid is denoted

$$\mathbf{EffReal}_r^{\text{anch}}(O; \mathfrak{d}). \quad (\text{II.114})$$

Effectivity does not mean that the coefficient band is trivial or that a single global Hilbert-space trivialization has been chosen. A nonneutral banded realization may be effective as a stack-local object.

Write  $\mathbf{CohReal}_r^{\text{cand,Cau}}(O; \mathfrak{d})$  for the homotopy-full finite-demand fiber of the P2–P3 candidate prestack.

There is a forgetful pseudofunctor

$$\mathbf{U}_{r,O;\mathfrak{d}} : \mathbf{EffReal}_r^{\text{anch}}(O; \mathfrak{d}) \longrightarrow \mathbf{CohReal}_r^{\text{cand,Cau}}(O; \mathfrak{d}) \quad (\text{II.115})$$

that forgets the effectivity and analytic-correction certificates while retaining the underlying chart-local packets, comparisons, cores, markings, and P2 anchor.

**Definition II.60** (Physically coherent anchored realization prestack). The two-groupoid

$$\mathbf{CohReal}_r^{\text{anch}}(O; \mathfrak{d}) \quad (\text{II.116})$$

is the homotopy-full essential image of (II.115). Passing over the directed system of finite demand data and using the restriction maps gives

$$\mathbf{CohReal}_r^{\text{anch}} \subseteq \mathbf{CohReal}_r^{\text{cand,Cau}}. \quad (\text{II.117})$$

This definition is independent of  $\mathbf{O}_{r,\text{pre}}$ : membership is witnessed by an actual effective anchored descent object, not by declaring the five defects to vanish.

**Theorem II.61** (P4 soundness). *Assume P1–P3. Every physically coherent anchored local positive-thickness realization has zero pre-incision defect. Equivalently, there is a homotopy-full inclusion of two-prestacks*

$$\boxed{\mathbf{CohReal}_r^{\text{anch}} \subseteq \ker \mathbf{O}_{r,\text{pre}}.} \quad (\text{II.118})$$

*Proof.* Fix an effective anchored descent object and a finite demand datum  $\mathfrak{d}$ .

*Interior Gauss component.* The decorated physical correspondences of the effective object carry the Gauss graph-dual maps through the declared graph-space comparisons. Hence, for every elementary comparison  $f$  and interior parameter  $\xi$ ,

$$(J_f^G)^* G_{t(f)}^{\text{int}} (c_f^{\mathfrak{g}} \xi) J_f^G = G_{s(f)}^{\text{int}}(\xi) \hat{\otimes} I_{L_f}. \quad (\text{II.119})$$

Comparison with Definition II.37 gives  $\Gamma_r^{G,\text{int}} = 0$ . This argument is at graph-form level and does not invoke a self-adjoint generator.

*Interior Ward component.* The effective object preserves the represented insertion algebra, the interior derivation, the graph-dual pairing, and the declared Ward core. Transporting the three terms in (II.73) therefore gives the same functional at the source and target. Thus

$$\Gamma_r^{W,\text{int}} = 0. \quad (\text{II.120})$$

No boundary character is used.

*Analytic component.* For every typed face, the effective realization contains a certificate (II.113) with  $A_F^{(\nu)} \in \mathfrak{A}_F^{T,(\nu)}$ . The actual defect therefore belongs to the declared admissible class, and its image in the pointed collapse quotient is the basepoint:

$$[\Gamma_r^T]_{\text{adm}} = 0. \quad (\text{II.121})$$

This does not require  $D_F^{(\nu)} = 0$ .

*Coherence component.* The effective object satisfies every generating pentagon, triangle, prism, and mixed cube in the complete P3 face ledger. The two boundary composites of each face are therefore equal after the declared associators and unitors are inserted, so

$$R_r^{\check{C}} = \mathbf{1}. \quad (\text{II.122})$$

A nontrivial banded Čech class may remain.

*Branch and anchor component.* The effective object carries the inherited root and marked branch through admissible marking equivalences, and its P2 anchor satisfies the regional, scale, mixed, and selected-Cauchy comparison modifications. Consequently

$$\Gamma_r^{B,\text{pre}} = 0. \quad (\text{II.123})$$

The five identities are precisely (II.88). P3 naturality shows that the argument is preserved under restriction and marking-preserving equivalence. Since  $\mathbf{CohReal}_r^{\text{anch}}$  is homotopy full in the candidate prestack, (II.118) follows.  $\square$

Soundness uses no new effectivity theorem: effectivity is part of the independent meaning of a coherent realization. The reverse inclusion is substantive. In particular, membership of each analytic defect in its admissible class does not by itself choose a composition-compatible family of corrections, and equality of generating residuals does not by itself prove that the corresponding descent object is effective.

**Hypothesis II.62** (P4: Effective Anchored Descent package). For every fixed selected scale object  $r$ , every finite demand datum  $\mathfrak{d}$ , and every selected testing region  $O$ , the following properties hold.

(EAD0) Primitive completeness of candidates. Every object of  $\mathbf{CohReal}_r^{\text{cand,Cau}}(O; \mathfrak{d})$  contains all primitive object-level data listed in (II.111), every comparison appearing in the P3 face ledger, the full P2 anchor packet, and no hidden post-incision datum. The candidate category is not allowed to omit a structure and then ask its vanishing defects to reconstruct that missing primitive.

(EAD1) Physical quotient and representation effectivity. The chart-local physical algebras are actual quotients by gauge-stable regional physical ideals. Their representations are nondegenerate on the declared Hilbert packets, and all candidate comparison maps descend through the ideals. The quotient-level represented packets, complete-ground projections, physical channels, support filters, Gram data, magnetic recompressions, and residual decompositions form a selected 2-fibration over the regional-chart-scale comparison category.

(EAD2) Gauss–Ward decoration effectivity. The P3 graph and Ward core maps are cartesian decorations of the 2-fibration in EAD1. A zero Gauss transport residual and a zero interior Ward residual are sufficient to make the already specified local graph forms and Ward derivations into a cartesian decorated descent object. No uniqueness of the local Ward derivation is asserted or required.

(EAD3) Simultaneous finite analytic-correction lifting. For every typed analytic face, let

$$\mathbf{Corr}_F^{(\nu)} \longrightarrow \mathfrak{D}_F^{(\nu)} \parallel \mathfrak{A}_F^{T,(\nu)} \quad (\text{II.124})$$

be the groupoid of admissible correction certificates. Over the basepoint, every finite compatible family of actual defects has a simultaneous lift to correction certificates. These lifts are compatible with composition, whiskering, regional restriction, chart refinement, scale transport, and the mixed cells. No strict zero representative is required.

(EAD4) Completeness of the coherence presentation. The list  $\mathcal{F}_{\mathfrak{d}}^{\check{C}}$  contains every generating 2-dimensional face for the selected bicategorical model. The only higher conditions are the equality of the two composites of the declared pentagons, triangles, prisms, and cubes. Thus the model is genuinely 2-truncated: no unlisted independent 3-cell datum is required. Identity of all residuals on the generating faces implies a well-defined decorated pseudofunctor on the whole finite comparison category.

(EAD5) Admissible finite descent effectivity. For every selected finite chart cover and finite common refinement, the descent pseudofunctor from effective stack-local objects of  $\mathbf{LocPkg}_r^{+, \text{pre}}$  to decorated descent presentations is essentially surjective and fully faithful. Effectivity is taken in the banded or correspondence-valued target and therefore does not imply neutrality of the coefficient gerbe.

(EAD6) Rooted branch and Cauchy-anchor effectivity. The marked-branch and P2-anchor decorations form cartesian groupoid-valued fibrations over the effective descent 2-fibration. A zero  $\Gamma_r^{B, \text{pre}}$  supplies an effective root-preserving marking and an effective selected-Cauchy anchor, including the regional, scale, mixed, and triple-comparison modifications.

(EAD7) Relative effectivity. The effectivity equivalences in EAD3–EAD6 commute pseudonaturally with admissible regional restriction, selected chart refinement, selected scale transport,

and selected admissible Cauchy comparison. Consequently the effective essential image is a two-prestack rather than an objectwise family of unrelated essential images.

(EAD8) Finite-demand locality and uniformity. All analytic constants required in EAD1–EAD3 are finite on the fixed demand datum and stable under its selected finite refinements. No cofinal-scale uniform estimate is assumed in P4; simultaneous cofinal lifting belongs to P5c.

*Remark II.63* (Why Hypothesis II.62 is not the conclusion). EAD0–EAD8 are structural properties of the candidate 2-fibration, the analytic correction fibrations, and the selected descent target. None says that a particular zero-defect candidate is realizable. The local-completeness theorem below derives realizability by combining the five zero conditions with those independent structural properties.

**Proposition II.64** (Zero defect gives a coherent descent presentation). *Assume P1–P3 and EAD0–EAD4 and EAD6. Let  $\mathfrak{P} \in \mathbf{CohReal}_r^{\text{cand}, \text{Cau}}(O; \mathfrak{d})$  satisfy (II.88). Then  $\mathfrak{P}$  determines a coherent anchored decorated descent presentation on  $\mathfrak{d}$ . This statement does not yet assert effectivity of that presentation.*

*Proof.* EAD0 supplies every primitive local packet and comparison. By EAD1 and DN1, the physical quotient and represented maps are already defined at quotient level. The equality  $\Gamma_r^{G, \text{int}} = 0$  makes the specified graph forms cartesian under every elementary comparison. The equality  $\Gamma_r^{W, \text{int}} = 0$  does the same for the specified interior Ward functional and derivation. These equations transport existing local data; they do not create a new derivation or a new unbounded operator.

The basepoint condition  $[\Gamma_r^T]_{\text{adm}} = 0$  says that every typed actual defect lies in its admissible class. EAD3 simultaneously lifts the finite family to composition-compatible correction certificates. The identity  $R_r^{\check{C}} = \mathbf{1}$ , together with the complete generating face statement EAD4, supplies the pseudofunctorial multiplication, reassociation, regional, scale, and mixed coherence laws on the entire finite comparison category. Finally,  $\Gamma_r^{B, \text{pre}} = 0$  and EAD6 supply the effective root-preserving marked comparison and the complete selected Cauchy-anchor comparison packet. The result is a coherent anchored decorated descent presentation.  $\square$

**Theorem II.65** (P4 local completeness). *Assume P1–P3 and Hypothesis II.62. Every zero-defect candidate is locally equivalent to the underlying candidate of an effective physically coherent anchored realization. Equivalently,*

$$\boxed{\ker \mathbf{O}_{r,\text{pre}} \subseteq \mathbf{CohReal}_r^{\text{anch}}.} \quad (\text{II.125})$$

*Proof.* Let  $\mathfrak{P}$  be a zero-defect candidate over a finite demand datum. Proposition II.64 turns  $\mathfrak{P}$  into a coherent anchored decorated descent presentation. EAD5 applies to that presentation and produces an effective stack-local object

$$\tilde{\mathfrak{R}} \in \mathbf{EffReal}_r^{\text{anch}}(O; \mathfrak{d}) \quad (\text{II.126})$$

and a marking-preserving equivalence

$$\mathbf{U}_{r,O;\mathfrak{d}}(\tilde{\mathfrak{R}}) \simeq \mathfrak{P} \quad (\text{II.127})$$

in the candidate two-groupoid. Full faithfulness in EAD5 shows that candidate equivalences between two such effective objects lift uniquely up to invertible modification. EAD7 makes (II.127) pseudonatural under every admissible restriction and selected comparison, while EAD8 guarantees that the finite analytic construction remains valid after selected finite refinement. Therefore  $\mathfrak{P}$  belongs to the homotopy-full essential image defining  $\mathbf{CohReal}_r^{\text{anch}}$ . This proves (II.125) as an inclusion of two-prestacks.  $\square$

**Theorem II.66** (P4 defect exactness). *Under P1–P3 and Hypothesis II.62, soundness and local completeness combine to give an equivalence of homotopy-full two-prestacks*

$$\boxed{\mathbf{CohReal}_r^{\text{anch}} \simeq \ker \mathbf{O}_{r,\text{pre}}.} \quad (\text{II.128})$$

*After choosing the homotopy-full representatives fixed throughout this front-end construction, the equivalence may be written as equality of the selected full subprestacks.*

*Proof.* Theorem II.61 gives the forward inclusion. Theorem II.65 gives the reverse inclusion. Both inclusions are pseudonatural under the admissible comparison system and are homotopy full on one- and two-morphisms. Their composites are connected to the respective identities by the effectivity equivalences from EAD5 and EAD7. Hence they define the displayed equivalence.  $\square$

*Remark II.67* (Substantive content of P4). Theorem II.66 is not obtained by defining  $\mathbf{CohReal}_r^{\text{anch}}$  to be the kernel. The realization prestack is the essential image of the independently defined effective anchored descent category. The reverse implication uses, in order, quotient effectivity, graph–Ward decoration effectivity, simultaneous analytic correction lifting, completeness of the generating coherence faces, admissible descent effectivity, and rooted-anchor effectivity.

*Remark II.68* (What zero Ward defect does and does not determine). The zero Ward defect transports the interior adjoint action already contained in the candidate. It does not determine that action uniquely from the physical observable algebra. A uniqueness statement would require an additional faithfulness or generation theorem for the insertion algebra and is not needed for P4 or for the later lifting prestack.

*Remark II.69* (Admissible analytic correction rather than flattening). The local-completeness proof uses EAD3 to lift the basepoint of the pointed collapse quotient to a coherent family of admissible correction certificates. It never replaces a controlled nonzero defect by zero. Thus Sector  $\text{II}_{\text{ctrl}}$  remains allowed, and a coherent nontrivial coefficient band remains allowed in Sector III.

*Remark II.70* (Categorical truncation). The target is a  $(2, 1)$ -categorical model. Objects, one-morphisms, and invertible two-morphisms are explicit. Pentagon and mixed-cube conditions are equalities between composites of two-morphisms; no independent three-cell is present. If a later model is upgraded to a genuine tricategory or  $\infty$ -category, EAD4 must be replaced by the corresponding higher coherence effectivity theorem.

*Remark II.71* (Circularity firewall). Neither soundness nor local completeness uses an obstruction collar, a boundary Ward character, entropy surgery, a post-incision normal-state construction, a closed thick form, a spectral threshold, invariant momentum, or a mass-gap estimate. Inherited forms and graph norms are transported only as pre-incision decorations. None of  $\hat{\kappa}_r$ ,  $\kappa_*$ ,  $m_0$ ,  $P_\mu$ ,  $P^2$ , or  $\Pi_0$  defines the effective realization category, the correction fibrations, or the kernel.

**Proposition II.72** (Finite-dimensional effectivity witness for P4). *Assume the finite-cellular P1 witness, the finite-cylinder P2 witness, and the finite-dimensional P3 witness. Suppose*

*that the selected chart cover and finite demand category are finite, every physical package is finite-dimensional, every comparison is a finite-dimensional unitary correspondence, and each admissible analytic class is represented by a nonempty finite-dimensional affine correction groupoid stable under all selected compositions. Then EAD0–EAD8 hold on that finite subsite, and  $P_4$  exactness follows. The coefficient-line cocycle may be nontrivial provided its multiplication and reassociation cells satisfy the  $P_3$  coherence residuals.*

*Proof.* Finite-dimensional quotient representations and correspondences give EAD0, EAD1, and EAD2. A finite compatible family of affine correction conditions has a simultaneous solution by the assumed nonempty composition-stable correction groupoid, giving EAD3. Mac Lane coherence for the selected bicategory and the explicitly finite face ledger give EAD4. Descent of finite-dimensional Hilbert spaces, matrix algebras, and invertible correspondences over a finite groupoid is effective by forming the finite 2-limit with its retained coefficient band, giving EAD5. Root and anchor labels are finite groupoid decorations, giving EAD6. Restriction is precomposition and hence gives EAD7. Every required bound is the maximum of finitely many norms, giving EAD8. Theorem II.66 therefore applies.  $\square$

*Remark II.73* (Status of retained Yang–Mills witnesses). The strict finite  $SU(2)$  control can seed the preceding witness only after the  $P_1$  regional,  $P_2$  Cauchy, and  $P_3$  common-core packages have been supplied. The linked-flux  $SU(3)$  realization supplies substantial post-incision evidence for coherent coefficient-line descent, causal propagation, and interior Gauss–Ward covariance. Neither calculation proves the general EAD3 simultaneous correction-lifting property or EAD5 effective pre-incision descent over the full selected site.

*Remark II.74* (Downstream dependency). The fiberwise lifting prestack in (II.138) must be formed from  $\mathbf{CohReal}_r^{\text{anch}}$ , not merely from the formal kernel, unless Theorem II.66 is available. Soundness alone would give only a map from physically coherent lifts into zero-defect candidates and would not justify habitat formation from the kernel.

**Corollary II.75** ( $P_4$  closure and remaining model obligation). *At the abstract front-end level, Theorems II.61 and II.66 close  $P_4$  under the explicit structural Hypothesis II.62. Soundness is proved without that additional hypothesis. For a model-specific Yang–Mills realization,*

verification of EAD0–EAD8 remains an explicit effectivity proof obligation unless a separate pre-incision decorated-descent theorem is supplied.

## K. Selected $SU(3)$ effective anchored descent

The general P4 theorem isolates EAD0–EAD8 because zero defect is not, in an arbitrary candidate bicategory, an effectivity theorem. On the selected retained  $SU(3)$  branch, however, the P1 regional packets are finite tensor products indexed by persistent carrier labels, the P2 spacetime packets are stationary suspensions of those same tensor products, and the P3 analytic classes contain the actual finite residuals together with their complete finite transport orbits. These facts permit a direct reconstruction that does not appeal to a later stackification theorem.

**Definition II.76** (Carrier-complete finite demand). Let  $\mathfrak{d}$  be a selected finite P3 demand datum over  $B \in \mathbf{SpReg}^{0, \text{dyad}}$ , and let  $r_{k_*}$  be the largest scale object appearing in its finite scale subcategory. A dyadic refinement  $\mathfrak{d}^\sharp$  is *carrier complete* when its chart cover is closed under the intersections used in the selected Cech nerve and, for every persistent label

$$\lambda \in E_{k_*}(B),$$

there is a chart  $U_\alpha$  with  $\widehat{Q}_\lambda \subseteq U_\alpha \subseteq B$ . The same chart then contains the control cube of that label at every earlier demanded scale at which the label is present.

**Lemma II.77** (Existence and cofinality of carrier-complete refinements). *Every selected finite demand datum admits a carrier-complete finite dyadic refinement. Any two such refinements admit a carrier-complete finite common refinement. The construction is compatible with regional restriction, finite enlargement of the selected scale demand, and protected P2 Cauchy comparison.*

*Proof.* The set  $E_{k_*}(B)$  is finite. For each  $\lambda \in E_{k_*}(B)$ , the compact control cube  $\widehat{Q}_\lambda$  lies in the open region  $B$ , hence has positive distance from  $\mathbb{R}^3 \setminus B$ . A sufficiently fine finite union of dyadic cubes gives an open dyadic chart  $U_\lambda$  satisfying  $\widehat{Q}_\lambda \subseteq U_\lambda \subseteq B$ . Add these charts to a finite dyadic common refinement of the original cover and close the result under the

finitely many intersections used by the demand nerve. This proves existence. Repeating the same construction after taking the common dyadic refinement of two finite covers proves the second assertion. Persistent labels and the fixed-control-cube selection rule make the construction natural under regional restriction and scale enlargement. Protected Cauchy comparison changes only the stationary slab presentation and not the spatial control cubes, so it preserves carrier completeness.  $\square$

For a carrier-complete demand  $\mathfrak{d}^\sharp$ , denote the canonical P1–P2 candidate by

$$\mathfrak{P}_{B,j,m;\mathfrak{d}^\sharp}^{\text{can}}$$

Its chart packet on  $U_\alpha$  is the stationary suspension of  $\mathcal{P}_{U_\alpha,j}^0$ , and every regional, chart, scale, Cauchy, and mixed comparison is the corresponding P1 or P2 decorated correspondence.

**Definition II.78** (Canonical selected effective enhancement). The canonical effective enhancement

$$\mathfrak{E}_{B,j,m;\mathfrak{d}^\sharp}^{\text{can}}$$

of  $\mathfrak{P}_{B,j,m;\mathfrak{d}^\sharp}^{\text{can}}$  is defined as follows.

1. The global regional object is the P2 stationary suspension of the P1 packet  $\mathcal{P}_{B,j}^0$ , with all complete-ground, support, core, Gauss, Ward, preform, coefficient-line, root, and marking decorations retained.
2. Its local chart objects and overlap correspondences are the exact restrictions of that global persistent-label packet.
3. For a typed analytic face  $F$ , let  $D_F^{(\nu)}$  be its actual P3 residual. Choose the correction certificate

$$\mathfrak{a}_{F,\text{can}}^{(\nu)} = (D_F^{(\nu)}, D_F^{(\nu)}, \mathbf{1}). \quad (\text{II.129})$$

In every exact slot this is  $(0, 0, \mathbf{1})$ ; in a selected scale-form slot, membership  $D_F^{(q)} \in \mathfrak{A}_F^{T,(q)}$  follows from Definition II.53.

4. The effectivity certificate is the identity identification of the chartwise descent presentation with the restrictions of the global persistent-label packet. No global Hilbert trivialization of the coefficient band is added.

5. The selected Cauchy anchor, coefficient lines, normalized reassociator, root, and marked band are exactly those of P1–P2.

Let  $\mathbf{EffReal}_{SU(3),m}^{\text{sel}}$  be the selected  $(2,1)$ -subprestack generated by these canonical enhancements, the admitted persistent-label-preserving P1–P2 one-morphisms, their marking-preserving decorated equivalences, and invertible decorated modifications.

Let

$$\mathbf{Z}_{SU(3),m}^{\text{sel}} := \ker \mathbf{O}_{SU(3),m,\text{pre}} \cap \mathbf{CohReal}_{SU(3),m}^{\text{dyad,stat}} \quad (\text{II.130})$$

be the selected P3 zero component. By Definition II.51, this is the homotopy-full selected component generated by the explicit P1–P2 packet and its admitted marking-preserving transports and equivalences. Fix once and for all a cleavage from this essentially small selected essential closure to the canonical persistent-label skeleton. Any two such cleavages are connected by a marking-preserving pseudonatural equivalence, so the resulting reconstruction biequivalence is independent of that auxiliary choice up to unique equivalence in the selected  $(2,1)$ -category.

**Theorem II.79** (Selected  $SU(3)$  effective anchored descent and inverse to extraction). *For every marked band  $m \in \mathbb{Z}_3$ , the selected retained P1–P3 branch verifies EAD0–EAD8 on the finite dyadic regional-chart-protected-Cauchy-selected-scale demand system. The forgetful pseudofunctor restricts to a pseudonatural biequivalence*

$$\boxed{\text{Ext}_{SU(3),m} : \mathbf{EffReal}_{SU(3),m}^{\text{sel}} \simeq \mathbf{Z}_{SU(3),m}^{\text{sel}}} \quad (\text{II.131})$$

A quasi-inverse

$$\text{Rec}_{SU(3),m} : \mathbf{Z}_{SU(3),m}^{\text{sel}} \longrightarrow \mathbf{EffReal}_{SU(3),m}^{\text{sel}} \quad (\text{II.132})$$

is obtained by carrier-complete finite refinement, persistent-label reconstruction, and the canonical correction certificates (II.129). There are pseudonatural unit and counit equivalences

$$\eta : \mathbf{1} \Longrightarrow \text{Ext}_{SU(3),m} \text{Rec}_{SU(3),m}, \quad \epsilon : \text{Rec}_{SU(3),m} \text{Ext}_{SU(3),m} \Longrightarrow \mathbf{1}, \quad (\text{II.133})$$

whose triangle modifications are identities on the canonical skeleton and canonical tensor-coherence modifications on its homotopy-full closure.

*The equivalence is homotopy full on one- and two-morphisms and preserves the physical quotient, complete-ground projection, support filter, P3 common core, Gauss graph form, Ward domain, preform and its admissible correction class, regional and scale compositors, mixed cells, coefficient lines, normalized reassociator, selected Cauchy anchor, roots, and marked band.*

*Proof.* We first construct reconstruction on the canonical skeleton and then pass to its homotopy-full selected closure.

*Persistent-label reconstruction on objects.* Choose a carrier-complete refinement  $\mathfrak{d}^\#$ . For each demanded scale  $r_j$ , the local chart packets contain precisely the tensor factors indexed by  $E_j(U_\alpha)$ . On an overlap, the common persistent labels are identified by the identity labelled correspondence. Carrier completeness implies

$$E_j(B) = \bigcup_{\alpha} E_j(U_\alpha)$$

for every demanded  $j$ . Form one copy of  $\mathcal{K}_{j,\lambda}$  for each label  $\lambda \in E_j(B)$  and order the factors by the fixed global label order. The overlap identifications ensure that a label occurring in several charts contributes only one factor. This reconstructs

$$\mathcal{H}_{B,j}^0 = \widehat{\bigotimes_{\lambda \in E_j(B)} \mathcal{K}_{j,\lambda}}, \quad \mathcal{W}_{B,j}^{0,\text{phys}} = \widehat{\bigotimes_{\lambda \in E_j(B)} \mathcal{A}_{j,\lambda}^{\text{ret}}},$$

and the labelwise tensor products of the complete-ground projections, channels, Gram data, magnetic recompressions, support filters, and common cores. The stationary P2 suspension reconstructs the corresponding global spacetime packet without changing any decoration. Restriction of the result to each chart is the original local packet, up to the canonical ordered reassociation. This is the required object-level effectivity certificate.

*Quotient, Gauss–Ward, and analytic decorations.* The physical ideal is zero, so quotient and representation effectivity are literal. The Gauss graph form and Ward derivation vanish on the full finite-dimensional common domain and therefore glue uniquely as zero cartesian decorations. In each exact analytic slot, the actual residual is zero. In a selected scale-form slot, the actual residual belongs to the fixed admissible class by P3. The canonical choice  $(D_F^{(q)}, D_F^{(q)}, \mathbf{1})$  therefore supplies a correction lift. DN4 and DN7 state precisely that the finite orbit of these residuals is closed under pullback, whiskering, composition, refinement,

and mixed transport. Thus the canonical certificates are simultaneous and composition compatible; no nonzero residual is flattened.

*Coherence, branch, and anchor.* P3 gives identity residual on the complete selected face ledger. Hence the canonical regional, chart, scale, Cauchy, and mixed correspondences define a decorated pseudofunctor on the whole finite demand category. The normalized cyclic reassociator is retained as the associator of the coefficient-line system; its cocycle identity supplies the pentagon without trivializing its class. The P2 stationary anchor, selected roots, and marked band are already primitive cartesian decorations and glue labelwise. Their regional, scale, mixed, and Cauchy modifications are the P1–P2 canonical cells.

*One-morphisms.* On the canonical skeleton, an admitted one-morphism is persistent-label preserving and is specified by its finite family of one-label decorated maps, coefficient-line maps, and canonical tensor compositors. A compatible chartwise family assigns the same one-label map wherever a label occurs, because overlap compatibility is part of the descent datum. Taking the ordered tensor product over  $E_j(B)$  gives a unique global decorated one-morphism whose restrictions are the given local morphisms. Regional, scale, and Cauchy naturality follows from the same persistent-label inclusion and the P1 Beck–Chevalley and P2 anchor cells.

*Two-morphisms.* An admitted two-morphism is an invertible decorated modification between the preceding labelwise correspondences. Overlap compatibility again identifies its one-label components. Their ordered tensor product gives the unique global modification, and restriction recovers the local family. Vertical and horizontal compositions are preserved because tensor product and whiskering are functorial. Thus extraction is fully faithful at both the one- and two-morphism levels.

*Independence of carrier-complete refinement.* Two carrier-complete refinements admit a carrier-complete common refinement by Lemma II.77. Both reconstruction procedures use one factor for each member of the same finite set  $E_j(B)$ , the same global label order, the same P1–P2 correspondences, and the same actual analytic residuals. Their pullbacks to the common refinement are therefore canonically equal up to ordered tensor reassociation. Mac Lane coherence and the P1–P3 face identities make these comparison cells transitive.

Reconstruction is consequently a pseudofunctor and is pseudonatural under every admitted finite refinement and selected comparison.

*Unit, counit, and essential closure.* On a canonical zero-defect candidate, extraction after reconstruction returns the same local packets, maps, cells, anchors, roots, and corrections; the unit is the identity after the fixed tensor reassociation. On a canonical effective enhancement, reconstruction after extraction returns the global persistent-label object and its canonical certificates; the counit is again the identity. The triangle identities therefore hold strictly on the canonical skeleton. The selected P3 candidate prestack and the selected P4 effective prestack are their homotopy-full closures under marking-preserving decorated equivalence. Using the fixed cleavage above, transport of the canonical enhancement gives the extension of  $\text{Rec}_{SU(3),m}$ ; a different cleavage produces a pseudonaturally equivalent reconstruction. Whiskering the identity unit and counit gives the displayed pseudonatural equivalences and triangle modifications on the closure. No new anchor or root is selected in this extension.  $\square$

**Corollary II.80** (Model-specific P4 closure for the selected retained branch). *On the selected finite dyadic regional-chart-protected-Cauchy-scale demand system, obligations P4.2–P4.8 are discharged for the retained pre-incision  $SU(3)$  branch. Together with P4 soundness and the finite-dimensional witness, this proves P4 defect exactness on that selected branch:*

$$\boxed{\text{CohReal}_{SU(3),m}^{\text{anch,sel}} \simeq \mathbf{Z}_{SU(3),m}^{\text{sel}}} \quad (\text{II.134})$$

*The result is fixed finite demand and positive thickness. It is not a P5 stackification or a cofinal pseudocone theorem.*

*Remark II.81* (Effectivity does not neutralize the selected band). The reconstruction glues one persistent Hilbert and observable factor per carrier label and retains the coefficient lines and their normalized cyclic reassociator on scale compositions. It does not choose a cochain that trivializes  $\Omega_m$ . Thus effective anchored descent is compatible with a nonneutral represented coefficient system.

*Remark II.82* (Enhancement fibers and retained automorphisms). Over a fixed canonical normal-form candidate, the added P4 data are terminal: the global persistent-label reconstruction, actual-residual correction certificates, stationary anchor, roots, and markings are

fixed. The candidate itself may still have nontrivial decorated automorphisms. Extraction and reconstruction preserve those automorphisms rather than quotienting them away. Hence no hidden anchor or root modulus is introduced by P4, but P4 does not claim trivial isotropy of the physical packet.

*Remark II.83* (Selected-branch firewall). The proof uses only the pre-incision P1 regional tensor packets, the P2 stationary suspension, the P3 finite defect calculus, carrier-complete dyadic refinement, and finite-dimensional tensor descent. It uses no obstruction carrier, incision, collar charge, boundary Ward character, entropy surgery, coercive constant, prescribed-incision Hamiltonian, thin limit, or spectral statement. It proves effectivity only on the selected generated branch; the general P4 theorem remains conditional on Hypothesis II.62 outside that branch.

## L. Fiberwise anchored lifting prestacks

P4 identifies the physically coherent anchored realization prestack with the zero locus of the pre-incision defect, under the Effective Anchored Descent package. The next operation is not another zero-locus construction. It is a homotopy fiber over the independently inherited regional time-zero branch. This distinction is essential: the defect tests physical coherence, whereas the homotopy fiber selects the prescribed time-zero realization type.

**Definition II.84** (Common pre-incision testing site). Let

$$\mathfrak{S}_{\text{pre}}^{\Gamma} = \left( \mathbf{Reg}_{\text{pre}}^{\Gamma, \text{cp}}(M), J_{\text{pre}}^{\Gamma} \right) \quad (\text{II.135})$$

be one fixed essentially small skeleton of the following common site. An object is a relatively compact ordinary Minkowski testing region belonging to every selected fixed-scale site used by the cofinal diagram and supporting every finite regional-chart-scale-Cauchy demand after a common admissible refinement. A cover is a finite compatible family which is a cover at every scale appearing in the relevant finite demand. No obstruction carrier, collar, tubular neighborhood, or incision-adapted atlas occurs in this definition.

The common site may be small. Its nonempty locus is determined below rather than presumed. All fixed-scale prestacks are restricted to  $\mathfrak{S}_{\text{pre}}^{\Gamma}$  before taking a homotopy limit.

**Definition II.85** (Prescribed marked time-zero branch). For each  $r \in \Gamma_{\text{cof}}$ , fix the inherited regional time-zero object

$$\mathfrak{h}_r^0 \in \mathbf{H}_r^{0,\text{phys}} \quad (\text{II.136})$$

carrying the complete-ground projection, physical Wilson channels, support filters, coefficient-line or correspondence data, roots, and the marked reconstruction branch  $\tau$ . For every selected scale arrow  $a : r \rightarrow s$ , fix the inherited marking-preserving comparison

$$\sigma_a^0 : \mathbb{T}_a^0 \mathfrak{h}_r^0 \xrightarrow{\cong} \mathfrak{h}_s^0 \quad (\text{II.137})$$

with the Paper-II scale compositor and rooted reassociation cells. These are part of the inherited marked branch; they are not selected from a later spacetime realization.

**Definition II.86** (Explicit fiberwise homotopy-fiber model). At fixed  $r$ , define

$$\mathfrak{L}_{r,\mathfrak{h}}^{\text{anch}} = \text{hofib}_{\mathfrak{h}_r^0} \left( \text{TZ}_r : \mathbf{CohReal}_r^{\text{anch}} \longrightarrow \mathbf{H}_r^{0,\text{phys}} \right). \quad (\text{II.138})$$

For  $O \in \mathfrak{S}_{\text{pre}}^\Gamma$ , its selected strict  $(2,1)$ -categorical model is as follows.

An object is a triple

$$\mathfrak{x} = (\mathfrak{R}, \mathfrak{c}, \vartheta), \quad \vartheta : \mathfrak{h}_r^0|_{B_\mathfrak{c}} \xrightarrow{\cong} \text{TZ}_r(\mathfrak{R}, \mathfrak{c}), \quad (\text{II.139})$$

where  $\mathfrak{R}$  is an object of  $\mathbf{CohReal}_r^{\text{anch}}(O)$ ,  $\mathfrak{c} = (N, S, \iota, B)$  is a selected admissible Cauchy presentation, and  $\vartheta$  is a marking-preserving anchor equivalence. The anchor preserves the full P2 list: physical quotient, represented action, complete-ground projection without a preferred vector, channels, Gram and recompression data, the declared core packet, coefficient data, roots, and  $\tau$ .

A one-morphism

$$(F, \lambda) : (\mathfrak{R}, \mathfrak{c}, \vartheta) \longrightarrow (\mathfrak{R}', \mathfrak{c}', \vartheta') \quad (\text{II.140})$$

consists of a marking-preserving equivalence  $F$  in the coherent realization prestack, including a selected Cauchy comparison when  $\mathfrak{c} \neq \mathfrak{c}'$ , and an invertible modification

$$\lambda : \text{TZ}_r(F) \circ \vartheta \Longrightarrow \vartheta'. \quad (\text{II.141})$$

A two-morphism  $\nu : (F, \lambda) \Rightarrow (G, \mu)$  is an invertible modification  $\nu : F \Rightarrow G$  satisfying

$$\mu \circ (\text{TZ}_r(\nu) * \vartheta) = \lambda. \quad (\text{II.142})$$

Composition is the ordinary pasted composition in the iso-comma  $(2, 1)$ -category. Thus the word “homotopy” is implemented by the anchor 2-cell and is not replaced by the strict equation  $\mathrm{TZ}_r(\mathfrak{R}) = \mathfrak{h}_r^0$ .

**Theorem II.87** (Fiberwise lifting is a two-prestack). *Assume P1–P4. The assignment  $O \mapsto \mathfrak{L}_{r, \mathfrak{h}}^{\mathrm{anch}}(O)$ , with the object, one-cell, and two-cell model in Definition II.86, is a contravariant two-prestack on  $\mathfrak{S}_{\mathrm{pre}}^\Gamma$ .*

*Proof.* Regional restriction in P4 carries a coherent realization on  $O$  to one on  $O' \Subset O$ . P2 regional pseudonaturality supplies the comparison between restriction followed by  $\mathrm{TZ}_r$  and  $\mathrm{TZ}_r$  followed by time-zero restriction. Whiskering  $\vartheta$  by this comparison defines the restricted anchor. The same pasting defines restriction of  $(F, \lambda)$  and  $\nu$ . The P1 regional compositors, P2 anchor comparison modifications, and P3–P4 relative naturality give the restriction compositor and its pentagon. Equation (II.142) is preserved because  $\mathrm{TZ}_r$  is a pseudofunctor. Hence the iso-comma model is a two-prestack.  $\square$

For diagnostic purposes set

$$M_{\mathrm{reg}}^r = \bigcup \left\{ O \in M : \mathfrak{L}_{r, \mathfrak{h}}^{\mathrm{anch}}(O) \neq \emptyset \right\}, \quad \Sigma_{\mathrm{obs}}^r = M \setminus M_{\mathrm{reg}}^r. \quad (\text{II.143})$$

These fixed-scale loci are diagnostic only. They are not used as the background on which the cofinal object is defined.

## M. The simultaneous cofinal anchored lifting prestack

**Definition II.88** (Scale transition on anchored homotopy fibers). Let  $a : r \rightarrow s$  be a selected scale arrow. P1 and P4 give a scale transport  $\mathbb{S}_a$  on coherent realization packets, while P2 gives an invertible scale comparison

$$\chi_a^{\mathrm{TZ}} : \mathbb{T}_a^0 \mathrm{TZ}_r \Longrightarrow \mathrm{TZ}_s \mathbb{S}_a. \quad (\text{II.144})$$

Define

$$a_* : \mathfrak{L}_{r, \mathfrak{h}}^{\mathrm{anch}} \longrightarrow \mathfrak{L}_{s, \mathfrak{h}}^{\mathrm{anch}} \quad (\text{II.145})$$

by

$$a_*(\mathfrak{R}, \mathfrak{c}, \vartheta) = \left( \mathbb{S}_a \mathfrak{R}, \mathfrak{c}_a, \chi_a^{\text{TZ}} \circ \mathbb{T}_a^0(\vartheta) \circ (\sigma_a^0)^{-1} \right), \quad (\text{II.146})$$

with the analogous whiskered action on one- and two-morphisms. The selected scale compositors, the P2 mixed comparison cube, and the rooted compositors of  $\sigma_a^0$  give invertible transformations

$$\kappa_{b,a} : b_* a_* \implies (ba)_*, \quad \kappa_r : \text{id} \implies (1_r)_*. \quad (\text{II.147})$$

**Proposition II.89** (Cofinal diagram of lifting prestacks). *The scale-transition functors and compositors define a pseudofunctor*

$$\mathfrak{L}_{-, \mathfrak{h}}^{\text{anch}} : \Gamma_{\text{cof}} \longrightarrow 2\text{PreStk} \left( \mathfrak{S}_{\text{pre}}^\Gamma \right). \quad (\text{II.148})$$

*The construction retains selected scale arrows, coefficient correspondences, analytic comparison cells, mixed regional-scale cells, and roots.*

*Proof.* For composable arrows  $a : r \rightarrow s$  and  $b : s \rightarrow t$ , the two anchors obtained by applying  $b_* a_*$  and  $(ba)_*$  are related by pasting the P1 scale compositors, the P2 scale pseudonaturality compositors, the mixed regional-scale comparison cell, and the rooted compositors for  $\sigma_b^0 \sigma_a^0$ . The equality of the two fourfold pastings is the selected mixed coherence cube already retained in P1–P4. This gives the pentagon for  $\kappa$ ; the unit triangle is analogous. Regional restriction commutes with these pastings by P2 and P4 relative naturality.  $\square$

**Definition II.90** (Explicit cofinal homotopy-limit model). Define

$$\boxed{\mathfrak{L}_{\Gamma, \mathfrak{h}}^{\text{anch}}(O) = \text{holim}_{r \in \Gamma_{\text{cof}}} \mathfrak{L}_{r, \mathfrak{h}}^{\text{anch}}(O)}. \quad (\text{II.149})$$

An object is a tuple

$$\mathfrak{X} = \left( \{\mathfrak{x}_r\}_r, \{\Phi_a : a_* \mathfrak{x}_r \xrightarrow{\cong} \mathfrak{x}_s\}_a, \{\Omega_{b,a}\}_{b,a} \right) \quad (\text{II.150})$$

where, for every composable pair  $r \xrightarrow{a} s \xrightarrow{b} t$ ,

$$\Omega_{b,a} : \Phi_b \circ b_*(\Phi_a) \implies \Phi_{ba} \circ \kappa_{b,a}(\mathfrak{x}_r) \quad (\text{II.151})$$

is invertible and the standard triple-arrow pentagon holds. Unit cells are included.

A one-morphism  $\mathfrak{F} : \mathfrak{X} \rightarrow \mathfrak{Y}$  is a family of one-morphisms  $F_r : \mathfrak{x}_r \rightarrow \mathfrak{y}_r$  together with invertible cells

$$\Lambda_a : \Psi_a \circ a_*(F_r) \Longrightarrow F_s \circ \Phi_a \quad (\text{II.152})$$

compatible with  $\Omega_{b,a}$ . A two-morphism is a family of modifications  $\nu_r : F_r \Rightarrow G_r$  satisfying the corresponding arrowwise square. This is the selected pseudocone model of the homotopy limit, not a set-theoretic intersection of  $\pi_0$ -solution sets.

**Theorem II.91** (The cofinal lifting object is a two-prestack). *Assume P1–P4. Equation (II.149), with Definition II.90, defines a contravariant two-prestack on  $\mathfrak{S}_{\text{pre}}^\Gamma$ . Evaluation at each scale gives a pseudonatural projection*

$$\text{ev}_r : \mathfrak{L}_{\Gamma, \mathfrak{h}}^{\text{anch}} \longrightarrow \mathfrak{L}_{r, \mathfrak{h}}^{\text{anch}}. \quad (\text{II.153})$$

*Proof.* Restriction is performed componentwise on  $\mathfrak{x}_r$ ,  $\Phi_a$ ,  $\Omega_{b,a}$ ,  $F_r$ , and their modifications. Proposition II.89 supplies the regional-scale interchange cells. All homotopy-limit equations are finite pasting equations and are preserved by restriction. The prestack compositor is obtained componentwise from Theorem II.87; its pentagon holds because it holds at every scale and on every selected scale arrow.  $\square$

## N. P5a: decorated admissible stackification

Ordinary stackification of the underlying groupoid-valued truncation is not the full claim needed here. The stackification must preserve the physical quotients, graph and Ward domains, admissible analytic-correction classes, coefficient correspondences, roots, and selected scale pseudocone cells.

**Hypothesis II.92** (P5a: Decorated Admissible Stackification package). The following conditions hold on  $\mathfrak{S}_{\text{pre}}^\Gamma$ .

- (DAS0) The site is essentially small, finite-cover generated, and every two selected covers admit a selected finite common refinement.
- (DAS1) The class of marking-preserving admissible equivalences contains identities, is closed under composition, inverse, whiskering, and regional restriction, and is local on source and target.

- (DAS2) Physical quotient algebras, represented Hilbert packets, complete-ground projections, support filters, core packets, Gauss forms, and Ward derivations satisfy effective finite descent whenever their comparison cells satisfy the P3–P4 coherence ledger.
- (DAS3) Admissible analytic-correction certificates form a stack of groupoids over the typed analytic-defect targets. Local correction certificates with compatible overlap modifications glue without requiring a zero representative.
- (DAS4) Coefficient lines or normal correspondences, multiplication maps, reassociators, mixed cells, roots, marked types, and P2 anchors satisfy effective finite bicategorical descent. Descent is allowed in a nonneutral band.
- (DAS5) Every analytic bound used in the local packets is cover-local and stable under the selected finite refinements. Stackification does not introduce a cofinal uniform bound not already present.
- (DAS6) The decorated descent construction is compatible with the scale-transition pseudofunctors  $a_*$ , their compositors, and the homotopy-limit pseudocone equations.

**Definition II.93** (Admissible plus construction). For a selected finite cover  $\mathcal{U} = \{U_i \rightarrow O\}$ , let

$$\mathrm{Desc}_{\mathcal{U}}^{\mathrm{adm}} \left( \mathfrak{L}_{\Gamma, \mathfrak{h}}^{\mathrm{anch}} \right) \quad (\text{II.154})$$

be the two-groupoid of local cofinal lift objects  $\mathfrak{X}_i$ , marking-preserving admissible equivalences on overlaps, invertible triple overlap modifications, and the quadruple coherence equation, with every physical, analytic, coefficient, root, and anchor decoration retained. Set

$$\mathfrak{L}_{\Gamma, \mathfrak{h}}^{\mathrm{anch}, +}(O) = 2 \operatorname{colim}_{\mathcal{U} \in \mathrm{Cov}_{\mathrm{adm}}(O)} \mathrm{Desc}_{\mathcal{U}}^{\mathrm{adm}} \left( \mathfrak{L}_{\Gamma, \mathfrak{h}}^{\mathrm{anch}} \right), \quad (\text{II.155})$$

where the transition functors are induced by selected common refinement. The admissible stackification is the separated effective completion of this plus construction, denoted

$$\left( \mathfrak{L}_{\Gamma, \mathfrak{h}}^{\mathrm{anch}} \right)^+. \quad (\text{II.156})$$

**Theorem II.94** (P5a decorated stackification). *Assume DAS0–DAS6. The prestack  $\mathfrak{L}_{\Gamma, \mathfrak{h}}^{\mathrm{anch}}$  admits an admissible stackification*

$$\mathfrak{j} : \mathfrak{L}_{\Gamma, \mathfrak{h}}^{\mathrm{anch}} \longrightarrow \left( \mathfrak{L}_{\Gamma, \mathfrak{h}}^{\mathrm{anch}} \right)^+, \quad (\text{II.157})$$

*which is locally fully faithful and locally essentially surjective. It is initial, up to equivalence, among marking-preserving pseudofunctors from the lifting prestack to decorated admissible two-stacks. Nonneutral coefficient bands and controlled admissible analytic defects are preserved.*

*Proof.* DAS0 makes the refinement two-category filtered at every finite stage. The first plus construction glues objects along admissible overlap equivalences; DAS1 guarantees closure of the morphism class. DAS2–DAS4 provide effectivity for every decoration appearing in a descent object. Therefore the local object, one-cell, and two-cell descent functors are essentially surjective and fully faithful after the separated completion. DAS5 preserves the finite analytic domains and estimates, while DAS6 transports the scale pseudocone cells and their pentagon. The usual refinement comparison supplies the universal factorization into any decorated stack; uniqueness follows from local full faithfulness. Since DAS3 glues correction certificates inside their admissible classes and DAS4 glues in the banded target, neither step forces strict analytic flatness or gerbe neutrality.  $\square$

## O. P5b: local connectedness

Stackification proves descent effectivity, not transitivity of the local realization groupoids. Local connectedness therefore requires an independent marked-comparison theorem.

**Hypothesis II.95** (P5b: Local Marked Transitivity package). For every  $O \in \mathfrak{S}_{\text{pre}}^\Gamma$  and every pair of objects  $\mathfrak{X}, \mathfrak{Y}$  of  $(\mathfrak{L}_{\Gamma, \mathfrak{h}}^{\text{anch}})^+(O)$ , the following hold whenever both objects are locally present.

(LMT0) The groupoid-valued presheaf

$$U \longmapsto \text{Eq}_U^{\tau, \text{anch}}(\mathfrak{X}|_U, \mathfrak{Y}|_U) \tag{II.158}$$

of marking-, root-, and anchor-preserving admissible equivalences is locally nonempty.

(LMT1) Its automorphism groups are the declared admissible band automorphisms; they act transitively on local equivalence choices.

(LMT2) Equivalence choices admit common finite regional and finite-shape refinements preserving all scale and mixed cells.

(LMT3) Local one-equivalences and their two-isomorphisms obey effective descent in the stackification of Theorem II.94.

(LMT4) The comparison never changes the marked reconstruction branch  $\tau$ , the prescribed time-zero roots, or the admissible analytic class.

**Theorem II.96** (P5b local connectedness). *Assume P5a and LMT0–LMT4. The stackification  $\left(\mathfrak{L}_{\Gamma, \mathfrak{h}}^{\text{anch}}\right)^+$  is locally connected by marking-preserving admissible equivalences on its locally nonempty locus.*

*Proof.* Given  $\mathfrak{X}$  and  $\mathfrak{Y}$ , LMT0 gives a finite selected cover on which the equivalence presheaf is inhabited. LMT1–LMT2 compare any two choices after common refinement, and LMT3 supplies the required overlap modifications and their effective descent. LMT4 keeps every equivalence inside the selected rooted marked component. This is precisely local connectedness of the decorated stack. No global equivalence on  $O$  is asserted.  $\square$

## P. P5c: simultaneous rooted lifting

Fiberwise nonemptiness does not imply cofinal nonemptiness. The obstruction is not merely set-theoretic: a cofinal lift must choose scale comparison one-cells and their rooted compositor modifications simultaneously. We adapt the directed compact finite-intersection mechanism of Paper II to selected presentation spaces containing the full lift data.

**Definition II.97** (Finite rooted lift demand). Fix  $U \in M$ . A finite rooted lift demand  $D$  consists of:

1. a finite set of selected scale objects and arrows, closed under every composition needed by the demand;
2. a finite admissible regional cover of  $U$  and a finite common chart refinement;
3. the selected Cauchy presentations and roots used on those charts;
4. every coefficient, analytic, regional–scale, chart–scale, and anchor face needed to state the pseudocone equations on that finite shape.

Demands are ordered by root-preserving common finite extension. Write  $\mathfrak{J}_{U,\text{fin}}^{\text{root}}$  for the resulting directed index category.

**Hypothesis II.98** (P5c: Simultaneous Rooted Lifting package). For every relatively compact

$$U \in \text{int} \left( \bigcap_{r \in \Gamma_{\text{cof}}} M_{\text{reg}}^r \right) \quad (\text{II.159})$$

which satisfies the declared finite compatibility condition, the following hold.

(SRL0) The category  $\mathfrak{J}_{U,\text{fin}}^{\text{root}}$  is directed under common finite regional-chart-scale extension and tests every finite part of  $\Gamma_{\text{cof}}$ .

(SRL1) For every demand  $D$ , there is a nonempty selected presentation space

$$\mathcal{X}_D^{\text{root}}(U) \quad (\text{II.160})$$

of finite effective anchored lifts. A point contains representatives of all objects, scale comparison one-cells, coefficient correspondences, admissible analytic-correction cells, mixed cells, roots, and all demanded coherence 2-cells. It is not merely an equivalence class of fiberwise objects.

(SRL2) Each  $\mathcal{X}_D^{\text{root}}(U)$  is compact Hausdorff in the selected finite-presentation topology.

(SRL3) For every inclusion  $D \subset D'$ , full restriction induces a coherent continuous map

$$\rho_{D',D} : \mathcal{X}_{D'}^{\text{root}}(U) \longrightarrow \mathcal{X}_D^{\text{root}}(U). \quad (\text{II.161})$$

(SRL4) Every map  $\rho_{D',D}$  is surjective. Extension preserves the prescribed root, branch  $\tau$ , complete-ground projections, coefficient band, admissible analytic classes, support controls, and all previously selected cells.

(SRL5) Finite local compatibility is effective: every finite family of fiberwise lifts and comparisons occurring in  $D$  admits, after a finite admissible regional and chart refinement of  $U$ , a point of  $\mathcal{X}_D^{\text{root}}(U)$ .

(SRL6) A compatible point of the inverse system  $\{\mathcal{X}_D^{\text{root}}(U), \rho_{D',D}\}$  determines an object of the homotopy-limit stackification on an admissible refinement of  $U$ . This is the selected-presentation recognition theorem for the 2-truncated pseudocone model.

(SRL7) The construction is relative under further regional restriction and common finite refinement; its constants are finite-demand constants only.

**Theorem II.99** (P5c simultaneous rooted-lifting theorem). *Assume SRL0–SRL7. For every  $U$  satisfying (II.159) and finite local compatibility, there exists an admissible cover  $\{U_i \rightarrow U\}$  such that*

$$\left(\mathfrak{L}_{\Gamma, \mathfrak{h}}^{\text{anch}}\right)^+(U_i) \neq \emptyset \quad (\text{II.162})$$

*for every  $i$ . The resulting objects preserve all selected scale arrows, coefficient data, analytic cells, mixed cells, roots, and the marked branch.*

*Proof.* Consider the compact product

$$K_U = \prod_{D \in \mathcal{I}_{U, \text{fin}}^{\text{root}}} \mathcal{X}_D^{\text{root}}(U), \quad (\text{II.163})$$

which is compact by SRL2. For every inclusion  $D \subset D'$ , let

$$C_{D', D} = \{(x_E)_E \in K_U : \rho_{D', D}(x_{D'}) = x_D\}. \quad (\text{II.164})$$

The set is closed because  $\rho_{D', D}$  is continuous and the target is Hausdorff.

The family  $\{C_{D', D}\}$  has the finite-intersection property. For a finite collection of bonding equations, SRL0 provides one common finite rooted demand  $E$  containing all shapes involved. SRL5 supplies a finite effective rooted lift on a common regional refinement, and SRL4 extends it whenever the common demand must be enlarged. Set every occurring coordinate equal to the restriction of the chosen point of  $\mathcal{X}_E^{\text{root}}(U)$ . The coherent restriction identities in SRL3 make all selected bonding equations hold. Remaining coordinates may be chosen because SRL1 makes every factor nonempty.

Compactness gives

$$\varprojlim_{D \in \mathcal{I}_{U, \text{fin}}^{\text{root}}} \mathcal{X}_D^{\text{root}}(U) \neq \emptyset. \quad (\text{II.165})$$

A point contains compatible representatives of every finite object, one-cell, and two-cell coordinate. SRL6 recognizes it as a pseudocone object of the cofinal homotopy-limit stack after an admissible regional refinement. SRL7 makes the construction local under further restriction. Since the root and marking are coordinates of every factor and are fixed by every bonding map, they survive the inverse limit.  $\square$

*Remark II.100* (Substantive content of P5c). Compactness is not the model-specific difficulty. SRL4–SRL6 are the serious claims: universal root-preserving finite extension, simultaneous compatibility of regional and scale data, and recognition of the inverse-limit point as an actual homotopy-limit object. Paper II proves the abstract finite-intersection mechanism but does not verify these three properties for the Paper-III spacetime lifting groupoids.

**Hypothesis II.101** (P5: consolidated admissible lifting package). Hypotheses II.92, II.95, and II.98 hold. Thus P5 is the conjunction of decorated admissible stackification, local marked transitivity, and simultaneous rooted lifting; none of these clauses is absorbed into the definition of the homotopy fiber or homotopy limit.

HYPOTHESIS / CONDITIONAL THEOREM PACKAGE

### Q. Fiberwise and cofinal realizability loci

Define

$$M_{\text{reg}}^{\Gamma_{\text{cof}}} = \bigcup \left\{ O \in M : \mathfrak{L}_{\Gamma, \mathfrak{h}}^{\text{anch}}(O) \neq \emptyset \right\}. \quad (\text{II.166})$$

Equivalently, the same open locus is obtained by asking that the stackification be locally nonempty, because every stackified object is locally represented by prestack objects under Theorem II.94. Set

$$\Sigma_{\text{obs}}^{\Gamma_{\text{cof}}} = M \setminus M_{\text{reg}}^{\Gamma_{\text{cof}}}. \quad (\text{II.167})$$

**Proposition II.102** (Comparison with fiberwise loci). *Assume P1–P4. Then*

$$M_{\text{reg}}^{\Gamma_{\text{cof}}} \subseteq \bigcap_{r \in \Gamma_{\text{cof}}} M_{\text{reg}}^r. \quad (\text{II.168})$$

*Assume in addition P5a and P5c, including the finite local compatibility condition on every relatively compact subregion of the interior. Then*

$$M_{\text{reg}}^{\Gamma_{\text{cof}}} = \text{int} \left( \bigcap_{r \in \Gamma_{\text{cof}}} M_{\text{reg}}^r \right). \quad (\text{II.169})$$

*Proof.* An object of the cofinal homotopy limit projects under (II.153) to a fiberwise object at every  $r$ , which proves (II.168). Since the left-hand side is open, it is contained in the interior of the intersection.

Conversely, let  $x$  lie in the interior on the right and choose a relatively compact testing region  $U$  around  $x$  with closure in that interior. The finite local compatibility clause supplies finite rooted lift presentations for every finite demand after admissible refinement. Theorem II.99 supplies a cover on which the cofinal stack is nonempty. By local representability of stackification, one may refine once more to regions carrying objects of the original cofinal lifting prestack. Hence  $x \in M_{\text{reg}}^{\Gamma_{\text{cof}}}$ .  $\square$

## R. The common cofinal obstruction carrier and lifting gerbe

**Theorem II.103** (Pre-incision habitat formation). *Assume P1–P4 and the cofinal lifting construction above. Then:*

1.  $M_{\text{reg}}^{\Gamma_{\text{cof}}}$  is open;
2.  $\Sigma_{\text{obs}}^{\Gamma_{\text{cof}}}$  is closed;
3.  $M_{\text{reg}}^{\Gamma_{\text{cof}}}$  is maximal among open subsets of  $M$  on which the selected marked branch is locally realizable by objects of the simultaneous cofinal lifting prestack; and
4.  $\Sigma_{\text{obs}}^{\Gamma_{\text{cof}}}$  is minimal among closed subsets  $K \subset M$  such that the same lifting problem is locally solvable on  $M \setminus K$ .

No P5c equality with the fiberwise intersection is needed for these four formal habitat statements.

*Proof.* The regular locus is a union of open testing regions, hence is open, and its complement is closed. If  $U \subset M$  is open and the simultaneous lifting problem is locally solvable on  $U$ , then every point of  $U$  lies in a testing region occurring in (II.166); therefore  $U \subseteq M_{\text{reg}}^{\Gamma_{\text{cof}}}$ . Taking complements gives minimality of the carrier.  $\square$

**Theorem II.104** (Admissible lifting gerbe on the regular habitat). *Assume P1–P5. Let*

$$\mathfrak{G}_{\Gamma, \mathfrak{h}} = \left( \mathfrak{L}_{\Gamma, \mathfrak{h}}^{\text{anch}} \right)^+ \Big|_{M_{\text{reg}}^{\Gamma_{\text{cof}}}}. \quad (\text{II.170})$$

*Then  $\mathfrak{G}_{\Gamma, \mathfrak{h}}$  is locally nonempty and locally connected by marking-preserving admissible equivalences. Hence it is an admissible lifting gerbe in the selected  $(2, 1)$ -categorical sense. Its*

*strict, controlled-scale-twisted, and fixed-thickness-gerbe-twisted objects retain the marked branch type  $\tau$ . The theorem does not assert neutrality or a global Hilbert-space trivialization.*

*Proof.* Local nonemptiness follows from the definition of the regular habitat and local essential surjectivity of the stackification map. Theorem II.96 gives local connectedness. DAS4, DAS6, and LMT4 preserve the coefficient band, roots, selected scale pseudocone, and marked branch. Therefore the resulting stack is gerbe-like without being forced to be neutral.  $\square$

*Remark II.105 (Exact claim boundary).* Theorems II.103 and II.104 do not prove that the carrier is nonempty, has a prescribed codimension, is stratified, carries finite defect energy or a Radon defect measure, admits collars, confines color, has scattering states, or produces a relativistic mass gap. Those are separate geometric, measure-theoretic, and dynamical questions.

*Remark II.106 (Circularity firewall for P5).* The homotopy fibers, cofinal homotopy limit, stackification, local transitivity, and compact rooted selection use only pre-incision regional physical packets, selected Cauchy anchors, the five pre-incision defect components, coefficient correspondences, admissible analytic classes, roots, and finite support controls. No obstruction collar, boundary Ward character, entropy surgery, post-incision normal-state calculus, invariant momentum, spectral threshold, mass estimate, confinement input, or scattering datum enters the construction of  $\Sigma_{\text{obs}}^{\Gamma_{\text{cof}}}$ .

**Proposition II.107** (Finite presentation witness for P5). *Suppose the common testing site is finite, every local realization and correspondence groupoid is finite, every analytic-correction groupoid is a finite nonempty groupoid closed under composition, and all restriction maps on rooted finite lift presentations are surjective. Then DAS0–DAS6, LMT0–LMT4, and SRL0–SRL7 hold. The resulting lifting gerbe may retain a nontrivial finite coefficient band.*

*Proof.* Finite groupoids have effective finite descent after the selected descent relations are imposed, proving DAS. The local equivalence groupoids are finite torsors under the declared automorphism bands, proving LMT. Finite presentation spaces are compact Hausdorff in the discrete topology; surjective restrictions, directed common extension, and finite-dimensional recognition give SRL. Nothing in these arguments trivializes the finite coefficient cocycle.  $\square$

*Remark II.108* (Status of the Yang–Mills witnesses). The Paper-I  $SU(2)$  package can seed finite strict presentation spaces after P1–P4. The selected Paper-III  $SU(3)$  realization provides post-incision evidence for nonneutral local descent and mixed regional–scale covariance. Neither witness proves universal root-preserving finite extension, compactness of the full pre-incision lift-presentation spaces, or the recognition theorem SRL6 over the full cofinal scale category.

**Corollary II.109** (P5 closure status). *The fiberwise homotopy fibers, scale diagram, cofinal homotopy limit, locus inclusion, and formal habitat theorem are constructed under P1–P4. P5a is conditionally proved under DAS0–DAS6; P5b is conditionally proved under LMT0–LMT4; P5c and the locus equality are conditionally proved under SRL0–SRL7. Model-specific Yang–Mills verification of these packages remains an explicit proof obligation.*

## S. Selected $SU(3)$ fixed-scale lifting stacks and cofinal section

The selected retained branch has two features that permit a direct P5 construction. First, the total persistent carrier forest is locally finite, so every relatively compact region meets only finitely many carrier control cubes over the entire dyadic scale spine. Second, the scale maps are the package-adapted isometries and complete-ground correspondences of P1. They are coherent one-cells, but they are not generally equivalences when the retained cutoff increases. The correct simultaneous object is therefore a normal correspondence-valued lax section of the selected scale diagram. An equivalence-valued homotopy-limit model would be too strong for this branch.

**Definition II.110** (Carrier-saturated selected topology). For  $B \in \mathbf{SpReg}^{0,\text{dyad}}$ , put

$$E_\infty(B) = \bigcup_{j \geq 2} E_j(B). \quad (\text{II.171})$$

A finite selected dyadic cover  $\mathcal{U} = \{B_i \hookrightarrow B\}$  is *carrier saturated* when

$$\forall \lambda \in E_\infty(B) \quad \exists i \quad \widehat{Q}_\lambda \subset B_i. \quad (\text{II.172})$$

The selected Grothendieck topology generated by such covers and their finite dyadic common refinements is denoted  $J_{SU(3)}^{\text{car}}$ .

At scale  $r_j$  and marked band  $m$ , let  $\mathfrak{r}_{B,j,m}^{\text{can}}$  be the P4 effective stationary lift of the canonical P1–P3 packet on  $B$ . Define

$$\mathfrak{L}_{j,m}^{\text{sel}}(B) \tag{II.173}$$

to be the homotopy-full  $(2, 1)$ -subcategory of the fixed-scale anchored homotopy fiber generated by  $\mathfrak{r}_{B,j,m}^{\text{can}}$  under marking-, root-, anchor-, ground-, and common-core-preserving decorated equivalences and invertible modifications.

**Lemma II.111** (Finite total carrier set and common carrier refinement). *For every relatively compact selected region  $B$ , the set  $E_\infty(B)$  is finite. Every finite family of carrier-saturated covers, regional inclusions, protected Cauchy presentations, and selected scale arrows admits a carrier-saturated finite common refinement.*

*Proof.* The family of fixed control cubes in the persistent carrier forest is locally finite. A compact set meeting infinitely many members of a locally finite family would possess a finite subcover by neighborhoods each meeting only finitely many members, a contradiction. Hence only finitely many control cubes meet  $\overline{B}$ , and  $E_\infty(B)$  is finite.

For finitely many covers, first take a common dyadic refinement. For each of the finitely many labels in  $E_\infty(B)$  choose one member of each original cover containing its fixed control cube whenever that cover is required on the corresponding region. The finite intersections of these chosen members have a selected dyadic protective refinement by Lemma II.10. Adding the ordinary dyadic refinement pieces away from the carrier cubes gives a finite common refinement that is again carrier saturated. Protected Cauchy presentations and finitely many scale arrows are incorporated using Lemma II.28.  $\square$

**Theorem II.112** (Selected fixed-scale anchored lifting stacks). *For every  $j \geq 2$  and  $m \in \mathbb{Z}_3$ , the assignment*

$$B \longmapsto \mathfrak{L}_{j,m}^{\text{sel}}(B) \tag{II.174}$$

*is a nonempty decorated two-stack on  $(\mathbf{SpReg}^{0,\text{dyad}}, J_{SU(3)}^{\text{car}})$ . Descent is effective on objects, one-morphisms, and two-morphisms and preserves the physical quotient, complete-ground projection, complementary corner, P3 common core, Gauss graph form, Ward domain, support filter, preform and actual-residual correction certificate, protected Cauchy anchor, coefficient line, normalized reassociator, selected root, and marked band.*

*Proof.* Nonemptiness is supplied on every  $B$  by  $\mathfrak{r}_{B,j,m}^{\text{can}}$ , obtained from P1, P2, P3, and the P4 reconstruction theorem.

Let  $\{B_i \rightarrow B\}$  be a carrier-saturated cover and let a compatible descent datum in  $\mathfrak{L}_{j,m}^{\text{sel}}$  be given. Every label  $\lambda \in E_j(B)$  has its entire fixed control cube in some  $B_i$ . Consequently the full one-label Hilbert packet, observable factor, complete-ground projection, common core, support tube, and all finite analytic and coefficient decorations are present in at least one local object. On overlaps, the P3 zero-defect equations and the P4 fully faithful extraction identify all occurrences of the same label. Taking the ordered tensor product over the finite set  $E_j(B)$  reconstructs the unique global object up to the canonical tensor reassociation.

The same argument applies to one-morphisms: overlap-compatible decorated correspondences have identical one-label components and therefore glue by ordered tensor product. Compatible invertible modifications glue labelwise at the two-morphism level. P4 proves that extraction and reconstruction are inverse on these object, one-cell, and two-cell data. Thus the descent functor is a biequivalence. The actual finite scale-form residuals are glued as their own admissible correction certificates; no nonzero residual is replaced by zero. All remaining decorations are tensor-local or constant under the P2 stationary suspension, and hence are retained exactly. Common-refinement independence follows from Lemma II.111 and Mac Lane coherence.  $\square$

**Definition II.113** (Normal correspondence-valued cofinal section). For  $j \leq k$ , let

$$\Phi_{kj}^{B,m} : \mathfrak{r}_{B,j,m}^{\text{can}} \longrightarrow \mathfrak{r}_{B,k,m}^{\text{can}} \quad (\text{II.175})$$

be the constant Cauchy suspension of the P1 decorated scale correspondence  $\mathbb{T}_{kj}^B$ , including the coefficient line  $\mathcal{L}_{kj}^{(m)}$ , the actual finite analytic residual, and the P1–P3 mixed cells. For  $j \leq k \leq \ell$ , let

$$\Omega_{\ell kj}^{B,m} : \Phi_{\ell k}^{B,m} \circ \Phi_{kj}^{B,m} \Longrightarrow \Phi_{\ell j}^{B,m} \quad (\text{II.176})$$

be the tensor compositor decorated by the normalized cyclic reassociator  $\Omega_m$ . Unit cells are the selected rooted identity cells.

Let  $\mathfrak{L}_{\Gamma,m}^{\text{sel,lax}}(B)$  be the  $(2,1)$ -category of normal marking-preserving correspondence-valued lax sections of the fixed-scale diagram  $j \mapsto \mathfrak{L}_{j,m}^{\text{sel}}(B)$ , with section one-morphisms and

modifications defined by the usual invertible pasting cells. The scale structure one-cells  $\Phi_{kj}$  themselves are not required to be equivalences.

**Theorem II.114** (Selected cofinal pseudocone and simultaneous rooted lifting). *For every selected region  $B$  and every  $m \in \mathbb{Z}_3$ , the tuple*

$$\mathfrak{X}_{B,m}^{\text{can}} = \left( \{\mathfrak{r}_{B,j,m}^{\text{can}}\}_{j \geq 2}, \{\Phi_{kj}^{B,m}\}_{j \leq k}, \{\Omega_{\ell kj}^{B,m}\}_{j \leq k \leq \ell} \right) \quad (\text{II.177})$$

*is an object of  $\mathfrak{L}_{\Gamma,m}^{\text{sel,lax}}(B)$ . It is pseudonatural under regional restriction, protected Cauchy comparison, selected chart refinement, and every admitted mixed regional–Cauchy–scale cell.*

*Moreover,*

$$B \longmapsto \mathfrak{L}_{\Gamma,m}^{\text{sel,lax}}(B) \quad (\text{II.178})$$

*is a nonempty decorated two-stack on the carrier-saturated selected topology. The canonical section preserves the selected root, marked band, coefficient lines, complete-ground projections, common cores, Gauss–Ward packet, support controls, and actual finite analytic correction cells at every scale. No compact inverse-limit selection theorem is required.*

*Proof.* For  $j \leq k \leq \ell$ , the selected one-cell isometry classes obey  $U_{\ell k} U_{kj} = U_{\ell j}$ , persistent labels obey  $\iota_{\ell k}^B \iota_{kj}^B = \iota_{\ell j}^B$ , and newly appearing labels are anchored in their complete-ground ranges. The tensor compositor therefore identifies the two composite physical correspondences. Its coefficient face is  $\Omega_m$ , whose normalized three-cocycle identity is exactly the triple-arrow pentagon. The P1 Beck–Chevalley cells, P2 stationary anchor cells, and P3 mixed naturality prove regional, chart, protected-Cauchy, and mixed pseudonaturality. Thus  $\mathfrak{X}_{B,m}^{\text{can}}$  is a normal lax section.

For stack descent, apply Theorem II.112 at every scale. A carrier-saturated cover sees every label in the finite set  $E_\infty(B)$ . Hence all scale one-cells and all compositor two-cells are reconstructed labelwise from their local restrictions. The P3 mixed-face ledger makes the reconstructed scale cells independent of the cover and compatible with the fixed-scale gluing. Object, one-morphism, and two-morphism descent is therefore effective for the section bicategory.

The simultaneous object is explicit at every scale and on every selected arrow. Every finite rooted demand is its restriction, and enlarging the demand uses the already fixed cells

$\Phi_{kj}^{B,m}$  and  $\Omega_{\ell kj}^{B,m}$ . Thus no compactness, surjective inverse system, or choice of new roots is needed. The construction uses finite-demand analytic constants only and makes no assertion of a uniform bound on a thinning tail.  $\square$

**Corollary II.115** (Selected P5 closure and derived regular locus). *On the selected retained pre-incision branch, obligations P5.2, P5.3, P5.5, and P5.9 are discharged in the correspondence-valued sense appropriate to the actual scale maps. Together with the previously verified finite local lift, branch dependence, and preservation clauses, the selected fixed-scale lifting stacks and simultaneous cofinal section are explicitly inhabited.*

*Every selected relatively compact Minkowski testing region admits the P2 stationary suspension of the canonical cofinal section. Consequently the regular locus of this selected retained branch is*

$$M_{\text{reg}, SU(3), m}^{\text{stat}} = M, \quad \Sigma_{\text{obs}, SU(3), m}^{\text{stat}} = \emptyset. \quad (\text{II.179})$$

*This is a derived conclusion, not an imposed incision.*

*Proof.* The P1 regional packet and P2 stationary suspension exist on every selected basis region, and Theorem II.114 supplies the simultaneous rooted scale section there. The protected selected basis covers every relatively compact Minkowski testing region by Lemma II.28. Hence the local realizability locus is all of  $M$ , and its complement is empty.  $\square$

*Remark II.116* (The empty carrier is a branch-separation theorem). Equation (II.179) shows that the retained stationary pre-incision branch cannot, by itself, generate the nonempty physical incision used by the linked-flux comparison realization. The formal marked coefficient band is compatible everywhere on this stationary branch. A nonempty dynamical obstruction carrier can arise only after an additional theorem connects the pre-incision lifting problem to the intended nonstationary linked-flux, Gauss–Ward, and analytic realization constraints. Thus P5.6 remains open for the intended physical branch even though it is trivially resolved by the empty carrier on the selected stationary witness.

*Remark II.117* (Scale arrows are correspondences, not equivalences). When  $L_k > L_j$ , the package-adapted map  $U_{kj} : \mathcal{K}_j \rightarrow \mathcal{K}_k$  need not be surjective. Therefore the selected cofinal structure must not be rewritten as a family of invertible scale identifications. The normal

lax-section theorem retains the actual scale correspondences and their invertible compositor two-cells. On a branch whose scale transports are equivalences, this model reduces to the usual pseudocone model of a homotopy limit.

*Remark II.118* (Selected P5 firewall). The proof uses only P1 persistent carriers and scale correspondences, the P2 stationary anchor, P3 naturality, P4 effectivity, and finite carrier-saturated descent. It uses no obstruction collar, prescribed incision, linked-flux Hamiltonian, boundary Ward character, entropy surgery, coercive constant, thin limit, covariance, spectrum, confinement, or scattering statement. Full multiparameter thinning-tail uniformity remains a Paper-IV obligation.

### Selected physical branch matching and the source-free dynamical carrier

The stationary P1–P5 construction proves that the selected front-end architecture is inhabited. It does not, merely by carrying a label  $m \in \mathbb{Z}_3$ , become the nonstationary linked-flux Yang–Mills branch. The physical problem must therefore be constructed separately, and its source-free vacuum subbranch must be distinguished from a broader candidate stack that permits primitive defect currents.

**Definition II.119** (Three-sector firewall). At a selected scale  $r$ , the branch label has the following three meanings.

1. *Sector I (strict/untwisted)*. The coefficient reassociator and all refinement holonomies are trivial.
2. *Sector II (abstract cyclic or intermediate twisted witness)*. A nonzero cyclic coefficient law  $\Omega_m$ ,  $m \in \{1, 2\}$ , is retained as marked descent data without a theorem identifying it with the holonomy of a linked physical flux system.
3. *Sector III (physical linked-flux scalar-twisted branch)*. The scalar class is selected by linked Wilson–’t Hooft center-flux data and coexists with the physical quotient, complete-ground corner, nonstationary Cauchy evolution, Gauss graph forms, Ward insertions, and the declared analytic controls.

The stationary P1–P5 object with formal label  $m$  is not identified with Sector III.

**Definition II.120** (Nonstationary physical regional packet). Let  $O \in M$  belong to the ordinary pre-incision testing site of Definition II.84. A nonstationary physical packet at scale  $r$  and linked type  $m \in \{1, 2\}$  is the typed datum

$$\begin{aligned} \mathfrak{P}_{r,m}^{\text{phys}}(O) = & (\mathfrak{W}_r^{\text{raw}}(O), \mathfrak{J}_r^{\text{phys}}(O), \mathfrak{W}_r^{\text{phys}}(O), \mathcal{H}_r(O), \pi_r(O), P_{\Omega,r}(O), Q_{\Omega,r}(O), \\ & \mathcal{D}_r(O), \mathfrak{C}_r(O), \mathcal{U}_r(O), \mathfrak{G}_r(O), \mathfrak{W}_r^{\text{ins}}(O), \mathfrak{D}_r^{\text{W}}(O), \\ & \mathfrak{F}_{r,m}(O), \mathfrak{A}_r(O), \mathfrak{R}_{r,m}(O), \mathfrak{M}_{r,m}(O)). \end{aligned} \quad (\text{II.180})$$

Here:

1.  $\mathfrak{W}_r^{\text{phys}}(O)$  is the represented quotient of the raw Wilson algebra by a restriction-stable physical ideal;
2.  $P_{\Omega,r}(O)$  is the complete-ground projection and  $Q_{\Omega,r}(O) = I - P_{\Omega,r}(O)$ ;
3.  $\mathcal{D}_r(O)$  is one invariant common core for the represented algebra, preform or Hamiltonian, Gauss generators, Ward insertions, and every admitted Cauchy propagator;
4.  $\mathcal{U}_r(O)$  contains only the selected propagation-adapted nonstationary Cauchy frames and their reversible evolution maps;
5.  $\mathfrak{G}_r(O)$  and  $(\mathfrak{W}_r^{\text{ins}}(O), \mathfrak{D}_r^{\text{W}}(O))$  retain the Gauss graph and Ward-domain data, with interior and possible collar terms typed separately;
6.  $\mathfrak{F}_{r,m}(O)$  contains the linked center-flux detector, coefficient line or normal correspondence, multiplication maps, triality marking, and reassociator  $\Omega_m$ ;
7.  $\mathfrak{A}_r(O)$  contains the actual fixed-positive-thickness form, entropy, Orlicz, stiffness, and comparison certificates invoked at this stage;
8.  $\mathfrak{R}_{r,m}(O)$  contains roots, markings, anchors, tensorators, and reassociators; and
9.  $\mathfrak{M}_{r,m}(O)$  contains regional, scale, chart, and Cauchy correspondences together with all required mixed cells. Scale arrows are not assumed invertible.

No datum constructed only on an already incised spacetime enters this packet without an explicit ordinary-region extension theorem.

**Definition II.121** (Physical lifting two-stack and intrinsic carrier). Let  $\mathbf{Phys}_{r,m}(O)$  be the bicategory of the preceding packets, marking-preserving normal correspondences and isometries, and compatible bounded intertwiners. Let  $\mathbf{TZ}_{r,m}^{\text{phys}}(O)$  be the linked-flux time-zero branch obtained by selected Cauchy extraction. The fixed-scale anchored lifting prestack is

$$\mathfrak{L}_{r,m}^{\text{phys}}(O) := \text{hofib}_{\mathcal{D}_{r,m}^{\text{phys},0}(O)} \left( \mathbf{Phys}_{r,m}(O) \xrightarrow{\text{Cau}_{r,m}^{\text{phys}}} \mathbf{TZ}_{r,m}^{\text{phys}}(O) \right). \quad (\text{II.181})$$

For a selected cofinal tail  $\Gamma' \subseteq \Gamma_{\text{cof}}$ , put

$$\mathfrak{L}_{\Gamma',m}^{\text{phys}}(O) := \text{LaxSect}_{\Gamma'} \left( \mathfrak{L}_{-,m}^{\text{phys}}(O) \right), \quad (\text{II.182})$$

and denote its decorated admissible stackification by  $\mathfrak{L}_{\Gamma',m}^{\text{phys},\#}(O)$ . Its objects are rooted cofinal families in which the linked flux, physical quotient, complete-ground transport, Cauchy dynamics, Gauss–Ward packet, analytic controls, and mixed cells coexist on one common core.

Define

$$M_{\text{reg},m}^{\text{phys}} := \{x \in M : \text{some } O \ni x \text{ and cofinal tail } \Gamma_x \text{ carry an object of } \mathfrak{L}_{\Gamma_x,m}^{\text{phys},\#}(O)\}, \quad (\text{II.183})$$

$$\Sigma_{\text{obs},m}^{\text{phys}} := M \setminus M_{\text{reg},m}^{\text{phys}}. \quad (\text{II.184})$$

**Lemma II.122** (Restriction stability). *For every admitted restriction  $V \Subset O$ , restriction of all packet components, anchors, scale correspondences, and mixed cells defines*

$$\text{res}_V^O : \mathfrak{L}_{\Gamma',m}^{\text{phys},\#}(O) \longrightarrow \mathfrak{L}_{\Gamma',m}^{\text{phys},\#}(V). \quad (\text{II.185})$$

*Consequently  $M_{\text{reg},m}^{\text{phys}}$  is open and  $\Sigma_{\text{obs},m}^{\text{phys}}$  is closed.*

*Proof.* The quotient ideal, represented algebra, complete-ground corner, common core, short-ened Cauchy slabs, Gauss graph forms, Ward domains, linked coefficient data, and fixed-stage analytic certificates all restrict rather than extend. The retained Beck–Chevalley cells commute restriction with scale and Cauchy transport up to the specified invertible two-cells. Thus restriction sends a rooted cofinal lax section to one on  $V$ . Openness and closedness follow from the neighborhood definition.  $\square$

**Proposition II.123** (Typed comparison of the three branches). *The selected stationary P1–P5 stack and the intended physical stack are distinct. They admit decoration-forgetting pseudofunctors to a common bare shadow retaining only the represented quotient, Hilbert packet, complete-ground corner, common core, ordinary preform, and regional restrictions:*

$$\mathfrak{L}_{\Gamma,m}^{\text{stat},\#} \longrightarrow \mathbf{Bare}_{\Gamma}, \quad \mathfrak{L}_{\Gamma,m}^{\text{phys},\#} \longrightarrow \mathbf{Bare}_{\Gamma}. \quad (\text{II.186})$$

*No equivalence between the two branches is asserted.*

*Let*

$$\Sigma_{\text{pres}} = \{(t, 0, 0, z) : t, z \in \mathbb{R}\}. \quad (\text{II.187})$$

*On every ordinary region  $O$  with  $\overline{O} \subset M \setminus \Sigma_{\text{pres}}$ , the explicit prescribed-incision witness embeds at fixed scale fully faithfully into the physical lifting stack and, across selected scales, by a normal correspondence-valued lax transformation. Hence*

$$\Sigma_{\text{obs}}^{\text{phys}} \subseteq \Sigma_{\text{pres}}. \quad (\text{II.188})$$

*Proof.* The first assertion follows because stationarity forgets precisely the nonstationary linked-flux provenance and Cauchy–Gauss–Ward compatibility required in Definition II.120. For the second, compact disjointness from  $\Sigma_{\text{pres}}$  gives positive clearance. On a late selected tail, every tube, collar, propagation enlargement, and chart footprint remains inside the ordinary region. The same operators, quotient, projection, common core, Cauchy unitaries, Gauss–Ward data, linked coefficient lines, and analytic certificates therefore define an ordinary-region packet. No operator is extended through the worldsheet. This proves local realizability off  $\Sigma_{\text{pres}}$  and the inclusion.  $\square$

**Definition II.124** (Transversely source-free refinement). The full decorated sub-two-stack  $\mathfrak{L}_{\Gamma',m}^{\text{phys,sf},\#}(O)$  consists of physical packets satisfying, near every  $p \in O \cap \Sigma_{\text{pres}}$ , a rooted diagonal meridional certificate with the following properties:

1. the absolutely continuous transverse currents obey one common superlinear de la Vallée–Poussin/Orlicz bound;
2. the worldsheet-supported singular modulus tends to zero; and

3. the joint Gauss, Ward, selected-Cauchy, scale, and mixed-cell remainder modulus tends to zero.

On regions disjoint from  $\Sigma_{\text{pres}}$  this condition is vacuous. The inclusion into the ambient physical-candidate stack is locally fully faithful, not an equivalence.

**Lemma II.125** (Diagonal removable-singularity estimate). *Let  $D_r$  be transverse disks with radius  $\rho_r \downarrow 0$ . Suppose a source-free certificate has matrix-element current densities  $j_{r;\phi,\psi}$  satisfying one uniform Orlicz bound*

$$\sup_r \sup_{\substack{\phi,\psi \in P_{\Omega,r} \mathcal{D}_r \\ \|\phi\|, \|\psi\| \leq 1}} \|j_{r;\phi,\psi}\|_{L^\Phi(D_0)} < \infty \quad (\text{II.189})$$

*for a Young function with  $\Phi(s)/s \rightarrow \infty$ , while the singular and typed remainder moduli vanish. Then the compressed meridional defect satisfies*

$$\Delta_r^{\text{mer}}(\gamma_r) \longrightarrow 0. \quad (\text{II.190})$$

*Proof.* Orlicz–Hölder gives

$$\left| \int_{D_r} j_{r;\phi,\psi} dA \right| \leq 2 \|j_{r;\phi,\psi}\|_{L^\Phi(D_0)} \|1_{D_r}\|_{L^\Psi(D_0)}, \quad (\text{II.191})$$

where  $\Psi$  is complementary to  $\Phi$ . Since  $|D_r| \rightarrow 0$ ,  $\|1_{D_r}\|_{L^\Psi} \rightarrow 0$ . The singular and Gauss–Ward–Cauchy–scale remainders vanish by hypothesis, so the complete-ground Stokes–Ward identity forces the meridional defect to zero.  $\square$

*Remark II.126* (Why fixed-scale smoothness is insufficient). The cofinal condition cannot be replaced by scale-by-scale smoothness. If  $\eta \in C_c^\infty(\mathbb{R}^2)$  has integral one and  $a_r \downarrow 0$ , then

$$j_r^{(m)}(u) = c_m a_r^{-2} \eta(u/a_r), \quad c_m = \omega^m - 1, \quad (\text{II.192})$$

is smooth at each scale, has fixed total variation  $|c_m| = \sqrt{3}$ , and converges weakly to  $c_m \delta_0$ . Thus exact fixed-stage Gauss and Ward identities with smooth currents do not imply meridional removability; a cofinal concentration-exclusion theorem is indispensable.

**Definition II.127** (Selected admissible vacuum Yang–Mills branch). The ambient physical-candidate stack of Definition II.121 may contain primitive collar or worldsheet source slots. Its selected vacuum Yang–Mills substack  $\mathfrak{L}_{\Gamma',m}^{\text{YM,vac},\#}(O)$  is the full decorated sub-two-stack for which:

1. the Gauss and Ward equations have no external collar, boundary, or worldsheet source on the common core;
2. every collar term is induced only by restriction of the interior source-free field and has zero primitive source class;
3. regional, scale, Cauchy, and mixed transport preserve the zero-source condition;
4. the cofinal transverse certificate of Definition II.124 holds; and
5. the inherited branch marking retains the nonzero controlled Sector-II strictification defect or the nonzero fixed-thickness Sector-III linked-flux detector, as applicable.

The inclusion into the ambient stack is locally fully faithful and is not asserted essentially surjective.

*Remark II.128* (Meaning of source free). Source-freeness removes an independent right-hand side  $J_\Sigma$  from the Yang–Mills/Gauss–Ward equations. It does not remove nonabelian self-interaction, curvature, center flux, coefficient twisting, roots, or reassociators. Source-freeness alone also does not exclude concentration; that is the role of the cofinal transverse certificate.

**Definition II.129** (Admissible point relaxation). Let  $p \in \Sigma_{\text{pres}}$ . An admissible point relaxation is a nested cap-complete marked Čech/refinement family on transverse disks contracting to  $p$ , with all regional and scale comparisons remaining admissible and with no primitive source or defect coefficient on the terminal point cell. The complete-ground corner, common core, root, and branch marking are preserved until that terminal cell.

**Lemma II.130** (Source-free filling produces point relaxation). *If an object of  $\mathfrak{L}_{\Gamma',m}^{\text{YM,vac},\#}(O)$  exists on an ordinary neighborhood meeting  $\Sigma_{\text{pres}}$ , then on a selected cofinal tail it determines an admissible point relaxation.*

*Proof.* Choose nested transverse disks and cap-complete finite marked covers. P4 effectivity reconstructs one regional object for each finite demand, while P5 rooted refinement supplies compatible comparison one-cells and compositor modifications on a common tail. The vacuum condition excludes primitive collar current and worldsheet measure, and its cofinal Orlicz and vanishing-remainder certificate invokes Lemma II.125. The meridional defect

therefore tends to zero. The terminal point cell has contractible corrected-cell nerve and the flat point channel. These compatible reconstructions form the required rooted relaxation. No scale correspondence is inverted.  $\square$

**Theorem II.131** (Source-free magnetic nonrelaxation). *The selected Paper-I/Paper-II twisted branches admit no admissible point relaxation.*

1. *In Sector II, such a relaxation would supply a uniformly local, band-preserving, scale-natural strictifying primitive, contradicting the retained nonzero controlled strictification defect.*

2. *In Sector III, such a relaxation would carry the fixed-thickness linked detector*

$$I_3(\Omega_m) = \omega^m \neq 1, \quad m \in \{1, 2\}, \quad (\text{II.193})$$

*into the trivial point channel. Equivalently it would contradict the retained positive detected magnetic certificate*

$$\overline{\mathfrak{B}}_\infty^x \geq a_\chi p_{\text{col}} \delta_\chi > 0. \quad (\text{II.194})$$

*Proof.* P4 effectivity realizes coherent twisted data but cannot provide the forbidden scale-natural strictification in Sector II. In Sector III, every admitted regional restriction, rooted refinement, selected Cauchy map, and scale correspondence preserves the linked detector. The terminal point cell has no meridional cycle and detector one. A marking-preserving relaxation would therefore identify  $\omega^m$  with 1, or drive the positive detected magnetic budget to zero, both impossible.  $\square$

**Interpretive guide.** The upper carrier inclusion says that the explicit linked-flux packet is realizable away from the prescribed worldsheet. The source-free filling lemma and magnetic nonrelaxation theorem supply the reverse implication: a vacuum lift through the worldsheet would force the retained detector into the flat point channel. The next theorem combines these two directions and is the precise point at which the incision becomes derived geometry rather than an imposed background choice.

**Theorem II.132** (Persistent linked-flux obstruction, derived carrier, and dynamical incision). *For the selected admissible source-free linked-flux  $SU(3)$  branch, the frustum/linked-flux*

witness and the cofinal concentration-exclusion estimate prove that the Sector-III obstruction cannot be removed by an admissible point relaxation. Its derived failure carrier is therefore the prescribed worldsheet, and

$$\boxed{M_{\text{reg}}^{\text{YM,vac}} = M \setminus \Sigma_{\text{pres}}, \quad \Sigma_{\text{obs}}^{\text{YM,vac}} = \Sigma_{\text{pres}}.} \quad (\text{II.195})$$

Consequently the selected incised spacetime is defined only after carrier identification by

$$M_{\text{inc}}^{\text{YM,vac}} := M \setminus \Sigma_{\text{obs}}^{\text{YM,vac}}. \quad (\text{II.196})$$

The stationary  $P1$ – $P5$  branch and the source-free Sector-III branch are distinct and comparable only by the typed pseudofunctors, embeddings, and correspondence-valued lax transformations above. No noninvertible correspondence is called an equivalence.

*Proof.* Every region compactly contained in  $M \setminus \Sigma_{\text{pres}}$  carries the ordinary-region extension of the explicit linked-flux witness. The source-free condition is vacuous there, so  $\Sigma_{\text{obs}}^{\text{YM,vac}} \subseteq \Sigma_{\text{pres}}$ . Conversely, a vacuum lift on a neighborhood meeting  $\Sigma_{\text{pres}}$  would yield an admissible point relaxation by Lemma II.130, contradicting Theorem II.131. This proves the reverse inclusion and equality. Equation (II.196) is then a definition made from the proved carrier, not an input to the nonrelaxation argument.  $\square$

*Remark II.133* (The incision is derived, not regulatory). The incision is not a cutoff, ultraviolet regulator, infrared regulator, boundary condition inserted to improve estimates, or geometric device chosen to force solvability. The pre-incision source-free lifting problem is posed first on ordinary Minkowski testing regions. The frustum/linked-flux witness, the retained nonneutral gerbe coherence, and the concentration-exclusion argument prove persistence of the obstruction and identify its failure carrier. Only after that identification is the complement  $M_{\text{inc}}^{\text{YM,vac}}$  formed. The incision therefore records a dynamically derived obstruction locus; it does not manufacture the obstruction or the positive lower bound.

**What has now been established.** The physical regular locus and obstruction carrier are now identified by a two-sided argument: explicit realization off the worldsheet and prohibited source-free relaxation through it. This is stronger than observing a nontrivial cocycle. The proved carrier equality is what licenses the formation of the incised spacetime and the transfer of the tubular detector, Whitney control, and finite-atlas data to the selected vacuum branch.

**Corollary II.134** (H1 and H2 propagation on the selected vacuum branch). *For the selected vacuum branch, the detector*

$$\delta(t, x, y, z) = (x, y) \quad (\text{II.197})$$

*and the existing prescribed-carrier tubular data are data of the dynamically derived carrier itself. Hence the smooth codimension-two stratum, Whitney–b regularity, normal tube, radial retraction, finite adapted atlas, and the already verified fixed-stage geometric margins satisfy the selected H1 and H2 obligations.*

*Proof.* The carrier equality identifies the two closed subsets, and Proposition II.296 supplies the explicit detector and controlled geometry. No data are transferred merely by analogy.  $\square$

*Remark II.135* (Claim boundary and next edge). The physical-branch and carrier subproblem has Outcome A, while the manuscript as a whole remains Outcome B. The theorem does not identify the stationary and physical dynamics, does not infer nonsolvability from twisting alone, and does not use post-incision existence as a proof of pre-incision failure. It asserts no arbitrary-Cauchy reconstruction, thin-limit uniformity, Haag–Kastler net, covariance, spectrum, no-leakage theorem, or mass gap. The selected physical H3 theorem below closes the simultaneous fixed-demand margin problem at the actual positive thicknesses. The next exact proof edge is controlled incised-category recovery on the selected physical basis.

## T. H1: regular support of the dynamically derived carrier

For the remainder of the article set

$$\boxed{\Sigma := \Sigma_{\text{obs}}^{\Gamma_{\text{cof}}}, \quad M_{\text{inc}} := M \setminus \Sigma = M_{\text{reg}}^{\Gamma_{\text{cof}}}.} \quad (\text{II.198})$$

Theorem II.103 proves that  $\Sigma$  is closed and minimal for the selected simultaneous lifting problem. Neither assertion implies that  $\Sigma$  is a submanifold, a stratified set, a tame set, or the zero set of a finite-dimensional section. The purpose of H1 is to state a checkable representability criterion under which the closed carrier acquires exactly the stratified tubular geometry needed later.

*Remark II.136* (The P3 defect is not yet a spacetime support map). The pre-incision defect  $\mathbf{O}_{r,\text{pre}}$  is defined on the two-groupoid of local realization candidates. Even after P4 exactness,

it is not a map  $M \rightarrow E$  into a finite-dimensional vector space, nor does it canonically assign a numerical defect to each spacetime point. Passing from the lifting prestack to a finite-dimensional or tame discriminant on spacetime is a new localization and representability statement. H1 records that statement without introducing a defect measure, a spectral parameter, or a post-incision collar construction.

**Definition II.137** (Faithful local support detector). Let  $U \Subset M$  be an ordinary pre-incision testing neighborhood. A *faithful local support detector* on  $U$  is a tuple

$$\mathfrak{d}_U = (Y_U, \mathscr{Y}_U, \Delta_U, \delta_U) \quad (\text{II.199})$$

with the following properties.

1.  $Y_U$  is a finite-dimensional smooth manifold, possibly with corners.
2.  $\mathscr{Y}_U$  is a locally finite Whitney–b stratification of  $Y_U$ .
3.  $\Delta_U \subseteq Y_U$  is a closed union of strata of  $\mathscr{Y}_U$ , called the local nonsolvability discriminant.
4.  $\delta_U : U \rightarrow Y_U$  is continuous and smooth on the presentation branch used below.
5. The detector is faithful in the germwise sense:

$$x \in \Sigma \cap U \iff \delta_U(x) \in \Delta_U. \quad (\text{II.200})$$

Thus the open set  $U \setminus \Sigma$  is the inverse image of  $Y_U \setminus \Delta_U$ . Equation (II.200) is a support-detection statement about local solvability. It does not assert that a representative of any controlled analytic defect vanishes.

**Definition II.138** (Finite joint detector). For a finite family  $I = \{U_1, \dots, U_m\}$  with nonempty overlap  $U_I = \bigcap_{i=1}^m U_i$ , define

$$Y_I = \prod_{i=1}^m Y_{U_i}, \quad \Delta_I = \prod_{i=1}^m \Delta_{U_i}, \quad \delta_I = (\delta_{U_1}, \dots, \delta_{U_m}). \quad (\text{II.201})$$

Faithfulness of the individual detectors gives

$$\Sigma \cap U_I = \delta_I^{-1}(\Delta_I). \quad (\text{II.202})$$

The product stratification of  $Y_I$  is denoted  $\mathscr{Y}_I$ .

**Definition II.139** (Controlled obstruction neighborhood and collar). Let  $\mathcal{S}_\Sigma$  be a locally finite Whitney stratification of the closed set  $\Sigma$ . Thom–Mather control data consist, for every stratum  $S \in \mathcal{S}_\Sigma$ , of a tube  $T_S$ , a smooth retraction  $\pi_S : T_S \rightarrow S$ , and a nonnegative control function  $\rho_S : T_S \rightarrow [0, \infty)$  with  $S = \rho_S^{-1}(0)$ , satisfying on incident tubes the control identities

$$\pi_S \pi_T = \pi_S, \quad \rho_S \pi_T = \rho_S \quad \text{whenever } S < T. \quad (\text{II.203})$$

For a compact stage  $K \Subset M$ , choose a finite control-radius vector  $\epsilon = (\epsilon_S)_{S \cap K \neq \emptyset}$  and let  $N_{K,\epsilon}(\Sigma)$  be the associated controlled regular neighborhood obtained by deleting the iterated inequalities  $\rho_S < \epsilon_S$  in increasing order of stratum depth. Its frontier in  $M \setminus \Sigma$  is

$$C_{K,\epsilon} = \text{Fr } N_{K,\epsilon}(\Sigma) \cap (M \setminus \Sigma). \quad (\text{II.204})$$

A *compact-stage obstruction collar* is a stratum-preserving embedding

$$\mathbf{c}_{K,\epsilon} : C_{K,\epsilon} \times [0, \eta) \longrightarrow M \setminus \Sigma, \quad \mathbf{c}_{K,\epsilon}(z, 0) = z, \quad (\text{II.205})$$

whose image is a neighborhood of the frontier on the exterior side. The frontier is allowed to be a stratified manifold with corners. This geometric collar is not yet a gauge collar, a boundary-charge space, or a Ward boundary domain.

**Hypothesis II.140** (H1: Regular-Support Presentation package). The dynamically derived carrier  $\Sigma$  admits a locally finite detector atlas  $\{(U_\alpha, \mathfrak{d}_\alpha)\}_{\alpha \in A}$  satisfying the following conditions. (RSC0) Faithful local support detection. The sets  $U_\alpha$  form a locally finite cover of a neighborhood of  $\Sigma$ , and every detector satisfies

$$\Sigma \cap U_\alpha = \delta_\alpha^{-1}(\Delta_\alpha). \quad (\text{II.206})$$

Outside that neighborhood the carrier is empty and no detector is required.

(RSC1) Finite-dimensional discriminants. Each target is finite-dimensional, each  $\Delta_\alpha$  is closed, and the target stratification is locally finite and Whitney–b. The discriminant may have strata of different codimensions. No pure-codimension condition is assumed.

(RSC2) Regular finite joint presentation. For every finite overlap  $U_I$ , one of the following two branches holds after possibly shrinking the overlap.

(T) The smooth joint detector  $\delta_I$  is transverse to every product stratum of  $\mathcal{Y}_I$ .

(A) The set  $\delta_I^{-1}(\Delta_I)$  and the graphs of the detector maps are locally subanalytic, or definable in one fixed o-minimal structure compatible with the smooth charts on  $M$ .

Different compact stages may use different branches, but every finite mixed family must admit one common branch after a selected shrinking.

(RSC3) Joint refinement compatibility. The pullback strata produced on a finite overlap are compatible with the individual detector stratifications: their connected components give a common locally finite refinement, the frontier relation is preserved, and restriction to a smaller overlap is refinement rather than a change of the detected carrier.

(RSC4) Compact-stage finiteness. For every compact  $K \subseteq M$ , only finitely many detector neighborhoods and only finitely many resulting strata meet  $K$ . The local presentation maps and target control data are bounded on the compact detector footprints used at that stage.

(RSC5) Comparison stability. Every admitted pre-incision regional restriction acts by restriction of the detector atlas. Every declared smooth background comparison which is used before the post-incision analysis carries detector presentations to a common finite joint presentation by a stratified target isomorphism or a selected Whitney common refinement. In particular it preserves  $\Sigma$  and its frontier relation.

(RSC6) Clean control radii on compact stages. After compatible Thom–Mather tubes are chosen, every compact stage admits a strictly depth-decreasing radius vector  $\epsilon$  such that all restricted control functions meet their selected levels transversely on every incident face. Two radius vectors and two compact stages admit a componentwise smaller common refinement.

(RSC7) Pre-incision dependence only. The detector atlas is constructed from the pre-incision lifting problem, its support filters, finite candidate presentation spaces, admissible analytic classes, and their local solvability germs. It is independent of obstruction collars, boundary Ward characters, entropy surgery, normal-state selection, invariant momentum, spectral thresholds, mass estimates, confinement input, and scattering input.

*Remark II.141* (What H1 actually assumes). RSC0 is the substantive localization statement: it represents failure of the simultaneous lifting problem by a finite-dimensional or tame

discriminant over spacetime. RSC2 is the regularity statement on that representation. The remaining clauses ensure that local stratifications glue, remain finite on the stages used in the manuscript, and carry compatible control neighborhoods. None of these clauses follows from the fact that  $\Sigma$  is closed.

**Theorem II.142** (Regular-support criterion for the derived carrier). *Assume Hypothesis II.140. Then the dynamically derived carrier  $\Sigma$  admits a locally finite Whitney- $b$  stratification*

$$\mathcal{S}_\Sigma = \{S_\lambda\}_{\lambda \in \Lambda} \quad (\text{II.207})$$

*with the following properties.*

1. *Every  $S_\lambda$  is a locally closed smooth embedded submanifold of  $M$ , and the frontier condition holds.*
2. *The stratification admits compatible Thom–Mather control data  $(T_S, \pi_S, \rho_S)$  satisfying (II.203).*
3. *Every compact stage  $K \Subset M$  admits a controlled regular neighborhood and a compact-stage obstruction collar (II.205).*
4. *For two compact stages, two detector presentations, or two choices of control radii, there is a common shrinking on which the resulting control data and collars are related by a control-preserving stratified isotopy.*
5. *Admitted comparisons covered by RSC5 preserve the stratification up to the selected common refinement and transport the control data after shrinking.*

*Consequently the geometric content formerly stated as “regularity of the derived obstruction carrier” follows from the Regular-Support Presentation package. The theorem does not assert that  $\Sigma$  is nonempty, has a fixed codimension, or is a smooth hypersurface.*

*Proof.* We divide the construction into six steps.

*Step 1: local strata in the transverse branch.* Fix a finite overlap  $U_I$  satisfying RSC2(T). For a product stratum  $Q = Q_1 \times \cdots \times Q_m$  of  $\mathcal{B}_I$ , transversality gives a smooth embedded

inverse-image stratum

$$S_Q = \delta_I^{-1}(Q) \subset U_I \quad (\text{II.208})$$

with

$$T_x S_Q = (D\delta_I(x))^{-1} (T_{\delta_I(x)} Q). \quad (\text{II.209})$$

The frontier relation is pulled back from the product target. To verify Whitney- $b$ , take strata  $Q < R$ , sequences  $x_n \in S_R$ ,  $y_n \in S_Q$  converging to  $y \in S_Q$ , and suppose the secant lines and tangent spaces converge. In local coordinates transverse to  $Q$ , the constant-rank splitting of  $D\delta_I$  identifies the normal behavior of the pullback pair with the normal behavior of  $(Q, R)$ , together with a fixed kernel factor. The Whitney- $b$  inclusion for  $(Q, R)$  therefore pulls back to the required inclusion for  $(S_Q, S_R)$ . Thus the connected components of the inverse images of the strata contained in  $\Delta_I$  form a Whitney- $b$  stratification of  $\Sigma \cap U_I = \delta_I^{-1}(\Delta_I)$ .

*Step 2: local strata in the tame branch.* On an overlap satisfying RSC2(A), the carrier is a closed locally subanalytic or definable subset of the smooth finite-dimensional manifold  $U_I$ . Finite tame families admit a Whitney- $b$  stratification simultaneously compatible with every member of the family, its boundary, and the graphs of the detector maps. Apply this to the finite family consisting of  $\delta_J^{-1}(\Delta_J)$  for  $J \subseteq I$  and to the detector graphs. The resulting source stratification is adapted to  $\Sigma \cap U_I$  and makes each detector map stratified after refinement. Local finiteness follows from tameness and the finite character of the family.

*Step 3: compatibility on overlaps.* By RSC3, the connected pullback strata from Steps 1 and 2 admit a common Whitney refinement on every finite overlap. Because the detector cover is locally finite, every point has a neighborhood meeting only finitely many presentations. Choose the common finite refinement there. On a smaller neighborhood the choice agrees with the refinement already selected on each sub-overlap. A locally finite induction over a countable exhaustion of the second-countable manifold  $M$  glues these refinements to a global locally finite Whitney- $b$  stratification  $\mathcal{S}_\Sigma$ . RSC4 guarantees that only finitely many modifications occur on each compact stage. This proves item 1.

*Step 4: compatible control data.* A locally finite Whitney- $b$  stratification of a closed subset of a smooth manifold admits Thom-Mather control data. For completeness, order the strata by depth and choose ordinary smooth tubular neighborhoods for the minimal strata. Apply

the controlled-tube extension step inductively: after the tubes for all lower-depth incident strata have been fixed, shrink the tube of the new stratum and adjust its retraction only outside smaller protected subtubes so that on every incident older tube

$$\pi_S \pi_T = \pi_S, \quad \rho_S \pi_T = \rho_S. \quad (\text{II.210})$$

Whitney- $b$  regularity is exactly the tangent-limit condition used in this extension step. Work along a locally finite exhaustion of the stratified set; at the next compact stage keep all control data fixed on a smaller neighborhood of the preceding stage. Local finiteness ensures that each point is changed only finitely many times, so the inductive limit is smooth and retains every previously imposed control identity. This proves item 2.

*Step 5: compact-stage regular neighborhoods and collars.* Fix  $K \Subset M$ . By RSC4, only finitely many strata meet a compact neighborhood of  $K$ . RSC6 permits the control radii to be chosen successively by depth so that every level  $\rho_S = \epsilon_S$  is transverse to all previously created incidence faces. The controlled deletion neighborhood  $N_{K,\epsilon}(\Sigma)$  then has frontier a finite union of smooth faces meeting cleanly. Hence  $C_{K,\epsilon}$  is a compact stratified manifold with corners. Choose inward-pointing vector fields on its faces, compatible on their intersections, and average them using a partition of unity constant along the control retractions. Their flow gives  $\mathbf{c}_{K,\epsilon}$  in (II.205). The flow preserves the incidence strata and the control projections after a further shrinking. This proves item 3.

*Step 6: independence and comparison stability.* Given two finite detector refinements or two radius vectors, RSC3 and RSC6 supply a common detector refinement and a componentwise smaller clean radius vector. The straight deformation between the two regular-neighborhood boundaries, performed one stratum depth at a time, has no tangency with the control levels. Isotopy extension therefore produces a control-preserving stratified isotopy between the two collars after shrinking. The same argument applies to nested compact stages. Finally, RSC5 transports the detector presentation through admitted comparisons; applying the preceding construction to the joint source and target presentation yields the selected common refinement and transported control data. This proves items 4 and 5.  $\square$

**Proposition II.143** (Smooth transverse-discriminant criterion). *Suppose a locally finite*

cover  $\{U_\alpha\}$  carries smooth maps

$$\delta_\alpha : U_\alpha \longrightarrow Y_\alpha \tag{II.211}$$

into finite-dimensional Whitney-stratified targets with closed discriminants  $\Delta_\alpha$ , and suppose:

1.  $\Sigma \cap U_\alpha = \delta_\alpha^{-1}(\Delta_\alpha)$ ;
2. every finite joint map is transverse to every product stratum;
3. the detector data are stable under the admitted comparisons; and
4. the compact-stage clean-radius condition RSC6 holds.

Then Hypothesis II.140 holds in branch RSC2(T), and Theorem II.142 applies.

*Proof.* RSC0–RSC1 are the first assumption and the target data. Joint transversality provides RSC2(T), and the inverse images of product strata give the common refinements required by RSC3. Local finiteness of the cover and target stratifications implies compact-stage finiteness. The last two assumptions are RSC5–RSC6. RSC7 is verified by auditing the provenance of the maps  $\delta_\alpha$ . The result follows from Theorem II.142.  $\square$

**Proposition II.144** (Tame discriminant criterion). *Assume that  $M$ , the detector maps, their graphs, and the discriminants are real analytic and locally subanalytic, or definable in one fixed o-minimal structure. Assume faithful detection, local finiteness, comparison stability, and RSC6. Then Hypothesis II.140 holds in branch RSC2(A).*

*Proof.* Finite products, inverse images, finite unions, finite intersections, and graph constructions preserve the chosen tame category. Hence every finite joint carrier  $\delta_I^{-1}(\Delta_I)$  is tame. A finite family of tame subsets admits a common Whitney– $b$  stratification compatible with all frontier relations and detector graphs. These stratifications are locally finite and refine under restriction. Thus RSC2(A)–RSC4 hold, and the remaining clauses are the stated assumptions.  $\square$

**Proposition II.145** (Finite-dimensional reduction criterion). *Suppose the local lifting obstruction is initially presented by a smooth Fredholm or tame-Fredholm section on a Banach manifold of candidate germs. Assume that near every point of  $M$  it admits a support-faithful*

*finite-dimensional reduction*

$$(X_\alpha, E_\alpha, s_\alpha, p_\alpha), \quad p_\alpha : X_\alpha \longrightarrow U_\alpha, \quad (\text{II.212})$$

where  $p_\alpha$  is proper on compact stages and the projected nonsolvability discriminant is either a transverse smooth discriminant or a tame discriminant satisfying Proposition II.143 or Proposition II.144. Then H1 follows.

*Proof.* The finite-dimensional reduction produces the faithful detector atlas required by RSC0. Properness on compact stages prevents new support components from escaping the reduced presentation and preserves local finiteness under projection. The reduced discriminant satisfies one of the two preceding criteria, so Theorem II.142 applies.  $\square$

*Remark II.146* (No automatic finite-dimensional reduction). P1–P5 do not prove the hypotheses of Proposition II.145. In particular, compactness of the finite rooted presentation spaces in P5c is a selection statement over finite demand shapes; it is not a Fredholm theorem and does not make the spacetime support of nonsolvability subanalytic. A Yang–Mills proof of H1 must construct the detector or the finite-dimensional reduction and prove its faithfulness.

**Proposition II.147** (Elementary inhabitation witness). *Let  $E \rightarrow M$  be a finite-rank smooth vector bundle, let  $s : M \rightarrow E$  be a smooth section transverse to the zero section, and define a toy lifting problem whose local realizability locus is  $M \setminus s^{-1}(0)$ . Then*

$$\Sigma = s^{-1}(0) \quad (\text{II.213})$$

*satisfies H1. More generally, the same holds for a smooth map transverse to a closed Whitney-stratified discriminant.*

*Proof.* Take  $Y = E$  in a local trivialization,  $\Delta$  equal to the zero section, and  $\delta = s$ . Transversality gives RSC2(T), and the remaining clauses are immediate on compact stages. The general statement is Proposition II.143.  $\square$

*Remark II.148* (Boundary between H1, H2, and H3). H1 produces a locally finite Whitney carrier, compatible tubes, control functions, and compact-stage geometric collars. It does not yet select a finite obstruction-adapted atlas for a prescribed finite family; that is H2. It also

does not supply positive obstruction, chart, spacelike, or cofinal-thickening margins; those are H3. The collar in H1 is only the geometric collar of a controlled regular neighborhood. Boundary gauge parameters, collar charges, full Ward characters, and entropy surgery remain post-H1 constructions.

*Remark II.149* (Status of the retained witnesses). The Paper-I SU(2) package contains no spacetime carrier and does not address H1. The retained Paper-III SU(3) branch is constructed on an already selected regular incised geometry; it shows that the downstream analysis can inhabit a regular carrier, but it neither derives  $\Sigma_{\text{obs}}^{\Gamma_{\text{cof}}}$  nor proves faithful detector representability for that dynamically defined carrier.

*Remark II.150* (Circularity firewall for H1). The H1 detector may use only pre-incision solvability germs, finite candidate presentation data, support filters, roots, coefficient correspondences, and admissible analytic classes. It may not use a collar charge or boundary Ward character to define the carrier whose collar geometry is being constructed. It also may not use entropy surgery, post-incision normal states, a closed spacetime form, invariant momentum, a spectral threshold, a mass estimate, confinement, or scattering. Controlled nonzero analytic defects and a nonneutral coefficient gerbe remain compatible with H1.

**Corollary II.151** (H1 closure status). *The geometric regularity conclusion of the former H1 is conditionally proved by Theorem II.142 under RSC0–RSC7. Closedness and minimality of the dynamically derived carrier are already proved in P5, but do not imply RSC0–RSC7. Model-specific Yang–Mills construction of a faithful transverse, tame, or finite-dimensionally reduced support detector remains an explicit proof obligation.*

## U. H2: finite obstruction-adapted atlases and common refinements

Hypothesis II.140 produces a locally finite Whitney– $b$  stratification of the dynamically derived carrier, compatible Thom–Mather control data, and compact-stage geometric collars. These data are local and locally finite. The later regional, chart, support, and Cauchy constructions are instead performed on finite test stages and require one finite chart system carrying all selected decorations simultaneously. The passage from local control data to such a finite chart system is the content of H2.

A minor but important point concerns Cauchy coordinates. There is no reason for one coordinate map to make every selected temporal function and every stratified control function into coordinate projections simultaneously. Accordingly, the basic H2 object is a *multi-presentation chart packet*: one underlying precompact patch carries one obstruction-control presentation and, separately, the finitely many temporal coordinate presentations needed for the selected Cauchy slabs meeting that patch. Refinement retains all of these presentations and their transition maps. No artificial transversality among unrelated temporal functions is imposed.

**Definition II.152** (Finite obstruction-control demand). A *finite obstruction-control demand* is a tuple

$$\mathfrak{F} = (\mathcal{O}_{\mathfrak{F}}, \mathcal{K}_{\mathfrak{F}}, \mathcal{C}_{\mathfrak{F}}, \mathcal{N}_{\mathfrak{F}}, \mathcal{D}_{\mathfrak{F}}) \quad (\text{II.214})$$

with the following entries.

1.  $\mathcal{O}_{\mathfrak{F}}$  is a finite family of relatively compact regular regions  $O \in M_{\text{inc}}$ , together with any finite family of already selected regional restrictions among them.
2.  $\mathcal{K}_{\mathfrak{F}}$  is a finite family of compact skeletal or thick support carriers, compact graph-core support footprints, and compact overlap carriers occurring in the selected finite test stage.
3.  $\mathcal{C}_{\mathfrak{F}}$  is a finite family of compact pieces of the H1 controlled-neighborhood frontiers and compact pieces of the exterior geometric collars. These are geometric footprints only; no gauge collar, boundary charge, or Ward boundary domain is included.
4.  $\mathcal{N}_{\mathfrak{F}}$  is a finite family of selected admissible Cauchy presentations  $\mathfrak{c} = (N, S, \iota, B)$ , compact Cauchy subslabs inside their comparison regions, and compact source or target footprints used by the P2 comparison groupoid.
5.  $\mathcal{D}_{\mathfrak{F}}$  is a finite diagram of admitted regional, chart, selected-Cauchy, selected-scale, and declared smooth background comparisons whose geometric footprints are to be represented in the same finite chart system.

For each compact footprint in the demand, a prescribed open neighborhood may be included as part of the datum. Let  $\mathcal{K}_{\mathfrak{F}}$  be the finite collection of all listed compact closures and

compact footprints. The total footprint

$$K_{\mathfrak{F}} = \bigcup_{K \in \mathcal{K}_{\mathfrak{F}}} K \quad (\text{II.215})$$

is therefore compact.

**Definition II.153** (Adapted chart packet). Fix a finite demand  $\mathfrak{F}$ . A chart packet subordinate to it is a tuple

$$\mathfrak{a} = (\tilde{U}, U, \varphi, \mathcal{T}_U, \mathcal{P}_U, \mathcal{Q}_U) \quad (\text{II.216})$$

with the following properties.

1.  $U \Subset \tilde{U} \Subset M$  are ordinary smooth open sets, and  $\varphi : \tilde{U} \rightarrow V \subset \mathbb{R}^4$  is a smooth coordinate chart. The closure of  $\tilde{U}$  lies in every prescribed open neighborhood to which the packet is declared subordinate.
2.  $\mathcal{T}_U$  is the finite list of H1 strata and Thom–Mather tubes meeting  $\tilde{U}$ , together with the restricted maps

$$\pi_S|_{\tilde{U} \cap T_S}, \quad \rho_S|_{\tilde{U} \cap T_S}. \quad (\text{II.217})$$

Every incident pair satisfies the H1 control identities on the packet. No requirement is made that all  $\pi_S$  and  $\rho_S$  be coordinate projections of the single map  $\varphi$ .

3.  $\mathcal{P}_U$  is the finite list of region, support, collar-frontier, and overlap footprints from  $\mathfrak{F}$  meeting  $U$ , with their inclusion and support labels retained.
4. For each selected Cauchy presentation or comparison slab  $\mathfrak{c} \in \mathcal{N}_{\mathfrak{F}}$  meeting  $U$ , the packet  $\mathcal{Q}_U$  contains a separate temporal coordinate presentation

$$\psi_{\mathfrak{c}, U} : \tilde{U} \cap N_{\mathfrak{c}} \longrightarrow I_{\mathfrak{c}, U} \times V_{\mathfrak{c}, U} \quad (\text{II.218})$$

on the relevant connected component, with

$$\tau_{\mathfrak{c}} = \text{pr}_1 \circ \psi_{\mathfrak{c}, U}, \quad S_t \cap \tilde{U} = \psi_{\mathfrak{c}, U}^{-1}(\{t\} \times V_{\mathfrak{c}, U}). \quad (\text{II.219})$$

The temporal presentations are stored as separate decorations on the same underlying patch. They need not coincide with  $\varphi$  or with one another.

5. Every arrow of the finite diagram  $\mathcal{D}_{\mathfrak{F}}$  whose source and target meet the packet is represented by the appropriate restricted geometric map, together with the H1 common-refinement data and the P2 Cauchy-comparison cells already selected for that arrow.

**Definition II.154** (Finite adapted atlas and refinement). A *finite obstruction-adapted atlas* for  $\mathfrak{F}$  is a finite family

$$\mathcal{A}_{\mathfrak{F}} = \{\mathfrak{a}_i\}_{i=1}^N \quad (\text{II.220})$$

of chart packets whose inner domains cover  $K_{\mathfrak{F}}$ , and such that every nonempty finite overlap occurring on the demand footprint is itself covered by finitely many chart packets obtained by restriction and retaining all control, support, temporal, and comparison decorations.

Let  $\mathcal{A}$  and  $\mathcal{B}$  be finite adapted atlases for the same demand. A refinement  $\mathcal{B} \preceq \mathcal{A}$  assigns to each packet of  $\mathcal{B}$  a packet of  $\mathcal{A}$  whose outer domain contains it, and requires:

1. restriction of all H1 strata, tubes, retractions, and control functions;
2. restriction of all support and collar-footprint labels;
3. restriction of every temporal presentation, with the smooth coordinate transition map displayed when two presentations of the same Cauchy slab are compared;
4. restriction of every arrow and two-cell in the finite comparison diagram.

A *common refinement* of  $\mathcal{A}$  and  $\mathcal{A}'$  is a finite adapted atlas refining both.

**Hypothesis II.155** (H2: Finite Adapted-Chart Compatibility package). The H1 control system and the selected P2 Cauchy-presentation system satisfy the following finite-stage compatibility conditions.

(FAC0) Compact finite demands. Every geometric stage used later in the manuscript can be represented by a finite obstruction-control demand in the sense of Definition II.152. In particular, only compact pieces of regions, supports, collar frontiers, and Cauchy slabs occur in one demand.

(FAC1) Local H1 control presentation. For every point of the total footprint and every finite family of H1 tubes meeting that point, there is an arbitrarily small precompact smooth chart on which all restricted control data are defined and continue to satisfy (II.203). The chart may be chosen subordinate to any finite family of prescribed open neighborhoods.

(FAC2) Selected temporal product presentations. Every selected finite Cauchy slab in the P2 comparison system is equipped with a smooth temporal function and admits arbitrarily small precompact product charts satisfying (II.219). Restriction of a selected Cauchy presentation restricts these temporal presentations.

(FAC3) Multi-presentation retention. When finitely many H1 control charts and finitely many selected temporal presentations meet the same point, their domains admit a common arbitrarily small precompact subdomain on which all presentations can be retained as separate decorations. No simultaneous-coordinate normal form is required.

(FAC4) Local comparison closure. A finite family of regional restrictions, H1 detector refinements, P2 Cauchy comparisons, and selected chart transitions restricts to every sufficiently small common subdomain and retains its declared composition and two-cell laws. The allowed chart-packet morphisms are closed under restriction and composition.

(FAC5) Finite background transport. For every finite diagram of admitted smooth background comparisons, the H1 control presentations and the selected temporal presentations transport to the target footprints up to the common refinements already allowed by RSC5 and the P2 comparison groupoid. No chart comparison is inserted for an unlisted background map.

(FAC6) Qualitative, not quantitative, subordination. The nested containments  $U \Subset \tilde{U}$  and subordination to prescribed neighborhoods are required, but H2 does not select one positive obstruction clearance, chart margin, spacelike margin, or thickening radius uniform over the demand. Those numerical choices belong to H3.

*Remark II.156* (Which H2 clauses are genuinely new). FAC1 is supplied by the H1 locally finite control system after shrinking. FAC2 is a selection requirement on the P2 finite Cauchy slabs. FAC3 is formal once chart packets are allowed to carry several coordinate presentations on one underlying domain. FAC4 and FAC5 record the finite restriction and comparison closure already required in P1, P2, and H1. Thus the substantive H2 burden is not a new global atlas theorem; it is the availability of the selected temporal product presentations and their retention under the admitted finite comparison system.

**Proposition II.157** (Local obstruction-Cauchy chart packet). *Assume H1 and FAC1–FAC4. Let  $x \in K_{\mathfrak{F}}$ , and let  $\mathcal{U}_x$  be any finite family of prescribed open neighborhoods of  $x$  occurring*

in the demand. Then there exists an obstruction-adapted multi-presentation chart packet

$$\mathfrak{a}_x = (\tilde{U}_x, U_x, \varphi_x, \mathcal{I}_x, \mathcal{P}_x, \mathcal{Q}_x) \quad (\text{II.221})$$

with

$$x \in U_x \subseteq \tilde{U}_x \subseteq \bigcap_{W \in \mathcal{U}_x} W. \quad (\text{II.222})$$

*Proof.* Local finiteness of the H1 stratification gives a neighborhood of  $x$  meeting only finitely many strata and tubes. FAC1 supplies a precompact smooth chart on a smaller neighborhood on which all restricted H1 control data remain defined. Only finitely many support, collar, and Cauchy footprints of  $\mathfrak{F}$  meet  $x$ . For each relevant selected Cauchy slab, FAC2 supplies a temporal product chart. Intersect the finitely many outer domains and apply FAC3 to retain the resulting coordinate presentations as separate decorations on one smaller precompact domain. FAC4 restricts every comparison arrow and two-cell to that domain. A final shrinking gives the nested pair in (II.222).  $\square$

**Theorem II.158** (H2 finite obstruction-adapted atlas theorem). *Assume P2, H1, and Hypothesis II.155. Every finite obstruction-control demand  $\mathfrak{F}$  admits a finite obstruction-adapted atlas  $\mathcal{A}_{\mathfrak{F}}$  with the following properties.*

1. *The inner chart domains cover the total footprint  $K_{\mathfrak{F}}$ , and every outer chart closure lies in all prescribed adapted neighborhoods attached to that footprint.*
2. *Each packet meets only finitely many H1 strata and carries the complete restricted Thom–Mather control data for those strata.*
3. *Every region, support carrier, overlap carrier, H1 collar footprint, selected Cauchy slab, and selected Cauchy source or target footprint is represented by finitely many packets.*
4. *Every nonempty finite overlap on the demand footprint admits a finite common adapted subcover retaining all control, support, temporal, and comparison decorations.*
5. *The finite regional, chart, Cauchy, scale, and admitted background diagram  $\mathcal{D}_{\mathfrak{F}}$  is represented by a finite refinement diagram whose ordinary transition maps satisfy their cocycle laws and whose retained P2 comparison cells satisfy their selected bicategorical coherence laws.*

6. Any two finite obstruction-adapted atlases for  $\mathfrak{F}$  admit a third finite obstruction-adapted atlas refining both.

Consequently the refinement preorder of atlas-equivalence classes

$$\pi_0 \text{Atl}_{\Sigma}^{\text{adm}}(\mathfrak{F}) \tag{II.223}$$

is nonempty and directed: every finite family of classes has a common refinement class. No equalizer statement for arbitrary parallel atlas morphisms is asserted.

*Proof.* We divide the proof into five steps.

*Step 1: finite cover of the footprint.* For every  $x \in K_{\mathfrak{F}}$ , apply Proposition II.157 with the prescribed open neighborhoods attached to the finitely many demand objects containing  $x$ . The inner domains form an open cover of the compact set  $K_{\mathfrak{F}}$ . Choose a finite subcover  $\{U_i\}_{i=1}^N$ . The corresponding packets form a finite family satisfying items 1–3.

*Step 2: finite overlap closure.* There are only finitely many nonempty intersections of the selected outer domains. On each compact portion of such an intersection occurring in the demand, apply Proposition II.157 with all parent chart domains included among the prescribed neighborhoods. Compactness gives a finite adapted subcover of that overlap. Add these packets to the atlas. Repeating this for the finitely many overlap depths appearing in the finite comparison diagram proves item 4. Restriction of the H1 controls preserves (II.203); restriction of the P2 temporal and comparison packets preserves the selected cocycle and compositor laws.

*Step 3: representation of the finite comparison diagram.* For each arrow of  $\mathcal{D}_{\mathfrak{F}}$ , add the finitely many source, target, and common-comparison packets required by FAC4. Since the arrow set is finite, the enlarged atlas remains finite. Scale arrows do not create an unlisted spacetime map; they retain the selected support and chart labels on the packets already representing their finite geometric footprint. FAC5 handles the finitely many admitted background arrows by transporting the packets and then taking the selected common refinements. This proves item 5.

*Step 4: common refinement of two atlases.* Let  $\mathcal{A}$  and  $\mathcal{A}'$  be finite adapted atlases for the same demand. For every pair of packets with nonempty intersection on  $K_{\mathfrak{F}}$ , cover the relevant

compact part of the intersection by local packets subordinate to both parent outer domains. The union over the finitely many pairs has a finite subcover. Add the finite overlap packets required for the comparison diagram as in Steps 2 and 3. The resulting atlas refines both  $\mathcal{A}$  and  $\mathcal{A}'$ , proving item 6.

*Step 5: directedness of atlas classes.* Nonemptiness is Step 1. Step 4 gives a common refinement for every pair of atlases, and induction gives one for every finite family. Passing to atlas-equivalence classes yields the directed preorder in (II.223).  $\square$

**Corollary II.159** (Finite chart independence). *Any construction on a finite demand which is natural under the H1 control restrictions, the P2 selected Cauchy comparisons, and the refinement maps of Definition II.154 is independent of the chosen H2 atlas up to its declared equivalence after passage to a finite common refinement.*

*Proof.* Choose a common refinement by Theorem II.158. Naturality identifies the two restricted constructions on that refinement. The refinement cocycle gives independence under a third choice.  $\square$

### 1. *Finite Čech nerves and refinement-controlled passage*

The finite overlap closure built into Definition II.154 and Theorem II.158 has a simplicial form. Two levels must be kept distinct. The ordinary Čech nerve is formed from the open chart domains themselves. The obstruction-adapted chart packets used to present a simplex are auxiliary finite presentations, compared only after passage to a common refinement. This distinction avoids imposing a functorial choice of adapted subcover on every overlap.

**Definition II.160** (Raw finite Čech nerve and demand footprint). Let  $\mathcal{A}_{\mathfrak{F}} = \{\mathbf{a}_i\}_{i=1}^N$  be a finite obstruction-adapted atlas for a demand  $\mathfrak{F}$ , and let  $U_i$  denote the inner domain of  $\mathbf{a}_i$ . The raw finite Čech nerve

$$\check{C}_{\bullet}(\mathcal{A}_{\mathfrak{F}}) \tag{II.224}$$

is the simplicial diagram whose degree- $p$  objects are indexed by ordered  $(p+1)$ -tuples  $(i_0, \dots, i_p)$ , repetitions allowed, and are the open sets

$$U_{i_0 \dots i_p} = U_{i_0} \cap \dots \cap U_{i_p}, \tag{II.225}$$

with empty intersections omitted. Its face maps omit an index and its degeneracy maps repeat an index, all maps being the evident inclusions or identity restrictions. The portion relevant to the finite demand is the compact relative footprint

$$K_{i_0 \dots i_p} = U_{i_0 \dots i_p} \cap K_{\mathfrak{F}}. \quad (\text{II.226})$$

The set  $K_{i_0 \dots i_p}$  is not asserted to be an open chart domain; it only records which part of the open simplex is used by the demand.

**Definition II.161** (Adapted presentation of a simplex). For a nonempty raw simplex  $U_{i_0 \dots i_p}$ , an *adapted presentation over the demand footprint* is a finite family of chart packets, each subordinate to every parent outer domain, whose inner domains cover  $K_{i_0 \dots i_p}$ , and which retain the restrictions of:

1. the H1 strata, tubes, retractions, radial functions, and control identities;
2. the regional, support, overlap-carrier, and geometric collar-footprint labels belonging to the demand;
3. every selected temporal product presentation meeting the footprint;
4. every admitted regional, chart, selected-Cauchy, selected-scale, and declared background comparison arrow and two-cell represented there.

Two adapted presentations of the same raw simplex are equivalent when they admit a finite common adapted refinement. Theorem II.158 ensures existence and equivalence of such presentations, but no compatible choice of one presentation for every simplex is included in the definition of the raw nerve.

**Proposition II.162** (Simplicial compatibility on the raw nerve). *Assume P2, H1, and Hypothesis II.155. Restriction along every face or degeneracy map of  $\check{C}_\bullet(\mathcal{A}_{\mathfrak{F}})$  carries an adapted presentation of the source footprint to an adapted presentation of the target footprint after, if necessary, passing to a finite common refinement. The H1 control identities, support and collar-footprint labels, selected temporal presentations, and declared P2 comparison cells are preserved. Hence the decoration attached to a raw simplex is well defined as a common-refinement class, and these classes are simplicially compatible.*

*Proof.* The open intersections in (II.225) form the ordinary simplicial Čech nerve, so the simplicial identities hold before any adapted presentation is chosen. Item 4 of Theorem II.158 supplies a finite adapted presentation of each compact footprint  $K_{i_0 \dots i_p}$ . Restriction preserves the H1 control identities and every retained label. FAC2–FAC4 preserve the selected temporal presentations, comparison arrows, and two-cells under further restriction. If the restricted packets do not coincide with a previously selected presentation of the target footprint, item 6 of Theorem II.158 supplies a finite common refinement. Therefore compatibility is asserted at the level of refinement classes, not through a globally functorial choice of overlap subcovers.  $\square$

**Definition II.163** (Specified atlas refinement). A specified refinement  $r : \mathcal{B} \rightarrow \mathcal{A}$  consists of a refinement  $\mathcal{B} \preceq \mathcal{A}$  together with the chosen parent-packet assignment

$$r_0 : \text{Ob}(\mathcal{B}) \longrightarrow \text{Ob}(\mathcal{A}) \quad (\text{II.227})$$

whose containments and decoration restrictions satisfy Definition II.154. Composition uses composition of the specified parent assignments. Different parent assignments are not declared equal; they may be compared only after passage to a common refinement carrying the induced comparison data.

**Proposition II.164** (Specified refinement induces a simplicial nerve map). *A specified refinement  $r : \mathcal{B} \rightarrow \mathcal{A}$  induces a simplicial map*

$$\check{C}_\bullet(r) : \check{C}_\bullet(\mathcal{B}) \longrightarrow \check{C}_\bullet(\mathcal{A}) \quad (\text{II.228})$$

*by applying the parent assignment componentwise to ordered tuples and using the inclusion of each refining intersection into its parent intersection. The map preserves the adapted decoration classes of Proposition II.162. Specified composable refinements induce strictly composable maps of the raw nerves.*

*Proof.* For an ordered tuple  $(j_0, \dots, j_p)$  of packets of  $\mathcal{B}$ , set

$$(j_0, \dots, j_p) \longmapsto (r_0(j_0), \dots, r_0(j_p)). \quad (\text{II.229})$$

Repetitions are permitted in the raw nerve, so no reduction of the target tuple is needed. The defining packet containments give

$$V_{j_0 \cdots j_p} \subset U_{r_0(j_0) \cdots r_0(j_p)}. \quad (\text{II.230})$$

Componentwise application of  $r_0$  commutes with omission and repetition of indices, proving simpliciality. The defining clauses of a specified refinement restrict all retained decorations, and Proposition II.162 identifies any two resulting adapted presentations after common refinement. Strict composability follows from the specified composition of parent assignments.  $\square$

**Proposition II.165** (Object-directed system of decorated nerves). *For a fixed finite demand  $\mathfrak{F}$ , finite obstruction-adapted atlases with specified refinements form a nonempty object-directed refinement system: every finite family of atlas objects admits a common refining atlas with specified maps to each member. Applying  $\check{C}_\bullet(-)$  gives an object-directed system*

$$\left\{ \check{C}_\bullet(\mathcal{A}) : \mathcal{A} \in \text{Atl}_\Sigma^{\text{adm}}(\mathfrak{F}) \right\} \quad (\text{II.231})$$

*of finite raw nerves equipped simplexwise with common-refinement classes of adapted presentations. No assertion is made that arbitrary parallel specified refinement maps are equalized.*

*Proof.* Nonemptiness and the existence of a common refining atlas follow from Theorem II.158. Choosing, for every packet of the common refinement, a containing parent packet in each target atlas supplies the specified refinement maps. The preceding proposition then supplies the maps of raw nerves. The final sentence matches the absence of an equalizer claim in Theorem II.158.  $\square$

**Corollary II.166** (Nervewise chart independence). *Let a finite-stage construction be defined on the raw nerve, depend on the H2 decorations only through their common-refinement classes, and be natural under specified refinement maps (II.228). Then its values obtained from any two H2 atlases agree up to the construction's declared equivalence after pullback to a finite common refinement. Thus descent data may be compared through (II.231), without identifying unrelated chartwise representatives.*

*Remark II.167* (Role in the continuum and selected-branch constructions). The phrase “continuum passage” in this manuscript refers, at the geometric atlas level, to compatibility along the object-directed system of finite raw Čech nerves, together with the refinement classes of adapted presentations, and then to the particular limits or completions explicitly introduced later. H2 does not assert that a bare inverse or direct limit of chart domains is the physical spacetime realization, does not promote the compact footprints (II.226) to open chart objects, and supplies no analytic convergence theorem. The selected  $SU(3)$  nervewise RID and CP4 arguments below add their marked branch, coefficient, and realization data to this raw simplicial carrier; they do not require a functorial global choice of H2 overlap subcovers.

**Proposition II.168** (Stronger single-coordinate criterion). *Suppose, in addition to H1, that on every relevant finite incident-stratum chain and selected Cauchy slab the combined map formed from the temporal function, the stratum coordinates, and the independent radial control functions has locally constant rank. Then the packets in Theorem II.158 may be chosen so that one coordinate chart simultaneously displays the temporal function and the independent control coordinates.*

*Proof.* Apply the constant-rank theorem to the combined finite map on each source stratum and then shrink compatibly along incident strata using the H1 control identities. Compactness and the proof of Theorem II.158 give a finite atlas of such stronger packets. This strengthening is optional; the main H2 theorem does not use it.  $\square$

*Remark II.169* (Why the stronger criterion is not the definition). Two unrelated Cauchy temporal functions need not be independent, and a singular incident-stratum chain may carry more control functions than can be inserted as independent coordinates in one chart. Requiring one master coordinate map would therefore add an unnecessary and sometimes false transversality condition. The multi-presentation packet retains all data needed for descent and comparison without making that demand.

**Proposition II.170** (Elementary H2 witness). *Let  $\Sigma \subset M$  be a closed smooth submanifold, or a locally finite Whitney-stratified subset with fixed Thom–Mather control data. Let  $\mathfrak{F}$*

*contain finitely many compact regular regions, compact support carriers, compact controlled-neighborhood frontiers, and compact subslabs of standard Minkowski Cauchy frames. Then H2 holds for  $\mathfrak{F}$ .*

*Proof.* Ordinary tubular charts and the H1 control charts give FAC1. Standard Minkowski time gives FAC2. Intersections of finitely many open chart domains give FAC3, and restriction gives FAC4. Poincare transformations preserving or explicitly transporting the selected carrier give FAC5 on the declared finite background diagram. Theorem II.158 applies.  $\square$

*Remark II.171 (Boundary between H2 and H3).* The nested containment  $U \subseteq \tilde{U}$  ensures qualitative precompact subordination. H2 does not choose one numerical lower bound for the distance of every footprint from the chart boundary or from  $\Sigma$ , does not preserve spacelike separation after positive thickening, and does not produce a scale-uniform thickness corridor. H3 begins by extracting positive obstruction, chart, and spacelike margins from a fixed finite H2 atlas and then proving their stability under the selected thickening and comparison operations.

*Remark II.172 (Circularity firewall for H2).* H2 uses the carrier and control data already obtained in H1, together with pre-incision support footprints and selected P2 Cauchy presentations. It does not use boundary Ward characters, gauge collar charges, entropy surgery, post-incision normal-state selection, invariant momentum, a spectral threshold, a mass estimate, confinement, or scattering. The geometric collar footprints retained in the demand are the H1 controlled regular-neighborhood frontiers, not post-incision physical boundary data.

**Corollary II.173** (H2 closure status). *Under H1 and FAC0–FAC6, finite obstruction-adapted atlases, finite overlap refinements, finite comparison diagrams, and common refinements are constructed by Theorem II.158. The construction requires no global preferred atlas and no simultaneous master coordinate system. Model-specific Yang–Mills work remains only in verifying that the selected P2 Cauchy slabs and admitted comparison arrows satisfy FAC2, FAC4, and FAC5. Positive finite-stage margins and thickening stability are not part of H2.*

## V. H3: positive finite-stage margins and selected thickening

Hypothesis II.155 supplies finite precompact chart packets and common refinements, but it intentionally does not attach a numerical buffer to those packets. The next step is quantitative. On each fixed finite demand one must separate compact carriers from the obstruction locus, chart boundaries, support boundaries, and Cauchy-slab boundaries, and one must separate every listed spacelike pair from the relative causal relation. The selected positive-thickness and propagation operations must then fit inside those buffers.

There are two distinct statements. First, compactness produces positive margins and a positive threshold for each fixed finite stage. Second, the actual selected thicknesses  $\varepsilon_r$  must lie below that threshold. Uniformity on a cofinal tail is stronger still and is isolated in Hypothesis II.185; it is not inferred from objectwise positivity.

**Definition II.174** (Control metric and one-sided displacement). Fix a smooth Riemannian control metric  $h$  on  $M$ , chosen to agree with the selected H1 control metric on a neighborhood of the compact footprint of a finite H2 demand. Write  $d_h$  for its distance and

$$\mathcal{N}_h(A; t) = \{x : d_h(x, A) < t\} \quad (\text{II.232})$$

for the open metric neighborhood of a subset  $A$ . For compact  $A$  and a set  $\hat{A}$ , define the one-sided outer displacement

$$\text{disp}_h(\hat{A} \mid A) = \sup_{x \in \hat{A}} d_h(x, A). \quad (\text{II.233})$$

Thus  $\text{disp}_h(\hat{A} \mid A) \leq t$  means  $\hat{A} \subseteq \overline{\mathcal{N}_h(A; t)}$ . No reverse Hausdorff bound is required.

**Definition II.175** (Protected finite-stage relations). Let  $\mathfrak{F}$  be a finite obstruction-control demand and let  $\mathcal{A}_{\mathfrak{F}}$  be one H2 atlas. A *protected quantitative stage* over  $\mathfrak{F}$  consists of the following finite lists.

1. A list  $\mathcal{P}_{\mathfrak{F}}$  of compact containments

$$(K, V), \quad K \Subset V, \quad (\text{II.234})$$

where  $V$  is one of:

- (a) the regular habitat  $M_{\text{inc}} = M \setminus \Sigma$ ;
- (b) an outer H2 chart domain  $\tilde{U}$ ;
- (c) a selected regular region or support container;
- (d) a selected globally hyperbolic Cauchy neighborhood or comparison slab.

The same compact footprint may occur with several containers.

2. A list  $\mathcal{T}_{\mathfrak{F}}$  of temporal containments

$$(K, I_{\mathfrak{c}}, L_{\tau, \mathfrak{c}, K}), \quad \tau_{\mathfrak{c}}(K) \Subset I_{\mathfrak{c}}, \quad |\tau_{\mathfrak{c}}(x) - \tau_{\mathfrak{c}}(y)| \leq L_{\tau, \mathfrak{c}, K} d_h(x, y) \quad (\text{II.235})$$

on the compact chart neighborhood used by the stage, for the finite Cauchy slabs retained by the H2 chart packets.

3. A list  $\mathcal{S}_{\mathfrak{F}}$  of declared spacelike pairs

$$(K, L; N), \quad K, L \Subset N, \quad (K \times L) \cap \mathcal{C}_N = \emptyset, \quad (\text{II.236})$$

where

$$\mathcal{C}_N = \{(x, y) \in N \times N : y \in J_N^+(x) \cup J_N^-(x)\} \quad (\text{II.237})$$

is the relative causal relation.

4. For every scale object and every geometric operation used in the finite diagram, a finite list of *operation records*

$$q = (A_q, \hat{A}_{q,r}, w_{q,r}), \quad \hat{A}_{q,r} \subseteq \overline{\mathcal{N}_h(A_q; w_{q,r})}. \quad (\text{II.238})$$

The base  $A_q$  is the compact unthickened carrier, causal shadow, or target support footprint against which the operation is measured. The output  $\hat{A}_{q,r}$  may be a selected thickening, a propagated thick carrier, a scale-transported support, or the image under a declared finite background comparison.

All compact outputs appearing in an operation record are included among the footprints of the H2 finite comparison diagram.

**Definition II.176** (Finite margin ledger). For a protected stage define the one-body margins

$$\beta_h(K; V) = d_h(K, M \setminus V), \quad (\text{II.239})$$

$$\beta_\tau(K; I_\mathfrak{c}) = d_{\mathbb{R}}(\tau_\mathfrak{c}(K), \mathbb{R} \setminus I_\mathfrak{c}), \quad (\text{II.240})$$

$$\beta_\Sigma(K) = d_h(K, \Sigma). \quad (\text{II.241})$$

The first formula includes chart, support-container, regular-region, and Cauchy-neighborhood margins by the choice of  $V$ . The obstruction margin is displayed separately because it is the geometric content of the incision.

Give  $N \times N$  the sum product metric

$$d_{h \oplus h}((x, y), (x', y')) = d_h(x, x') + d_h(y, y'). \quad (\text{II.242})$$

For a declared spacelike pair define

$$\mu_N(K, L) = d_{h \oplus h}(K \times L, \mathcal{C}_N). \quad (\text{II.243})$$

Define the three finite one-body collections

$$\mathcal{B}_\mathfrak{F}^h = \{\beta_h(K; V) : (K, V) \in \mathcal{P}_\mathfrak{F}\},$$

$$\mathcal{B}_\mathfrak{F}^\tau = \{\beta_\tau(K; I) : (K, I, L_\tau) \in \mathcal{T}_\mathfrak{F}\},$$

$$\mathcal{B}_\mathfrak{F}^\Sigma = \text{the finite collection of } \beta_\Sigma(K) \text{ for protected } K.$$

Finally set

$$\beta_\mathfrak{F} = \min(\mathcal{B}_\mathfrak{F}^h \cup \mathcal{B}_\mathfrak{F}^\tau \cup \mathcal{B}_\mathfrak{F}^\Sigma), \quad (\text{II.244})$$

$$\mu_\mathfrak{F} = \min\{\mu_N(K, L) : (K, L; N) \in \mathcal{S}_\mathfrak{F}\}. \quad (\text{II.245})$$

An empty minimum is omitted from the ledger. These numerical values depend on the chosen finite atlas and control metric; positivity and threshold admissibility, not the raw numbers, are the invariant conclusions.

**Definition II.177** (Total geometric cost and safety ratio). For each protected compact footprint  $A$ , let

$$W_{\mathfrak{F}, r}(A) \quad (\text{II.246})$$

be the maximum of the displacement bounds of all operation records in the finite diagram whose base is  $A$ , including the finite composites actually used by that diagram. The operation ledger is required to use the sum of successive displacement moduli for a composite. For a spacelike pair set

$$W_{\mathfrak{F},r}(K, L) = W_{\mathfrak{F},r}(K) + W_{\mathfrak{F},r}(L). \quad (\text{II.247})$$

The finite-stage safety ratio is

$$\Theta_{\mathfrak{F}}(r) = \max \left\{ \frac{W_{\mathfrak{F},r}(K)}{\beta_h(K; V)}, \frac{W_{\mathfrak{F},r}(K)}{\beta_{\Sigma}(K)}, \frac{L_{\tau, \mathfrak{c}, K} W_{\mathfrak{F},r}(K)}{\beta_{\tau}(K; I)}, \frac{W_{\mathfrak{F},r}(K, L)}{\mu_N(K, L)} \right\}, \quad (\text{II.248})$$

where the maximum ranges only over relations declared in the protected stage. A scale object is *margin-admissible for  $\mathfrak{F}$*  when

$$\Theta_{\mathfrak{F}}(r) < 1. \quad (\text{II.249})$$

A quantitative safety factor is any  $\vartheta_{\mathfrak{F},r} \in (0, 1)$  with  $\Theta_{\mathfrak{F}}(r) \leq \vartheta_{\mathfrak{F},r}$ .

**Hypothesis II.178** (H3: Positive-Margin Selected-Thickening package). Every finite geometric stage used in the positive-thickness reconstruction admits a protected quantitative presentation satisfying the following conditions.

(PMG0) Finite protected position. Every compact containment, temporal containment, and spacelike relation used by the stage is explicitly entered in  $\mathcal{P}_{\mathfrak{F}}$ ,  $\mathcal{T}_{\mathfrak{F}}$ , or  $\mathcal{S}_{\mathfrak{F}}$ . In particular, no pair is called spacelike merely because its chosen base points are spacelike.

(PMG1) Compatible finite control metrics. Each finite H2 atlas carries a smooth control metric on a neighborhood of its compact footprint. Two choices and two H2 atlases admit a common refinement on which the metrics are uniformly bi-Lipschitz on the compact stage. Strict controlled background embeddings are isometries for the selected source and image metrics; a more general admitted background equivalence carries an explicit finite bi-Lipschitz constant.

(PMG2) Controlled selected thickenings. For every protected compact carrier  $A$ , scale object  $r$ , and auxiliary radius  $t$  used on the stage, the selected thickening has a nondecreasing modulus

$$\omega_{A,r}^{\text{th}}(t), \quad \omega_{A,r}^{\text{th}}(0) = 0, \quad \omega_{A,r}^{\text{th}}(t) \longrightarrow 0 \quad (t \downarrow 0), \quad (\text{II.250})$$

and satisfies

$$A_r^{+t} \subseteq \overline{\mathcal{N}_h \left( A; \omega_{A,r}^{\text{th}}(t) \right)}. \quad (\text{II.251})$$

The actual positive-thickness carrier uses  $t = \varepsilon_r$ . The raw metric neighborhood need not itself be an admissible region.

(PMG3) Controlled Cauchy propagation. For every selected finite Cauchy comparison and every compact source carrier, its propagated unthickened shadow is compact and retained in the H2 diagram. Propagation of a selected thickening satisfies an operation record with a finite modulus

$$\text{Sh}_{s_2, s_1} \left( A_r^{+t} \right) \subseteq \overline{\mathcal{N}_h \left( \text{Sh}_{s_2, s_1}(A); \omega_{A, r; s_2, s_1}^{\text{Cau}}(t) \right)}, \quad (\text{II.252})$$

continuous at  $t = 0$ . The moduli compose subadditively along the selected Cauchy comparison cells and respect the exact shadow-composition law on the unthickened carriers.

(PMG4) Regional, chart, support, and scale cost ledger. Regional restriction and H2 chart refinement act by restriction and require no new displacement beyond the recomputed target margin. Every support-filter operation, selected scale arrow, and mixed regional-scale operation occurring in the finite demand has an explicit operation record. For composable arrows, the recorded cost of the chosen composite is bounded by the sum of the costs of the factors after transporting them to one common H2 refinement. The bicategorical comparison cells do not silently identify distinct geometric carriers; their declared source and target carriers both occur in the ledger.

(PMG5) Finite comparison stability. The finite operation ledger is closed under every composition, inverse, and mixed face actually used on the demand. Under an admitted background arrow, all containments, causal relations, operation records, and moduli transport to the target finite stage with the isometric or bi-Lipschitz constants declared in PMG1. No stability is asserted for an unlisted background map, arbitrary Cauchy slice, or arbitrary scale comparison.

(PMG6) Actual fixed-stage admissibility. Every scale object  $r$  at which the finite stage is evaluated carries an actual admissibility witness. The general displacement witness is

$$\Theta_{\mathfrak{F}}(r) < 1. \quad (\text{II.253})$$

On a branch whose actual endpoint outputs are explicitly constructed, PMG6 may instead be certified by the exact residual-margin ratio of Definition II.194. If the stage uses finitely many scale objects and arrows, the relevant finite maximum is strictly smaller than one. This is an actual estimate on the selected positive-thickness outputs, not a consequence of the fact that  $\varepsilon_r > 0$ .

(PMG7) Pre-spectral provenance. The margins and operation moduli are obtained from H1–H2 geometry, pre-incision support filters, selected positive-thickness profiles, and the P2 Cauchy comparison system. They do not use boundary Ward characters, gauge collar charges, entropy surgery, post-incision normal states, invariant momentum, a spectral threshold, a mass estimate, confinement, or scattering.

*Remark II.179* (The substantive H3 burden). Compactness proves positivity of the undeformed margin ledger. PMG2–PMG5 supply the quantitative displacement estimates. PMG6 is the nonformal model-specific assertion that the selected positive thickness and every finite comparison composite fit inside the available margins. It may fail at coarse thickness even when it holds on a smaller cofinal tail.

**Lemma II.180** (Finite undeformed margins are positive). *Assume H1–H2 and PMG0–PMG1. Every nonempty entry of the finite margin ledger is strictly positive. Hence*

$$\beta_{\mathfrak{F}} > 0, \quad \mu_{\mathfrak{F}} > 0 \tag{II.254}$$

*whenever the corresponding lists are nonempty.*

*Proof.* For  $K \Subset V$ , the compact set  $K$  is disjoint from the closed set  $M \setminus V$ ; therefore  $d_h(K, M \setminus V) > 0$ . The same argument applies to  $K$  and the closed carrier  $\Sigma$ . The temporal image  $\tau_{\mathfrak{c}}(K)$  is compact and is compactly contained in the open interval  $I_{\mathfrak{c}}$ , so its distance from the complement is positive.

A globally hyperbolic comparison neighborhood is causally simple, hence  $\mathcal{C}_N$  is closed in  $N \times N$ . The compact product  $K \times L$  is disjoint from that relation by PMG0, so  $\mu_N(K, L) > 0$ . Only finitely many quantities occur, and taking a finite minimum preserves positivity.  $\square$

**Lemma II.181** (Margin loss under controlled displacement). *Let  $\widehat{K} \subseteq \overline{\mathcal{N}_h(K; a)}$  and  $\widehat{L} \subseteq \overline{\mathcal{N}_h(L; b)}$ . Then*

$$d_h(\widehat{K}, M \setminus V) \geq d_h(K, M \setminus V) - a, \quad (\text{II.255})$$

$$d_h(\widehat{K}, \Sigma) \geq d_h(K, \Sigma) - a, \quad (\text{II.256})$$

$$\mu_N(\widehat{K}, \widehat{L}) \geq \mu_N(K, L) - a - b. \quad (\text{II.257})$$

*If  $|\tau(x) - \tau(y)| \leq L_\tau d_h(x, y)$  on the compact chart stage, then*

$$d_{\mathbb{R}}(\tau(\widehat{K}), \mathbb{R} \setminus I) \geq \beta_\tau(K; I) - L_\tau a. \quad (\text{II.258})$$

*Proof.* Distance to a fixed closed subset is one-Lipschitz. Approximate every point of  $\widehat{K}$  by a point of  $K$  within  $a$  and apply this fact with  $M \setminus V$  and with  $\Sigma$ . On the product, distance to the closed set  $\mathcal{C}_N$  is one-Lipschitz for the sum metric (II.242). Approximate the two coordinates separately to obtain (II.257). The temporal estimate follows by combining the local Lipschitz bound for  $\tau$  with the one-dimensional distance-to-complement estimate.  $\square$

**Theorem II.182** (H3 positive-margin and selected-thickening theorem). *Assume H1, H2, and Hypothesis II.178. Let  $\mathfrak{F}$  be a protected finite stage and let  $r$  be one of its selected scale objects. Then every selected thickening, propagated thick carrier, support-filter image, scale transport, and finite composite represented by the operation ledger satisfies:*

1. *positive obstruction clearance;*
2. *positive chart, support-container, regular-region, and Cauchy-neighborhood margins;*
3. *positive temporal distance from the endpoints of every selected compact Cauchy slab;*
4. *preservation of every declared spacelike relation;*
5. *stability under every regional, H2 chart, selected Cauchy, selected scale, mixed, and admitted background comparison in the finite diagram.*

*More precisely, if  $\Theta_{\mathfrak{F}}(r) \leq \vartheta < 1$ , then every protected one-body margin with denominator  $\beta$  has residual margin at least*

$$(1 - \vartheta)\beta > 0, \quad (\text{II.259})$$

and every protected spacelike pair with initial margin  $\mu$  has residual margin at least

$$(1 - \vartheta)\mu > 0. \quad (\text{II.260})$$

The assertion holds simultaneously for a finite family of scale objects after replacing  $\vartheta$  by the finite maximum of their safety factors.

*Proof.* If PMG6 is certified by an exact endpoint residual-margin witness, the stated positive residual conclusions follow directly from the definition of that witness; the selected physical implementation is Theorem II.197. For the general displacement witness, Lemma II.180 gives positive denominators in the safety ratio. PMG2–PMG5 assign to every selected output a base compact carrier and a total displacement bound no larger than  $W_{\mathfrak{F},r}$ . Apply Lemma II.181 to every protected containment and spacelike pair. Definition II.177 and  $\Theta_{\mathfrak{F}}(r) \leq \vartheta$  give

$$W_{\mathfrak{F},r}(K) \leq \vartheta\beta, \quad L_{\tau,\mathfrak{c},K}W_{\mathfrak{F},r}(K) \leq \vartheta\beta_{\tau}, \quad W_{\mathfrak{F},r}(K, L) \leq \vartheta\mu \quad (\text{II.261})$$

for the corresponding containment, temporal, and pair denominators. Substitution yields (II.259) and (II.260).

The operation ledger is closed under the finite compositions and mixed faces actually used, so the same estimate applies to each composite rather than only to primitive thickenings. Regional restrictions and chart refinements are handled on the common H2 refinement; their protected margins are recomputed there and remain positive by compact containment. Cauchy and scale operations use PMG3–PMG4. Background transport uses PMG1 and PMG5. Finally, the maximum of finitely many numbers strictly below one is strictly below one, proving the simultaneous finite-scale statement.  $\square$

**Proposition II.183** (Derived finite-stage threshold). *Assume PMG0–PMG5, but do not assume PMG6. Suppose every primitive operation modulus is continuous at zero and every finite composite cost is obtained by a finite sum of those moduli. Then there exists*

$$\delta_{\mathfrak{F}} > 0 \quad (\text{II.262})$$

*such that replacing every auxiliary thickening radius in the finite diagram by numbers satisfying  $0 \leq t < \delta_{\mathfrak{F}}$  makes the entire stage margin-admissible. If  $\varepsilon_r < \delta_{\mathfrak{F}}$  and the actual selected profiles use that radius, PMG6 follows for  $r$ .*

*Proof.* There are finitely many positive denominators in the margin ledger and finitely many primitive moduli. Every finite composite cost tends to zero when all its radius arguments tend to zero. Choose a radius on which every one-body cost is less than one half of its corresponding margin and every pair cost is less than one half of its spacelike margin. The minimum of the finitely many resulting radii is positive and has the stated property.  $\square$

*Remark II.184* (Why the derived threshold does not settle the selected theory). Proposition II.183 proves that sufficiently small auxiliary radii work. It does not prove that the already selected physical thickness  $\varepsilon_r$  is below that threshold, nor that the comparison constants remain bounded as  $r$  runs to finer scales. Those are PMG6 and the cofinal package below.

**Hypothesis II.185** (Cofinal Uniform Margin package). For every protected finite demand  $\mathfrak{F}$ , there is a cofinal tail  $\Gamma_{\mathfrak{F}} \subseteq \Gamma_{\text{cof}}$  and one number  $\vartheta_{\mathfrak{F}} \in (0, 1)$  such that

$$\sup_{r \in \Gamma_{\mathfrak{F}}} \Theta_{\mathfrak{F}}(r) \leq \vartheta_{\mathfrak{F}} < 1. \quad (\text{II.263})$$

The operation ledgers on the tail are closed under every finite scale composite required by the demand, and the same bound holds after admitted regional, chart, Cauchy, mixed, and background comparison. The tail and constant may depend on  $\mathfrak{F}$ ; no uniform margin over all regions or all finite demands is asserted.

**Corollary II.186** (Cofinal positive-margin stability). *Assume H1–H3 and Hypothesis II.185. For every protected finite demand, all selected scales in the cofinal tail  $\Gamma_{\mathfrak{F}}$  preserve the declared obstruction, chart, support, Cauchy-slab, and spacelike margins with the common residual factor  $1 - \vartheta_{\mathfrak{F}} > 0$ . In particular, the finite diagram satisfies the geometric admission conditions for the controlled incised regional category throughout that tail.*

*Proof.* Apply Theorem II.182 with the uniform safety factor from (II.263).  $\square$

**Proposition II.187** (Linear-profile sufficient criterion). *Suppose that on a protected finite stage every operation record satisfies*

$$W_{\mathfrak{F},r}(A) \leq C_{\mathfrak{F},r}(A) \varepsilon_r \quad (\text{II.264})$$

with finite constants, and similarly the total pair cost is bounded by the sum of the two one-body bounds. Then PMG6 follows whenever

$$\max_A \frac{C_{\mathfrak{F},r}(A)\varepsilon_r}{\beta_A} < 1, \quad \max_{(K,L;N)} \frac{C_{\mathfrak{F},r}(K)\varepsilon_r + C_{\mathfrak{F},r}(L)\varepsilon_r}{\mu_N(K,L)} < 1, \quad (\text{II.265})$$

where  $\beta_A$  ranges over every protected one-body denominator attached to  $A$ , and temporal entries use the weighted numerator  $L_{\tau,c,A}C_{\mathfrak{F},r}(A)\varepsilon_r$ . If the left sides are bounded uniformly below one on a cofinal tail, Hypothesis II.185 holds.

*Proof.* The displayed ratios dominate every entry in (II.248). The conclusions are therefore immediate from the definitions.  $\square$

**Proposition II.188** (Standard Minkowski finite-stage witness). *Let a protected stage lie in finitely many standard constant-time Minkowski charts. Use Euclidean spatial thickening on each selected slice, assume the propagated shadow of a spatial  $t$ -thickening equals the  $t$ -thickening of the propagated shadow, and let every declared background arrow be a carrier-preserving Poincare isometry. Then PMG0–PMG5 hold with linear profile constants equal to one for primitive spatial thickening and standard Cauchy shadow transport. Every selected scale satisfying (II.265) verifies H3 on that finite stage.*

*Proof.* Euclidean metric neighborhoods satisfy the monotonicity and additive semigroup laws, so the primitive displacement is at most the thickening radius. In a constant-time Minkowski frame, causal shadows commute with this spatial thickening. The relative causal relation is closed in each selected globally hyperbolic slab, and Poincare isometries preserve the control and causal distances on the transported image. The linear-profile criterion applies.  $\square$

*Remark II.189* (Refinement and numerical noncanonicity). Two H2 atlases may give different numerical chart margins. Pass to a common H2 refinement, recompute the finite ledger there, and choose the smaller admissible threshold. The numerical margin is not claimed to be canonical; the existence of a positive common threshold and the validity of the selected finite diagram are independent of the atlas after this refinement procedure.

*Remark II.190* (Boundary after H3). H1 supplies the regular carrier and geometric control neighborhoods. H2 supplies finite adapted chart packets and common refinements. H3

supplies the positive quantitative margins needed for selected positive-thickness support and causal operations. These three edges now permit recovery of the controlled incised regional category. They do not yet identify the stackified lifting gerbe with the existing Paper-III regional realization; that is the subsequent regional-identification theorem.

*Remark II.191* (Circularity firewall for H3). H3 measures the selected positive-thickness geometry against the dynamically derived carrier. It does not use a coercive constant, invariant momentum, spectrum condition, spectral threshold, mass estimate, confinement result, or scattering input to choose the incision or its margins. Geometric H1 collar frontiers may occur as protected footprints, but gauge collar charges, boundary Ward characters, entropy surgery, and post-incision normal-state selection remain downstream structures.

**Corollary II.192** (H3 closure status). *For each fixed protected finite stage, H1–H2 and PMG0–PMG5 give positive undeformed margins and a positive auxiliary-radius threshold. Under PMG6, the actual selected scales preserve all declared margins and comparisons. Cofinal uniform stability requires Hypothesis II.185. Model-specific Yang–Mills verification of the thickening, propagation, support-filter, scale, comparison, and mixed-cell endpoint geometry remains an explicit proof obligation in general. It is discharged on the selected source-free physical branch by Corollary II.198.*

### Selected physical simultaneous margins at the actual thicknesses

The general displacement ledger of Definition II.177 is a convenient sufficient criterion, but it is intentionally conservative. On the selected physical branch it can overcount in two ways. First, a spatial support displacement may be charged against a temporal margin even when the selected Cauchy comparison preserves the temporal coordinate of the endpoint carrier. Second, two factorizations of one regional-chart–Cauchy-scale comparison may be charged as distinct geometric motions although their mixed comparison cell identifies the same endpoint support family. The selected model therefore admits a sharper certificate formed from the actual residual margins of the endpoint-normal geometric outputs.

**Definition II.193** (Endpoint-normalized physical operation ledger). Let  $\mathfrak{d}$  be a finite selected

physical regional-chart-Cauchy-scale demand on the source-free linked-flux  $SU(3)$  branch. Pass once and for all to a common H2 refinement supplied by Corollary II.134.

Before any positive-thickness endpoint is evaluated, fix a finite *obligation registry*

$$\mathcal{O}(\mathfrak{d}) = \mathcal{O}_1(\mathfrak{d}) \sqcup \mathcal{O}_{\text{sp}}(\mathfrak{d}). \quad (\text{II.266})$$

The one-body part  $\mathcal{O}_1(\mathfrak{d})$  records every chart, support-container, regular-region, obstruction-clearance, and temporal containment that each endpoint operation is required to preserve. Each such record includes a preselected open protective cell  $W_{q,A}$  with

$$A_q \Subset W_{q,A} \Subset V, \quad \overline{W_{q,A}} \cap \Sigma_{\text{pres}} = \emptyset, \quad (\text{II.267})$$

and, when temporal control is required,  $\tau_c(\overline{W_{q,A}}) \Subset I_c$ .

The pair part  $\mathcal{O}_{\text{sp}}(\mathfrak{d})$  records every spacelike relation required by the original finite demand, together with preselected open neighborhoods  $W_{q,K}, W_{q,L} \Subset N$  satisfying

$$K_q \Subset W_{q,K}, \quad L_q \Subset W_{q,L}, \quad \overline{W_{q,K}} \times \overline{W_{q,L}} \cap \mathcal{C}_N = \emptyset. \quad (\text{II.268})$$

These obligations and witnesses are fixed from the unthickened selected geometry on the common H2 refinement; they are not chosen after inspecting the thick endpoint outputs.

For every primitive arrow, every finite composite used by  $\mathfrak{d}$ , and every boundary path of a listed mixed cell, retain:

1. its unthickened endpoint carrier  $A_q$ , propagated shadow, or target support footprint;
2. its actual selected positive-thickness endpoint output  $\hat{A}_q$ ;
3. every predeclared obligation in  $\mathcal{O}(\mathfrak{d})$  attached to that endpoint operation.

The selected endpoint construction is required to realize each attached witness by

$$\hat{A}_q \subset W_{q,A}, \quad \hat{K}_q \subset W_{q,K}, \quad \hat{L}_q \subset W_{q,L}. \quad (\text{II.269})$$

The ledger is *endpoint normalized* when the following convention is used.

1. A regional restriction or chart refinement is recorded by its actual restricted endpoint support.

2. A selected Cauchy arrow is recorded relative to the propagated unthickened shadow at the target slice.
3. A selected scale arrow is recorded relative to the target-scale skeletal carrier or target support footprint. A genuinely new persistent label is entered with its own target-scale skeletal carrier; it is not treated as a displacement of an unrelated source label.
4. A selected carrier-preserving controlled background arrow is recorded relative to the transported endpoint carrier.
5. When an invertible comparison cell identifies the two boundary paths with the same endpoint support family, that cell has zero additional geometric cost. The common endpoint output is entered once.

Thus a syntactically longer zigzag does not acquire an artificial geometric cost when the selected coherence system reduces it to the same endpoint support operation.

**Definition II.194** (Exact residual-margin safety ratio). For an endpoint-normalized ledger, define the output margins

$$\hat{\beta}_h(q; V) = d_h(\hat{A}_q, M \setminus V), \quad (\text{II.270})$$

$$\hat{\beta}_\Sigma(q) = d_h(\hat{A}_q, \Sigma_{\text{pres}}), \quad (\text{II.271})$$

$$\hat{\beta}_\tau(q; I_\mathfrak{c}) = d_{\mathbb{R}}(\tau_\mathfrak{c}(\hat{A}_q), \mathbb{R} \setminus I_\mathfrak{c}). \quad (\text{II.272})$$

For a protected spacelike pair with endpoint outputs  $(\widehat{K}_q, \widehat{L}_q)$ , put

$$\hat{\mu}_N(q; K, L) = d_{h \oplus h}(\widehat{K}_q \times \widehat{L}_q, \mathcal{C}_N). \quad (\text{II.273})$$

Write  $[x]_+ = \max\{x, 0\}$ . The relation-specific loss ratios are

$$\theta_h(q; K, V) = \frac{[\beta_h(K; V) - \hat{\beta}_h(q; V)]_+}{\beta_h(K; V)}, \quad (\text{II.274})$$

$$\theta_\Sigma(q; K) = \frac{[\beta_\Sigma(K) - \hat{\beta}_\Sigma(q)]_+}{\beta_\Sigma(K)}, \quad (\text{II.275})$$

$$\theta_\tau(q; K, I_\mathfrak{c}) = \frac{[\beta_\tau(K; I_\mathfrak{c}) - \hat{\beta}_\tau(q; I_\mathfrak{c})]_+}{\beta_\tau(K; I_\mathfrak{c})}, \quad (\text{II.276})$$

$$\theta_{\text{sp}}(q; K, L; N) = \frac{[\mu_N(K, L) - \hat{\mu}_N(q; K, L)]_+}{\mu_N(K, L)}. \quad (\text{II.277})$$

The *exact physical safety ratio* of the finite demand is

$$\Theta_{\mathfrak{d}}^{\text{phys}} = \max \{ \theta_h, \theta_{\Sigma}, \theta_{\tau}, \theta_{\text{sp}} \}, \quad (\text{II.278})$$

where the maximum ranges over every protected relation and every endpoint operation, finite composite, and mixed boundary path actually used by  $\mathfrak{d}$ .

This is an exact residual-margin certificate. It does not replace the one-sided displacement criterion of Definition II.177; it supplies a sharper alternative whenever the selected model gives the endpoint outputs explicitly.

**Lemma II.195** (Selected physical endpoint normal form). *Every finite selected physical demand  $\mathfrak{d}$  admits an endpoint-normalized operation ledger. In that ledger:*

1. *regional restriction and chart refinement introduce no support motion beyond passage to the restricted endpoint support;*
2. *a selected scale comparison carries every old persistent label to its declared target descendant inside the same fixed control cell and adds only the separately declared target supports of new labels;*
3. *selected Cauchy propagation carries an actual spatial thickening into the thickening of the propagated unthickened shadow with no increase of its transverse radius;*
4. *selected carrier-preserving controlled background comparisons transport the control metric, the carrier  $\Sigma_{\text{pres}}$ , and the causal relation isometrically on the finite stage; and*
5. *every listed regional-chart, regional-scale, chart-scale, and regional-scale-Cauchy mixed cell has the same endpoint support family on its two boundary paths.*

*Consequently every finite composite used by  $\mathfrak{d}$  has a direct endpoint operation record, and comparison cells contribute zero additional geometric loss.*

*Proof.* The persistent-label convention in the selected  $SU(3)$  regional construction assigns each label a fixed control cell. Regional restriction deletes precisely the labels not retained by the target region. A selected scale arrow retains each old label, replaces its support by the declared nested target-scale tube in the same control cell, and enters genuinely new

labels with their own target supports. Hence the geometric endpoint of a scale comparison is determined by the target label set and does not depend on a chosen factorization.

The selected Cauchy frames are propagation adapted. In the standard Minkowski product charts used on the physical comparison system, causal shadow and spatial thickening commute. Thus the propagated thick output is measured relative to the propagated unthickened shadow with the same transverse radius.

The admitted controlled background comparisons preserve the oriented carrier data and are strict controlled isometries on the selected finite stages. Finally, the regional-scale Beck–Chevalley cells, chart-scale prisms, and regional-scale–Cauchy cubes identify the same persistent target labels and the same added target labels. Their two boundary paths therefore have the same support output. Associators, tensor permutations, coefficient lines, and rooted marking cells act on the represented or coefficient data and do not move the geometric carrier. This gives the claimed endpoint normal form.  $\square$

**Lemma II.196** (Positive residual margins for every selected endpoint). *Let  $\mathfrak{d}$  be a finite selected physical demand and use the common H2 refinement and endpoint ledger of Lemma II.195. Then every output margin appearing in Definition II.194 is strictly positive:*

$$\hat{\beta}_h(q; V) > 0, \quad \hat{\beta}_\Sigma(q) > 0, \quad \hat{\beta}_\tau(q; I_\mathfrak{c}) > 0, \quad \hat{\mu}_N(q; K, L) > 0. \quad (\text{II.279})$$

*The statement holds at the actual selected thickness of every scale object appearing in  $\mathfrak{d}$ .*

*Proof.* Theorem II.132 identifies the physical obstruction carrier with  $\Sigma_{\text{pres}}$ , and Corollary II.134 supplies one finite family of standard product charts, normal tubes, exterior collars, selected Cauchy slabs, and common refinements for the whole demand.

At each selected scale, every physical tube support is a compact actual positive-thickness carrier contained in its declared fixed control cell and in the regular locus  $M \setminus \Sigma_{\text{pres}}$ . Regional and chart restrictions only reduce this support. Scale transport replaces it by the declared target-scale support in the same protected cell, and a new label is entered only with its own protected target cell. The compact endpoint supports are therefore disjoint from the relevant chart, support-container, regular-region, and Cauchy-neighborhood complements and from  $\Sigma_{\text{pres}}$ . Their containment and obstruction distances are positive.

The selected product shrinkings place the compact endpoint supports strictly inside the retained temporal intervals. Hence the temporal distances from the slab endpoints are positive. Propagation-adapted Cauchy transport preserves the selected transverse thickness and remains inside the declared globally hyperbolic comparison slab.

The obligation registry was fixed before the thick endpoints were evaluated. For every required spacelike pair, the witness neighborhoods satisfy (II.268), and the selected endpoint construction satisfies (II.269). Hence

$$\widehat{K}_q \times \widehat{L}_q \subset W_{q,K} \times W_{q,L} \subset (N \times N) \setminus \mathcal{C}_N.$$

Because the comparison neighborhood is causally simple,  $\mathcal{C}_N$  is closed. The compact endpoint product is therefore at positive distance from it. Likewise, each one-body endpoint lies in its preselected protective cell  $W_{q,A}$ , whose compact closure is already contained in the required chart, support container, regular region, and Cauchy slab and is disjoint from  $\Sigma_{\text{pres}}$ . Thus all containment, obstruction, and temporal output margins are positive. Background transport carries the witnesses isometrically, and mixed cells do not change the endpoint support family. Since the obligation registry is finite, the argument applies simultaneously to every obligation originally required by the demand.  $\square$

**Theorem II.197** (Selected physical H3 simultaneous-margin theorem). *Fix  $m \in \{1, 2\}$ . Let  $\mathfrak{d}$  be any finite selected physical regional-chart-Cauchy-scale demand on the canonical source-free linked-flux  $SU(3)$  branch, including every comparison map, finite composite, and mixed cell used by that demand. Evaluate each scale object at its actual selected positive thickness.*

*Then the exact physical safety ratio satisfies*

$$\boxed{\Theta_{\mathfrak{d}}^{\text{phys}} < 1.} \tag{II.280}$$

*Consequently there is one number*

$$\vartheta_{\mathfrak{d}}^{\text{phys}} \in (0, 1) \tag{II.281}$$

*such that every protected endpoint relation in the entire finite demand has the simultaneous*

*residual bounds*

$$\hat{\beta}_h(q; V) \geq (1 - \vartheta_{\mathfrak{d}}^{\text{phys}}) \beta_h(K; V), \quad (\text{II.282})$$

$$\hat{\beta}_{\Sigma}(q) \geq (1 - \vartheta_{\mathfrak{d}}^{\text{phys}}) \beta_{\Sigma}(K), \quad (\text{II.283})$$

$$\hat{\beta}_{\tau}(q; I_{\mathfrak{c}}) \geq (1 - \vartheta_{\mathfrak{d}}^{\text{phys}}) \beta_{\tau}(K; I_{\mathfrak{c}}), \quad (\text{II.284})$$

$$\hat{\mu}_N(q; K, L) \geq (1 - \vartheta_{\mathfrak{d}}^{\text{phys}}) \mu_N(K, L). \quad (\text{II.285})$$

*In particular:*

1. *the actual selected supports retain positive obstruction, chart, support-container, regular-region, Cauchy-neighborhood, temporal, and spacelike margins;*
2. *all selected regional, chart, Cauchy, scale, background, and mixed comparisons preserve those margins on the common finite refinement;*
3. *comparison-map distortion and mixed-cell displacement are included in the one common ratio; and*
4. *obligations H3.8, H3.9, and H3.10 are discharged on the selected physical branch.*

*Proof.* By Lemma II.180, every denominator in Definition II.194 is positive. By Lemma II.196, every corresponding output margin is also positive.

For any positive initial margin  $\beta$  and positive output margin  $\hat{\beta}$ ,

$$0 \leq \frac{[\beta - \hat{\beta}]_+}{\beta} < 1.$$

The same statement applies to every spacelike margin. The endpoint-normalized ledger is finite because the demand contains finitely many regions, charts, Cauchy presentations, scale objects, arrows, composites, and mixed faces. Therefore  $\Theta_{\mathfrak{d}}^{\text{phys}}$  is the maximum of finitely many numbers strictly below one and is itself strictly below one.

Choose

$$\Theta_{\mathfrak{d}}^{\text{phys}} \leq \vartheta_{\mathfrak{d}}^{\text{phys}} < 1.$$

The definitions of the four loss ratios give (II.285). The endpoint-normal form includes every finite composite and both boundary paths of every mixed cell, while assigning zero additional geometric loss to a cell whose boundary paths have the same support endpoint. Hence

the common factor is independent of the syntactic length or factorization of the selected comparison zigzag.

All quantities are evaluated on the already selected positive-thickness outputs. No auxiliary replacement of  $\varepsilon_r$ , no later thinning argument, and no spectral datum enters the proof.  $\square$

**Corollary II.198** (Selected physical H3 closure). *On the canonical source-free physical  $SU(3)$  branch,  $H1$ ,  $H2$ , and the fixed-demand  $H3$  package are verified at every actual selected positive-thickness scale. In particular,  $H3.1$ – $H3.10$  are discharged.*

*The following statements are not consequences of this corollary:*

1. *one safety factor uniform on a cofinal tail for a fixed demand;*
2. *one safety factor uniform over all finite demands;*
3. *admissible thin descent, zero-thickness persistence, or cofinal independence.*

*Those are the  $H3$ -UMG and thin-reconstruction obligations retained for Paper IV.*

*Remark II.199* (Why this is not a tautological redefinition). The proof does not declare an arbitrary output admissible and then infer its admissibility. The obligation registry and its protective witnesses are fixed from the unthickened finite demand before the positive-thickness outputs are inspected. The endpoint outputs are fixed independently by the selected physical  $SU(3)$  regionalization, its propagation-adapted Cauchy system, persistent scale descendants, carrier-preserving controlled background maps, and mixed coherence cells. The verification is therefore that those independently constructed outputs land in the predeclared witnesses. Lemmas II.195 and II.196 verify, from that explicit geometry, that every endpoint used by the finite demand has positive residual margins. Definition II.194 merely compresses those finitely many verified inequalities into one common numerical certificate.

*Remark II.200* (Circularity and scope firewall). The simultaneous-margin theorem uses only the derived carrier, the  $H1$ – $H2$  geometric control system, actual selected tube supports, propagation-adapted Cauchy shadows, persistent scale labels, carrier-preserving controlled background transport, and the already constructed mixed support cells. It uses no coercive lower bound, boundary Ward character, gauge collar charge, entropy surgery, post-incision

state selector, closed spacetime form, invariant momentum, spectral threshold, mass estimate, confinement input, scattering input, or thin-limit theorem.

## W. Recovery of the controlled incised regional category

The dynamically derived carrier and the H1–H3 control system now supply the geometry needed to work on an incised spacetime. A category-identification statement nevertheless requires more care than the assertion that the underlying open sets are the same. Four distinctions must be retained.

First, an H2 atlas, an H3 margin ledger, and a selected Cauchy presentation are choices decorating a region; the regional base category should not depend on those choices. Second, regional arrows are inclusions. Chart changes, Cauchy comparisons, selected scale arrows, and background maps are comparison data over the regional category and are not additional regional arrows. Third, H3 is fixed-stage and finite-demand unless the separate cofinal package II.185 is assumed. Fourth, the existing Paper-III notation “Cauchy presentable” includes an observable-algebra independence clause. That final clause is not geometric and is proved only after the regional realization has been identified. We therefore recover a geometric Cauchy-presentable category first.

**Definition II.201** (Controlled geometric regional certificate). Fix  $r \in \Gamma_{\text{cof}}$  and write  $\varepsilon_r = e(r)$ . A *controlled geometric regional certificate* at  $r$  is a tuple

$$\mathfrak{z} = (O, \mathfrak{c}, \mathfrak{F}, \mathcal{A}_{\mathfrak{F}}, \mathfrak{q}_{\mathfrak{F},r}) \quad (\text{II.286})$$

with the following entries.

1.  $O$  is an object of the common pre-incision testing site  $\mathfrak{S}_{\text{pre}}^\Gamma$ , and  $\overline{O}$  is included among the compact footprints of the finite demand below.
2.  $\mathfrak{c} = (N, S, \iota, B)$  is a selected P2 admissible Cauchy presentation of  $O$ .
3.  $\mathfrak{F}$  is an H2 finite obstruction-control demand containing  $\overline{O}$ , the selected Cauchy comparison footprints needed for  $\mathfrak{c}$ , the selected region-level thickening of  $O$ , and every geometric comparison used to evaluate this regional stage.

4.  $\mathcal{A}_{\mathfrak{F}}$  is an H2 finite obstruction-adapted atlas for  $\mathfrak{F}$ .
5.  $\mathfrak{q}_{\mathfrak{F},r}$  is an H3 protected quantitative presentation whose operation ledger contains the region thickening, the selected Cauchy-shadow operations, and the regional comparison faces used at  $r$ , and satisfies

$$\Theta_{\mathfrak{F}}(r) < 1. \quad (\text{II.287})$$

The certificate is geometric. It carries no regional observable algebra, normal representation, coefficient-line trivialization, boundary charge, Ward character, entropy-surgery map, or closed spacetime form.

**Definition II.202** (Geometric refinement of certificates). Let  $\mathfrak{z}$  and  $\mathfrak{z}'$  be certificates over the same underlying region  $O$ . A geometric refinement  $\mathfrak{z}' \preceq \mathfrak{z}$  consists of:

1. a selected P2 geometric Cauchy comparison between the two Cauchy presentations, after common restriction if required;
2. an enlargement to one finite demand containing both original demands;
3. an H2 atlas refining both original atlases and retaining every temporal presentation and comparison footprint;
4. one H3 protected ledger on that common refinement, evaluated at the same scale object  $r$ , containing the union of the two operation ledgers and satisfying (II.287); and
5. the evident restriction maps from the common certificate to the two original certificates.

Two certificates are *geometrically equivalent* when they admit a common refinement. Write

$$\text{Cert}_r(O) \quad (\text{II.288})$$

for the preorder of geometric-equivalence classes of certificates under refinement.

**Hypothesis II.203** (Controlled Incision Recognition package). The selected pre-incision region system and the H1–H3 data satisfy the following recognition conditions.

(CIR0) Regional regularity. Every region carrying a certificate is nonempty and relatively compact in  $M$ , has the boundary regularity required by the selected support and thickening

calculus, and is causally convex in the selected globally hyperbolic comparison neighborhood occurring in its P2 presentation.

(CIR1) Controlled-background recognition. The H1 stratification, tubes, retractions, control functions, and compact-stage geometric collars, together with one fixed complete smooth control metric on  $M$ , form a controlled incision structure on the finite stages used below. The H2 chart packets are adapted to that structure. Replacing the control metric by another H3-compatible metric changes the finite numerical ledger but not the admitted region or refinement class.

(CIR2) Directed region-thickening laws. On the certified regional basis, the selected region-level operations

$$\mathsf{T}_t(O) = O^{+t}, \quad 0 \leq t < r_{\text{th}}(O), \quad (\text{II.289})$$

satisfy identity, extensivity, regional monotonicity, thickness monotonicity, finite controlled composition, H2 chart compatibility, and naturality under the admitted strict controlled background maps. H3 supplies the quantitative assertion that  $t = \varepsilon_r$  and every selected finite composite used in the certificate remain inside the admitted region class. The directed laws are not inferred from a displacement estimate alone.

(CIR3) Geometric Cauchy presentability. For every certified region, the selected P2 Cauchy-presentation groupoid is nonempty; every two selected presentations are joined by a finite zigzag of the admitted geometric comparison moves; and every such finite zigzag admits a common finite H2–H3 refinement at the scale under consideration. This is the geometric content of CP1–CP3. No observable-algebra comparison independence is included in CIR3.

(CIR4) Regional-arrow discipline. A regional arrow is an open inclusion  $i : O \hookrightarrow O'$ . It is admitted precisely when one finite joint certificate contains  $O$ ,  $O'$ , the inclusion, and all regional support and Cauchy footprints used by that arrow. Chart transitions, Cauchy comparisons, selected scale arrows, and background maps remain decorations or comparison arrows over this inclusion; they do not become additional arrows in the regional base category.

(CIR5) Finite refinement effectivity. Every finite family of certificates and admitted regional arrows at one selected scale admits a common certificate refinement. Composition of regional arrows is represented on a common finite refinement, and the result is independent of the

chosen common refinement after passing to geometric certificate equivalence.

*Remark II.204* (What H1–H3 do and do not supply). H1 supplies the controlled carrier, H2 supplies finite adapted chart packets, and H3 proves the required positive finite-stage margins. CIR0 records the ordinary causal and boundary regularity of the selected regional basis. CIR2 records the functorial laws of the selected region thickening, which are stronger than the H3 displacement bounds. CIR3 identifies exactly the geometric portion of Cauchy presentability. These clauses must be displayed because none follows merely from the closedness of  $\Sigma$  or from the inequality  $\Theta_{\mathfrak{z}}(r) < 1$ .

**Definition II.205** (Geometric Cauchy-presentable incised category). Let

$$\mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{gcp}}(M, \Sigma) \quad (\text{II.290})$$

be the poset category whose objects are nonempty relatively compact open regions  $O \Subset M \setminus \Sigma$  satisfying:

1. the selected causal-convexity and boundary-regularity conditions;
2. finite H1-adapted chart coverage;
3.  $\varepsilon_r$ -admissibility for the selected directed thickening; and
4. geometric Cauchy presentability CP1–CP3 through the selected P2 comparison system.

The arrows are admitted inclusions. The suffix “gcp” means *geometrically Cauchy presentable*. The category  $\mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{cp}}(M, \Sigma)$  used later in Paper III additionally requires CP4: independence of the induced observable-algebra comparison. That extra condition is not part of Definition II.205.

**Definition II.206** (Decorated pre-incision regional category). Define

$$\widetilde{\mathbf{Reg}}_{\text{pre}}^{r, \text{ctrl}, \text{Cau}} \quad (\text{II.291})$$

as follows. An object is a pair  $(O, \mathfrak{z})$ , where  $\mathfrak{z}$  is a controlled geometric regional certificate over  $O$ . A morphism

$$(i, \mathfrak{d}) : (O, \mathfrak{z}) \longrightarrow (O', \mathfrak{z}') \quad (\text{II.292})$$

consists of an admitted inclusion  $i : O \hookrightarrow O'$  and a geometric equivalence class of finite joint certificates refining  $\mathfrak{z}$  and  $\mathfrak{z}'$  and representing  $i$ . Composition is induced by CIR5 on one common finite refinement.

Let  $W_r$  be the class of vertical refinement arrows whose underlying regional inclusion is  $\text{id}_O$ . Define the *recovered unadorned regional category*

$$\mathbf{Reg}_{\text{inc,rec}}^{\varepsilon_r, \text{ctrl}, \text{Cau}}(M, \Sigma) \quad (\text{II.293})$$

to have as objects the regions  $O$  for which  $\text{Cert}_r(O) \neq \emptyset$ , and a unique arrow  $O \rightarrow O'$  precisely when  $O \subseteq O'$  and the inclusion admits a joint certificate in the sense of CIR4.

**Proposition II.207** (Recognition of certified regions). *Assume H1–H3 and Hypothesis II.203. The assignment which forgets the certificate and remembers the underlying region defines a fully faithful functor*

$$J_r : \mathbf{Reg}_{\text{inc,rec}}^{\varepsilon_r, \text{ctrl}, \text{Cau}}(M, \Sigma) \hookrightarrow \mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{gcp}}(M, \Sigma). \quad (\text{II.294})$$

*Proof.* Let  $O$  carry a certificate. The equality  $M_{\text{inc}} = M \setminus \Sigma$  and the protected obstruction margin in H3 put  $\overline{O}$  compactly in the derived regular habitat. H1 gives the locally finite Whitney control system and compact-stage controlled neighborhoods. H2 gives the required finite adapted chart cover. CIR0 gives relative compactness, causal convexity in the selected globally hyperbolic neighborhood, and the declared boundary regularity. CIR2 and Theorem II.182 show that the actual selected  $\varepsilon_r$ -thickening remains relatively compact, disjoint from  $\Sigma$ , subordinate to the selected charts and Cauchy neighborhood, and inside the selected region class. CIR3 supplies geometric CP1–CP3. Hence  $O$  is an object of  $\mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{gcp}}(M, \Sigma)$ .

By CIR4, the arrows on both sides are the same admitted open inclusions. The target is a poset, so equality of underlying inclusions gives faithfulness; every admitted inclusion in the recovered source maps to the corresponding inclusion in the target, giving fullness onto the displayed image.  $\square$

**Lemma II.208** (Directedness and contractibility of certificate choices). *Assume H1–H3 and CIR3–CIR5. For every recovered region  $O$ , the preorder  $\text{Cert}_r(O)$  is nonempty and directed. Its nerve is weakly contractible. The same statement holds for the preorder of joint certificates lifting any fixed admitted inclusion  $O \subseteq O'$ .*

*Proof.* Nonemptiness is the definition of a recovered object or arrow. Given finitely many certificate classes, CIR5 supplies one finite joint demand. Theorem II.158 gives a common H2 refinement, CIR3 gives a common geometric Cauchy refinement, and H3 together with CIR5 supplies an actual margin-admissible protected ledger for the combined finite stage. Hence one class refines all classes in the finite family. A nonempty directed preorder has contractible nerve: every finite simplex diagram is coned by a common upper refinement. The arrow statement is identical with the underlying inclusion retained in the finite demand.  $\square$

**Interpretive guide.** At this stage the carrier, the protected geometry, and the finite certificate system have all been constructed, but a category has not yet been recovered: certificate choices still decorate each region and inclusion. The theorem below removes only those weakly contractible presentation choices. It does not yet identify observable algebras or represented vacuum data, which remain a separate realization-identification problem.

**Theorem II.209** (Recovery of the controlled incised regional category). *Assume P2, H1–H3, and Hypothesis II.203. Localization at vertical geometric refinements gives a canonical equivalence*

$$\widetilde{\mathbf{Reg}}_{\text{pre}}^{r,\text{ctrl},\text{Cau}}[\mathbf{W}_r^{-1}] \simeq \mathbf{Reg}_{\text{inc,rec}}^{\varepsilon_r,\text{ctrl},\text{Cau}}(M, \Sigma). \quad (\text{II.295})$$

*Under this equivalence the functor to the ordinary incised geometric category is the fully faithful inclusion (II.294).*

*Proof.* Let

$$\mathbf{U}_r : \widetilde{\mathbf{Reg}}_{\text{pre}}^{r,\text{ctrl},\text{Cau}} \longrightarrow \mathbf{Reg}_{\text{inc,rec}}^{\varepsilon_r,\text{ctrl},\text{Cau}}(M, \Sigma) \quad (\text{II.296})$$

forget all certificate choices. It sends every arrow of  $\mathbf{W}_r$  to an identity and therefore factors through the localization.

Every recovered region has a certificate, so the factored functor is essentially surjective. Fix recovered regions  $O$  and  $O'$ . If there is no admitted inclusion, neither category has a morphism between them. If  $O \subseteq O'$  is admitted, Lemma II.208 shows that the category of decorated lifts of that inclusion is nonempty and directed. After vertical refinements are inverted, all lifts represent one morphism. Thus the factored functor is full and faithful.

Composition agrees because CIR5 represents every finite composite on a common refinement and makes the result independent of that refinement. This proves the equivalence.  $\square$

*Remark II.210* (Why localization is necessary). Before localization, one region can carry many H2 atlases, Cauchy presentations, control metrics, and H3 margin ledgers. The decorated category is therefore not literally the incised regional poset. H2 common refinement, P2 geometric comparison, and H3 finite-stage effectivity show that these choices form contractible refinement systems. The regional category is recovered only after those vertical choices are forgotten or inverted.

**Hypothesis II.211** (Selected admissible finite Čech nerve). Fix a selected scale object  $r$ , a target region  $U$ , and a finite selected cover  $\mathcal{U} = \{U_i \hookrightarrow U\}_{i \in I}$ . The cover is *CIR-admissible* when every nonempty finite intersection  $U_{i_0 \dots i_k}$  is an object of  $\mathbf{Reg}_{\text{inc,rec}}^{\varepsilon_r, \text{ctrl}, \text{Cau}}(M, \Sigma)$ , every face inclusion admits a joint certificate, and all chart, Cauchy, scale, background, and mixed comparison data used on the intersections occur in one finite common refinement. Any root, branch marking, or carrier marking is retained on the resulting Čech diagram.

This is a hypothesis on one selected admissible finite cover. It is not a hypothesis that every open cover is admissible, and it does not assert saturation of an ambient Grothendieck topology.

**Corollary II.212** (Recovery of the selected geometric Čech nerve). *Assume the hypotheses of Theorem II.209 and Hypothesis II.211. Let  $\check{\mathbf{Reg}}_{\bullet, \text{inc}}^{\varepsilon_r, \text{gcp}}(\mathcal{U})$  denote the geometric Čech nerve of  $\mathcal{U}$ , and let  $\widetilde{\mathbf{Reg}}_{\bullet, \text{pre}}^{r, \text{ctrl}, \text{Cau}}(\mathcal{U})$  be its decorated certificate lift. Then levelwise localization gives a pseudonatural equivalence*

$$\boxed{\widetilde{\mathbf{Reg}}_{\bullet, \text{pre}}^{r, \text{ctrl}, \text{Cau}}(\mathcal{U})[\mathbf{W}_{\bullet, r}^{-1}] \simeq \check{\mathbf{Reg}}_{\bullet, \text{inc}}^{\varepsilon_r, \text{gcp}}(\mathcal{U}).} \quad (\text{II.297})$$

*Proof.* Apply Theorem II.209 to every nonempty finite intersection in the selected cover. Because the cover is finite, CIR5 gives one common certificate for all nondegenerate face inclusions and comparison cells. Lemma II.208 makes the levelwise equivalences compatible, up to a unique coherent comparison after localization, with all face and degeneracy maps.  $\square$

*Remark II.213* (The CP4 boundary). Corollary II.212 recovers a geometric Čech nerve. It does not yet identify that nerve with the full physical Cauchy-presentable realization diagram.

The missing CP4 assertion is that the regional observable-algebra comparison is independent of the selected chart, Cauchy frame, anchor, representative, and refinement zigzag on every simplex and is coherent with the face and degeneracy maps. H1–H3 contain no observable algebra and cannot prove that statement. CP4 is a consequence of the subsequent regional realization-identification theorem together with its evaluation, overlap, higher-intersection, and comparison-independence laws.

**Corollary II.214** (Upgrade to the physical Čech realization diagram). *Assume the hypotheses of Corollary II.212 and, in addition, the regional realization-identification theorem proves CP4 on the same selected Čech nerve. Then*

$$\boxed{\widetilde{\mathbf{Reg}}_{\bullet, \text{pre}}^{r, \text{ctrl}, \text{Cau}}(\mathcal{U})[\mathbf{W}_{\bullet, r}^{-1}] \simeq \mathbf{Reg}_{\bullet, \text{inc}}^{\varepsilon_r, \text{CP}}(\mathcal{U}).} \quad (\text{II.298})$$

*Proof.* The additional theorem identifies the geometric and physical realization objects on every Čech simplex and proves compatibility with all face, degeneracy, and higher coherence maps. Compose that nervewise identification with Corollary II.212.  $\square$

**Definition II.215** (Finite-demand cofinal recovered basis). Assume Hypothesis II.185 for a protected finite demand  $\mathfrak{F}$ . Define

$$\mathbf{Reg}_{\text{inc}, \text{rec}}^{\Gamma_{\mathfrak{F}}, \text{ctrl}, \text{Cau}}(M, \Sigma) \quad (\text{II.299})$$

to be the selected category whose objects and arrows admit one finite certificate diagram valid for every  $r \in \Gamma_{\mathfrak{F}}$ , with the common safety factor supplied by UMG. This is a demand-dependent cofinal regional basis. It is not an intersection over all possible finite demands.

**Proposition II.216** (Cofinal-tail category stability). *Assume H1–H3, CIR0–CIR5, and Hypothesis II.185 on  $\mathfrak{F}$ . Every object and arrow of  $\mathbf{Reg}_{\text{inc}, \text{rec}}^{\Gamma_{\mathfrak{F}}, \text{ctrl}, \text{Cau}}(M, \Sigma)$  is admitted at every scale in the selected tail. Regional, chart, Cauchy, mixed, and scale comparisons in the finite demand preserve the recovered category. No tail uniform over all finite demands is concluded.*

*Proof.* Corollary II.186 gives one residual positive margin for every protected relation and operation on the finite diagram throughout the tail. CIR2 retains the directed thickening laws, CIR3 retains the geometric Cauchy comparisons, and CIR4–CIR5 retain the regional arrows and their finite composites. The demand dependence is exactly the dependence allowed in Hypothesis II.185.  $\square$

**Proposition II.217** (Elementary recovery witness). *Let  $\Sigma \subset \mathbb{R}^{1,3}$  be a closed smooth submanifold or a locally finite Whitney-stratified set with fixed Thom–Mather control data. Use a complete Euclidean control metric, standard constant-time Cauchy slabs, and a selected directed metric thickening which remains in the causally convex regular-region class. On every finite family of compactly contained regular regions satisfying the H3 threshold inequalities, CIR0–CIR5 hold. Hence the decorated pre-incision region system recovers the corresponding selected geometric incised regional basis after refinement localization.*

*Proof.* Ordinary regular regions and standard globally hyperbolic slabs give CIR0. The fixed stratified tubes and Euclidean metric give CIR1. Metric neighborhoods satisfy the directed thickening laws whenever the selected regularization stays in the region class, giving CIR2. Standard Cauchy slabs and their common restrictions give CIR3. Inclusions and finite common chart refinements give CIR4–CIR5. H2 and the H3 threshold theorem supply the finite certificates and their positive margins.  $\square$

*Remark II.218* (No regional-realization identification in the recovery proof). The recovery theorem compares geometric region categories only. It does not identify the stackified lifting gerbe with the positive-thickness regional observable or represented realization. In particular, it does not prove chart-independent von Neumann algebras, local-normal representation descent, coefficient-line comparison, Gauss or Ward descent, entropy surgery, or closed form descent. Those structures are the content of the next regional realization-identification edge.

*Remark II.219* (Circularity firewall for category recovery). The category recovery uses the dynamically derived carrier, H1 geometric control data, H2 finite chart packets, H3 margin certificates, selected pre-incision supports, and P2 geometric Cauchy presentations. It uses no boundary Ward character, gauge collar charge, entropy-surgery selector, post-incision normal-state choice, invariant momentum, spectral threshold, mass estimate, confinement input, or scattering input. The category is formed before any physical comparison theorem is used to upgrade gcp to CP4.

**Corollary II.220** (Controlled-incised-category recovery status). *Under H1–H3 and CIR0–CIR5, the refinement localization of the decorated pre-incision regional category is equivalent to the recovered selected controlled incised geometric category. CIR coverage upgrades this to*

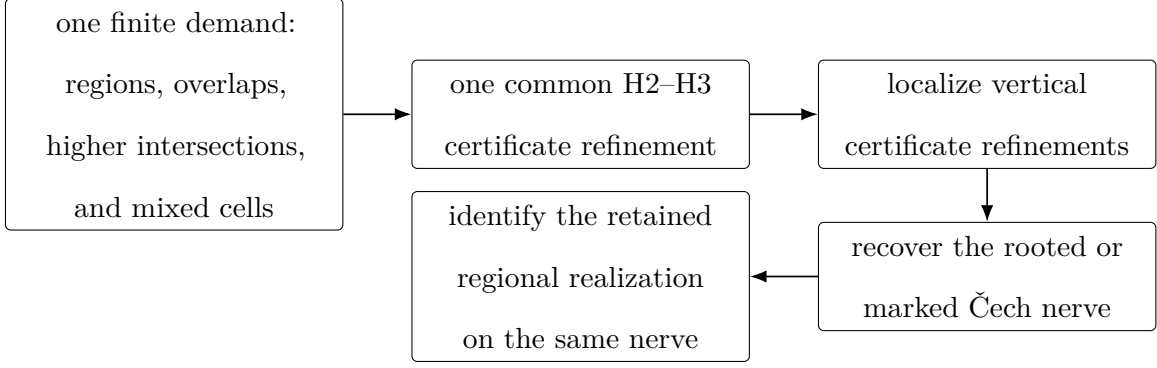


FIG. 2. Finite-diagram recovery. Vertices, overlap arrows, higher cells, and control data are refined and localized together; isolated chartwise limits are never glued afterward.

*the full intended geometric Cauchy-presentable basis. Identification with the physical category  $\mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{CP}}(M, \Sigma)$  remains conditional on the subsequent  $CP_4$  regional realization-identification theorem. Cofinal-tail stability additionally requires Hypothesis II.185 and remains finite-demand dependent.*

**What has now been established.** The geometric category is now recovered independently of chart, atlas, Cauchy-presentation, and margin-ledger choices. The finite-diagram rule is load bearing: one common refinement must carry every vertex, overlap arrow, higher intersection, and mixed comparison used in the demand. Observable  $CP_4$  and represented realization identification are proved only afterward on that same recovered nerve.

### 1. *Selected physical controlled-incised-category recovery*

The abstract recovery theorem separates the formal certificate-localization argument from the model-specific question whether the selected physical branch actually supplies CIR0–CIR5 and covers the regional basis used by the positive-thickness realization. We now answer that question for the source-free nonzero-flux  $SU(3)$  branch. The target is the selected propagation-adapted regional basis, not the poset of all relatively compact open subsets of  $M \setminus \Sigma_{\text{pres}}$ .

**Definition II.221** (Selected protected physical Cauchy diamonds). Fix  $m \in \{1, 2\}$ , a selected

scale object  $r$ , and  $\varepsilon_r = e(r)$ . A *selected protected physical Cauchy-diamond datum* is

$$\mathfrak{d}_O = (N, I_t, t; B^- \Subset B \Subset B^+; O) \quad (\text{II.300})$$

with the following properties.

1.  $B^-$ ,  $B$ , and  $B^+$  are relatively compact objects of the selected spatial dyadic site, with  $\overline{B^-} \subset B$  and  $\overline{B} \subset B^+$ .

2. The tuple

$$\mathfrak{c}_O = (N, I_t, t, B^+; S_t, \iota_t, B_t)$$

is a selected propagation-adapted P2 Cauchy presentation.

3. The regional object is

$$O = \text{int } D_N(\overline{B_t^-}), \quad \overline{O} \Subset D_N(B_t). \quad (\text{II.301})$$

4. Every actual linked-flux support footprint attached to  $O$ , and every endpoint support required by the selected finite comparison stage, lies in the protected packet determined by  $B^- \Subset B \Subset B^+$ .

5. At the actual selected thickness, the obstruction, chart, support-container, regular-region, Cauchy-slab, temporal, and declared spacelike endpoint margins are positive.

Finite intersections and precompact common shrinkings of such diamonds are again admitted when retained with their separate presentation decorations.

Let

$$\mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{phys}, \text{sel}}(M, \Sigma_{\text{pres}}) \quad (\text{II.302})$$

be the poset whose objects are the underlying admitted protected diamonds and their admitted finite intersections. Its arrows are open inclusions only. Chart changes, protected Cauchy comparisons, selected scale arrows, carrier-preserving controlled background maps, and mixed cells are comparison decorations over this poset and are not additional regional arrows.

*Remark II.222* (Regional base versus comparison data). The regional base is generated by selected regions and admitted inclusions. Chart, Cauchy, scale, background, and mixed

comparisons generate its decorated presentation system. Conflating these two layers would violate CIR4 and would falsely turn one-way scale correspondences into regional equivalences.

**Definition II.223** (Selected directed diamond thickening). For a datum  $\mathfrak{d}_O$ , set

$$r_{\text{th}}(\mathfrak{d}_O) = d_{\mathbb{R}^3}(\overline{B^-}, \mathbb{R}^3 \setminus B) > 0. \quad (\text{II.303})$$

For  $0 \leq s < r_{\text{th}}(\mathfrak{d}_O)$ , define

$$\mathsf{T}_s^{\mathfrak{d}_O}(O) = \text{int } D_N(\overline{(B^-)_t^{+s}}), \quad (\text{II.304})$$

where  $(B^-)^{+s}$  is spatial Euclidean thickening on the selected constant-time slice. Only values of  $s$  whose complete endpoint ledger is certified are admitted. In particular,  $s = \varepsilon_r$  is admitted by Theorem II.197. Different protected presentations define vertically equivalent decorated thickenings after common refinement; no literal equality between arbitrary presentation choices is asserted before localization.

**Lemma II.224** (Functorial directed-thickening laws). *The operations  $\mathsf{T}_s^{\mathfrak{d}_O}$  satisfy CIR2 on the selected decorated physical basis:*

1. *identity and extensivity;*
2. *regional and thickness monotonicity;*
3. *finite controlled composition,*

$$\mathsf{T}_u \mathsf{T}_s(O) = \mathsf{T}_{s+u}(O), \quad (\text{II.305})$$

*whenever all three operations occur in one protected certificate;*

4. *compatibility with selected H2 chart restriction;*
5. *compatibility with propagation-adapted P2 Cauchy comparison; and*
6. *naturality under admitted strict carrier-preserving controlled background maps.*

*The identities hold in one protected presentation or after a declared common refinement. They do not promote selected scale correspondences to equivalences.*

*Proof.* Spatial metric thickenings satisfy

$$(B^-)^{+0} = B^-, \quad B_1^- \subseteq B_2^- \implies (B_1^-)^{+s} \subseteq (B_2^-)^{+s},$$

and

$$\left((B^-)^{+s}\right)^{+u} = (B^-)^{+(s+u)}.$$

Monotonicity of domains of dependence gives identity, extensivity, both monotonicity laws, and (II.305).

The standard product charts use the same spatial Euclidean control metric on common slice restrictions, so chart restriction commutes with the displayed operation. In the selected Minkowski Cauchy frames, causal shadow and spatial thickening commute on the protected compact footprints. Protected Cauchy transport therefore preserves the transverse radius and carries the thickened diamond to the corresponding target presentation.

The admitted strict background maps preserve  $\Sigma_{\text{pres}}$ , its transverse orientation, the selected Cauchy structure, and the control metric on the finite stage. They carry the spatial  $s$ -neighborhood and its domain of dependence to the corresponding target operation. The H3 theorem supplies the separate quantitative assertion that every actual operation and finite composite in the demand remains inside the admitted region class.  $\square$

**Lemma II.225** (Finite selected physical certificate refinement). *Fix one finite family of selected scale objects and arrows. Every finite family of selected physical regional certificates, admitted inclusions, chart comparisons, protected Cauchy comparisons, carrier-preserving background comparisons, and mixed cells admits one common selected physical certificate refinement at their actual selected positive thicknesses.*

*Proof.* Take the finite union of all regional closures, spatial footprints, protective envelopes, chart intersections, Cauchy-shadow footprints, scale endpoints, background images, and both boundary paths of every mixed cell. Corollary II.134 supplies one finite family of standard product charts, common smaller normal and collar radii, and precompact common Cauchy-slab refinements for this union.

Apply Lemma II.195. Regional and chart restriction introduce no extra carrier motion; persistent labels remain in their fixed control cells; new labels enter with their own target footprints; protected Cauchy transport preserves transverse thickness; strict controlled background maps are finite-stage isometries; and the two boundary paths of every mixed cell have the same endpoint support family.

Theorem II.197 gives one exact residual-margin safety ratio below one for the entire combined finite demand at the actual selected thickness of every scale object appearing in it. Retain the resulting joint H2 atlas, P2 comparison presentation, and H3 endpoint ledger. This certificate restricts to every original certificate and represents every original inclusion and comparison, so it is the required common refinement.  $\square$

**Theorem II.226** (Selected physical CIR package). *For  $m = 1, 2$ , every fixed selected scale object  $r$ , and every finite selected physical comparison demand, the branch*

$$\mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{phys, sel}}(M, \Sigma_{\text{pres}})$$

*satisfies CIR0–CIR5. More precisely:*

1. *CIR0 follows from the selected protected-diamond geometry;*
2. *CIR1 follows from the transverse detector  $\delta(t, x, y, z) = (x, y)$ , the standard normal control system, and the H2 product atlases;*
3. *CIR2 is Lemma II.224;*
4. *CIR3 follows from the selected P2 comparison groupoid, the physical propagation-adapted Cauchy system, and common H2–H3 refinement;*
5. *CIR4 is the regional-arrow discipline of Definition II.221; and*
6. *CIR5 is Lemma II.225.*

*Proof.* The protected diamonds are nonempty, relatively compact, causally convex in their selected globally hyperbolic comparison neighborhoods, and have the boundary regularity used by the selected support calculus. This proves CIR0.

Theorem II.132 identifies the physical carrier with  $\Sigma_{\text{pres}}$ , while Corollary II.134 supplies the controlled detector, normal tubes, retractions, collars, product charts, selected Cauchy compatibility, and strict carrier-preserving finite-refinement system. This proves CIR1.

CIR2 and CIR5 are the preceding lemmas. For CIR3, every admitted object has a selected P2 presentation. The selected geometric Cauchy comparisons obey identity, inverse, and composition laws, and H2 supplies one common precompact product refinement for every

finite zigzag. The selected H3 theorem certifies the combined endpoint ledger at the actual thicknesses. CIR4 holds by construction because the regional arrows are inclusions and all other comparisons remain decorations over them.  $\square$

**Theorem II.227** (Contractibility of selected physical certificate fibers). *For every object  $O \in \mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{phys}, \text{sel}}(M, \Sigma_{\text{pres}})$ , the preorder  $\text{Cert}_r^{\text{phys}}(O)$  of selected physical certificates is nonempty and directed, and its nerve is weakly contractible. The same holds for the preorder of joint certificates lifting any fixed admitted inclusion  $O \subseteq O'$ .*

*Proof.* Nonemptiness follows from Theorem II.226. Given finitely many objects in either certificate preorder, Lemma II.225 gives one object refining all of them. Hence each preorder is directed.

Let  $K$  be a finite simplicial complex and

$$K \longrightarrow N \text{Cert}_r^{\text{phys}}(O)$$

a simplicial map. Only finitely many certificate vertices occur in its image. A common upper refinement cones the image of  $K$ . Every map from a finite sphere into the nerve is therefore null-homotopic. The argument for a fixed admitted inclusion is identical with that inclusion retained in the common finite demand.  $\square$

**Definition II.228** (Selected admissible physical cover and its Čech nerve). Fix  $m \in \{1, 2\}$ , a selected scale object  $r$ , and a selected positive-thickness target region  $U$  equipped with one protected H1–H3 comparison certificate. A *selected admissible physical cover* of  $U$  is a finite family

$$\mathcal{U} = \{O_i \hookrightarrow U\}_{i \in I} \tag{II.306}$$

of selected protected physical Cauchy diamonds satisfying:

1.  $U = \bigcup_{i \in I} O_i$ ;
2. every nonempty finite intersection  $O_{i_0 \dots i_k} = O_{i_0} \cap \dots \cap O_{i_k}$  is retained as an admitted protected object with its own certificate;
3. all face inclusions among these intersections, together with the selected chart, Cauchy, scale, background, and mixed comparison data used on them, occur in one finite joint-certificate refinement; and

4. any branch marking or carrier marking is retained on the intersections and restriction maps rather than forgotten by passage to the cover.

No assertion is made that every open cover is admissible. The cover is chosen from the finite propagation-adapted cover class certified by the selected H1–H3 data.

Write

$$\check{\mathcal{C}}_k(\mathcal{U}) = \coprod_{\substack{(i_0, \dots, i_k) \in I^{k+1} \\ O_{i_0 \dots i_k} \neq \emptyset}} O_{i_0 \dots i_k} \quad (\text{II.307})$$

for its geometric Čech nerve, with face maps induced by open inclusions and degeneracies by repeated indices. When a root or marking is part of the physical packet,  $\check{\mathcal{C}}_\bullet(\mathcal{U})$  is understood as the corresponding rooted or marked Čech nerve.

**Definition II.229** (Decorated certificate Čech nerve). For a selected admissible cover  $\mathcal{U}$ , let  $\check{\mathcal{C}}_k^{r,m,\text{phys}}(\mathcal{U})$  be the disjoint union of the selected physical certificate categories over the nonempty  $k$ -fold intersections  $O_{i_0 \dots i_k}$ . Let  $W_k^{r,m,\text{phys}}$  be the vertical certificate-refinement arrows. Restriction along face maps and repetition along degeneracies define a normal pseudofunctor

$$\check{\mathcal{C}}_\bullet^{r,m,\text{phys}}(\mathcal{U}) : \Delta^{\text{op}} \rightsquigarrow \mathbf{Cat}, \quad (\text{II.308})$$

whose coherence cells are supplied by finite joint-certificate refinement. Different refinement choices are related by vertical arrows.

**Theorem II.230** (Selected physical Čech-nerve recovery). *Fix one selected admissible finite physical cover  $\mathcal{U}$  of  $U$ . For  $m = 1, 2$  and every fixed selected scale object  $r$ , levelwise localization at vertical geometric certificate refinements gives a pseudonatural equivalence*

$$\boxed{\check{\mathcal{C}}_\bullet^{r,m,\text{phys}}(\mathcal{U})[(W_\bullet^{r,m,\text{phys}})^{-1}] \simeq \check{\mathcal{C}}_\bullet(\mathcal{U}).} \quad (\text{II.309})$$

*The equivalence forgets only weakly contractible geometric certificate choices. It is compatible with all face and degeneracy maps, and it preserves all retained root, branch, and carrier markings as external descent data.*

*Proof.* At every nonempty intersection  $O_{i_0 \dots i_k}$ , Theorem II.227 identifies the localized certificate fiber with the underlying geometric intersection. Because  $\mathcal{U}$  is finite, its nondegenerate

Čech nerve contains only finitely many intersection and face-inclusion demands. Apply Lemma II.225 once to that combined finite demand. The resulting joint certificate represents all face maps, degeneracies, selected comparisons, and mixed cells simultaneously.

Any two such choices admit a common vertical refinement. Contractibility of the object and fixed-inclusion certificate fibers makes the induced comparison unique up to coherent homotopy after localization. Hence the levelwise equivalences assemble pseudonaturally over  $\Delta^{\text{op}}$ , proving (II.309).  $\square$

**Corollary II.231** (Selected physical CIR closure on an admissible Čech nerve). *On the selected source-free nonzero-flux  $SU(3)$  branch, CIR0–CIR5 and weak contractibility of geometric certificate fibers are discharged at every fixed selected positive thickness. For each fixed selected admissible finite cover  $\mathcal{U}$ , the decorated pre-incision data recover the geometric rooted or marked Čech nerve of  $\mathcal{U}$  after localization at vertical certificate refinements.*

*This corollary does not assert recovery for every open cover, generation or saturation of an ambient Grothendieck topology, equality with all relatively compact regular opens, observable presentation independence CP4, identification with the retained represented realization, H3-UMG, or any thin-limit statement. The next front-end edge is realization identification and observable CP4 formulated on the same selected Čech nerve: local packet identification on vertices, comparison on pairwise intersections, and coherent two-cell/effectivity comparison on higher intersections.*

**Remark II.232** (Circularity and scope firewall). The proof uses only the dynamically derived carrier, H1 geometric control, H2 finite chart packets, H3 positive endpoint margins, selected P2 geometric Cauchy presentations, and pre-incision support and comparison footprints. It uses no boundary Ward character, gauge collar charge, entropy-surgery selector, post-incision normal state, invariant momentum, spectral threshold, mass estimate, confinement input, or scattering input. The regional category is recovered before any physical comparison theorem upgrades gcp to CP4.

## X. Regional realization identification and observable CP4

The category-recovery theorem identifies the selected geometric testing basis. It does not identify the two physical realization systems carried over that basis. On one side is the stackified simultaneous lifting gerbe constructed from the pre-incision anchored lifting problem. On the other side is the positive-thickness regional realization assembled in the later Paper-III construction from presented regional algebras, chart-local represented packets, complete-ground projections, coefficient lines or correspondences, scale transport, and mixed coherence.

There are two reasons that the comparison must be typed carefully. First, the retained Paper-III object distinguishes an ordinary descended observable algebra from its possibly twisted chart-local Hilbert realization. Second, the old realization object does not visibly store the P2 time-zero anchor, root certificate, or marked-branch witness as part of every object. We therefore compare the lifting gerbe first with a minimally anchored enlargement of the retained realization prestack and remove the additional decorations only after proving that their fibers are contractible.

**Definition II.233** (Retained fixed-scale Paper-III realization packet). Fix a selected scale object  $r$ , write  $\varepsilon_r = e(r)$ , and let  $O$  be an object of the recovered geometric category  $\mathbf{Reg}_{\text{inc,rec}}^{\varepsilon_r, \text{ctrl}, \text{Cau}}(M, \Sigma)$ . A retained fixed-scale Paper-III realization packet on  $O$  is a tuple

$$\mathfrak{R}_r^{\text{III}}(O) = \left( \mathcal{A}_r^W(O), \mathcal{E}_r(O), \mathcal{H}_r(O), \mathcal{L}_r(O), \mathcal{S}_r(O), \mathcal{M}_r(O), \mathbf{An}_r(O), \tau_r \right), \quad (\text{II.310})$$

where the entries have the following meanings.

1.  $\mathcal{A}_r^W(O)$  is the ordinary descended regional  $W^*$ -algebra, with normal unital evaluation isomorphisms

$$\text{ev}_{p,r}^O : \mathcal{A}_r^W(O) \xrightarrow{\cong} \mathcal{A}_{p,r}^W(O) \quad (\text{II.311})$$

for every selected regional Cauchy presentation  $p$  of  $O$ .

2.  $\mathcal{E}_r(O)$  is the finite-presentation comparison system: anchor changes, common frame restrictions, adapted chart comparisons, same-thickness representative comparisons, containing-slab maps, and their ordinary observable-algebra cocycle relations.

3.  $\mathcal{H}_r(O)$  is the chart-local represented packet consisting of the physical Hilbert spaces, faithful normal regional representations, complete-ground projections, support filters, and their selected ground-preserving Cauchy comparison maps. No preferred ground vector is included.
4.  $\mathcal{L}_r(O)$  is the fixed-thickness coefficient datum: Hermitian lines or the selected normal correspondences, overlap intertwiners, multiplication maps, tensorators, and fixed-thickness reassociation cells. Its adjoint action on  $\mathcal{A}_r^W(O)$  is ordinary even when the represented datum is nonneutral.
5.  $\mathcal{S}_r(O)$  is the selected scale datum on every scale arrow appearing in the finite stage: ordinary algebra maps or normal correspondences, representation-scale coefficient lines or ground-pointed correspondences, scale tensorators, and their finite composition cells.
6.  $\mathcal{M}_r(O)$  is the mixed chart-scale datum, including the comparison lines or correspondences, mixed tensorators, prism cells, and the selected chart-scale reassociation laws.
7.  $\mathbf{An}_r(O)$  is the typed admissible analytic comparison datum on the propagation, representation, complete-ground, common-core, interior Gauss, and interior Ward faces that occur before the later collar, state-surgery, and closed-form constructions.
8.  $\tau_r$  is the retained strict, controlled-scale-twisted, or fixed-thickness-gerbe-twisted branch marking.

One-morphisms are marking-preserving regional equivalences carrying the ordinary algebra, represented packet, coefficient or correspondence data, analytic classes, and all declared coherence cells. Two-morphisms are invertible modifications. Restriction along an admitted regional inclusion is given by the retained regional restriction maps.

**Definition II.234** (Retained and anchored realization prestacks). Let

$$\mathfrak{Real}_{\text{III}}^{+,r} \tag{II.312}$$

be the selected  $(2,1)$ -prestack of packets from Definition II.233 on the recovered geometric regional category.

An *anchored enhancement* of an object  $\mathfrak{R}_r^{\text{III}}(O)$  consists of:

1. a selected P2 admissible Cauchy presentation  $\mathfrak{c} = (N, S, \iota, B)$ ;
2. a marking-preserving anchor

$$\eta_{\mathfrak{c},r}^{\text{III}} : \mathfrak{h}_r^0(B) \xrightarrow{\simeq} \text{Data}_{\mathfrak{c},r}(\mathfrak{R}_r^{\text{III}}(O)); \quad (\text{II.313})$$

3. a root and marked-branch certificate compatible with  $\tau_r$ ; and
4. the regional, selected-Cauchy, scale, and mixed comparison modifications required by P2.

The resulting anchored prestack is denoted

$$\mathfrak{Real}_{\text{III}}^{+,r,\text{anch}}. \quad (\text{II.314})$$

There is a forgetful pseudofunctor

$$\text{For}_r : \mathfrak{Real}_{\text{III}}^{+,r,\text{anch}} \longrightarrow \mathfrak{Real}_{\text{III}}^{+,r}. \quad (\text{II.315})$$

No assertion that  $\text{For}_r$  is an equivalence is built into this definition.

**Definition II.235** (Observable shadow). The observable shadow of the retained realization is the pseudofunctor

$$\text{Obs}_r : \mathfrak{Real}_{\text{III}}^{+,r,\text{anch}} \longrightarrow \mathbf{W}^* \mathbf{Alg}_{\mathbf{n}} \quad (\text{II.316})$$

which retains  $\mathcal{A}_r^W(O)$ , its normal regional inclusions, and the normal algebra maps induced by presentation comparisons, while forgetting Hilbert implementers, coefficient lines, roots, and all higher represented coherence. The target is an ordinary one-category, regarded as locally discrete in dimension two.

**Definition II.236** (Fixed-scale and cofinal comparison stacks). On the recovered regional basis define the fixed-scale lifting stack

$$\mathfrak{G}_{r,\mathfrak{h}} = \left( \mathfrak{L}_{r,\mathfrak{h}}^{\text{anch}} \right)^+ \Big|_{\mathbf{Reg}_{\text{inc,rec}}^{\varepsilon_r, \text{ctrl}, \text{Cau}}(M, \Sigma)}. \quad (\text{II.317})$$

Define the retained cofinal realization prestacks by the selected homotopy limits

$$\mathfrak{Real}_{\text{III}}^{+,\Gamma,\text{anch}} = \text{holim}_{r \in \Gamma_{\text{cof}}} \mathfrak{Real}_{\text{III}}^{+,r,\text{anch}}, \quad \mathfrak{Real}_{\text{III}}^{+,\Gamma} = \text{holim}_{r \in \Gamma_{\text{cof}}} \mathfrak{Real}_{\text{III}}^{+,r}, \quad (\text{II.318})$$

using only the selected scale transition functors, compositors, and mixed cells. These definitions do not replace the full scale category by the numerical thickness poset.

**Hypothesis II.237** (Regional Identification package). The pre-incision lifting gerbe and the retained Paper-III realization system satisfy the following conditions on one fixed selected admissible geometric Čech nerve.

(RID0) Common selected Čech nerve. For every selected scale object, both prestacks are restricted to the same selected admissible rooted or marked Čech nerve from Hypothesis II.211. Every chart, support, Cauchy, scale, and mixed finite demand used on its vertices and intersections has a common H2–H3 refinement, and both realization systems use the same face, degeneracy, root, branch, and carrier-marking data.

(RID1) Local packet dictionary. For every adapted chart packet and fixed scale  $r$ , there is a decorated biequivalence

$$D_{\alpha,r} : \mathbf{LocPkg}_r^{+,pre}(U_\alpha) \simeq \mathbf{LocPkg}_r^{\text{III}}(U_\alpha) \quad (\text{II.319})$$

which identifies the physical Wilson quotient, represented observable action, complete-ground projection, support filter, common-core packet, interior Gauss and Ward data, admissible analytic structure, and marked branch. It preserves declared domains rather than identifying merely algebraic cores.

(RID2) Observable-core and comparison-presentation compatibility. Under  $D_{\alpha,r}$ , the P4 physical regional quotient and its support-controlled generators map to the retained presented regional algebra with ultraweakly dense image. Regional Cauchy, chart, representative, and containing-slab comparisons induce the same normal algebra maps on both sides. The elementary maps satisfy the strict observable Cauchy, chart, representative, refinement, and mixed relations. Moreover, those elementary relations are complete for the selected finite presentation system: after a common H2–H3 refinement, any two finite zigzags with the same endpoints are related by finitely many of the displayed generator relations.

(RID3) Fixed-thickness represented descent. The local dictionary carries P4 chart correspondences, coefficient lines or normal correspondences, multiplication cells, and fixed-thickness reassociators to the retained overlap system. It preserves the selected band and does not replace a nonneutral coefficient system by scalar equalities.

(RID4) Cauchy anchor, root, and marking compatibility. The local dictionary intertwines P2 Cauchy-data extraction with the retained Paper-III extraction. It carries the P2 anchor,

complete-ground projection, root, and branch marking to the anchored enhancement (II.313), including pairwise, triple, and mixed comparison modifications.

(RID5) Analytic-domain fidelity. Propagation, representation, complete-ground, Gauss, Ward, and other pre-incision analytic comparisons are transported in their correct typed categories. Equality on a common algebraic core is not promoted to equality of closed operators, graph domains, spectra, or state spaces. Controlled nonzero admissible analytic representatives remain allowed.

(RID6) Regional pseudonaturality. For every admitted inclusion  $i : O \hookrightarrow O'$ , the local dictionary and its inverse commute with restriction up to specified invertible modifications. These modifications satisfy identity and composition laws on every finite H2 refinement diagram.

(RID7) Scale and mixed pseudonaturality. For every selected scale arrow and finite composite, the dictionary carries ordinary algebra transport, representation-scale lines or correspondences, scale tensorators, mixed chart-scale cells, and rooted reassociation data to the retained Paper-III datum. The chart, scale, and mixed prisms and cubes commute after the selected finite common refinement.

(RID8) Effectivity comparison. Applying the local dictionary chartwise sends an effective anchored descent object of Definition II.59 to an effective retained realization object, and the inverse dictionary sends an effective retained object to an effective P4 descent object. These operations commute with admissible stackification up to a pseudonatural equivalence. The local unit and counit satisfy coherent descent and induce invertible triangle modifications.

(RID9) Contractible enhancement fibers. For every retained realization object, the  $(2, 1)$ -groupoid of admissible Cauchy-anchor, root, and marked-branch enhancements is locally nonempty and has weakly contractible nerve. The analogous enhancement groupoid over each one-morphism is locally nonempty and weakly contractible. This condition is not replaced by mere connectedness.

(RID10) Cofinal pseudocone recognition. The fixed-scale comparison pseudofunctors preserve the selected scale transition functors and their compositors. Hence compatible fixed-scale comparisons assemble to pseudofunctors on the selected cofinal homotopy limits. No comparison

is asserted for scales or corridors outside the selected cofinal system.

*Remark II.238* (Exact substantive burden). RID1 is the local object dictionary. RID2 is the load-bearing observable comparison theorem and contains the input that proves CP4. RID8 is the actual descent/effectivity comparison. RID9 is required before the anchored target can be replaced by the older undecorated realization prestack. None of these statements follows from the fact that both constructions use similar notation for charts, Hilbert spaces, or coefficient lines.

**Theorem II.239** (Fixed-scale anchored realization identification). *Assume P1–P5, H1–H3, category recovery, and RID0–RID9. For every selected scale object  $r$ , there are marking-preserving pseudofunctors*

$$F_r : \mathfrak{G}_{r,\mathfrak{h}} \longrightarrow \mathfrak{Real}_{\text{III}}^{+,r,\text{anch}}, \quad G_r : \mathfrak{Real}_{\text{III}}^{+,r,\text{anch}} \longrightarrow \mathfrak{G}_{r,\mathfrak{h}} \quad (\text{II.320})$$

*with pseudonatural equivalences*

$$G_r F_r \simeq \text{id}_{\mathfrak{G}_{r,\mathfrak{h}}}, \quad F_r G_r \simeq \text{id}_{\mathfrak{Real}_{\text{III}}^{+,r,\text{anch}}}, \quad (\text{II.321})$$

*and invertible triangle modifications. Thus*

$$\boxed{\mathfrak{G}_{r,\mathfrak{h}} \simeq \mathfrak{Real}_{\text{III}}^{+,r,\text{anch}}} \quad (\text{II.322})$$

*as  $(2,1)$ -stacks on the recovered geometric regional basis.*

*Proof.* We construct the two pseudofunctors and then compare their composites.

*Construction of  $F_r$ .* An object of  $\mathfrak{G}_{r,\mathfrak{h}}$  is locally represented by an anchored effective descent object because the gerbe is the admissible stackification of the fixed-scale homotopy fiber. Apply  $D_{\alpha,r}$  to every chart-local realization packet. RID3, RID5, and RID8 transport the chart correspondences, coefficient system, analytic correction certificates, multiplication maps, and reassociation cells to an effective retained descent presentation.

RID2 forms the ordinary regional algebra as the compatible-family descent of the presented algebras. Evaluation in any selected presentation gives the maps (II.311). The comparison-presentation completeness in RID2 makes this ordinary algebra independent of the local

descent representative, even when the represented Hilbert packet has nontrivial coefficient-band isotropy. RID4 transports the anchor, root, complete-ground projection, and branch marking. RID6 makes the assignment pseudonatural in regional restriction. This defines  $F_r$ . *Construction of  $G_r$ .* Start with an anchored retained realization. Evaluate its ordinary regional algebra in the selected local Cauchy and chart presentations. Use the inverse local dictionary from RID1 on the chart-local represented packets. RID2 identifies the resulting physical regional quotient and support-controlled core with the P4 local packet; RID3 and RID5 recover the coefficient, coherence, common-domain, interior Gauss, Ward, and analytic data. RID4 recovers the P2 anchor and root. RID8 supplies an effective anchored descent object, and P4 exactness places its underlying candidate in  $\mathbf{CohReal}_r^{\text{anch}}$ . Taking the homotopy fiber over  $\mathfrak{h}_r^0$  and applying the admissible stackification gives an object of  $\mathfrak{G}_{r,\mathfrak{h}}$ . RID6 gives regional pseudonaturality.

*Unit, counit, and triangles.* On every chart packet the two composites are related to the identity by the unit and counit of  $D_{\alpha,r}$ . RID8 says that these local cells respect effective descent and stackification. RID2 identifies the ordinary observable algebra reconstructed after either composite with the original one by evaluation, while RID3–RID5 identify all represented and anchored decorations. The local triangle identities descend to invertible triangle modifications. H2 common refinement and RID8 make the result independent of the selected local representative. This proves the biequivalence.  $\square$

**Theorem II.240** (Simultaneous cofinal anchored identification). *Assume the hypotheses of Theorem II.239 and RID10. The fixed-scale comparison pseudofunctors assemble to a marking-preserving pseudonatural biequivalence*

$$\boxed{\mathfrak{G}_{\Gamma,\mathfrak{h}} \Big|_{\mathbf{Reg}_{\text{inc,rec}}^{\Gamma,\text{ctrl,Cau}}} \simeq \mathfrak{Real}_{\text{III}}^{+,\Gamma,\text{anch}}(M, \Sigma).} \quad (\text{II.323})$$

*The restriction is taken on the selected finite-demand cofinal bases for which the relevant UMG certificates are available. The theorem does not produce a category uniform over all finite demands.*

*Proof.* RID10 gives pseudonatural comparison transformations between the fixed-scale functors and the selected scale transition functors. Their compositor cells satisfy the required

pseudocone equations. Apply the fixed-scale functors objectwise to a cofinal pseudocone and transport every scale and mixed cell by RID7. This defines comparison pseudofunctors between the two selected homotopy limits. The fixed-scale units, counits, and triangle modifications are compatible with the scale compositors by RID10, so they assemble to the cofinal unit, counit, and triangles. Restriction to a finite-demand tail uses H3–UMG and the cofinal category-stability proposition.  $\square$

**Proposition II.241** (Contractible removal of anchor and root decorations). *Assume RID9. The forgetful pseudofunctors*

$$\text{For}_r : \mathfrak{Real}_{\text{III}}^{+,r,\text{anch}} \longrightarrow \mathfrak{Real}_{\text{III}}^{+,r} \quad (\text{II.324})$$

*and, under RID10,*

$$\text{For}_\Gamma : \mathfrak{Real}_{\text{III}}^{+,\Gamma,\text{anch}} \longrightarrow \mathfrak{Real}_{\text{III}}^{+,\Gamma} \quad (\text{II.325})$$

*are local biequivalences after admissible stackification.*

*Proof.* RID9 gives local essential surjectivity on objects. Over a fixed retained object, the enhancement  $(2,1)$ -groupoid has weakly contractible nerve, so two enhancements are connected after admissible refinement and every two comparison paths are coherently identified. The corresponding statement for one-morphisms gives local full faithfulness. Hence the forgetful map is a local biequivalence. Stackification converts it into a biequivalence of the selected stacks. RID10 applies the same argument to compatible cofinal pseudocones.  $\square$

**Theorem II.242** (Regional realization identification). *Assume P1–P5, H1–H3, category recovery, and RID0–RID10. On the recovered selected cofinal regional basis, the stackified simultaneous lifting gerbe is marking-preservingly pseudonaturally equivalent to the retained positive- thickness Paper-III realization prestack:*

$$\boxed{\mathfrak{G}_{\Gamma,\mathfrak{h}} \Big|_{\mathbf{Reg}_{\text{inc,rec}}^{\Gamma,\text{ctrl,Cau}}} \simeq \mathfrak{Real}_{\text{III}}^{+,\Gamma} \left( M, \Sigma_{\text{obs}}^{\Gamma,\text{cof}} \right)}. \quad (\text{II.326})$$

*The equivalence preserves support filters, ordinary regional observable algebras, selected admissible Cauchy presentations, chart-local faithful normal representations, complete-ground projections, coefficient lines or normal correspondences, multiplication maps, fixed-thickness and scale tensorators, analytic comparison classes, mixed cells, roots before forgetting, and the marked sector type.*

*Proof.* Compose the cofinal anchored equivalence (II.323) with the forgetful biequivalence of Proposition II.241. The preservation statement is componentwise RID1–RID7. Contractibility in RID9 shows that forgetting the added anchor and root does not introduce a new choice class.  $\square$

*Remark II.243* (Why identification is separate). The category equivalence (II.295) identifies testing regions after vertical geometric certificate choices are inverted. It does not identify two prestacks of physical realizations. Theorem II.242 requires actual comparison pseudofunctors, unit and counit equivalences, triangle modifications, effectivity comparison, and a contractible-fiber theorem for the extra anchored decorations.

**Theorem II.244** (Observable comparison independence and CP4 on the selected Čech nerve). *Assume RID0–RID8 at a fixed selected scale  $r$ . Every vertex and nonempty intersection of the selected admissible Čech nerve satisfies CP4: the normal observable-algebra comparison between any two selected regional presentations is independent of the chosen refinement, common slab, and finite zigzag. These identifications commute with every face and degeneracy map. Equivalently, the observable shadow of each presentation comparison groupoid is connected with trivial isotropy, coherently over the nerve.*

*Consequently, on each recovered simplex,*

$$\boxed{\mathbf{Reg}_{\text{inc,rec}}^{\varepsilon_r, \text{ctrl}, \text{Cau}}(M, \Sigma) \subseteq \mathbf{Reg}_{\text{inc}}^{\varepsilon_r, \text{CP}}(M, \Sigma),} \quad (\text{II.327})$$

*and on the selected nerve from Hypothesis II.211,*

$$\boxed{\check{\mathbf{Reg}}_{\bullet, \text{inc}}^{\varepsilon_r, \text{gCP}}(\mathcal{U}) = \check{\mathbf{Reg}}_{\bullet, \text{inc}}^{\varepsilon_r, \text{CP}}(\mathcal{U})} \quad (\text{II.328})$$

*as simplicial regional diagrams.*

*Proof.* Fix one nonempty intersection  $O_{i_0 \dots i_k}$ . By CIR3, its selected presentation groupoid is nonempty and connected. By RID2, each elementary comparison induces the normal algebra map of the retained presentation system. This applies separately to anchor, frame, chart, representative, and containing-slab comparisons. The observable Cauchy, chart, representative, refinement, and mixed relations are strict. Two finite zigzags with the same

endpoints therefore agree after a common H2–H3 refinement, and every isotropy loop acts identically on the observable algebra. This is CP4 on the simplex.

RID0 supplies the same selected Čech nerve on both sides, while RID2–RID8 prove compatibility with all face, degeneracy, and higher coherence maps. Hence the geometric and physical Čech diagrams agree.  $\square$

**Corollary II.245** (Upgrade of the Čech-nerve recovery theorem). *Assume the hypotheses of Corollary II.212 and Theorem II.244. Then*

$$\boxed{\widetilde{\mathbf{Reg}}_{\bullet, \text{pre}}^{r, \text{ctrl}, \text{Cau}}(\mathcal{U})[W_{\bullet, r}^{-1}] \simeq \mathbf{Reg}_{\bullet, \text{inc}}^{\varepsilon_r, \text{cp}}(\mathcal{U})}. \quad (\text{II.329})$$

Thus the conditional upgrade stated in Corollary II.214 is discharged under RID.

*Proof.* Combine the geometric Čech-nerve recovery equivalence (II.297) with the nervewise CP4 equality (II.328).  $\square$

**Corollary II.246** (Canonical ordinary regional observable functor). *Under the hypotheses of Theorem II.244, the assignments*

$$O \longmapsto \mathcal{A}_r^W(O), \quad (O \subseteq O') \longmapsto \iota_{O, O'}^r \quad (\text{II.330})$$

*define an ordinary functor*

$$\mathcal{A}_r^W : \mathbf{Reg}_{\text{inc}, \text{rec}}^{\varepsilon_r, \text{ctrl}, \text{Cau}}(M, \Sigma) \longrightarrow \mathbf{W}^* \mathbf{Alg}_n. \quad (\text{II.331})$$

*Evaluation in any selected presentation is a normal unital isomorphism, and the inclusion maps are independent of the presentation used to compute them.*

*Proof.* CP4 makes the compatible-family algebra and its evaluations independent of the presentation zigzag. Evaluate the inclusion  $O \subseteq O'$  in one joint presentation; it is a concrete inclusion of von Neumann algebras. Independence follows from the commuting evaluation squares. Identity and composition reduce in a common presentation to identity and composition of ordinary inclusions.  $\square$

*Remark II.247* (CP4 does not trivialize the gerbe). CP4 is a statement about the ordinary observable shadow. A coefficient-line or normal-correspondence isotropy may act trivially by

adjoint action on the regional algebra while remaining nontrivial on the represented Hilbert, complete-ground, and scale-coherence data. Therefore (II.328) does not imply neutrality of  $\mathfrak{G}_{\Gamma, \mathfrak{h}}$ , a global Hilbert-space realization, or vanishing of the fixed-thickness or total chart-scale class.

**Proposition II.248** (Finite strict and banded witnesses). *Suppose a finite recovered regional site carries finite-dimensional physical quotients and Hilbert packets, a finite presentation groupoid with trivial observable isotropy, finite coefficient-line or correspondence groupoids, and exact finite descent. Suppose further that the local  $P_4$  and retained Paper-III packet dictionaries are literal identities on the displayed data and that the anchored enhancement groupoids are nonempty directed groupoids. Then RID0–RID10 hold. The witness may retain a nontrivial finite coefficient band even though  $CP_4$  holds.*

*Proof.* The identity dictionaries verify RID1–RID7. Finite effective descent gives RID8, directed enhancement groupoids give RID9, and finite compatible scale diagrams give RID10. Trivial observable isotropy proves  $CP_4$ , whereas a nontrivial coefficient cocycle remains visible in the represented descent system.  $\square$

*Remark II.249* (Status of retained Yang–Mills witnesses). The retained explicit chart-local and coefficient-line constructions show that the target realization architecture is inhabited after an incision has already been selected. They do not by themselves prove that the new pre-incision lifting gerbe is carried to that target by RID1, that the two effectivity notions agree as required by RID8, or that all anchored extensions have contractible fibers as required by RID9. Those are the model-specific realization-identification obligations.

*Remark II.250* (Reintroduction boundary). Only after the derived carrier, H1–H3 geometry, category recovery,  $CP_4$ , and Theorem II.242 have been established may the manuscript identify the inherited complete-ground projections and reintroduce the Paper-I and Paper-II coercive constants on the retained positive-thickness realization. No lower bound, spectral threshold, invariant momentum, or mass datum is used to prove RID.

*Remark II.251* (Circularity firewall for realization identification). RID uses the pre-incision physical quotients, support filters, selected Cauchy anchors, chart-local represented packets,

complete-ground projections, coefficient or correspondence data, admissible analytic classes, and selected scale/mixed coherence. It does not use gauge collar charges, boundary Ward characters, entropy surgery, post-incision normal-state selection, closed spacetime forms, invariant momentum, spectral thresholds, mass estimates, confinement, or scattering. Later post-incision structures may be transported through the identification only after they have been separately constructed.

**Corollary II.252** (Regional identification closure status). *The fixed-scale anchored equivalence is conditionally proved under RID0–RID9. The cofinal anchored equivalence and the unanchored retained identification are conditionally proved under RID10 and the contractible fiber condition RID9. CP4 and the ordinary regional observable functor are conditionally proved under RID2 and the finite comparison compatibility clauses. Model-specific Yang–Mills verification of the local packet dictionary, observable comparison equality, effectivity comparison, and contractible enhancement fibers remains an explicit proof obligation.*

### 1. Selected $SU(3)$ realization identification on an admissible Čech nerve

The abstract RID architecture separates geometric recovery from realization identification. The preceding theorem recovers, for one chosen admissible finite physical cover  $\mathcal{U}$ , its rooted or marked geometric Čech nerve. We now compare the two realization systems only on that same simplicial diagram. The source is the selected source-free physical lifting stack, not the stationary P1–P5 suspension with empty carrier. The target is the retained positive-thickness  $SU(3)$  regional realization.

**Definition II.253** (Nervewise source and target realization diagrams). Fix  $m \in \{1, 2\}$ , a selected scale object  $r$ , and a selected admissible physical cover  $\mathcal{U} = \{O_i \hookrightarrow U\}_{i \in I}$  as in Definition II.228. For every nonempty intersection  $O_{i_0 \dots i_k}$ , let

$$\mathfrak{G}_{r,m}^{\text{vac}}(O_{i_0 \dots i_k}) := \text{ev}_r \mathfrak{L}_{\Gamma_{\mathfrak{D}},m}^{\text{YM,vac},\#}(O_{i_0 \dots i_k}), \quad (\text{II.332})$$

where  $\Gamma_{\mathfrak{D}}$  is any selected finite scale diagram or selected tail containing every scale object and arrow in the finite RID demand  $\mathfrak{D}$ . Different admissible choices are compared only on a common later selected diagram; no universal tail is asserted.

Let

$$\mathfrak{Real}_{r,m}^{SU(3)}(O_{i_0 \cdots i_k}) \subset \mathfrak{Real}_{\text{III}}^{+,r}(O_{i_0 \cdots i_k}) \quad (\text{II.333})$$

be the full selected subcategory generated by the retained linked-flux  $SU(3)$  packet, its selected Cauchy presentations, normal regional inclusions, center-background chart maps, coefficient lines or normal correspondences, roots, markings, and all selected fixed-scale and mixed coherence cells. Its anchored enhancement is denoted  $\mathfrak{Real}_{r,m}^{SU(3),\text{anch}}$ .

Taking products over the nonempty  $k$ -fold intersections gives normal pseudofunctors

$$\mathfrak{G}_{r,m,\bullet}^{\text{vac}}(\mathcal{U}), \quad \mathfrak{Real}_{r,m,\bullet}^{SU(3)}(\mathcal{U}) : \Delta^{\text{op}} \rightsquigarrow (2,1)\text{-}\mathbf{Cat}, \quad (\text{II.334})$$

whose face maps are regional restrictions and whose degeneracies repeat an index. Roots, branch markings, and carrier markings are retained as simplicial descent data. Scale arrows remain normal correspondences or isometries and are not declared equivalences.

**Definition II.254** (Finite nervewise RID demand). A finite nervewise RID demand consists of the finite nondegenerate part of  $\check{\mathcal{C}}_{\bullet}(\mathcal{U})$ , together with finitely many:

1. selected scale objects, arrows, and finite composites;
2. adapted charts, selected Cauchy frames, anchors, roots, and common refinements on its vertices and intersections;
3. objects, one-morphisms, and two-morphisms in the source and target fibers; and
4. regional, chart, Cauchy, scale, background, and mixed cells needed to compare those data through the required simplicial degree.

All geometric and analytic endpoints are evaluated at their actual selected positive thicknesses. Theorem II.197 and Lemma II.225 provide one joint certificate for this finite demand. This statement does not produce one margin or one scale tail valid for all possible demands.

**Lemma II.255** (Nervewise local packet dictionary). *For every nonempty intersection  $O_{i_0 \cdots i_k}$ , every finite nervewise RID demand, and every selected scale object in that demand, the off-carrier source-free extension determines a marking-preserving decorated biequivalence*

$$\mathbf{D}_{r,m;i_0 \cdots i_k}^{\text{vac}} : \mathfrak{G}_{r,m}^{\text{vac}}(O_{i_0 \cdots i_k}) \simeq \mathfrak{Real}_{r,m}^{SU(3),\text{anch}}(O_{i_0 \cdots i_k}). \quad (\text{II.335})$$

*On the canonical one-copy representatives it identifies:*

1. the physical Wilson quotient and ordinary regional  $W^*$ -factor;
2. the faithful normal represented packet, complete-ground projection, complementary corner, support filters, and common analytic core;
3. the selected nonstationary Cauchy trace, solution, and evolution maps;
4. the interior Gauss graph forms and Ward derivations on their declared domains;
5. the linked-flux coefficient line or normal correspondence, multiplication cells, reassociator  $\Omega_m$ , normalized root, and Sector-III marking; and
6. every selected regional, chart, Cauchy, scale, and mixed comparison in the finite demand.

No operator is extended across  $\Sigma_{\text{pres}}$ , and no equality on a common algebraic core is promoted to equality of unrelated closed operators, domains, spectra, or state spaces.

*Proof.* The physical branch-matching construction retains, off  $\Sigma_{\text{pres}}$ , the operators, Hilbert packet, complete-ground corner, common core, nonstationary Cauchy evolution, interior Gauss–Ward packet, linked coefficient data, roots, markings, and selected comparison cells of the prescribed-incision witness. The retained regional construction uses the same finite persistent carrier labels and the same physicalized one-cell  $SU(3)$  seed. Its regional Hilbert packet is the tensor product over the same one-copy label set, and its ordinary observable algebra and complete-ground projection are the same finite factor and tensor-product projection. Thus extraction and reconstruction are inverse after canonical tensor ordering. Tensor reorderings and overlap implementers supply the required invertible coherence cells. Source-freeness is preserved because only the interior Gauss–Ward packet is used; no primitive collar or worldsheet source is inserted.  $\square$

**Lemma II.256** (Simplicial, represented, and mixed pseudonaturality). *The dictionaries of Lemma II.255 form a marking-preserving pseudonatural equivalence over  $\Delta^{\text{op}}$ . They intertwine:*

1. every face restriction and degeneracy repetition of the chosen Čech nerve;
2. selected chart and Cauchy comparisons;
3. fixed-thickness coefficient lines, multiplication cells, and the nonneutral reassociator  $\Omega_m$ ;

4. *selected scale correspondences and their tensorators; and*

5. *regional-scale, chart-scale, and Cauchy-scale mixed cells.*

*All required squares, prisms, cubes, and higher-intersection boundaries commute after the one finite joint refinement supplied for the demand. No scale correspondence is inverted.*

*Proof.* Regional restrictions and one-copy carrier additions are represented by the same tensor restriction and inclusion maps on both sides. Chart and Cauchy comparisons use the same spatial implementers. General selected scale arrows use the same normal correspondences or isometries, coefficient lines, and tensorators, while the off-carrier extension retains their mixed cells. The selected H3 endpoint ledger admits every cell in the finite demand simultaneously at its actual endpoint thickness. Therefore each pseudonaturality diagram is the same finite typed diagram on the two sides. Compatibility with face and degeneracy operators follows from restriction functoriality and the joint certificate on the finite Čech nerve.  $\square$

**Lemma II.257** (Observable generator completeness on the chosen nerve). *At each vertex and nonempty intersection of the chosen Čech nerve, the selected presentation comparison groupoid is generated by chart changes, same-thickness representative changes, anchor changes, selected Cauchy-frame changes, containing-slab comparisons, and common refinements. After one finite H2–H3 refinement:*

1. *every generator acts by the same normal map on the source and retained ordinary observable algebras;*
2. *all generator relations and all simplicial face identities hold strictly on the observable shadow; and*
3. *every closed presentation loop acts trivially on the ordinary observable algebra.*

*Hence two finite presentation zigzags with the same endpoints induce the same normal observable-algebra map.*

*Proof.* The selected presentation system is generated by the displayed elementary moves. Chart maps, Cauchy maps, regional inclusions, and refinement maps are the same spatial or tensor maps under the local dictionary. A closed linked Hilbert-comparison loop may

carry the coefficient scalar  $\omega^q I$ , but  $\text{Ad}(\omega^q I) = \text{id}$  on the ordinary regional factor. Thus represented isotropy can remain nontrivial while observable isotropy is trivial. Common finite refinement places any two zigzags and every required face restriction into one diagram; their difference reduces to the listed relations.  $\square$

**Proposition II.258** (Nervewise effectivity and contractible enhancements). *The local dictionary carries effective rooted source-free descent objects on  $\check{\mathcal{C}}_\bullet(\mathcal{U})$  to effective retained  $SU(3)$  realization objects and conversely. Extraction and one-copy reconstruction commute with the selected fixed-scale stackification on this nerve up to pseudonatural equivalence, with unit, counit, and invertible triangle modifications.*

*For every retained object and one-morphism on the nerve, the groupoid of selected Cauchy-anchor, normalized-root, and marked-branch enhancements is locally nonempty and has weakly contractible nerve. The nontrivial  $\mu_3$  coefficient isotropy belongs to the retained realization itself, not to the fiber of forgotten anchor/root choices.*

*Proof.* P4 reconstruction puts one copy of each persistent carrier label into the regional packet and identifies repeated chart appearances by the selected overlap implementer. The same labelwise argument applies to one- and two-morphisms; coefficient multiplication and the scalar reassociator remain coherence cells rather than observable elements. P4 effectivity and P5 fixed-scale stackification therefore descend the local identity unit and counit, whose triangle identities are the ordinary identity and tensor associativity triangles on each label.

For enhancement fibers, choose the canonical adapted-chart refinement, carrier ordering, propagation-adapted Cauchy frame, and normalized root. Every other enhancement admits a common finite refinement with it by Theorem II.227 and the selected P2 and P5 comparison cells. Any two such refinement arrows agree after a further common refinement. After vertical refinement is localized, the enhancement fiber is a directed thin category with no nontrivial automorphism preserving the normalized root and anchor, so its nerve is weakly contractible. The same construction applies over a fixed one-morphism.  $\square$

**Theorem II.259** (Selected  $SU(3)$  nervewise RID closure). *For each fixed selected admissible physical cover  $\mathcal{U}$ , each selected scale object  $r$ , and  $m = 1, 2$ , the source-free physical lifting diagram and the retained positive-thickness  $SU(3)$  realization diagram are marking-preservingly*

*pseudonaturally biequivalent over the same rooted or marked Čech nerve:*

$$\boxed{\mathfrak{G}_{r,m,\bullet}^{\text{vac}}(\mathcal{U}) \simeq \mathfrak{Real}_{r,m,\bullet}^{SU(3)}(\mathcal{U})}. \quad (\text{II.336})$$

*The equivalence is implemented by mutually inverse pseudofunctors with pseudonatural unit and counit equivalences and invertible triangle modifications. It preserves the ordinary observable algebras, represented Hilbert packets, complete-ground projections, common cores, interior Gauss–Ward data, selected Cauchy evolution, coefficient lines or normal correspondences, roots, branch and carrier markings, and all selected simplicial, scale, and mixed coherence cells.*

*For any finite selected scale diagram included in the RID demand, these biequivalences commute with its transition correspondences and compositors. They are compatible under enlargement of finite demands after passage to a common admissible selected diagram. No one tail or one margin valid for all demands is asserted.*

*Proof.* Apply Lemma II.255 levelwise. Lemma II.256 assembles the levelwise dictionaries over  $\Delta^{\text{op}}$  and over every finite selected scale diagram. Proposition II.258 descends the unit, counit, and triangle modifications and permits the auxiliary anchor/root enhancements to be forgotten. Compatibility under enlargement follows by restricting a joint refinement of the larger finite demand.  $\square$

**Theorem II.260** (Observable CP4 after descent on the chosen Čech nerve). *Under the hypotheses of Theorem II.259, the ordinary observable shadow is presentation independent on the selected rooted or marked Čech nerve. For every nonempty intersection, normal comparison between any two admitted regional Cauchy presentations is independent of chart, anchor, Cauchy frame, containing slab, representative, common refinement, and finite comparison zigzag.*

*The compatible-family algebra*

$$\mathcal{A}_{r,m}^W(U; \mathcal{U}) := \text{Eq} \left( \prod_{i \in I} \mathcal{A}_{r,m}^W(O_i) \rightrightarrows \prod_{\substack{(i,j) \in I^2 \\ O_{ij} \neq \emptyset}} \mathcal{A}_{r,m}^W(O_{ij}) \right) \quad (\text{II.337})$$

*is therefore independent, up to the canonical normal isomorphism supplied by the nervewise RID equivalence, of every admitted presentation choice used to compute it. Higher intersec-*

tions supply the coherence required for the two equalizer arrows and their compositions. This is observable  $CP_4$  on the chosen admissible cover.

*Proof.* Lemma II.257 proves trivial observable isotropy and zigzag independence at every simplicial level. Theorem II.259 identifies the source and retained observable diagrams and preserves all face maps. Therefore the equalizer and its evaluation maps are unchanged by replacing any chart, Cauchy, anchor, slab, representative, or common-refinement presentation with another admitted one. Triple and higher intersections guarantee coherence of the overlap comparisons; the conclusion is descent on this fixed nerve, not a claim about arbitrary covers.  $\square$

*Remark II.261* (CP4 does not neutralize the represented gerbe). Theorem II.260 concerns the ordinary adjoint observable shadow. A linked presentation loop may still carry  $\omega^m$  and the nontrivial represented reassociator  $\Omega_m$ . Since its adjoint action on the ordinary algebra is trivial, observable CP4 and nonneutral represented gerbe descent coexist. No global Hilbert-space trivialization follows.

**Corollary II.262** (Selected nervewise RID/CP4 closure). *For the selected admissible source-free  $SU(3)$  vacuum branch, RID and observable  $CP_4$  are discharged on every fixed selected admissible finite rooted or marked Čech nerve certified by Corollary II.231. The source-free physical lifting stack and retained positive-thickness realization agree on vertices, pairwise overlaps, and all higher intersections required by that nerve.*

*This corollary does not assert the result for every open cover, saturation of an ambient Grothendieck topology, equality with all regular opens, one universal scale tail, inversion of physical scale correspondences, or neutrality of the represented gerbe. The inherited complete-ground coercive provenance is transported through this selected nervewise descent and RID equivalence in Theorem II.292.*

*Remark II.263* (Circularity and claim boundary). The RID/CP4 proof uses P1–P5, the independently derived physical carrier, H1–H3, and the selected Čech-nerve recovery. It does not use collar charge, entropy surgery, closed-form coercivity, invariant momentum, spectral threshold, confinement, scattering, or any thin-limit conclusion. The stationary empty-carrier

branch remains distinct from the source-free physical branch, and the canonical comparison Weyl net is not substituted for the physical Yang–Mills observable system.

## Y. Reintroduction of complete-ground coercive provenance

The obstruction carrier, its regular support, the controlled incised regional category, and the regional realization identification have now been obtained without using a coercive lower bound. Only at this point may the finite Hamiltonian information inherited from Papers I and II be attached to the retained positive-thickness realization. This attachment is a provenance statement. It does not reprove finite-scale coercivity, and it does not infer that deleting the obstruction carrier improves an operator spectrum.

Four distinctions are essential. Paper I supplies an objectwise estimate for each verified finite physical package. Paper II supplies one common numerical profile only on the selected compatible family for which that profile has been proved. The explicit tensor-depth witness of Paper II is uniform only at its fixed stratum unless a separate full multiparameter extension theorem is available. Finally, the lower bound is initially a statement about a represented time-zero preform and its complete-ground projection; closability, closed spacetime forms, associated operators, and spectral consequences remain later constructions.

**Definition II.264** (Inherited complete-ground coercive certificate). Fix a selected scale object  $r$ , a selected spatial region  $B$ , and the regional Paper-I package  $\mathcal{P}_{B,r}^0$  of (II.5). An *inherited complete-ground coercive certificate* is the tuple

$$\mathfrak{C}_{B,r}^{0,\text{coer}} = \left( \mathcal{H}_{B,r}^0, \mathcal{C}_{B,r}^0, q_{B,r}^0, P_{\Omega,B,r}^0, Q_{\Omega,B,r}^0, \mathbf{q}_{B,r}, \hat{\kappa}_{B,r} \right), \quad Q_{\Omega,B,r}^0 = I - P_{\Omega,B,r}^0, \quad (\text{II.338})$$

where  $\mathcal{C}_{B,r}^0$  is the declared dense form core,  $P_{\Omega,B,r}^0$  is the complete exact ground projection, and  $\mathbf{q}_{B,r}$  is the verified finite threshold profile. The certificate requires

$$P_{\Omega,B,r}^0 \mathcal{C}_{B,r}^0 \subseteq \mathcal{C}_{B,r}^0, \quad (\text{II.339})$$

$$q_{B,r}^0[P_{\Omega,B,r}^0 \psi, \phi] = 0 \quad (\psi, \phi \in \mathcal{C}_{B,r}^0), \quad (\text{II.340})$$

$$q_{B,r}^0[\psi] \geq \hat{\kappa}_{B,r} \left\| Q_{\Omega,B,r}^0 \psi \right\|^2, \quad \hat{\kappa}_{B,r} > 0. \quad (\text{II.341})$$

The projection, rather than a vector in its range, is part of the certificate. No constant rank is required unless it is separately imposed by the selected Paper-II stratum.

**Definition II.265** (Paper-II common-profile certificate). Let  $\mathcal{F}$  be a selected compatible family of inherited certificates. A *Paper-II common-profile certificate* on  $\mathcal{F}$  consists of numbers

$$c_{\text{ch},*}, c_{\text{res},*}, c_{\text{UV},*} > 0, \quad 0 \leq \rho_{\text{sr},*} < 1, \quad 0 \leq \rho_{\text{mix},*} < 1, \quad (\text{II.342})$$

which dominate the corresponding channel, residual, ultraviolet, and mixed entries of every  $\mathbf{q}_{B,r}$  in  $\mathcal{F}$ . Define

$$c_{\text{IR},*} = \frac{1 - \rho_{\text{sr},*}}{2} \min\{c_{\text{ch},*}, c_{\text{res},*}\}, \quad \kappa_* = \frac{1 - \rho_{\text{mix},*}}{2} \min\{c_{\text{UV},*}, c_{\text{IR},*}\}. \quad (\text{II.343})$$

Thus  $\kappa_* > 0$ . The statement that the family is covered by this profile means that Paper II's profilewise coercivity theorem applies and gives

$$\hat{\kappa}_{B,r} \geq \kappa_* \quad \text{for every certificate in } \mathcal{F}. \quad (\text{II.344})$$

Equation (II.343), not an unproved equality  $\kappa_* = \inf \hat{\kappa}_{B,r}$ , is the definition of the inherited common constant. The inequality

$$\inf_{(B,r) \in \mathcal{F}} \hat{\kappa}_{B,r} \geq \kappa_* > 0 \quad (\text{II.345})$$

is a consequence when  $\mathcal{F}$  is profile-covered.

**Definition II.266** (Transported regional provenance packet). Let  $O$  be an object of the recovered regional category, let  $r$  be a selected scale object, and let  $p = (\alpha, \mathbf{c}, B)$  be a selected chart–Cauchy presentation used by the retained Paper-III realization. A *transported regional provenance packet* is

$$\mathfrak{C}_{p,r;O}^{\text{III,coer}} = \left( \mathcal{H}_{p,r;O}, \mathcal{C}_{p,r;O}^{\text{prov}}, q_{p,r;O}^{\text{prov}}, P_{\Omega,p,r;O}, Q_{\Omega,p,r;O}, \hat{\kappa}_{B,r} \right), \quad Q_{\Omega,p,r;O} = I - P_{\Omega,p,r;O}, \quad (\text{II.346})$$

obtained by the P2 anchor, effective P4–P5 descent, and the RID comparison pseudofunctor from  $\mathfrak{C}_{B,r}^{0,\text{coer}}$ .

In the coefficient-line branch, a local representative is a unitary

$$U_{p,r;O}^{\text{prov}} : \mathcal{H}_{B,r}^0 \hat{\otimes} L_{p,r;O} \longrightarrow \mathcal{H}_{p,r;O} \quad (\text{II.347})$$

which intertwines the complete-ground projection after tensoring with the identity on the line. In the normal-correspondence branch, the same notation means the declared ground-pointed correspondence together with its represented Hilbert realization on the chart. No global Hilbert-space object is inferred from these local packets.

**Hypothesis II.267** (Complete-Ground Coercive Provenance package). The selected Paper-I and Paper-II data and the repaired Paper-III front end satisfy the following conditions.

(CPR0) Verified Paper-I source certificates. Every regional package used by the selected branch carries an inherited certificate in the sense of Definition II.264. Its projection is the complete exact ground spectral projection of the finite Hamiltonian, not merely a stable low-energy island.

(CPR1) Typed Paper-II profile status. The selected compatible family is explicitly classified as one of:

- (a) an objectwise family with no common profile;
- (b) a profile-covered compatible family;
- (c) a full multiparameter cofinal profile-covered family; or
- (d) the explicit fixed-stratum tensor-depth family.

Only cases (b) and (c) carry the common constant  $\kappa_*$ . Case (d) carries its depth-uniform constant but is not promoted to full-scale cofinal uniformity without a separate extension theorem.

(CPR2) Regional-extension fidelity. The P1 regional extension retains the form core, shifted preform, complete- ground projection, physical channel, block decomposition, threshold profile, and objectwise coercive certificate. Regional bordisms carry these as typed decorations. They do not replace the complete-ground projection by a vector or infer a lower bound for a regional package not already covered by a source certificate.

(CPR3) Anchor and Cauchy fidelity. The P2 anchor and every selected geometric Cauchy comparison preserve the complete-ground corner and carry the source form core to the declared transported core. In every exact represented branch,

$$U_{p,r;O}^{\text{prov}}(P_{\Omega,B,r}^0 \hat{\otimes} I) = P_{\Omega,p,r;O} U_{p,r;O}^{\text{prov}}, \quad (\text{II.348})$$

$$q_{p,r;O}^{\text{prov}} [U_{p,r;O}^{\text{prov}}(\psi \otimes \ell), U_{p,r;O}^{\text{prov}}(\phi \otimes \ell)] = q_{B,r}^0[\psi, \phi] \quad (\text{II.349})$$

for normalized  $\ell$ . In a correspondence-valued branch, these equations are read in the selected represented correspondence chart. Phase changes in  $\ell$  do not change the projection, form, or

norm.

(CPR4) Descent and stackification effectivity. The property of carrying a certificate (II.338) is local for the admissible descent topology, is preserved by the P4 effective-descent equivalence, and descends through the P5 stackification. Local units, counits, roots, and two-cells preserve the projection, core, preform, and numerical profile.

(CPR5) RID provenance fidelity. The local RID dictionary and its inverse identify the transported P1–P5 certificate with the retained Paper-III packet (II.346). They preserve the complete-ground projection, shifted preform, common core, physical channel, threshold profile, and marked branch. The RID unit and counit preserve these data after effective descent and stackification.

(CPR6) Exact and controlled comparison discipline. Exact chart, selected-Cauchy, representative, and exact scale seams are form isometries intertwining the complete-ground projection, so the same numerical constant is transported through them. At a controlled analytic seam, both endpoints must separately carry their inherited certificate. A one-sided bounded form defect is not used to derive the target lower bound from the source lower bound. Any later degraded comparison estimate must display its own two-sided graph bounds and defect budget.

(CPR7) Sector and coefficient invariance. A central coefficient-line phase or reassociation twist acts trivially on the ordinary finite Hamiltonian, shifted form, complete-ground projection, and threshold profile. Hence strict and center-banded globalizations of the same finite analytic family carry the same objectwise and profilewise constants. For a noncentral normal-correspondence branch, equality of constants is asserted only when the endpoint profile certificates and the ground-pointed comparison have been separately verified.

(CPR8) No hidden cofinal promotion. The notation  $\Gamma_{\text{cof}}$  carries a uniform constant only if the selected Paper-II family on that branch is profile-covered in the full multiparameter sense. Cofinality of thickness values alone, objectwise positivity of  $\hat{\kappa}_{B,r}$ , or depth-uniformity at fixed stratum does not imply  $\kappa_* > 0$  on the full branch.

*Remark II.268* (Substantive provenance burden). CPR0 is the imported Paper-I theorem. CPR1 is the Paper-II profile-status audit. CPR3 and CPR5 are the load-bearing transport

statements: they identify which retained projection and preform are the images of the inherited finite ones. CPR4 prevents a local certificate from disappearing under stackification. CPR6 separates exact transport from controlled comparison defects. None of these statements follows merely from equality of notation or from the existence of the incised habitat.

**Proposition II.269** (Objectwise transport of the Paper-I certificate). *Assume P1–P5, H1–H3, category recovery, RID0–RID10, and CPR0–CPR7. Let  $O$  be a recovered region,  $r$  a selected scale object, and  $p$  a selected regional presentation whose anchor region is  $B$ . Then the retained packet carries a transported preform and complete-ground projection satisfying*

$$P_{\Omega,p,r;O} \mathcal{C}_{p,r;O}^{\text{prov}} \subseteq \mathcal{C}_{p,r;O}^{\text{prov}}, \quad (\text{II.350})$$

$$q_{p,r;O}^{\text{prov}}[P_{\Omega,p,r;O} \Psi, \Phi] = 0, \quad (\text{II.351})$$

$$q_{p,r;O}^{\text{prov}}[\Psi] \geq \hat{\kappa}_{B,r} \|Q_{\Omega,p,r;O} \Psi\|^2 \quad (\Psi, \Phi \in \mathcal{C}_{p,r;O}^{\text{prov}}). \quad (\text{II.352})$$

Consequently,

$$\ker \left( q_{p,r;O}^{\text{prov}} \Big|_{\mathcal{C}_{p,r;O}^{\text{prov}}} \right) = P_{\Omega,p,r;O} \mathcal{C}_{p,r;O}^{\text{prov}}. \quad (\text{II.353})$$

*This is a core/preform statement. It does not assert closability or identify an associated positive operator.*

*Proof.* Choose a local effective anchored descent representative of the lifting-gerbe object and apply the RID comparison pseudofunctor. CPR2–CPR5 identify the source core, preform, and complete-ground projection with the data in (II.346). In the line-unitary branch, (II.348) gives

$$Q_{\Omega,p,r;O} U_{p,r;O}^{\text{prov}} = U_{p,r;O}^{\text{prov}} (Q_{\Omega,B,r}^0 \hat{\otimes} I).$$

Apply (II.341) and (II.349). The represented correspondence branch is the same argument in the chosen chart realization. CPR4 and CPR5 descend the result and make it independent of the local effective representative. The nullity identity is transported from (II.340). It gives one inclusion in (II.353); the lower bound gives the reverse inclusion.  $\square$

**Theorem II.270** (Inherited uniform complete-ground coercive provenance). *Assume the hypotheses of Proposition II.269 and suppose that the selected compatible family is covered*

by one Paper-II profile in the sense of Definition II.265. Then every retained regional presentation in that family satisfies

$$\boxed{q_{p,r;O}^{\text{prov}}[\Psi] \geq \kappa_* \|(I - P_{\Omega,p,r;O})\Psi\|^2, \quad \kappa_* > 0.} \quad (\text{II.354})$$

The constant is independent of the selected regional presentation, admissible H2 refinement, Cauchy anchor, and coefficient-line representative. It is independent of the selected scale object precisely on the profile-covered family asserted by Paper II.

*Proof.* Equation (II.344) gives  $\hat{\kappa}_{B,r} \geq \kappa_*$  at every source package in the selected family. Insert this inequality into (II.352). Presentation and refinement independence follow from CPR3–CPR5 and the contractible certificate and anchored-enhancement fibers. CPR7 removes the choice of normalized coefficient-line representative. CPR8 restricts the scale-uniform conclusion to the family on which the common profile is actually proved.  $\square$

**Corollary II.271** (Strict and twisted analytic-shadow equality). *Suppose the strict and center-banded branches use the same finite Hamiltonians, complete-ground projections, physicalized packages, and threshold profile, and differ only in their coefficient-line reassociation law. Then their retained regional provenance packets have identical numerical constants  $\hat{\kappa}_{B,r}$  and  $\kappa_*$ . The represented descent classes may nevertheless be inequivalent.*

*Proof.* By CPR7, multiplication of a line representative or associator by a central unit phase leaves the shifted form, projection, norms, threshold entries, and therefore the Paper-I and Paper-II coercive formulas unchanged. The RID identification preserves the marked coefficient class rather than strictifying it, so equality of constants does not identify the represented globalizations.  $\square$

**Proposition II.272** (Exact seams versus controlled seams). *Let  $a$  be a selected comparison between two retained provenance packets.*

1. *If  $a$  is exact on the represented form core and intertwines the complete-ground projections, then it carries (II.352) to the target with the same constant.*
2. *If  $a$  has only a one-sided bounded graph-form defect, the target estimate is justified by its independent endpoint certificate, not by transporting the source inequality through that defect.*

3. *A degraded lower bound across a controlled seam may be asserted only after a separate two-sided graph comparison and an explicit subcritical defect estimate are supplied.*

*Proof.* The first assertion follows by substituting the exact form and projection intertwining identities. A one-sided estimate controls only one direction of form comparison and cannot bound the target norm from below. The second and third assertions are therefore logical necessities; the later controlled-form calculus may provide the missing inverse and defect estimates, but they are not part of provenance reintroduction.  $\square$

**Theorem II.273** (Complete provenance chain). *Under Hypothesis II.267, the complete-ground coercive datum follows the chain*

$$\begin{aligned}
& \text{Paper-I verified finite package} \longrightarrow \text{Paper-II selected physical higher pseudosection} \\
& \longrightarrow P1 \text{ regional time-zero package} \longrightarrow P2 \text{ anchored Cauchy extraction} \\
& \longrightarrow P4 \text{ effective coherent realization} \longrightarrow P5 \text{ fixed-scale and cofinal lifting stacks} \\
& \longrightarrow RID \text{ retained positive-thickness realization.}
\end{aligned} \tag{II.355}$$

*At every stage the transported ground datum is the complete projection, the transported analytic datum is the declared preform on its declared core, and the numerical label is objectwise  $\hat{\kappa}_{B,r}$  or profilewise  $\kappa_*$ . No stage replaces this datum by a preferred vector, an unidentified low-energy subspace, or a spectral statement on a new operator.*

*Proof.* CPR0 supplies the first certificate, CPR1 supplies the exact status of the common profile, CPR2 and CPR3 transport it through the regional and Cauchy front end, CPR4 transports it through effective descent and stackification, and CPR5 transports it through RID. CPR6 and CPR7 prove invariance under the allowed exact and coefficient changes, while CPR8 fixes the scale scope.  $\square$

**Corollary II.274** (Input to the later closure theorem). *For every retained regional presentation covered by Theorem II.270, the quadruple*

$$\left( \mathcal{C}_{p,r;O}^{\text{prov}}, q_{p,r;O}^{\text{prov}}, P_{\Omega,p,r;O}, \kappa_* \right) \tag{II.356}$$

*satisfies the complete-ground nullity and lower-bound clauses required by the later abstract closure package. The remaining closability, graph-completion, closed-form descent, and associated-operator hypotheses must still be proved in their stated locations.*

*Proof.* The ground invariance, nullity, and lower bound are (II.350), (II.351), and (II.354). No closability conclusion appears in the proof of Theorem II.270.  $\square$

*Remark II.275* (Objectwise, profilewise, and cofinal meanings). The word “uniform” has three distinct scopes.

1. Paper-I objectwise coercivity means only  $\widehat{\kappa}_{B,r} > 0$  for one verified package.
2. Paper-II profilewise uniformity means one  $\kappa_*$  works on one explicitly specified compatible family.
3. Full cofinal uniformity means that compatible family is cofinal in the full multiparameter demand category, not merely in thickness or tensor depth.

The explicit  $SU(2)$  tensor family is an inhabitation witness for the finite package and for depth-uniform provenance at fixed stratum. It is not by itself a witness for thinning, cutoff removal, or the full  $\Gamma_{\text{cof}}$  profile.

*Remark II.276* (The incision does not create the lower bound). The derived incision removes the carrier on which the simultaneous realization problem is not locally solvable. The lower bound is inherited from the finite physical package and is attached to the retained regional realization only after RID. Thus the incision is a habitat-selection mechanism, not a spectral projection argument and not a source of coercivity.

*Remark II.277* (No spectral or mass promotion). Equation (II.354) is a positive-thickness preform bound above a complete-ground projection in a chart-local represented packet. It does not construct a closed form, a self-adjoint regional Hamiltonian, a translation generator, a Poincare spectrum, a relativistic mass operator, a thin lower bound, or a mass gap. Those conclusions require the later closure, covariance, thin-reconstruction, and no-leakage theorems.

*Remark II.278* (Circularity firewall for coercive provenance). The source certificates are reintroduced only after the carrier, H1–H3, category recovery, CP4, and RID have been proved without them. The provenance theorem does not use a gauge collar charge, boundary Ward character, entropy surgery, post-incision normal-state selector, closed spacetime form,

invariant momentum, spectral threshold, confinement, or scattering. Conversely, the existence of  $\kappa_*$  is not cited retroactively to prove nonemptiness or regularity of the derived habitat.

**Corollary II.279** (Coercive provenance closure status). *Objectwise Paper-I coercive provenance is conditionally transported under CPR0 and CPR2–CPR7. The common retained estimate (II.354) is conditionally transported when the selected family carries the Paper-II common profile CPR1. Full cofinal uniformity is asserted only under CPR8. Model-specific verification of anchor/form fidelity, RID form fidelity, descent of the certificate, and the exact status of the selected common profile remains an explicit proof obligation.*

**The incision removes the nonrealizable carrier but does not erase the complete-ground coercive provenance of the realizable branch.**

The retained memory is encoded by the lifting gerbe, coefficient-line or correspondence transport, fixed-thickness and controlled chart–scale classes, rooted branch markings, and the inherited complete-ground coercive certificate. Only after support regularity may later obstruction collars and their separately constructed admissible boundary data be added.

### 1. *Selected $SU(3)$ complete-ground coercive provenance on the marked Čech nerve*

This subsection discharges the remaining coercive-provenance obligations for the selected admissible source-free Sector-III branch. It is downstream of the carrier theorem, H1–H3, controlled category recovery, the nervewise RID biequivalence, and observable CP4. The certificate is therefore attached to an already recovered habitat. It is not used to create the incision, select the physical branch, prove local solvability, or identify the retained realization.

Fix  $m \in \{1, 2\}$ , one chosen admissible finite rooted or marked cover  $\mathcal{U} = \{O_i \hookrightarrow U\}_{i \in I}$ , and its full nonempty intersection nerve  $\check{\mathcal{C}}_\bullet(\mathcal{U})$ . Let

$$\gamma_j^\# = (\mathcal{O}_j, \varepsilon_j^\#, h_j, L_j, \theta_j, \mathfrak{b}_j), \quad h_j = 2^{-j-1}, \quad \varepsilon_j^\# = 2^{-2j-8}, \quad L_j = j + 2, \quad (\text{II.357})$$

be the margin-adapted selected positive-thickness branch. Every finite subdiagram of this branch is an admissible finite demand for Theorem II.197. Denote by  $\mathcal{F}_{m, \mathcal{U}}^\#$  the family of all finite regional source packages and retained regional presentations obtained on the vertices

and nonempty intersections of this nerve by admitted face restrictions, degeneracies, selected chart changes, selected propagation-adapted Cauchy arrows, effective P4–P5 descent, rooted coefficient changes, RID comparisons, and finite-demand common refinements. Membership records the independently certified source endpoint; it does not identify distinct scale objects or invert a physical scale correspondence.

**Proposition II.280** (Margin-adapted common-profile inheritance). *The family  $\mathcal{F}_{m,\mathcal{U}}^\sharp$  is profile-covered in the full multiparameter sense relative to the declared dyadic separated-carrier positive-thickness demand category restricted to  $\check{\mathcal{C}}_\bullet(\mathcal{U})$ . Every source endpoint carries the same resolved constant*

$$\kappa_{\text{cof}} = \frac{1}{2} \left( \frac{17}{5} - \sqrt{\frac{557}{75}} \right) = 0.337404437602020 \dots > \frac{337}{1000}. \quad (\text{II.358})$$

*The assertion is confined to this declared profile-covered family. It is not a statement about every object of an ambient scale category, every open cover, or a zero-thickness form.*

*Proof.* At one retained character cutoff  $L \geq 2$ , the verified source package has complete exact ground projection  $P_{\Omega,L}$ , Gram and block bounds

$$\frac{9}{16}I \leq G_L \leq I, \quad B_{\text{mag},L}^{\text{phys}} \geq \frac{5}{6}I, \quad c_{\text{UV},L} \geq 3, \quad \|K_L\|^2 \leq \frac{1}{6}, \quad (\text{II.359})$$

and zero residual block. The exact two-block estimate gives (II.358). Finite separated regional tensor packages have tensor-product complete-ground projection, orthogonal one-excitation channels, and the same endpoint profile; sectors with at least two non-ground factors satisfy the stronger direct lower bound. The profile is therefore independent of the number of retained carriers, the chosen intersection of the nerve, and  $L \geq 2$ .

The declared demand category varies the selected incised region, carrier thickness, mesh, character cutoff, center background, and finite regional presentation. The sequence (II.357) has  $\varepsilon_j^\sharp \downarrow 0$ ,  $h_j \downarrow 0$ ,  $L_j \uparrow \infty$ , and  $h_j/\varepsilon_j^\sharp = 2^{j+7} > 8$ . Together with the selected regional exhaustion and background refinement, it is cofinal in that declared demand category. Passing to a vertex, nonempty intersection, face restriction, or finite common refinement changes none of the finite Hamiltonian, Gram, magnetic, ultraviolet, mixed-block, or complete-ground estimates. Hence every source endpoint in  $\mathcal{F}_{m,\mathcal{U}}^\sharp$  carries the same profile and the same constant  $\kappa_{\text{cof}}$ .  $\square$

**Lemma II.281** (Source and regional-extension fidelity). *Every finite source package in  $\mathcal{F}_{m,\mathcal{U}}^\#$  carries the certificate*

$$\mathfrak{C}_{B,j}^{0,\text{coer}} = \left( \mathcal{H}_{B,j}^0, \mathcal{C}_{B,j}^0, q_{B,j}^0, P_{\Omega,B,j}^0, I - P_{\Omega,B,j}^0, \mathbf{q}_{B,j}, \kappa_{\text{cof}} \right), \quad (\text{II.360})$$

with

$$q_{B,j}^0[\psi] \geq \kappa_{\text{cof}} \|(I - P_{\Omega,B,j}^0)\psi\|^2. \quad (\text{II.361})$$

*The selected P1 regional extension retains the declared form core, shifted preform, complete-ground projection and complementary corner, physical channel, block decomposition, and profile label. Every admitted face restriction and regional enlargement on the chosen nerve is an exact tensor-ground map on these data.*

*Proof.* The verified one-cell certificate and regional tensor theorem give (II.360)–(II.361). A regional enlargement adds separated carrier factors in their complete-ground sectors. It therefore sends the source ground projection to the corresponding tensor-product projection, carries the declared finite form core into the target core, and preserves the shifted tensor-sum preform. Face restrictions delete exactly the labels not present on the target intersection. These are the selected P1 decorations and exact regional bordisms of Theorem II.13. No lower bound is assigned to an endpoint lacking its own source certificate.  $\square$

**Lemma II.282** (Selected anchor and Cauchy fidelity). *Let  $p = (\alpha, \mathfrak{c}, B)$  be a selected chart–Cauchy presentation of a retained intersection  $O$  of the chosen nerve. The selected anchor and every selected propagation-adapted Cauchy arrow carry the complete-ground corner and source form core exactly. In a represented coefficient-line chart there is a unitary*

$$U_{p,j;O}^{\text{prov}} : \mathcal{H}_{B,j}^0 \widehat{\otimes} L_{p,j;O} \longrightarrow \mathcal{H}_{p,j;O} \quad (\text{II.362})$$

such that, for normalized  $\ell \in L_{p,j;O}$ ,

$$U_{p,j;O}^{\text{prov}}(P_{\Omega,B,j}^0 \widehat{\otimes} I) = P_{\Omega,p,j;O} U_{p,j;O}^{\text{prov}}, \quad (\text{II.363})$$

$$q_{p,j;O}^{\text{prov}} \left[ U_{p,j;O}^{\text{prov}}(\psi \otimes \ell), U_{p,j;O}^{\text{prov}}(\varphi \otimes \ell) \right] = q_{B,j}^0[\psi, \varphi]. \quad (\text{II.364})$$

*The same statement is read in the selected represented chart for a normal correspondence. A phase change of  $\ell$  changes neither projection, form, norm, nor numerical profile.*

*Proof.* The protected P2 extraction and exact return preserve the complete-ground projection, common-core packet, and inherited preform. On the matched physical branch, selected nonstationary Cauchy evolution is unitary and satisfies exact conjugacy for the shifted finite Hamiltonian and its spectral projection. Lemma II.255 and the selected Cauchy part of Lemma II.256 identify these source maps with the retained propagation-adapted maps on every nonempty intersection. Equations (II.363)–(II.364) follow. The scalar coefficient line is external to the ordinary analytic shadow.  $\square$

**Definition II.283** (Nervewise coercive-certificate decoration). On a nonempty intersection  $O_{i_0 \dots i_k}$ , let  $\text{Cert}_{j,m}^{\text{coer}}(O_{i_0 \dots i_k})$  be the groupoid whose objects are tuples

$$(\mathcal{H}, \mathcal{C}, q, P_\Omega, I - P_\Omega, \mathbf{q}, \kappa_{\text{cof}}) \quad (\text{II.365})$$

identified with a source certificate by an exact represented chart as in Lemma II.282. Its arrows are the selected exact regional, chart, Cauchy, tensor-reordering, coefficient-line, and rooted comparison cells that intertwine  $P_\Omega$ , carry  $\mathcal{C}$  onto the declared core, preserve  $q$ , and fix the profile label. A general noninvertible scale correspondence is recorded together with independently certified source and target objects; it is not made into an arrow transporting a lower bound by inversion.

**Definition II.284** (Intrinsic Yang–Mills descent datum and ancillary witnesses). On the selected admissible site and its marked Čech nerves, the intrinsic positive-thickness Yang–Mills descent datum is the common-refinement class

$$\mathfrak{D}_{\text{YM}, \mathfrak{d}}^{\text{thick}, \text{fin}} = \left( \mathcal{P}, \text{res}, \mathbb{T}, \mu, \eta, \alpha, \beta, [\omega]_{\text{adm}}, \mathbf{q}, P_\Omega, \kappa_{\text{cof}} \right). \quad (\text{II.366})$$

Here  $\mathcal{P}$  is the rooted regional physical packet on each retained region and scale;  $\text{res}$  denotes its admissible regional restriction maps;  $\mathbb{T}$  denotes the selected regional scale-bordism correspondences;  $\mu$  and  $\eta$  are their compositor and unit cells;  $\alpha$  is the coherent reassociator;  $\beta$  denotes the mixed regional–scale comparison cells;  $[\omega]_{\text{adm}}$  is the retained admissible coefficient class; and  $(\mathbf{q}, P_\Omega, \kappa_{\text{cof}})$  is the represented complete-ground coercive packet. The tuple is understood only on the selected positive-thickness demand category and modulo the specified vertical and common-refinement equivalences.

The atlas charts, chosen collars, cutoff functions, partitions of unity, parent assignments, protective neighborhoods, numerical margin values, intermediate gluing factorizations, and representative cocycles used to construct or verify (II.366) are *admissibility witnesses*. They are not additional components of the intrinsic datum when changing them preserves the displayed common-refinement class, the sector detector, the represented form, the complete-ground projection, and the certified lower-bound profile.

**Theorem II.285** (Intrinsic reduction of the selected Yang–Mills handoff). *Within the selected source-free positive-thickness  $SU(3)$  branch, the nervewise coercive-certificate decoration of Definition II.283 factors through the intrinsic datum (II.366). More precisely:*

1. *every face, degeneracy, selected Cauchy, exact regional, exact-scale, and mixed comparison map needed for the Paper-IV handoff is determined, up to the declared coherent equivalences, by  $\mathfrak{D}_{\text{YM},\mathfrak{d}}^{\text{thick,fin}}$ ;*
2. *the linked Sector-III distinction is retained by  $[\omega]_{\text{adm}}$  and its fixed detector, whereas the ordinary analytic shadow is retained by  $(\mathbf{q}, P_{\Omega}, \kappa_{\text{cof}})$ ;*
3. *replacing any admissibility witness by another witness with the same common-refinement class leaves the intrinsic datum and all induced Paper-IV handoff morphisms unchanged; and*
4. *no zero-thickness object, effective thin descent, or limiting spectral conclusion is included in this minimality statement.*

*Thus the information that makes the selected construction Yang–Mills is the rooted physical packet, its admissible regional and scale transport, the higher coefficient coherence, and the complete-ground coercive form packet. The remaining finite atlas and margin machinery certifies existence and well-definedness but is ancillary to the intrinsic finite positive-thickness datum and its Čech-compatible handoff class.*

*Proof.* The regional packet and restriction maps are the retained physical quotient, Gauss–Ward, linked-flux, and complete-ground data identified nervewise by the RID dictionary. The selected bordisms and their compositor, unit, reassociator, and mixed cells supply all scale and regional transport used by P4 extraction/reconstruction and P5 stackification. The

coefficient class and fixed detector retain the nontrivial Sector-III branch without altering the ordinary represented analytic packet. Lemma II.282 shows that exact regional, chart, and selected-Cauchy changes intertwine the form and complete-ground projection. The common-refinement and contractibility statements proved in the preceding CIR and RID closure results identify competing atlas, collar, parent, and representative choices by the declared vertical equivalences. Consequently those choices affect a presentation or a proof of admissibility, not the class (II.366). Conversely, deleting any of the four structural blocks—physical packet, regional/scale transport, higher coefficient coherence, or complete-ground coercive packet—removes, respectively, the observable content, the descent maps, the selected sector class, or the positive-thickness analytic certificate required by the stated handoff. This proves the asserted factorization and intrinsic reduction at the selected finite positive-thickness scope.  $\square$

**Theorem II.286** (Incised habitat and coherent finite-thick realization). *Fix the selected source-free physical branch. Let  $\mathfrak{F}_{\text{adm}}, \preceq$  be the declared directed preorder of selected finite admissible demands on the recovered incised regional category, and fix  $\mathfrak{d} \in \mathfrak{F}_{\text{adm}}$ . The carrier theorem, H1–H3 geometry, CIR recovery, nervewise RID equivalence, and complete-ground provenance determine an intrinsic finite positive-thickness datum*

$$\mathfrak{D}_{\text{YM}, \mathfrak{d}}^{\text{thick}, \text{fin}} \quad (\text{II.367})$$

*represented on the admissible site of the incised spacetime  $M_{\text{inc}} = M \setminus \Sigma_{\text{obs}}^{\text{YM}, \text{vac}}$ . It carries coherent regional restriction, protected-Cauchy comparison, marked-Čech compatibility, selected glued scale transport, the retained Sector-III coefficient class, and the inherited complete-ground coercive packet.*

*If  $\mathfrak{d}_1 \preceq \mathfrak{d}_2$  are selected finite admissible demands, common refinement and the selected comparison cells induce a coherent morphism*

$$\mathfrak{D}_{\text{YM}, \mathfrak{d}_1}^{\text{thick}, \text{fin}} \longrightarrow \mathfrak{D}_{\text{YM}, \mathfrak{d}_2}^{\text{thick}, \text{fin}}, \quad (\text{II.368})$$

*well defined up to the declared vertical and common-refinement equivalences. Thus the inhabited Paper-III front end produces a coherent directed system of finite positive-thickness realizations on the incised spacetime. No object at zero thickness, effective thin descent, thin observable net, or limiting spectral conclusion is constructed by this theorem.*

*Proof.* The source-free carrier theorem identifies the derived obstruction carrier and therefore the incised spacetime on which the retained physical packets are allowed to be represented. H1–H3 provide the regular support, finite adapted atlases, and simultaneous positive margins required at every selected finite demand. CIR recovery identifies the localized decorated pre-incision presentation with the selected controlled incised regional category, while the nervewise RID theorem identifies the source-free lifting stack with the retained positive-thickness realization on each chosen admissible protected cover. The cartesian complete-ground decoration supplies the represented form, projection, and uniform profilewise lower bound without being used to create the habitat. The habitat itself is the complement of the already proved persistent-obstruction carrier; it is not a regulator introduced to secure the estimate.

For  $\mathfrak{d}_1 \preceq \mathfrak{d}_2$  in the declared directed demand preorder, the selected demand comparison is represented on a common finite refinement supplied by the certificate fibers. Regional, Cauchy, chart, scale, and mixed comparison cells then induce (II.368); the established unit, compositor, reassociator, face, degeneracy, and triangle identities give coherence under iterated refinement. Contractibility of the certificate fibers makes the resulting morphism independent, up to the declared equivalences, of atlas, collar, cutoff, parent, and representative choices. Every construction remains at selected positive thickness, so no thin object is invoked.  $\square$

*Remark II.287* (Three layers of finite-stage information). The finite-stage package separates into intrinsic data, presentation data, and control data. The intrinsic layer is  $\mathfrak{D}_{\text{YM},\mathfrak{d}}^{\text{thick,fin}}$ ; it determines a refinement-compatible equivalence class and is transported coherently under selected finite-demand refinement. Atlas, collar, cutoff, ordering, and representative choices are presentation data identified under admissible common refinement. Margin, defect, and concentration-exclusion quantities are control data: they certify that the refinement remains inside the admissible class. Paper IV may use the control layer to justify the thick-to-thin passage, but it must separately prove which finite-stage quantities vanish, are quotiented, converge, or leave an obstruction in the thin reconstruction.

**Lemma II.288** (Cartesian locality and P4–P5 fidelity). *The assignment*

$$[i_0 \cdots i_k] \longmapsto \text{Cert}_{j,m}^{\text{coer}}(O_{i_0 \cdots i_k}) \quad (\text{II.369})$$

is a cartesian simplicial decoration of the selected physical lifting diagram on  $\check{\mathcal{C}}_{\bullet}(\mathcal{U})$ . It is preserved by P4 extraction and one-copy reconstruction, by P5 fixed-scale effective descent, and by the simultaneous rooted correspondence-valued lax section. Every local unit, counit, triangle, root, face, degeneracy, and stackification modification preserves the complete-ground projection, form core, shifted preform, endpoint profile, and constant  $\kappa_{\text{cof}}$ .

*Proof.* The selected P4 persistent-label packet includes the represented Hilbert packet, complete-ground projection, declared common core, inherited form, and actual analytic correction class. Extraction merely lists these labels. One-copy reconstruction takes one occurrence of each persistent label and glues identical restrictions; it neither averages projections nor alters the form or numerical profile. Therefore the P4 unit and counit are identities on the certificate decoration, and the triangle modifications are generated by canonical tensor reassociation and deletion of repeated labels, both analytic isometries.

On every simplicial face, restriction deletes precisely the absent carrier labels and preserves the tensor-ground certificate on the remaining factors. Degeneracies repeat an intersection and act by the rooted identity. Hence the certificate square for every face or degeneracy is cartesian.

The P5 fixed-scale two-stack glues the same one-label packets and their exact restrictions. Its root-preserving scale one-cells carry the ground-pointed analytic labels and their endpoint profile. For a noninvertible scale correspondence, the target certificate is its independently verified endpoint object; no one-sided residual is used to infer it. The compositor and mixed cells are tensorators, rooted identities, actual-residual bookkeeping cells, or central coefficient reassociators. Each acts trivially on the ordinary finite analytic packet. Thus effective descent and stackification preserve the certificate labelwise without making a scale correspondence invertible. This proves the master-ledger descent/stackification obligation CPR.3 on the chosen nerve.  $\square$

**Lemma II.289** (Nervewise RID provenance fidelity). *The pseudonatural biequivalence of Theorem II.259 and its inverse identify the transported P1–P5 certificate with the retained Paper-III provenance packet on every vertex and nonempty intersection. They identify the shifted preform, its declared core, the complete-ground null space, the profile (II.359), and*

$\kappa_{\text{cof}}$ . *Every RID unit, counit, triangle, face, degeneracy, selected Cauchy, finite-scale, and mixed coherence cell preserves these data.*

*Proof.* Off the derived carrier, the source-free extension and retained incised packet are one-copy realizations of the same finite  $SU(3)$  Wilson package. By Lemma II.255, their Hilbert packets, finite factors, complete-ground projections, common cores, channel and ultraviolet decompositions, and selected Cauchy maps agree on every intersection. The shifted preform is the represented quadratic form of that same finite Hamiltonian on that same core, so the local dictionary extends uniquely to the full tuple (II.365).

By Proposition II.258, the RID unit and counit after descent are generated by chart and selected-Cauchy unitaries, tensor reorderings, vertical certificate refinements, and the declared correspondence tensorators. Lemma II.288 shows that each generator preserves the certificate. The triangle cells are pasting composites of the same generators. Simplicial pseudonaturality is built from face restrictions and rooted degeneracies, and finite-scale mixed cells use the same independently certified endpoint packets. Therefore the full coercive-provenance decoration is invariant under the nervewise RID biequivalence. No lower bound is transported through a one-sided form defect. This proves the master-ledger RID obligation CPR.4.  $\square$

**Proposition II.290** (Seam discipline and coefficient invariance). *For the selected source-free branch on the chosen nerve:*

1. *exact regional, chart, selected-Cauchy, representative, and package-adapted exact-scale seams are form isometries intertwining the complete-ground projections and preserve  $\kappa_{\text{cof}}$  exactly;*
2. *a general selected scale arrow remains a normal correspondence or isometry with its declared analytic comparison class; the target lower bound is supplied by its independent endpoint certificate and is not inferred from a one-sided form defect; and*
3. *the central coefficient scalar and reassociator  $\Omega_m$  act trivially on the finite Hamiltonian, shifted preform, complete-ground projection, threshold profile, and numerical constant.*

*Consequently the strict analytic shadow and the two linked Sector-III coefficient branches*

carry the same objectwise and profilewise constants while their represented descent classes may remain inequivalent.

*Proof.* The first item is substitution into the exact projection and form intertwining identities. For the second, a one-sided graph-form estimate has no inverse inequality and therefore cannot transport a lower bound; endpoint profile coverage supplies the target certificate directly. For the third, central phases multiply only coefficient-line representatives and reassociation maps. Their adjoint action on the ordinary finite analytic package is the identity. Thus the analytic constants agree without strictifying the coefficient gerbe.  $\square$

**Proposition II.291** (Exact positive-thickness cofinal scope). *The constant  $\kappa_{\text{cof}}$  is uniform over  $\mathcal{F}_{m,\mathcal{U}}^\sharp$ , including all selected scale objects, finite separated regional packages, face restrictions, selected Cauchy presentations, effective descent representatives, and normalized coefficient-line representatives on the chosen nerve. This is full multiparameter cofinal uniformity only in the declared positive-thickness dyadic separated-carrier demand category. It does not imply a common constant on an arbitrary enlargement, a closed thin form, or spectral nonleakage.*

*Proof.* Proposition II.280 proves profile coverage at every source endpoint. Lemmas II.281–II.289 preserve the certificate through the complete nervewise front-end chain. Proposition II.290 removes presentation and coefficient choices and supplies endpoint discipline at controlled seams. Nothing in the argument forms a zero-thickness object or extends the declared demand category.  $\square$

**Theorem II.292** (Selected  $SU(3)$  nervewise CPR closure). *For every retained regional presentation  $p = (\alpha, \mathfrak{c}, B)$  over a vertex or nonempty intersection  $O$  of the chosen nerve, every selected margin-adapted scale object  $\gamma_j^\sharp$ , and all  $\Psi, \Phi$  in the transported provenance core,*

$$P_{\Omega,p,j;O} \mathcal{C}_{p,j;O}^{\text{prov}} \subseteq \mathcal{C}_{p,j;O}^{\text{prov}}, \quad (\text{II.370})$$

$$q_{p,j;O}^{\text{prov}}[P_{\Omega,p,j;O} \Psi, \Phi] = 0, \quad (\text{II.371})$$

$$q_{p,j;O}^{\text{prov}}[\Psi] \geq \kappa_{\text{cof}} \|(I - P_{\Omega,p,j;O})\Psi\|^2, \quad \kappa_{\text{cof}} > \frac{337}{1000}. \quad (\text{II.372})$$

*The packet and numerical constant are independent of the H2 refinement, selected Cauchy presentation and anchor, effective descent representative, finite comparison zigzag, simplicial*

representative, and normalized coefficient-line representative. They are independent of the selected scale object exactly on the source-certified family  $\mathcal{F}_{m,\mathcal{U}}^\sharp$ . The master-ledger obligations CPR.3 and CPR.4, and hence the selected positive-thickness CPR0–CPR8 package, are discharged on every chosen admissible finite rooted or marked Čech nerve.

*Proof.* Proposition II.280 and Lemma II.281 supply the source profile and certificate. Lemma II.282 transports it through regional and Cauchy anchoring. Lemma II.288 proves cartesian locality and P4–P5 preservation. Lemma II.289 proves invariance under the full nervewise RID unit, counit, triangle, simplicial, finite-scale, and mixed coherence system. Transporting (II.361) through the exact represented source-to-target maps gives (II.370)–(II.372). At controlled seams the target estimate holds because each endpoint already belongs to the profile-covered source family, not because the seam is inverted.  $\square$

**Corollary II.293** (Paper-III front-end closure). *For the selected admissible source-free Sector-III  $SU(3)$  branch, the full Paper-III front-end chain is inhabited on each chosen admissible finite rooted or marked Čech nerve:*

$$\boxed{\begin{aligned} \text{P1} - -\text{P5} &\implies \Sigma_{\text{obs}}^{\text{YM,vac}} = \Sigma_{\text{pres}} \implies \text{H1} - -\text{H3} \\ &\implies \text{CIR} \implies \text{RID/CP4} \implies \text{CPR0} - -\text{CPR8}. \end{aligned}} \quad (\text{II.373})$$

The remaining Paper-III work is the model-specific downstream H4–H19 closure audit and final synthesis on this already matched branch. No missing front-end obligation is transferred to Paper IV.

*Remark II.294* (Exact handoff and nonclaims). The front-end output is the quadruple

$$\left( \mathcal{C}_{p,j;O}^{\text{prov}}, q_{p,j;O}^{\text{prov}}, P_{\Omega,p,j;O}, \kappa_{\text{cof}} \right). \quad (\text{II.374})$$

It supplies complete-ground invariance, nullity, and the positive-thickness preform lower bound. It does not prove the result for every open cover, neutralize the represented  $\mu_3$  gerbe, invert a noninvertible scale correspondence, prove closability or graph completion, construct a new self-adjoint operator, take a thin limit, prove spectral nonleakage, confinement, scattering, or a Yang–Mills mass gap.

## Z. Model-verification closure and inhabited-branch boundary

**Definition II.295** (Canonical prescribed-incision witness category). Fix  $\varepsilon > 0$  and the smooth timelike worldsheet

$$\Sigma_0 = \{(t, 0, 0, z) : t, z \in \mathbb{R}\}.$$

Let  $\mathbf{Reg}_\varepsilon^{\text{can}}(M \setminus \Sigma_0)$  be the regional poset generated by the globally labelled separated tube cells of the explicit  $SU(3)$  regionalization and their admitted tensor-factor inclusions. Selected center-background chart changes and propagation-adapted Cauchy maps belong to the decorated presentation system over this poset; they are not additional regional arrows. The selected physical CIR theorem below identifies the corresponding protected-diamond basis with the geometric category recovered from the dynamically derived carrier at its stated finite-demand scope.

**Proposition II.296** (Regular geometry of the prescribed carrier). *The chosen carrier  $\Sigma_0$  satisfies the geometric conclusions of H1 and admits the finite-stage H2 atlas and H3 fixed-stage margin witnesses on  $\mathbf{Reg}_\varepsilon^{\text{can}}(M \setminus \Sigma_0)$ .*

*Proof.* In global Minkowski coordinates define  $\delta(t, x, y, z) = (x, y)$  and  $\Delta = \{0\} \subset \mathbb{R}^2$ . Then  $\Sigma_0 = \delta^{-1}(\Delta)$  and  $d\delta$  is surjective everywhere, so the single smooth stratum is transverse, Whitney- $b$ , and has the standard normal tube, radial retraction, and collar control. A compact demand meets finitely many standard product charts and propagation-adapted Cauchy slabs; finite intersections admit precompact common shrinkings. Finally, every canonical regional object obeys  $\text{dist}(\overline{O}, \Sigma_0) > 4\varepsilon$  by definition, while the retained tube radius is at most  $\varepsilon/20$ . Taking the minimum over the finitely many chart, tube, collar, Cauchy, and separation clearances gives a positive fixed-stage margin. This proves only the prescribed-carrier witness. It does not identify  $\Sigma_0$  with the dynamically derived carrier.  $\square$

**Proposition II.297** (Ordinary-shadow CP4 on canonical presentations). *On  $\mathbf{Reg}_\varepsilon^{\text{can}}(M \setminus \Sigma_0)$  the normal observable-algebra comparison between two canonical presentations is independent of the finite zigzag. The statement applies only to the ordinary observable shadow and does not trivialize the linked coefficient gerbe.*

*Proof.* The global cell labels make every regional inclusion the unique tensor-factor map  $A \mapsto A \otimes I$ . On chart overlaps, comparison is  $\text{Ad}(U_{\alpha\beta})$ , and the center-background transition unitaries satisfy the exact Cech relation  $U_{\alpha\beta}U_{\beta\gamma} = U_{\alpha\gamma}$ . The selected Cauchy unitaries satisfy identity, inverse, and composition and intertwine the chart maps. Hence every canonical zigzag reduces, by these generator relations, to the same tensor inclusion followed by the same endpoint chart/Cauchy conjugation. Its adjoint action is therefore endpoint dependent only. The coefficient-line reassociator is external to the adjoint observable map and may remain nontrivial.  $\square$

**Proposition II.298** (Exact complete-ground coercive provenance on the canonical witness). *For every object and exact tensor, chart, or selected Cauchy arrow of  $\mathbf{Reg}_\varepsilon^{\text{can}}(M \setminus \Sigma_0)$ , the source complete-ground projection, declared core, preform, and valid Paper-I/II lower constant are transported exactly. In particular, on the explicit regional  $SU(3)$  tensor branch,*

$$q_O[\psi] \geq \kappa_{SU(3)} \|(I - P_{\Omega,O})\psi\|^2$$

*with the source-certified regional constant, and no one-sided form defect is used in the transport.*

*Proof.* The regional Hamiltonian is a finite tensor sum of identical one-cell Hamiltonians and its complete ground projection is the tensor product of the one-cell complete-ground projections. Decomposition by the first excited tensor factor gives the one-cell lower bound on the orthogonal complement of the complete tensor ground space. Regional enlargement adds factors in their complete ground sectors and therefore intertwines the shifted preform and projection exactly. Chart and selected Cauchy maps are unitary conjugacies of the full package, so they preserve the core, null space, and lower bound with the same constant. This proposition supplies the exact source endpoint. Nervewise preservation through P4–P5 and RID is proved separately in Theorem II.292.  $\square$

**Theorem II.299** (Model-verification closure). *The available Paper-I–III sources establish the following inhabited thick subpackages:*

1. *finite physical Wilson packets and their complete-ground coercive certificates;*

2. *selected  $SU(3)$  P1–P5: ordinary pre-incision regional extension, protected stationary Cauchy anchoring, common-core defect naturality, effective anchored descent, fixed-scale decorated lifting stacks, and a rooted correspondence-valued normal lax section;*
3. *the stationary conclusion  $M_{\text{reg}, SU(3), m}^{\text{stat}} = M$  and  $\Sigma_{\text{obs}, SU(3), m}^{\text{stat}} = \emptyset$ ;*
4. *an independently typed nonstationary physical lifting stack containing the physical quotient, complete-ground corner, selected Cauchy evolution, Gauss–Ward data, linked center flux, analytic certificates, roots, markings, scale correspondences, and mixed cells;*
5. *the selected admissible source-free vacuum substack, including the cofinal transverse Orlicz/concentration-exclusion certificate;*
6. *the source-free magnetic nonrelaxation theorem and carrier identity*

$$M_{\text{reg}}^{\text{YM}, \text{vac}} = M \setminus \Sigma_{\text{pres}}, \quad \Sigma_{\text{obs}}^{\text{YM}, \text{vac}} = \Sigma_{\text{pres}}; \quad (\text{II.375})$$

7. *H1 and H2 on that selected vacuum branch via the detector  $\delta(t, x, y, z) = (x, y)$  and the existing tubular and finite-atlas data;*
8. *the selected physical H3 endpoint-normalized ledger and one exact finite-demand residual-margin ratio below one at every actual selected positive thickness, discharging H3.1–H3.10;*
9. *the selected physical CIR0–CIR5 package and, for each chosen finite admissible protected cover, weakly contractible certificate fibers and recovery of its rooted or marked Čech nerve after localization at vertical refinements;*
10. *the selected source-free physical lifting stack and retained positive-thickness  $SU(3)$  realization identified simplicially on each chosen admissible rooted or marked Čech nerve, with observable  $CP_4$  after descent and nonneutral represented coefficient data;*
11. *the prescribed-incision positive-thickness  $SU(3)$  regional system, selected propagation-adapted dynamics, linked coefficient descent, and exact tensor/unitary coercive provenance on the already verified exact seams;*

12. the cartesian complete-ground coercive-certificate decoration on each chosen rooted or marked Čech nerve, preserved by  $P_4$ – $P_5$  descent, stackification, and the full nervewise RID coherence system, with  $\kappa_{\text{cof}} > 337/1000$  on the declared profile-covered family; and
13. the strict  $SU(2)$  fixed-stratum/depth control and the  $SU(3)$  linked-flux witnesses at their declared scopes.

The stationary and source-free physical branches remain distinct. The carrier identity is not inferred from twisting alone: it uses the typed physical lifting problem, a cofinal concentration-exclusion certificate,  $P_4$ – $P_5$  compatible point relaxation, and magnetic nonrelaxation.

The available sources now establish the selected physical  $H_3$  simultaneous finite-demand margin theorem, controlled-incised-category recovery, nervewise RID, observable  $CP_4$ , and complete-ground coercive-provenance transport on each chosen admissible finite rooted or marked Čech nerve at the actual positive thicknesses. The Paper-III front-end obligation list is therefore closed. The correct current classification is

***The selected positive-thickness front end is inhabited; the model-specific downstream  $H_4$ – $H_{19}$  synthesis remains conditional.***

The physical branch/carrier subproblem and the front-end reconstruction have Outcome A at their declared scope; the full manuscript remains Outcome B.

*Remark II.300* (First unresolved proof edge). The next proof edge is the model-specific downstream  $H_4$ – $H_{19}$  closure on the already matched selected source-free branch. The audit must identify the primitive Cauchy propagation, physical Wilson quotient, local-normal completion, Gauss–Ward domains, entropy surgery, positive-thickness form closure, and restricted background covariance with the objects produced by the now-closed front end. No closed form, associated operator, covariance, or spectral threshold may be inferred merely from the preform certificate. Genuinely limiting  $H_3$ -UMG, thin-tail uniformity, and zero-thickness reconstruction remain assigned to Paper IV.

### III. REMAINING DOWNSTREAM HYPOTHESES AND CONDITIONAL RECONSTRUCTION THEOREMS

**Reader’s guide.** This section is an interface contract, not a claim that the entire downstream architecture has already been inhabited. The Paper-III front end proves the selected carrier, incised site, finite-nerve recovery, realization identification, and complete-ground coercive provenance at positive thickness. H4–H19 record the additional assumptions under which the later observable, represented, gauge, entropy, and form packages coexist. Their conditional status is preserved throughout.

#### A. Downstream hypotheses H4–H19

The preceding section formulates and proves the conditional implications for the regional time-zero prestack, selected Cauchy anchor, simultaneous lifting carrier, controlled incised category, regional realization identification, observable CP4, and complete-ground coercive provenance. The model-verification closure theorem, Theorem II.299, records which parts are inhabited and which remain hypotheses for the intended Yang–Mills branch. The remaining assumptions govern the positive-thickness observable propagation, normal completion, represented coherence, gauge and Ward domains, entropy surgery, form closure, and background covariance. The H4–H19 statements below are retained verbatim from the v1.2.0 journal source.

**Hypothesis III.1** (H4: primitive Cauchy propagation). Primitive Wilson generators admit reversible, composable Cauchy propagation on the physical quotient. In the exact branch carriers remain in the prescribed causal shadows; in the quasilocal branch a stated leakage modulus is available. For the canonical local  $SU(3)$  branch this hypothesis is proved in Appendix I.

**Hypothesis III.2** (H5: physical Wilson quotient and common core). Retracing, subdivision, gauge, support, adjoint, and product relations are preserved by propagation on a common invariant physical core, and reverse propagation is inverse on the quotient.

**Hypothesis III.3** (H6: faithful normal representations). Each propagated physical Wilson core has a faithful normal representation whose bicommutant is the regional von Neumann algebra used below.

**Hypothesis III.4** (H7: normal chart descent and observable isotropy). Chart changes and same-thickness representative comparisons extend to normal spatial isomorphisms. Observable isotropy loops act trivially on an ultraweakly generating physical Wilson core.

**Hypothesis III.5** (H8: normal scale transport). Each selected scale arrow extends to a predual-continuous normal regional homomorphism or, in the weak branch, to a normal  $W^*$ -correspondence with the stated source and target actions.

**Hypothesis III.6** (H9: bicommutant transfer of locality). Algebraic commutator and leakage estimates are uniform on represented unit balls, so Kaplansky approximation transfers them to the von Neumann closures.

**Hypothesis III.7** (H10: complete-ground transport). The complete spectral projections  $P_{\Omega, \alpha, r}$  are transported by local Cauchy maps, chart comparisons, and selected scale comparisons. No preferred ground vector is part of the invariant data.

**Hypothesis III.8** (H11: fixed-thickness coefficient-line reduction). The selected ground-preserving overlap torsors reduce, when asserted, to Hermitian lines  $L_{\alpha\beta, r}$  with normalized multiplication maps. If this central reduction fails, the corresponding statements remain ground-pointed correspondence-valued.

**Hypothesis III.9** (H12: scale coefficient correspondences). Selected scale arrows carry coefficient lines or normal intertwiner correspondences, represented implementers, multiplication maps, and reassociation cells compatible with the observable scale maps.

**Hypothesis III.10** (H13: mixed chart-scale coherence). Chart and scale comparisons possess mixed cells satisfying the complete coherence cube, including chart triangles, scale triangles, and the two mixed composition identities.

**Hypothesis III.11** (H14: controlled equivalence category). For rooted refinement branches, admissible coefficient equivalences are finite-shell local, band preserving, natural under

control-preserving shell isomorphisms, and uniformly Lipschitz in the declared shell metrics. The resulting controlled cochains form a subcomplex of the ordinary normalized cochains.

**Hypothesis III.12** (H15: Gauss graph forms). Compactly supported interior gauge parameters define bounded graph-dual Gauss forms on common energy graph spaces. Closability, self-adjointness, spectral calculus, and exponentiation are invoked only in the branches where their separate domain hypotheses hold.

**Hypothesis III.13** (H16: Ward domains and collar charges). Ward insertion domains are invariant under the required regional, chart, and scale operations. Collar-nontrivial parameters are retained as charge classes modulo interior constraints, and affine zero-point shifts are tracked separately from adjoint Ward derivations.

**Hypothesis III.14** (H17: entropy surgery). Regional normal-state spaces carry faithful normal semifinite reference weights and idempotent entropy-nonincreasing state selections compatible with restriction and selected scale arrows, exactly or with explicit predual defects satisfying the stated composition law.

**Hypothesis III.15** (H18: closed forms and branchwise uniform coercivity). Propagated preforms are closable; their closures have kernel  $\text{Ran } P_{\Omega, \alpha, r}$  and retain the Paper-I/Paper-II branchwise bound (II.354). Exact or controlled form transports satisfy the selected bordism composition laws.

**Hypothesis III.16** (H19: controlled background covariance). Strict controlled background embeddings act normally on observables, and invertible controlled background arrows act pseudoequivariantly on the represented, coefficient, gauge, entropy, and form data while preserving the declared control metrics.

## B. Observable and representation-descent reconstruction

**Theorem III.17** (Conditional observable and controlled-descent reconstruction). *Assume the front-end packages of Hypotheses II.3–II.101, II.203, and II.237, the selected admissible Čech-nerve condition of Hypothesis II.211, Hypotheses II.140–III.11, and the observable part*

of Hypothesis III.16. Cofinal statements are restricted to the selected finite-demand tails on which Hypothesis II.185 is verified. Then:

1. For each  $\varepsilon \in E_{\text{cof}}$  there is an ordinary functor

$$\mathfrak{A}_\varepsilon^W : \mathbf{Reg}_{\text{inc}}^{\varepsilon, \text{cp}}(M, \Sigma) \longrightarrow \mathbf{W}^* \mathbf{Alg}_n, \quad (\text{III.1})$$

canonically independent of adapted presentation.

2. Exact thick locality holds:

$$\mathcal{O}_1^{+\varepsilon} \perp \mathcal{O}_2^{+\varepsilon} \implies [\mathcal{A}_\varepsilon^W(\mathcal{O}_1), \mathcal{A}_\varepsilon^W(\mathcal{O}_2)] = 0, \quad (\text{III.2})$$

and thin-spacelike pairs satisfy the descended leakage modulus of Section IV.

3. At fixed thickness the ground-pointed chart representations form a banded two-presheaf

$$\mathbf{Vac}_\varepsilon^\Omega : \mathbf{Chart}_*(M_{\text{inc}})^{\text{op}} \longrightarrow \mathbf{2Gpd}, \quad (\text{III.3})$$

with intrinsic class  $[\Omega_\varepsilon]_{\text{band}}$ .

4. The chart-scale data define both an unrestricted ordinary class  $[\hat{\Omega}]_{\text{cs,ord}}$  and, on the controlled equivalence complex, a class  $[\hat{\Omega}]_{\text{cs,ctrl}}$ . On a rooted pure-scale category the ordinary scale component is exact; controlled nontriviality measures the absence of a uniformly local, scale-natural primitive.

5. The controlled sector trichotomy is

$$\begin{aligned} \text{I} : \quad & [\Omega_{e(r)}]_{\text{band}} = 0 \ \forall r, \quad [\hat{\Omega}]_{\text{cs,ctrl}} = 0, \\ \text{II}_{\text{ctrl}} : \quad & [\Omega_{e(r)}]_{\text{band}} = 0 \ \forall r, \quad [\hat{\Omega}]_{\text{cs,ord}} = 0, \quad [\hat{\Omega}]_{\text{cs,ctrl}} \neq 0, \\ \text{III} : \quad & [\Omega_{e(r_0)}]_{\text{band}} \neq 0 \text{ for some } r_0. \end{aligned} \quad (\text{III.4})$$

Vanishing of the controlled coefficient class is equivalent to controlled coefficient strictification, not to effective Hilbert descent.

6. The observable net is covariant under strict controlled background embeddings, and the two representation classes and their fiber restrictions are invariant under the invertible controlled background groupoid.

*Proof architecture.* Theorems II.230 and II.259, together with Theorem II.260, identify the selected source-free physical realization and its presentation-independent observable shadow on each chosen admissible rooted or marked Čech nerve. Hypotheses II.140–III.2 then give support-controlled algebraic Cauchy transport. Hypotheses III.3–III.6 give normal equalizer descent and transfer locality to bicommutants. Hypotheses III.7–III.10 produce the fixed and total coefficient systems. Hypothesis III.11 identifies the controlled subcomplex and therefore the distinction between ordinary root contraction and controlled strictification. Background covariance follows from Hypothesis III.16. Sections IV and V give the detailed constructions.  $\square$

### C. Gauge, entropy, and thick-form compatibility

**Theorem III.18** (Conditional analytic compatibility). *Assume Hypothesis II.267, Hypotheses III.7–III.16, and the observable system of Theorem III.17. Then:*

1. *In the self-adjoint Gauss branch,*

$$\pi_{\alpha,r}(\mathcal{A}_{e(r)}^W(\mathcal{O})) \subseteq \{G_{\alpha,r}(\xi)\}'_{\text{str}}, \quad P_{\Omega,\alpha,r} \leq E_{G_{\alpha,r}(\xi)}(\{0\}). \quad (\text{III.5})$$

*The ground kernels glue in the same coefficient type as the representations.*

2. *Collar charges descend as quotient classes modulo interior constraints. Ward derivations descend whenever the scalar zero-point changes cancel in adjoint commutators.*

3. *The entropy-surgery defect*

$$\mathfrak{d}_a^{\text{Surg}}(\omega_1) = \mathbb{T}_{a,*} \mathcal{S}_{r_1}^{\text{st}}(\omega_1) - \mathcal{S}_{r_0}^{\text{st}} \mathbb{T}_{a,*}(\omega_1) \quad (\text{III.6})$$

*satisfies the strict or coherence-corrected one-cocycle law, and bordism entropy production is nonnegative and additive under gluing.*

4. *Every closed thick form satisfies*

$$\ker \bar{q}_{\alpha,r;\mathcal{O}} = \text{Ran } P_{\Omega,\alpha,r}, \quad \bar{q}_{\alpha,r;\mathcal{O}}[\Psi] \geq \kappa_* \|(I - P_{\Omega,\alpha,r})\Psi\|^2. \quad (\text{III.7})$$

*The associated positive operator is not identified with a relativistic mass operator.*

5. *These analytic objects are pseudoequivariant under invertible controlled background arrows in the exact, controlled, or weak sense declared by their domains.*

*Proof architecture.* Hypotheses III.12 and III.13 give the graph-form, spectral-commutant, kernel, charge, and Ward conclusions. Hypothesis III.14 gives the predual surgery calculus. Corollary II.274 supplies the declared preform core, complete-ground projection, and inherited lower-bound input. Hypothesis III.15 separately gives closability, kernel identification after closure, and the closed-form result. Hypothesis III.16 transports the complete analytic package. Section VI contains the domain-level proofs.  $\square$

#### D. Dependency ledger and claim boundary

|                                                                                 |                                                                              |
|---------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| <b>Regional time-zero</b>                                                       | P1.                                                                          |
| <b>prestack.:</b><br><b>Selected Cauchy</b>                                     | P2.                                                                          |
| <b>anchoring.:</b><br><b>Defect naturality and</b>                              | P3–P4.                                                                       |
| <b>exactness.:</b><br><b>Common lifting carrier.:</b>                           | P5.                                                                          |
| <b>Controlled incised</b>                                                       | H1–H3.                                                                       |
| <b>geometry.:</b><br><b>Recovered selected Čech</b><br><b>descent diagram.:</b> | CIR on a chosen admissible finite cover and its rooted or marked Čech nerve. |
| <b>Physical realization and</b>                                                 | Nervewise RID and CP4 on the same selected Čech nerve.                       |
| <b>observable CP4.:</b><br><b>Regional <math>W^*</math>-net.:</b>               | H4–H8 after effective descent on the selected Čech nerve.                    |
| <b>Exact or asymptotic</b>                                                      | H3–H5 and H9.                                                                |
| <b>locality.:</b><br><b>Fixed-thickness</b>                                     | H10–H11.                                                                     |
| <b>representation descent.:</b><br><b>Scale and mixed</b>                       | H12–H13; cofinal geometric stability additionally uses UMG.                  |
| <b>coherence.:</b><br><b>Controlled trichotomy.:</b>                            | H11–H14.                                                                     |
| <b>Complete-ground</b>                                                          | CPR.                                                                         |
| <b>lower-bound</b>                                                              |                                                                              |
| <b>provenance.:</b>                                                             |                                                                              |

**Gauss kernels and Ward** H10 and H15–H16.

**actions.:**

**Entropy surgery and** H8, H13, and H17.

**glued bordisms.:**

**Closed forms and** CPR, H10, H12–H13, and H18.

**uniform thick bound.:**

**Background covariance.:** H2, RID, H7–H8, and H10–H19.

Neither theorem constructs a zero-thickness algebra, arbitrary-union additivity at zero thickness, a continuum Hilbert representation, Poincaré covariance, a final spectrum condition, non-leakage of  $\kappa_*$ , confinement, or a Yang–Mills mass gap.

*Remark III.19* (Realized subpackages and habitat boundary). Section VIII verifies complementary subpackages. The selected retained  $SU(3)$  branch now verifies P1 on the ordinary pre-incision dyadic time-zero basis, P2 on the protected stationary Cauchy basis, P3 on the selected finite dyadic regional-chart–Cauchy–scale demand system, and P4 by carrier-complete persistent-label reconstruction. The prescribed-incision  $SU(3)$  witness also proves a distinct nontrivial Cauchy dynamics, linked-flux, and canonical-comparison statements, and the  $SU(2)$  control verifies complete finite physicalization and strict fixed-stratum depth. Theorem II.299 shows that selected P1–P5, the dynamically derived carrier, H1–H3, and selected physical CIR recovery on a chosen admissible finite Čech nerve are now closed at their stated positive-thickness finite-demand scopes. Nervewise RID effectivity and observable CP4 after descent are also closed on that same nerve. CPR transport of the inherited complete-ground certificate through the resulting simplicial equivalence remains open. Thus the witnesses inhabit a larger portion of the architecture without yet proving the full analytic reconstruction.

## IV. THE THICK OBSERVABLE NET AND LOCALITY

**Reader’s guide.** This section changes the level at which the recovered regional data are read. Support filters and Cauchy propagation first produce regional algebraic cores; normal compatible-family descent then removes presentation choices and yields the ordinary thick observable net. The represented ground-pointed geometry is not collapsed in this process.

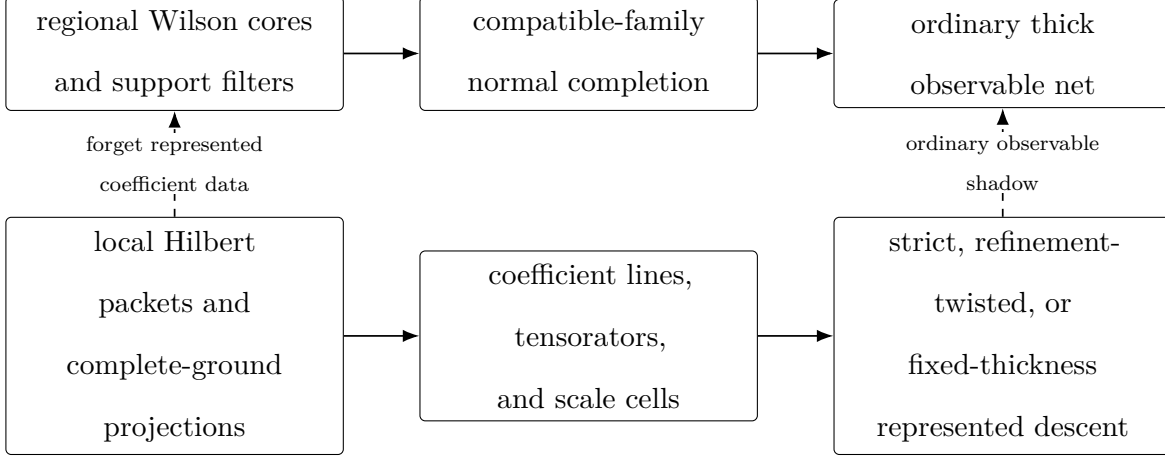


FIG. 3. Observable and represented descent are different layers. The observable net can be ordinary even when the selected ground-pointed representation retains nontrivial coefficient-line gluing.

Its coefficient lines and higher cells are retained for the separate vacuum-descent analysis of the next section.

### A. Controlled regions and support filters

Fix an incised background  $(M, \Sigma)$  and a thickness  $\varepsilon \in E_{\text{cof}}$ . An admissible region is a relatively compact causally convex open set  $\mathcal{O} \Subset M_{\text{inc}}$  with controlled boundary and positive clearance from the incision. The selected thickening operation satisfies

$$\mathcal{O} \subseteq \mathcal{O}^{+\varepsilon} \Subset M_{\text{inc}}, \quad \mathcal{O}_1 \subseteq \mathcal{O}_2 \implies \mathcal{O}_1^{+\varepsilon} \subseteq \mathcal{O}_2^{+\varepsilon}. \quad (\text{IV.1})$$

The construction is required only on the cofinal set of selected thicknesses; no interpolation to every positive real number is used.

A support pair is

$$\mathbf{K} = (K^0, K^\varepsilon), \quad K^0 \subseteq K^\varepsilon \subseteq (K^0)^{+\varepsilon}, \quad (\text{IV.2})$$

with compact skeletal and thick carriers. For a physical Wilson-tube element  $A$ , its support is the upward-closed filter

$$\text{Supp}_\varepsilon(A) = \{\mathbf{K} : A \text{ has an admissible presentation carried by } \mathbf{K}\}. \quad (\text{IV.3})$$

The filter, not a minimal compact set, is the invariant object. It obeys the expected monotone laws: if  $\mathbf{K}_A \in \text{Supp}(A)$  and  $\mathbf{K}_B \in \text{Supp}(B)$ , then a common upper bound  $\mathbf{K}_A \vee \mathbf{K}_B$  supports

$A + B$ ,  $AB$ , and  $[A, B]$ ; the same carriers support adjoints. Support-controlled physical equivalence ensures that these statements survive retracing, subdivision, gauge contraction, and passage to the physical quotient.

For an admissible region  $\mathcal{O}$ , define the regional algebraic core by

$$\mathfrak{A}_\varepsilon^{\text{alg}}(\mathcal{O}) = \{A : \text{some } \mathbf{K} \in \text{Supp}_\varepsilon(A) \text{ satisfies } K^\varepsilon \Subset \mathcal{O}^{+\varepsilon}\}. \quad (\text{IV.4})$$

Upward closure gives isotony on the algebraic level. Chart and same-thickness comparison maps preserve this regional localization. A noninvertible background embedding gives only the forward inclusion of support filters; equality is reserved for support-reflecting equivalences.

## B. Cauchy propagation

Let  $N \Subset M_{\text{inc}}$  be an incision-adapted globally hyperbolic neighborhood with local Cauchy slices  $\Sigma_s$ . For a compact  $K \subseteq \Sigma_{s_1}$ , its causal shadow on  $\Sigma_{s_2}$  is

$$\text{Sh}_{s_2, s_1}^N(K) = J_N(K) \cap \Sigma_{s_2}, \quad (\text{IV.5})$$

with the appropriate future or past branch. The shadows compose:

$$\text{Sh}_{s_3, s_2}^N(\text{Sh}_{s_2, s_1}^N(K)) = \text{Sh}_{s_3, s_1}^N(K). \quad (\text{IV.6})$$

The propagation-adapted thickening hypothesis is

$$\text{Sh}_{s_2, s_1}^N(K^{+\varepsilon}) \subseteq \text{Sh}_{s_2, s_1}^N(K)^{+c_N\varepsilon} \quad (\text{IV.7})$$

for a finite-stage constant  $c_N$ ; in the exact normalized branch  $c_N = 1$ .

Primitive assignments on elementary Wilson labels extend multiplicatively to the formal tube algebra and preserve the physical relations. They therefore descend to algebraic Cauchy isomorphisms

$$\alpha_{s_2, s_1}^{\text{alg}} : \mathfrak{A}_{\varepsilon, s_1}^{\text{alg}} \xrightarrow{\cong} \mathfrak{A}_{\varepsilon, s_2}^{\text{alg}}. \quad (\text{IV.8})$$

The support-exact branch satisfies

$$\mathbf{K} \in \text{Supp}_{s_1}(A) \implies \mathfrak{J}_{s_2, s_1}(\mathbf{K}) \in \text{Supp}_{s_2}(\alpha_{s_2, s_1}^{\text{alg}} A), \quad (\text{IV.9})$$

where the thick carrier of  $\mathfrak{J}_{s_2, s_1}(\mathbf{K})$  is contained in the causal shadow of  $K^\varepsilon$ . Composition of (IV.8) follows from (IV.6) and the primitive composition rule.

In the quasilocal branch, one has approximants  $A_R$  localized in successively enlarged shadows with

$$\|\alpha_{s_2, s_1}(A) - A_R\| \leq \lambda_{s_2, s_1}(R)\|A\|, \quad \lambda_{s_2, s_1}(R) \rightarrow 0. \quad (\text{IV.10})$$

This branch yields an asymptotic commutator estimate but does not prove exact positive-thickness locality.

### C. Normal completion and compatible-family descent

Choose a presentation  $p$  of  $\mathcal{O}$  consisting of an adapted chart, a local Cauchy frame, a slice, and a same-thickness representative. Let

$$\mathcal{M}_{\varepsilon, p}(\mathcal{O}) = \pi_{\varepsilon, p}(\mathfrak{A}_{\varepsilon, p}^{\text{alg}}(\mathcal{O}))''. \quad (\text{IV.11})$$

The presentation groupoid  $\mathcal{G}_\varepsilon(\mathcal{O})$  has these presentations as objects and the selected chart, Cauchy, and representative comparisons as arrows. Each arrow  $\nu : p \rightarrow q$  induces a normal \*-isomorphism

$$\Phi_\nu : \mathcal{M}_{\varepsilon, p}(\mathcal{O}) \longrightarrow \mathcal{M}_{\varepsilon, q}(\mathcal{O}). \quad (\text{IV.12})$$

The descended algebra is the normal equalizer

$$\mathcal{A}_\varepsilon^W(\mathcal{O}) = \{(X_p)_p \in \prod_p^{\ell^\infty} \mathcal{M}_{\varepsilon, p}(\mathcal{O}) : \Phi_\nu(X_p) = X_q \text{ for all } \nu : p \rightarrow q\}. \quad (\text{IV.13})$$

It is ultraweakly closed because it is an intersection of kernels of ultraweakly continuous difference maps. Hence it is a von Neumann algebra.

For a connected groupoid, evaluation at  $p$  identifies (IV.13) with the fixed-point algebra of the isotropy action:

$$\text{ev}_p : \mathcal{A}_\varepsilon^W(\mathcal{O}) \xrightarrow{\cong} \mathcal{M}_{\varepsilon, p}(\mathcal{O})^{\text{Iso}(p)}. \quad (\text{IV.14})$$

Hypothesis III.4 makes the observable isotropy trivial on an ultraweakly generating Wilson core, and therefore on the full algebra. Evaluation is consequently a normal \*-isomorphism onto  $\mathcal{M}_{\varepsilon, p}(\mathcal{O})$ .

A regional inclusion  $\mathcal{O}_1 \subseteq \mathcal{O}_2$  induces a functor between presentation groupoids and a natural family of normal inclusions between the presented algebras. The equalizer construction gives a normal unital injection

$$\iota_{\mathcal{O}_1, \mathcal{O}_2}^\varepsilon : \mathcal{A}_\varepsilon^W(\mathcal{O}_1) \longrightarrow \mathcal{A}_\varepsilon^W(\mathcal{O}_2). \quad (\text{IV.15})$$

Composition is strict because it is strict at every presented component. This proves the functor in (III.1). Normal additivity over arbitrary unions would require the presented identity

$$\mathcal{M}_{\varepsilon, p}(\mathcal{O}) = \left( \bigcup_i \mathcal{M}_{\varepsilon, p_i}(\mathcal{O}_i) \right)'', \quad (\text{IV.16})$$

and is not inferred from (IV.13).

## D. Locality

At the algebraic level, disjoint thick carriers commute. Let  $A$  and  $B$  have support pairs  $\mathbf{K}_A$  and  $\mathbf{K}_B$ . In a common Cauchy presentation,

$$K_A^\varepsilon \perp K_B^\varepsilon \implies [A, B] = 0. \quad (\text{IV.17})$$

Support-exact Cauchy propagation preserves the condition. If  $\mathcal{O}_1^{+\varepsilon} \perp \mathcal{O}_2^{+\varepsilon}$ , every pair of algebraic core elements therefore commutes in a common ambient presented algebra.

To pass from the core to the bicommutant, let  $X$  and  $Y$  be norm-bounded strong-\* limits of algebraic elements  $A_i$  and  $B_j$ . On bounded sets multiplication is separately strong-\* continuous. If

$$\|[A_i, B_j]\| \leq \eta \|A_i\| \|B_j\|, \quad (\text{IV.18})$$

then lower semicontinuity of the operator norm gives

$$\|[X, Y]\| \leq \eta \|X\| \|Y\|. \quad (\text{IV.19})$$

This is the required Kaplansky-normal extension. Ultraweak density alone would not suffice because multiplication is not jointly ultraweakly continuous.

For exact thick separation,  $\eta = 0$  in (IV.19), proving (III.2). For a thin-spacelike pair, let  $A_0$  and  $B_0$  be localized approximants with

$$\|A - A_0\| \leq \lambda_A \|A\|, \quad \|B - B_0\| \leq \lambda_B \|B\|, \quad [A_0, B_0] = 0. \quad (\text{IV.20})$$

A direct expansion yields

$$\|[A, B]\| \leq 2 \min\{1, \lambda_A + \lambda_B + 3\lambda_A\lambda_B\} \|A\| \|B\|. \quad (\text{IV.21})$$

The right-hand side defines the quasilocal contribution to  $\eta_\varepsilon^{\text{desc}}$ . Static collar removal gives an analogous estimate through conditional localization maps. On a finite family with thin spacelike margin  $\mu_* > 0$  and thickening constant  $C_*$ ,

$$2C_*\varepsilon < \mu_* \quad (\text{IV.22})$$

forces exact thick separation for every pair in the family. Therefore the geometric contribution to the locality defect vanishes identically below a finite-stage threshold.

**Proposition IV.1** (Fixed-thickness net and locality). *Under Hypotheses II.140–III.6, the construction above gives the ordinary functor (III.1). In the support-exact branch it satisfies exact thick locality (III.2). In the quasilocal branch it satisfies (IV.19) with the intrinsic defect obtained from (IV.21). Evaluation in any presentation is a normal isomorphism, and the commutator relation is independent of the common ambient region.*

*Proof.* The only point not already proved above is ambient-region independence. Given two common ambient regions, choose a larger jointly presentable region containing both. Normal isotony carries both commutators into the larger algebra, where their images agree. Since the regional maps are injective, vanishing and the operator norm of the commutator are independent of the chosen ambient algebra.  $\square$

*Remark IV.2.* The theorem is positive-thickness and basis-local. It does not prove a zero-thickness net, arbitrary-union additivity, or a time-slice generation equality. Those are separate reconstruction obligations.

## V. VACUUM DESCENT AND THE CHART-SCALE OBSTRUCTION

**Reader's guide.** The observable net is already globally defined, but the selected local-normal representations need not glue strictly. We therefore compare chart overlap lines, scale-transport lines, and their mixed coherence, then distinguish ordinary triviality from

controlled triviality. The resulting three-sector classification concerns represented descent architecture; it is not a classification of the local observable algebras or of finite spectra.

### A. Ground-pointed overlap correspondences

Fix a selected scale object  $r$  and suppress its thickness from the notation when no confusion can arise. On an adapted chart  $\alpha$ , the represented local datum is

$$\mathfrak{H}_{\alpha,r} = (\mathcal{H}_{\alpha,r}, \pi_{\alpha,r}, P_{\Omega,\alpha,r}). \quad (\text{V.1})$$

Let  $U_{\alpha\beta}$  be a nonempty overlap. A spatial overlap implementer  $W$  must satisfy

$$W\pi_{\beta,r}(A)W^* = \pi_{\alpha,r}(\chi_{\alpha\beta,r}A), \quad WP_{\Omega,\beta,r}W^* = P_{\Omega,\alpha,r}. \quad (\text{V.2})$$

If  $W_0$  is one such implementer, all others are

$$\mathcal{J}_{\alpha\beta,r} = \mathcal{N}_{\alpha\beta,r}^{\Omega} W_0, \quad (\text{V.3})$$

where

$$\mathcal{N}_{\alpha\beta,r}^{\Omega} = \{Z \in \mathcal{U}(\mathcal{M}'_{\alpha\beta,r}) : ZP_{\Omega,\alpha,r}Z^* = P_{\Omega,\alpha,r}\}. \quad (\text{V.4})$$

Thus spatial equivalence alone gives a torsor under a possibly nonabelian commutant group. Hypotheses III.7–III.10 selects a central one-dimensional reduction. The reduced torsor is the unit circle of a Hermitian line  $L_{\alpha\beta,r}$ , and the overlap comparison becomes

$$\mathbb{U}_{\alpha\beta,r} : \mathcal{H}_{\beta,r} \hat{\otimes} L_{\alpha\beta,r} \longrightarrow \mathcal{H}_{\alpha,r}. \quad (\text{V.5})$$

It intertwines the local observable algebras and carries

$$P_{\Omega,\beta,r} \otimes I_{L_{\alpha\beta,r}} \longmapsto P_{\Omega,\alpha,r}. \quad (\text{V.6})$$

The inverse line is canonically  $L_{\beta\alpha,r} \cong \overline{L_{\alpha\beta,r}}$ .

On a triple overlap there is a unitary line multiplication

$$m_{\alpha\beta\gamma,r} : L_{\alpha\beta,r} \otimes L_{\beta\gamma,r} \longrightarrow L_{\alpha\gamma,r}. \quad (\text{V.7})$$

The composite overlap route is

$$\mathbb{C}_{\alpha\beta\gamma,r} = \mathbb{U}_{\alpha\beta,r}(\mathbb{U}_{\beta\gamma,r} \otimes I)(I \otimes m_{\alpha\beta\gamma,r}^{-1}), \quad (\text{V.8})$$

while the direct route is  $\mathbb{D}_{\alpha\beta\gamma,r} = \mathbb{U}_{\alpha\gamma,r}$ . Their quotient lies in the ground-normalizing commutant. The selected triple central reduction gives

$$\mathbb{C}_{\alpha\beta\gamma,r} = \vartheta_{\alpha\beta\gamma,r} \mathbb{D}_{\alpha\beta\gamma,r}, \quad \vartheta_{\alpha\beta\gamma,r} \in U(1). \quad (\text{V.9})$$

Comparison of the two routes on a quadruple overlap yields

$$\vartheta_{\alpha\beta\gamma,r} \vartheta_{\alpha\gamma\delta,r} = \vartheta_{\beta\gamma\delta,r} \vartheta_{\alpha\beta\delta,r}. \quad (\text{V.10})$$

Changing normalized line representatives by phases  $z_{\alpha\beta,r}$  gives

$$\vartheta'_{\alpha\beta\gamma,r} = z_{\alpha\beta,r} z_{\beta\gamma,r} z_{\alpha\gamma,r}^{-1} \vartheta_{\alpha\beta\gamma,r}. \quad (\text{V.11})$$

These data define a banded two-presheaf

$$\mathbf{Vac}_{e(r)}^{\Omega} : \text{Chart}_*(M_{\text{inc}})^{\text{op}} \longrightarrow \mathbf{2Gpd}. \quad (\text{V.12})$$

Its intrinsic class is denoted  $[\Omega_{e(r)}]_{\text{band}}$ . This notation is degree-free. Under a good cover and a constant  $U(1)$ -band,  $\vartheta$  represents a class in  $\check{H}^2(\mathcal{U}; U(1))$ , with the corresponding integral degree-three image when the exponential sequence is used.

**Proposition V.1** (Fixed-thickness strictification). *The following are equivalent:*

1.  $[\Omega_{e(r)}]_{\text{band}} = 0$ ;
2. *there exist unit vectors  $\ell_{\alpha\beta,r} \in L_{\alpha\beta,r}$  satisfying*

$$\ell_{\alpha\gamma,r} = \vartheta_{\alpha\beta\gamma,r} m_{\alpha\beta\gamma,r} (\ell_{\alpha\beta,r} \otimes \ell_{\beta\gamma,r}); \quad (\text{V.13})$$

3. *the scalarized overlap maps*

$$U_{\alpha\beta,r}^{\ell} \psi = \mathbb{U}_{\alpha\beta,r} (\psi \otimes \ell_{\alpha\beta,r}) \quad (\text{V.14})$$

*form a strict unitary Čech descent datum.*

*The equivalence does not imply effective Hilbert descent without a separate effectivity theorem for the target category and the selected regional system.*

*Proof.* A family  $\ell_{\alpha\beta,r}$  is precisely a normalized 1-cochain whose coboundary cancels  $\vartheta$ , so the first two conditions are equivalent. Substitution of (V.13) into (V.8) gives strict composition of (V.14), proving the equivalence with the third condition. Conversely, strict composition determines the required line equation. The last statement follows because coefficient strictification only removes the central line ambiguity; it does not construct a colimit or global object in the Hilbert representation category.  $\square$

## B. Arrow-level scale transport

Let  $a : r_0 \rightarrow r_1$  be a selected scale-bordism arrow, with  $e(r_1) < e(r_0)$ . The support-controlled algebraic comparison extends to a normal regional map

$$\mathbb{T}_a^{\mathcal{O}} : \mathcal{A}_{e(r_0)}^W(\mathcal{O}) \longrightarrow \mathcal{A}_{e(r_1)}^W(\mathcal{O}) \quad (\text{V.15})$$

through the predual. Normality alone does not imply injectivity. In the strong representation branch there is a Hermitian scale line and unitary

$$\mathbb{V}_{\alpha;a} : \mathcal{H}_{\alpha,r_0} \hat{\otimes} L_{\alpha;a} \longrightarrow \mathcal{H}_{\alpha,r_1} \quad (\text{V.16})$$

implementing  $\mathbb{T}_a$  and carrying the complete ground projection. Such unitary implementation forces  $\mathbb{T}_a$  to be faithful. In the weak branch (V.16) is replaced by an ultraweakly closed normal intertwiner correspondence.

For composable arrows

$$r_0 \xrightarrow{a} r_1 \xrightarrow{b} r_2, \quad (\text{V.17})$$

there is a line multiplication

$$\mu_{\alpha;b,a} : L_{\alpha;a} \otimes L_{\alpha;b} \longrightarrow L_{\alpha;b \circ a}. \quad (\text{V.18})$$

The algebra maps may compose only up to a unitary cell  $\Xi_{b,a}$ . After inserting its represented action, direct and composite Hilbert routes differ by

$$\varphi_{\alpha;b,a} \in U(1). \quad (\text{V.19})$$

Associativity gives

$$\varphi_{\alpha;c,b} \varphi_{\alpha;c \circ b,a} = \varphi_{\alpha;b,a} \varphi_{\alpha;c, b \circ a}. \quad (\text{V.20})$$

The chart and scale routes are related by a mixed line map

$$\lambda_{\alpha\beta;a} : L_{\beta;a} \otimes L_{\alpha\beta,r_1} \longrightarrow L_{\alpha\beta,r_0} \otimes L_{\alpha;a}. \quad (\text{V.21})$$

The direction in (V.21) is fixed by the pullback convention. It produces the mixed chart identity

$$(\delta\lambda_a)_{\alpha\beta\gamma} = \vartheta_{\alpha\beta\gamma,r_0}^{-1} \vartheta_{\alpha\beta\gamma,r_1}, \quad (\text{V.22})$$

and hence

$$\Phi_a^*[\Omega_{e(r_1)}]_{\text{band}} = [\Omega_{e(r_0)}]_{\text{band}}. \quad (\text{V.23})$$

The mixed scale-composition relation is

$$\lambda_{\alpha\beta;b\circ a}^{\text{two}} = \varphi_{\alpha;b,a}^{-1} \varphi_{\beta;b,a} \lambda_{\alpha\beta;b\circ a}^{\text{dir}}. \quad (\text{V.24})$$

Equations (V.10), (V.20), (V.22), and (V.24) are the faces of the chart–scale coherence cube.

### C. Ordinary and controlled total classes

The total chart–scale indexing bicategory is the Grothendieck construction

$$\mathfrak{I}_{\text{cs}} = \int_{r \in \mathbf{Bor}_{\text{sc}}} \text{Chart}_*(r). \quad (\text{V.25})$$

Its objects are chart–scale pairs  $(\alpha, r)$ . The generating one-morphisms are fixed-thickness chart changes and selected scale arrows; the mixed cells and the algebraic  $\Xi$ -cells generate the relevant two-morphisms. The line and correspondence data

$$\widehat{\Omega}_{mcs} = (\mathfrak{H}, L^{\text{ch}}, L^{\text{sc}}, \vartheta, \varphi, \lambda) \quad (\text{V.26})$$

form a banded descent object over  $\mathfrak{I}_{\text{cs}}$ .

For unrestricted coefficient equivalences, its ordinary class is

$$[\widehat{\Omega}]_{\text{cs,ord}} \in \pi_0 \mathbf{Desc}_{\widehat{\mathcal{B}}_{\Omega}}^{\text{ord}}(\mathfrak{I}_{\text{cs}}, \widehat{\mathbf{Vac}}_{\text{cs}}^{\Omega}). \quad (\text{V.27})$$

For the physical refinement problem we also restrict equivalences to the controlled subcomplex of Hypothesis III.11: cochains are finite-shell local, band preserving, natural under control-preserving shell isomorphisms, and uniformly Lipschitz in the decorated-shell metrics. The corresponding class is

$$[\widehat{\Omega}]_{\text{cs,ctrl}} \in \pi_0 \mathbf{Desc}_{\widehat{\mathcal{B}}_{\Omega}}^{\text{ctrl}}(\mathfrak{I}_{\text{cs}}, \widehat{\mathbf{Vac}}_{\text{cs}}^{\Omega}). \quad (\text{V.28})$$

Restriction to the fiber over  $r$  gives

$$j_r^*[\widehat{\Omega}]_{\text{cs,ctrl}} = [\Omega_{e(r)}]_{\text{band}}, \quad (\text{V.29})$$

and similarly for the ordinary class.

A controlled trivialization consists of chart units  $\ell_{\alpha\beta,r}$  and scale units  $s_{\alpha;a}$  satisfying the chart equation (V.13), the scale equation

$$s_{\alpha;b\circ a} = \varphi_{\alpha;b,a} \mu_{\alpha;b,a}(s_{\alpha;a} \otimes s_{\alpha;b}), \quad (\text{V.30})$$

and the mixed equation

$$\lambda_{\alpha\beta;a}(s_{\beta;a} \otimes \ell_{\alpha\beta,r_1}) = \ell_{\alpha\beta,r_0} \otimes s_{\alpha;a}, \quad (\text{V.31})$$

with the units selected by controlled cochains.

The distinction between the two classes is essential on a rooted refinement category. If  $r_0$  is initial and the residual reassociator is a constant-coefficient pure-scale three-cocycle  $\Omega$ , the cone formula

$$\lambda(f, g) = \Omega(\iota_{s(f)}, f, g) \quad (\text{V.32})$$

produces an unrestricted primitive  $\delta\lambda = \Omega$ . In the cyclic batch model this primitive depends on absolute depth and is not natural under translation of identical renormalized shell packages. Thus the ordinary class can vanish while the controlled class remains nonzero.

**Theorem V.2** (Controlled total obstruction and sector trichotomy). *The controlled class (V.28) vanishes if and only if a simultaneous controlled chart–scale coefficient trivialization exists. The alternatives in (I.12) are mutually exclusive and exhaustive:*

1. *Sector I: all fixed-thickness classes vanish and the controlled total class vanishes;*
2. *controlled Sector II: all fixed-thickness classes vanish, the unrestricted ordinary total class vanishes, but the controlled total class is nonzero;*
3. *Sector III: at least one fixed-thickness restriction is nonzero.*

*A nonzero fixed-thickness class cannot be removed by scale transport. A pure-scale class on a rooted constant-coefficient nerve cannot be absolutely nonzero in unrestricted ordinary cohomology; its nontriviality must be controlled, relative, mixed, or carried by a nonconstant higher coefficient system.*

*Proof.* Controlled triviality is exactly the existence of controlled units satisfying the three displayed equations. Restriction gives (V.29), so a nonzero fiber class yields Sector III. If all fiber classes vanish, the residual controlled class either vanishes or lies in the kernel of the forgetful map to ordinary cohomology. The cone primitive (V.32) proves ordinary exactness of a pure-scale cocycle on the rooted category, while Appendix B proves that the cyclic primitive is excluded from the uniformly controlled subcomplex. These alternatives are disjoint and exhaustive.  $\square$

**What has now been established.** The branch type is now determined by where simultaneous chart–scale strictification fails. Sector I admits controlled strict gluing; Sector II is fiberwise strict but retains a controlled refinement obstruction; Sector III already carries a nontrivial fixed-thickness class. In every case the ordinary observable net remains available, while the represented object sent downstream must retain the corresponding coefficient-line or correspondence-valued structure.

*Remark V.3* (Observable versus representation descent). The ordinary regional observable net is already globally defined in all three sectors. The classes above obstruct only the selected ground-pointed representation descent. They neither convert the observable algebra into a gerbe nor imply that no other global representation exists.

*Remark V.4* (Weak branch). If central line reduction or strong unitary scale transport is unavailable, the total object remains a ground-pointed normal  $W^*$ -correspondence diagram. The scalar classes and the three-sector theorem are then replaced by the corresponding correspondence-valued descent statement; phases are not inserted artificially.

## VI. GAUGE CONSTRAINTS, WARD IDENTITIES, ENTROPY SURGERY, AND CLOSED FORMS

**Reader’s guide.** The descent classification does not itself supply the analytic structures needed at positive thickness. This section constructs them in a typed order: Gauss graph forms, complete-ground invariance, collar Ward identities, entropy surgery on the scale bordism, and finally the closed thick forms with the inherited coercive constant. These

structures are required to coexist with the selected branch, but none is used retroactively to create the obstruction carrier or neutralize represented twisting.

### A. Gauge-parameter spaces and Gauss graph forms

Let  $P \rightarrow M_{\text{inc}}$  be the selected principal gauge bundle and  $\text{ad } P$  its adjoint bundle. For an admissible region  $\mathcal{O}$ , the compactly supported interior parameters are

$$\mathfrak{g}_c(\mathcal{O}) = \Gamma_c^\infty(\mathcal{O}, \text{ad } P). \quad (\text{VI.1})$$

A collar resolution gives a larger Fréchet Lie algebra  $\mathfrak{g}_{\text{adm}}(\mathcal{O})$  of smooth resolved sections and a boundary trace sequence

$$0 \longrightarrow \mathfrak{g}_{\text{int}}(\mathcal{O}) \longrightarrow \mathfrak{g}_{\text{adm}}(\mathcal{O}) \xrightarrow{\text{tr}_\partial} \mathfrak{g}_\partial(\mathcal{O}) \longrightarrow 0. \quad (\text{VI.2})$$

The compactly supported algebra  $\mathfrak{g}_c$  is a safe initial core but need not equal the full trace-zero ideal  $\mathfrak{g}_{\text{int}}$ .

For a chart  $\alpha$ , scale object  $r$ , and local Cauchy slice, let

$$\mathcal{V}_{\alpha,r}^G = D(\bar{q}_{\alpha,r}^{\text{aux}}) \hookrightarrow \mathcal{H}_{\alpha,r} \hookrightarrow (\mathcal{V}_{\alpha,r}^G)^* \quad (\text{VI.3})$$

be the inherited energy graph triple. A real interior parameter  $\xi$  defines a Hermitian graph form

$$g_{\alpha,r}(\xi) : \mathcal{V}_{\alpha,r}^G \times \mathcal{V}_{\alpha,r}^G \longrightarrow \mathbb{C} \quad (\text{VI.4})$$

with support-stage estimate

$$|g_{\alpha,r}(\xi)[\Psi, \Phi]| \leq C_{K,m} p_{K,m}(\xi) \|\Psi\|_G \|\Phi\|_G. \quad (\text{VI.5})$$

Thus

$$\mathbb{G}_{\alpha,r}(\xi) : \mathcal{V}_{\alpha,r}^G \longrightarrow (\mathcal{V}_{\alpha,r}^G)^* \quad (\text{VI.6})$$

is bounded. This graph-dual realization is the primary object.

A Hilbert-space operator exists only on the pivot domain

$$D(G_{\alpha,r}^{\text{piv}}(\xi)) = \{\Psi \in \mathcal{V}_{\alpha,r}^G : \mathbb{G}_{\alpha,r}(\xi)\Psi \in j^* \mathcal{H}_{\alpha,r}\}. \quad (\text{VI.7})$$

Graph boundedness does not imply that this domain is dense. If a densely defined realization is symmetric, it is closable; self-adjointness and exponentiation require additional hypotheses<sup>36</sup>. The construction therefore retains the hierarchy

$$\begin{aligned}
&\text{graph form} \Rightarrow \text{closed symmetric operator (conditional),} \\
&\quad \Rightarrow \text{self-adjoint operator (stronger),} \\
&\quad \Rightarrow \text{unitary gauge group.}
\end{aligned} \tag{VI.8}$$

No reverse implication is used.

Cauchy transport is defined first by a canonical graph unitary. A selected Hilbert implementer may differ from it by a unitary in the observable commutant. Such commutant membership alone does not preserve an unbounded graph domain. Path independence therefore requires the additional isotropy conditions

$$Z\mathcal{V}_{\alpha,r}^G = \mathcal{V}_{\alpha,r}^G, \quad g_{\alpha,r}(\xi)[Z\Psi, Z\Phi] = g_{\alpha,r}(\xi)[\Psi, \Phi]. \tag{VI.9}$$

This distinction prevents a common circularity in unbounded constraint descent.

## B. Gauge invariance and the complete ground sector

On a raw Wilson transport label  $U_\gamma$ , the infinitesimal interior gauge derivation is

$$\delta_\xi^{\text{raw}} U_\gamma = \xi(\gamma(1))U_\gamma - U_\gamma \xi(\gamma(0)). \tag{VI.10}$$

Endpoint terms cancel in gauge-invariant Wilson loops and invariant network contractions. Hence

$$\delta_\xi^{\text{raw}} A = 0 \tag{VI.11}$$

for the physical Wilson core. The Gauss form implements the derivation by

$$i\{g(\xi)[\pi(A)\Psi, \Phi] - g(\xi)[\Psi, \pi(A^*)\Phi]\} = \langle \pi(\delta_\xi A)\Psi, \Phi \rangle. \tag{VI.12}$$

Thus physical observables satisfy graph-form commutation.

If  $G(\xi)$  is self-adjoint, form invariance implies resolvent commutation and therefore commutation with every spectral projection. The spectral strong commutant

$$\{G(\xi)\}'_{\text{str}} = \{B \in B(\mathcal{H}) : BE_G(\Delta) = E_G(\Delta)B \text{ for all Borel } \Delta\} \tag{VI.13}$$

is a von Neumann algebra. Since it contains the represented Wilson core, it contains the regional bicommutant:

$$\pi_{\alpha,r}(\mathcal{A}_{e(r)}^W(\mathcal{O})) \subseteq \{G_{\alpha,r}(\xi)\}'_{\text{str}}. \quad (\text{VI.14})$$

This is the rigorous extension of gauge invariance to the  $W^*$ -net. It does not rely on taking ultraweak limits of an unbounded commutator.

The complete-ground Gauss condition is

$$\mathbb{G}_{\alpha,r}(\xi)P_{\Omega,\alpha,r}^{V,G} = 0. \quad (\text{VI.15})$$

In the self-adjoint branch it yields

$$P_{\Omega,\alpha,r} \leq E_{G_{\alpha,r}(\xi)}(\{0\}). \quad (\text{VI.16})$$

Every normal state supported in  $P_{\Omega,\alpha,r}$  is therefore gauge invariant. No preferred density operator or vacuum vector is selected.

In Sector III, the correct global object is the family

$$\{P_{\Omega,\alpha,r}\mathcal{H}_{\alpha,r}\}_{\alpha} \quad (\text{VI.17})$$

glued by the restrictions of  $\mathbb{U}_{\alpha\beta,r}$ . A formula such as  $G_r(\xi)P_{\Omega,r} = 0$  is stated only after fixed-thickness coefficient strictification and effective descent of the unbounded Gauss system. In controlled Sector II it may hold separately at each thickness without a controlled scale-natural choice.

### C. Collar charges and Ward identities

Choose a linear trace splitting  $s^\partial : \mathfrak{g}_\partial \rightarrow \mathfrak{g}_{\text{adm}}$ . Every admissible parameter is

$$\zeta = \eta + s^\partial \lambda, \quad \eta \in \mathfrak{g}_{\text{int}}, \quad \lambda \in \mathfrak{g}_\partial. \quad (\text{VI.18})$$

The intrinsic collar charge is not the chosen representative  $\mathbb{G}^{\text{adm}}(s^\partial \lambda)$ , but its class modulo interior constraints:

$$\mathbf{Q}^\partial(\lambda) = [\mathbb{G}^{\text{adm}}(\xi)] \in \mathfrak{Q}^{G,\partial}, \quad \text{tr}_\partial \xi = \lambda. \quad (\text{VI.19})$$

Changing the splitting changes a representative by an interior Gauss term. On the joint interior kernel

$$\mathcal{K}^{G,\text{int}} = \bigcap_{\eta \in \mathfrak{g}_{\text{int}}} \ker \mathbb{G}^{\text{int}}(\eta), \quad (\text{VI.20})$$

the reduced boundary form is splitting independent. Since the complete ground sector lies in  $\mathcal{K}^{G,\text{int}}$ , it carries an intrinsic collar-charge action. It need not be collar neutral.

For a Ward-admissible observable  $A$ , the interior Ward functional is

$$\begin{aligned} \mathfrak{w}^{\text{int}}(\xi; A)[\Psi, \Phi] &= ig^{\text{int}}(\xi)[\pi(A)\Psi, \Phi] - ig^{\text{int}}(\xi)[\Psi, \pi(A^*)\Phi] \\ &\quad - \langle \pi(\delta_\xi A)\Psi, \Phi \rangle. \end{aligned} \quad (\text{VI.21})$$

The exact identity is  $\mathfrak{w}^{\text{int}} = 0$ . On the complete ground sector it reduces to

$$\langle \Psi, \pi(\delta_\xi A)\Phi \rangle = 0. \quad (\text{VI.22})$$

The boundary Ward functional is defined analogously using the reduced charge form. It is independent of  $s^\partial$  because two splittings differ by an interior Ward identity.

Boundary charge zero-points can change by scalar characters on coefficient lines. If

$$Q_\alpha^\partial(\lambda)\mathbb{U}_{\alpha\beta} = \mathbb{U}_{\alpha\beta}(Q_\beta^\partial(\lambda) \otimes I) + \chi_{\alpha\beta}^{\text{ch}}(\lambda)\mathbb{U}_{\alpha\beta}, \quad (\text{VI.23})$$

then the scalar term cancels from the adjoint commutator. Consequently a nonzero boundary-character class can obstruct global charge zero-points while leaving the Ward action globally well defined. Continuous infinitesimal characters factor through the abelianization of  $\mathfrak{g}_\partial$ ; they vanish on a topologically perfect boundary algebra.

For self-adjoint charge representatives the current-algebra defect is

$$\mathfrak{a}^\partial(\lambda, \mu) = \frac{1}{i}[Q^\partial(\lambda), Q^\partial(\mu)] - Q^\partial([\lambda, \mu]). \quad (\text{VI.24})$$

If it is scalar, the charge representation is projective but the adjoint Ward derivations remain exact because scalars commute with observables.

## D. Entropy surgery on the scale bordism

Let

$$\mathcal{M}_r(\mathcal{O}) = \pi_{\alpha,r}(\mathcal{A}_{e(r)}^W(\mathcal{O}))'' \quad (\text{VI.25})$$

be a chart realization of the intrinsic regional algebra, and choose a faithful normal semifinite reference weight  $\varpi_{r;\mathcal{O}}$ . The finite-entropy normal-state domain is

$$\mathfrak{D}_r^{\text{Ent}}(\mathcal{O}) = \{\omega \in \mathfrak{S}_n(\mathcal{M}_r(\mathcal{O})) : S(\omega \| \varpi_{r;\mathcal{O}}) < \infty\}. \quad (\text{VI.26})$$

Relative entropy is monotone under regional restriction and normal scale pullback. Hence it is intrinsic on the ordinary regional algebra even when density matrices or GNS vectors remain twisted.

The state surgery map is

$$\mathcal{S}_r^{\text{st}} : \mathfrak{D}_r^{\text{Ent}} \longrightarrow \mathfrak{D}_r^{\text{adm,Ent}}, \quad (\mathcal{S}_r^{\text{st}})^2 = \mathcal{S}_r^{\text{st}}, \quad (\text{VI.27})$$

with entropy removal

$$\Gamma_r^{\text{Surg}}(\omega) = \mathcal{E}_r(\omega) - \mathcal{E}_r(\mathcal{S}_r^{\text{st}}\omega) \geq 0. \quad (\text{VI.28})$$

The map acts on normal states. It does not define a linear Hilbert-space or graph-domain map unless an additional represented correspondence is supplied.

For a scale arrow  $a : r_0 \rightarrow r_1$ , the predual map is contravariant:

$$\mathbb{T}_{a,*} : \mathfrak{D}_{r_1}^{\text{Ent}} \longrightarrow \mathfrak{D}_{r_0}^{\text{Ent}}. \quad (\text{VI.29})$$

The surgery defect is (III.6). For strict composition  $r_0 \xrightarrow{a} r_1 \xrightarrow{b} r_2$ , one computes

$$\mathfrak{d}_{b \circ a}^{\text{Surg}}(\omega_2) = \mathbb{T}_{a,*} \mathfrak{d}_b^{\text{Surg}}(\omega_2) + \mathfrak{d}_a^{\text{Surg}}(\mathbb{T}_{b,*}\omega_2). \quad (\text{VI.30})$$

In the pseudofunctorial branch a predual action of  $\Xi_{b,a}$  is inserted. The pentagon gives threefold consistency.

The bordism entropy production is

$$\Delta_a^{\text{Bor,Ent}}(\omega_1) = \mathcal{E}_{r_1}(\omega_1) - \mathcal{E}_{r_0}(\mathbb{T}_{a,*}\omega_1) \geq 0. \quad (\text{VI.31})$$

Data processing gives nonnegativity, and strict composition gives additivity:

$$\Delta_{b \circ a}^{\text{Bor,Ent}}(\omega_2) = \Delta_b^{\text{Bor,Ent}}(\omega_2) + \Delta_a^{\text{Bor,Ent}}(\mathbb{T}_{b,*}\omega_2). \quad (\text{VI.32})$$

If surgery is exactly natural, then

$$\Delta_a^{\text{Bor,Ent}}(\mathcal{S}_{r_1}^{\text{st}}\omega_1) - \Delta_a^{\text{Bor,Ent}}(\omega_1) = \Gamma_{r_0}^{\text{Surg}}(\mathbb{T}_{a,*}\omega_1) - \Gamma_{r_1}^{\text{Surg}}(\omega_1). \quad (\text{VI.33})$$

The intermediate terms telescope on finite glued chains.

The exact state seam is the graph

$$\mathfrak{C}_a^{\text{Ent}} = \{(\omega_0, \omega_1) : \omega_0 = \mathbb{T}_{a,*}\omega_1\}. \quad (\text{VI.34})$$

For strict composition,

$$\mathfrak{C}_{b \circ a}^{\text{Ent}} \cong \mathfrak{C}_a^{\text{Ent}} \times_{\mathfrak{D}_{r_1}^{\text{Ent}}} \mathfrak{C}_b^{\text{Ent}}. \quad (\text{VI.35})$$

After controlled surgery, the seam mismatch is exactly  $-\mathfrak{D}_a^{\text{Surg}}$ . A small predual defect does not by itself imply a small entropy-balance residual; a later limit theorem needs an entropy-continuity modulus on the selected tight state sets.

Surgery is defined over the existing coefficient-line diagram. It therefore preserves

$$[\Omega_r]_{\text{band}}, \quad [\widehat{\Omega}]_{\text{CS,ord}}, \quad [\widehat{\Omega}]_{\text{CS,ctrl}}, \quad [\chi^\partial]_{\text{CS}}. \quad (\text{VI.36})$$

It cannot move a system between Sector I, controlled Sector II, and Sector III.

## E. Closed thick forms and uniform coercivity

Let  $q_{\alpha,r;\mathcal{O}}^{\text{pre}}$  be the propagated regional preform on a dense common core and assume

$$q_{\alpha,r;\mathcal{O}}^{\text{pre}}[\Psi] \geq \kappa_* \|(I - P_{\Omega,\alpha,r})\Psi\|^2, \quad q^{\text{pre}}[P_\Omega \Psi, \Phi] = 0. \quad (\text{VI.37})$$

Closure gives  $\bar{q}_{\alpha,r;\mathcal{O}}$ . Since  $P_\Omega$  is continuous in the graph norm, the form domain splits as

$$\mathcal{V}_{\alpha,r;\mathcal{O}}^q = P_{\Omega,\alpha,r} \mathcal{H}_{\alpha,r} \widehat{\oplus} (I - P_{\Omega,\alpha,r}) \mathcal{V}_{\alpha,r;\mathcal{O}}^q. \quad (\text{VI.38})$$

The bound survives closure with the same constant, and

$$\ker \bar{q}_{\alpha,r;\mathcal{O}} = P_{\Omega,\alpha,r} \mathcal{H}_{\alpha,r}. \quad (\text{VI.39})$$

The associated positive self-adjoint operator  $K_{\alpha,r;\mathcal{O}}$ <sup>33,34,37</sup> has

$$K_{\alpha,r;\mathcal{O}} = 0_{P_\Omega \mathcal{H}} \oplus K_{\alpha,r;\mathcal{O}}^\perp, \quad K_{\alpha,r;\mathcal{O}}^\perp \geq \kappa_* I. \quad (\text{VI.40})$$

Therefore

$$\text{spec}(K_{\alpha,r;\mathcal{O}}) \subseteq \{0\} \cup [\kappa_*, \infty). \quad (\text{VI.41})$$

This is a positive-thickness form spectrum, not a relativistic mass spectrum.

Exact scale transport is graph-unitary. In the controlled branch the form defect is

$$d_a^q(\Psi, \Phi) = \bar{q}_{r_1}[V_a^q\Psi, V_a^q\Phi] - \bar{q}_{r_0}[\Psi, \Phi]. \quad (\text{VI.42})$$

It satisfies the strict or  $\Xi$ -corrected cocycle law

$$d_{b \circ a}^q(\Psi, \Phi) = d_b^q(V_a^q\Psi, V_a^q\Phi) + d_a^q(\Psi, \Phi). \quad (\text{VI.43})$$

The exact vector seams glue by fiber product, while the diagonal controlled seam mismatch equals  $d_a^q(\Psi, \Psi)$ .

**Theorem VI.1** (Sectorial analytic package). *Under Hypotheses III.7–III.15, the Gauss graph forms, complete-ground kernels, Ward functionals, entropy-surgery maps, and closed thick forms are compatible with regional restriction, chart descent, selected scale arrows, and finite glued-bordism composition. In Sector III they form coefficient-line-banded chart-local objects; in controlled Sector II they may descend globally at each fixed thickness without a controlled scale-natural trivialization; in Sector I they admit a global scale-coherent realization only after the corresponding effective Hilbert, domain, and form-descent hypotheses are supplied. Every local closed form retains the same constant  $\kappa_*$ .*

*Proof.* Regional and chart compatibility follow from the graph-form covariance law  $\mathbb{G}_1 J = J^{-*} \mathbb{G}_0$ , the corresponding invariance of Ward insertions, and unitary invariance of relative entropy. The scale cocycle formulas are (VI.30) and (VI.43); their corrected versions insert the same algebraic coherence cells already present in the representation-scale diagram. The sectorial statement follows because all represented analytic objects live over the coefficient-line system of Section V. The persistence of  $\kappa_*$  is the closure argument leading to (VI.39). Effectivity remains separate from coefficient strictification.  $\square$

## VII. BACKGROUND COVARIANCE AND THE MATHEMATICAL ENDPOINT

### A. Strict embeddings and the incision groupoid

A controlled incised background is denoted

$$\mathbb{M} = (M, g, \Sigma, \mathfrak{c}), \quad (\text{VII.1})$$

where  $\mathfrak{c}$  collects the tubular, collar, temporal, and thickening controls. The category

$$\mathbf{Bg}_{\text{inc}}^{\text{thick}} \quad (\text{VII.2})$$

has strict control-preserving isometric embeddings as morphisms. Its maximal subgroupoid

$$\mathbf{Gpd}_{\text{inc}}^{\text{thick}} = \text{Core}(\mathbf{Bg}_{\text{inc}}^{\text{thick}}) \quad (\text{VII.3})$$

contains the invertible controlled background arrows. This distinction is essential. A noninvertible embedding transports bounded observables, but it does not canonically identify Hilbert spaces, complete ground sectors, or unbounded graph domains.

For every fixed thickness, the regional observable construction is covariant:

$$\mathfrak{A}_{\varepsilon}^{W, \text{bg}} : \mathbf{RegBg}_{\text{inc}}^{\varepsilon, \text{cp}} \longrightarrow \mathbf{W}^* \mathbf{Alg}_{\mathfrak{n}}. \quad (\text{VII.4})$$

If  $\psi : \mathbb{M} \rightarrow \mathbb{M}'$  is a strict embedding, the normal map

$$\alpha_{\varepsilon}^W(\psi) : \mathcal{A}_{\varepsilon}^{W, \mathbb{M}}(\mathcal{O}) \longrightarrow \mathcal{A}_{\varepsilon}^{W, \mathbb{M}'}(\psi\mathcal{O}) \quad (\text{VII.5})$$

is induced on Wilson cores and extended normally. It is compatible with regional inclusions and with the selected scale-bordism arrows:

$$\alpha_{e(r_1)}^W(\psi) \mathbb{T}_a = \mathbb{T}_{\psi \text{Bor} a} \alpha_{e(r_0)}^W(\psi). \quad (\text{VII.6})$$

When  $\psi$  is invertible, the represented system carries a coefficient line  $L_{\psi; \alpha, r}^{\text{bg}}$  and a unitary

$$\mathbb{U}_{\psi; \alpha, r}^{\text{bg}} : \mathcal{H}_{\alpha, r}^{\mathbb{M}} \widehat{\otimes} L_{\psi; \alpha, r}^{\text{bg}} \longrightarrow \mathcal{H}_{\psi\alpha, \psi \text{Bor} r}^{\mathbb{M}'} \quad (\text{VII.7})$$

Composition is governed by background line multiplications and scalar tensorators. These satisfy the expected cocycle law and define a band-preserving pseudoequivalence of total chart-scale descent objects.

Consequently,

$$\widehat{\Psi}_\psi^*[\widehat{\Omega}^{\mathbb{M}'}]_{\text{cs,ord}} = [\widehat{\Omega}^{\mathbb{M}}]_{\text{cs,ord}}, \quad \widehat{\Psi}_\psi^*[\widehat{\Omega}^{\mathbb{M}'}]_{\text{cs,ctrl}} = [\widehat{\Omega}^{\mathbb{M}}]_{\text{cs,ctrl}}, \quad (\text{VII.8})$$

with analogous identities for  $[\Omega_\varepsilon]_{\text{band}}$  and the boundary-character class. The controlled three-sector classification is invariant under the incision groupoid.

The analytic package is covariant at the strength established in Section VI. Exact background unitaries transport the complete ground projections, Gauss graph forms and self-adjoint spectral measures, Ward functionals, entropy reference weights and surgery maps, closed form domains, and associated positive operators. The coercive constant is unchanged:

$$\bar{q}_{\psi\alpha,\psi r}^{\mathbb{M}'}[\Psi'] \geq \kappa_* \|(I - P_{\Omega,\psi\alpha,\psi r}^{\mathbb{M}'})\Psi'\|^2. \quad (\text{VII.9})$$

Controlled defects retain the same finite-stage moduli.

Strict background maps preserve relative causal developments and causal shadows:

$$\psi(D_N(B)) = D_{\psi N}(\psi B), \quad \psi(J_N(B)) = J_{\psi N}(\psi B). \quad (\text{VII.10})$$

Thus background, scale, chart, and Cauchy transport form one coherence diagram: the bounded observable faces commute strictly, while represented faces commute through the selected coefficient-line cells. This compatibility does not prove a time-slice generation equality.

## B. Thin-readiness versus thin reconstruction

The complete output of the present article is the positive-thickness datum

$$\mathfrak{D}_{\text{III}}^{\text{YM,thick}} = (\mathbf{Geom}, \mathbf{Obs}, \mathbf{Vac}, \mathbf{GaussWard}, \mathbf{EntSurg}, \mathbf{Q}, \mathbf{Def}). \quad (\text{VII.11})$$

It contains the incised geometry, the ordinary regional observable functors, the chart-scale representation descent, the gauge and Ward systems, entropy-surgery and seam data, closed forms, and all exact or controlled comparison maps.

A zero-thickness reconstruction requires substantially more than the existence of (VII.11). On each finite test stage one needs at least:

1. cofinal corridor control for the full arrow-level scale-bordism system;
2. compactness or tightness of regional observables and normal states in chosen comparison carriers;
3. sectorially typed compactness of represented ground data;
4. recovery of common graph domains for Gauss and form limits;
5. lower semicontinuity and continuity moduli for relative entropy on the selected tight state sets;
6. convergence of state and vector seam data;
7. vanishing of locality, Gauss, Ward, surgery, and form defects;
8. stability of causal development and strict background covariance;
9. effective descent in the correct sector rather than forced strictification;
10. convergence or tightness of the complete ground projections;
11. a recovery or upper-identification theorem for the limiting energy;
12. independence of cofinal extraction and comparison carriers.

These are readiness conditions, not conclusions of the present theorem.

The objectwise bound (III.7) gives only

$$\liminf_r \bar{q}_r[\Psi_r] \geq \kappa_* \liminf_r \|(I - P_{\Omega,r})\Psi_r\|^2. \quad (\text{VII.12})$$

It does not identify a limiting projection or form. Even if a candidate form  $q_\infty$  satisfies the usual Mosco lower-bound inequality, the direction of that inequality is not enough to deduce

$$q_\infty[\Psi] \geq \kappa_* \|(I - P_\infty)\Psi\|^2. \quad (\text{VII.13})$$

One also needs a recovery sequence or equivalent upper identification, together with control of  $P_{\Omega,r}$ . Appendix C gives an explicit counterexample to the naive transfer.

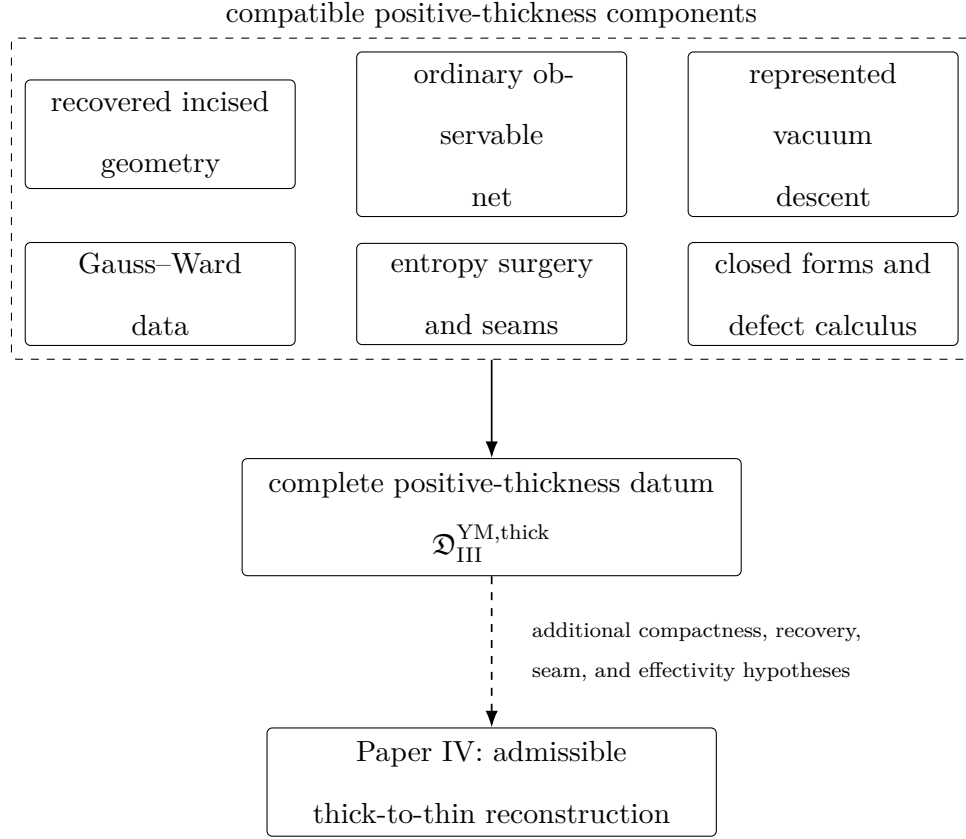


FIG. 4. Paper III packages compatible positive-thickness data. The dashed edge is deliberately not taken in this article.

**Interpretive guide.** The geometric, observable, represented, gauge, entropy, and closed-form components have now been established separately at positive thickness. The endpoint theorem records that they occupy one compatible finite diagram and that the exact and controlled faces retain their proper types. It packages the Paper-IV intake; it does not take a zero-thickness limit.

**Theorem VII.1** (Positive-thickness endpoint). *Under the assumptions of Theorems III.17 and III.18, the complete diagram (VII.11) is well defined and compatible on every fixed finite background–scale–chart–Cauchy gluing pattern. Its exact faces remain exact, its controlled defects satisfy composition laws and retain their finite-stage decay moduli, and its sector class is preserved. The datum is therefore suitable as input to a separate thin-reconstruction theorem, conditional on the compactness, recovery, seam, and effectivity conditions listed above.*

*Proof.* Every block of (VII.11) was constructed in Sections IV–VI. The compatibility cells are those of the total chart–scale system, and the background squares are (VII.6). The defect cocycles imply that a composite finite diagram has a transported sum of edge defects; hence directed-uniform edge control gives directed-uniform control of the composite. Sector preservation follows from (VII.8) and from the fact that analytic surgery and defects do not modify the coefficient lines or tensorators. The final assertion is a statement of typed input: no limiting object is defined in this proof.  $\square$

### C. Explicit nonclaims

For clarity, the following statements are outside the scope of Theorem VII.1:

- construction of a zero-thickness regional  $W^*$ -net or proof of all Haag–Kastler axioms;
- arbitrary-union additivity and a thin time-slice theorem;
- a continuum Hilbert representation or a unique vacuum vector;
- a strongly continuous Poincaré representation, translation generators, or a spectrum condition;
- a thin Gauss/Ward system, anomaly cancellation, or a global boundary-charge representation in every sector;
- Mosco convergence, norm-resolvent convergence, or spectral-projection convergence;
- survival of  $\kappa_*$ , vacuum non-leakage, confinement, or a Yang–Mills mass gap.

The contribution is the complete positive-thickness reconstruction and the identification of the precise additional hypotheses required to formulate the next limit problem without collapsing the strict and twisted sectors.

## VIII. EXPLICIT REALIZATIONS AND THE PHYSICAL TRICHOTOMY

**Reader’s guide.** The examples play different logical roles. The local Friedrichs construction shows that the positive-thickness comparison net is analytically realizable; the linked-flux

$SU(3)$  packet realizes the physical Sector-III detector and controlled scale data; and the independent  $SU(2)$  model supplies a strict finite-stratum control. These witnesses test separate components of the architecture and are not combined into a zero-thickness theory.

The general theorems are conditional, but their principal mechanisms are realized in complementary models. The  $SU(3)$  construction has three logically separate layers:

$$\begin{aligned} & \text{a local Friedrichs comparison net,} \\ & \text{a fixed-thickness linked-flux class,} \\ & \text{a controlled scale class on a dyadic thickness orbit.} \end{aligned} \tag{VIII.1}$$

The independent  $SU(2)$  model is a strict finite/depth control. None is promoted to a zero-thickness Yang–Mills theory.

#### A. Local Friedrichs assembly of the $SU(3)$ comparison net

Let

$$\ell = \{(0, 0, z) : z \in \mathbb{R}\}, \quad X_{\text{inc}} = \mathbb{R}^3 \setminus \ell, \quad M_{\text{inc}} = \mathbb{R} \times X_{\text{inc}}. \tag{VIII.2}$$

For  $m \in \mathbb{Z}_3$ , let  $E_m \rightarrow X_{\text{inc}}$  be the flat Hermitian excitation bundle with positive meridional holonomy  $\omega^m I_3$ . The causal canonical branch uses the local operator

$$A_m = \eta \nabla_m^* \nabla_m + V_m, \quad V_m \in C^\infty(X_{\text{inc}}; \text{End}_{\text{Herm}} E_m), \tag{VIII.3}$$

with

$$A_m \geq \kappa_{\text{cof}} I, \quad \kappa_{\text{cof}} = \frac{1}{2} \left( \frac{17}{5} - \sqrt{\frac{557}{75}} \right) = 0.337404437602020 \dots \tag{VIII.4}$$

The internal term is pointwise. An abstract nonlocal  $h_{\text{int}}$  is not included in the causal theorem. Put  $B_m = A_m^{1/2}$ .

The use of one global operator is load bearing. Independently chosen regional Friedrichs operators need not satisfy

$$e_{21} A_m (D_1)^{1/2} = A_m (D_2)^{1/2} e_{21}; \tag{VIII.5}$$

therefore positive-frequency embeddings are constructed by localizing compactly supported Cauchy data inside the one global energy space.

Let  $\mathbf{Reg}_{\text{inc}}^{\text{can}}$  be the collar-Cauchy admissible region category of Appendix I. For

$$P_m = \partial_t^2 + A_m, \quad (\text{VIII.6})$$

Theorem I.3 gives retarded and advanced Green operators and

$$\text{supp}(E_m f) \subseteq J_{\sqrt{\eta}}(\text{supp } f). \quad (\text{VIII.7})$$

Define

$$\mathcal{S}_m(\mathcal{O}) = C_c^\infty(\mathcal{O}; E_m)_{\mathbb{R}} / P_m C_c^\infty(\mathcal{O}; E_m)_{\mathbb{R}}, \quad (\text{VIII.8})$$

with symplectic form

$$\sigma_{m,\mathcal{O}}([f], [g]) = \int \text{Re} \langle f, E_m g \rangle \, d\text{vol}. \quad (\text{VIII.9})$$

**Theorem VIII.1** ( $SU(3)$  local-normal comparison assembly). *For each  $m \in \mathbb{Z}_3$ :*

1.  $\mathcal{O} \mapsto \mathcal{S}_m(\mathcal{O})$  is an injective causal symplectic functor;
2. the Weyl algebras

$$\mathcal{A}_m^{\text{can}}(\mathcal{O}) = \text{CCR}(\mathcal{S}_m(\mathcal{O}), \sigma_{m,\mathcal{O}}) \quad (\text{VIII.10})$$

*are isotone, causally local, additive, and satisfy time-slice;*

3. the global positive-frequency map

$$K_m(q, p) = 2^{-1/2} (B_m^{1/2} q + i B_m^{-1/2} p) \quad (\text{VIII.11})$$

*defines one regular Fock representation, and*

$$\mathcal{M}_m^{\text{can}}(\mathcal{O}) = \pi_m(\mathcal{A}_m^{\text{can}}(\mathcal{O}))'' \quad (\text{VIII.12})$$

*forms a local-normal  $W^*$ -net with normal unital inclusions;*

4. cofinal exhaustion and intermediate Cauchy-slice choices give the same represented global closure and normal Cauchy evolution;

5. the canonical Hamiltonian  $H_m^{\text{can}} = d\Gamma(B_m)$  obeys

$$H_m^{\text{can}} \geq \sqrt{\kappa_{\text{cof}}}(I - P_{\Omega,m}), \quad \sqrt{\kappa_{\text{cof}}} = 0.580865249091405 \dots \quad (\text{VIII.13})$$

*Proof.* Appendix I proves the fixed-collar energy estimate, uniform collar positivity, Mosco limit, compact-time wave-group convergence, global support theorem, and exact regional Green sequence. These give causality, quotient injectivity, and time-slice. The one global positive-frequency map makes represented isotony literal. The threshold follows from  $B_m \geq \sqrt{\kappa_{\text{cof}}}I$  and second quantization.  $\square$

The number  $\sqrt{\kappa_{\text{cof}}}$  is a canonical comparison threshold. It is not identified with a Poincaré mass or a Yang–Mills mass gap.

## B. Physical fixed-thickness linked flux

On the universal angular cover, the strict lifted  $2\pi/3$  corridors satisfy  $\Gamma_3 = T$ , where the deck transformation acts through the meridional center  $\omega^m I_3$ . Choosing a coefficient-preserving skeleton gives a fixed-thickness chart band with underlying groupoid  $B\mathbb{Z}_3$ . The canonical endpoint package contains declared rank-one, complete-ground, center-natural Wilson/collar lines  $L_{m,\alpha}$  of triality  $\alpha$ .

The meridional carry acts on these lines by

$$C_m|_{L_{m,\alpha}} = \omega^{m\tilde{\alpha}} I. \quad (\text{VIII.14})$$

Consequently, the carry created by the inner right-associated product crosses the first triality line with precisely this character.

**Theorem VIII.2** (Physical fixed-thickness  $SU(3)$  class). *The actual two parenthesized products of the selected coefficient lines differ by*

$$\Omega_m^{\text{fib}}(\alpha, \beta, \gamma) = \omega^{m\tilde{\alpha}\epsilon_3(\beta,\gamma)}. \quad (\text{VIII.15})$$

*The marked triple and detector are*

$$\Omega_m^{\text{fib}}(1, 2, 1) = \omega^m, \quad I_3(\Omega_m^{\text{fib}}) = \omega^m. \quad (\text{VIII.16})$$

*Hence*

$$[\Omega_m^{\text{fib}}] = mu_3, \quad (\text{VIII.17})$$

so  $m = 1, 2$  are Sector III at every selected positive thickness. The class is invariant under complete-ground compression, admissible coefficient rephasing, and genuinely neutral stabilization. The observable adjoint maps nevertheless compose strictly.

*Proof.* This is Theorem I.5. The phase is the induced central conjugation action on a physical intertwiner line, not an assigned graded flip. Scalar reassociation cancels under adjoint action.  $\square$

### C. Physical controlled scale class and total coordinates

Let  $\varepsilon_n = 2^{-n}\varepsilon_0$ . Exact spatial dilation gives renormalized one-particle operators

$$\hat{A}_{m,n} = \varepsilon_n^2 A_{m,n}, \quad \hat{B}_{m,n} = \varepsilon_n A_{m,n}^{1/2}, \quad (\text{VIII.18})$$

which are unitarily equivalent along the exact dyadic orbit and retain the same positive lower bound. On a retained collar module of triality  $a$ , choose

$$q_3 = \exp\left(\frac{2\pi i}{9} \text{diag}(1, 1, -2)\right), \quad q_3^3 = \omega I_3, \quad (\text{VIII.19})$$

and corridor grade  $\nu \in \mathbb{Z}_3$ . Reducing batch length modulo three yields

$$\Omega_{a,\nu}^{\text{sc}}(k, \ell, p) = \omega^{a\nu \tilde{k}\epsilon_3(\ell,p)}. \quad (\text{VIII.20})$$

The full integer batch implementation is strict; the cocycle appears only after reduced representatives retain the discarded three-step center carry.

The rooted depth category has an initial object. Therefore

$$\lambda_{a,\nu;n}^{\text{root}}(k, \ell) = \Omega_{a,\nu}^{\text{sc}}(n, k, \ell) \quad (\text{VIII.21})$$

is an unrestricted primitive. It is excluded from the controlled complex because it changes by  $\omega^{a\nu}$  under translation of identical renormalized shell packages.

**Theorem VIII.3** (Physical controlled  $SU(3)$  trichotomy). *Put  $r = a\nu \pmod{3}$ . On the exact dilation orbit,*

$$[\Omega_{a,\nu}^{\text{sc}}]_{\text{cs,ord}} = 0, \quad [\Omega_{a,\nu}^{\text{sc}}]_{\text{cs,ctrl}} = ru_3. \quad (\text{VIII.22})$$

For  $r = 1, 2$ , every controlled attempted strictification has marked defect at least  $\pi/3$ . The fixed-thickness and scale coefficient factors have strict mixed interchange, so the total controlled class has coordinates

$$(m, 0, r). \quad (\text{VIII.23})$$

Consequently,

| $m$  | $r$     | $sector$                  |
|------|---------|---------------------------|
| 0    | 0       | I                         |
| 0    | 1, 2    | $\text{II}_{\text{ctrl}}$ |
| 1, 2 | 0, 1, 2 | III.                      |

(VIII.24)

The observable shadow remains strict in every branch.

*Proof.* The scale reassociator is the physical center carry produced by  $q_3^3 = \omega I_3$ . The root primitive proves ordinary exactness. Appendix B reduces the exact controlled complex to normalized cochains on  $B\mathbb{Z}_3$ , where the cyclic detector gives the nonzero class and sharp defect. The fixed and scale lines are independent external factors, so no mixed coordinate occurs. □

The full Phase-VI supplement proves stability under asymptotically self-similar shell perturbations and transfers the coordinates to a canonical dyadic physical branch under explicit finite-window comparison estimates. This is still a positive-thickness comparison system, not a zero-thickness reconstruction.

#### D. Independent physicalized $SU(2)$ strict control

The linked-flux  $SU(3)$  branch should not be the only evidence for the strict sector. An independent retained  $SU(2)$  character model verifies that complete physicalization, genuine channel-ultraviolet leakage, center covariance, arbitrary tensor depth, and globally strict coefficient descent coexist in one concrete system.

Let

$$\mathcal{K}^{(2)} = \text{span}\{\chi_0, \chi_{1/2}, \chi_1, \chi_{3/2}\} \cong \mathbb{C}^4 \quad (\text{VIII.25})$$

with Hamiltonian

$$H^{(2)} = \begin{pmatrix} 1 & -\frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 4 & -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} & 9 & -\frac{1}{2} \\ 0 & 0 & -\frac{1}{2} & 16 \end{pmatrix}. \quad (\text{VIII.26})$$

For the selected normalized raw Wilson vector

$$\phi^W = \frac{-16\chi_0 + 97\chi_{1/2} + 10\chi_1}{\sqrt{9765}}, \quad (\text{VIII.27})$$

let  $P_{\text{vac}}^{(2)}$  be the complete ground spectral projection and apply the Paper-I physicalization in the order

$$T = (I - P_{\text{vac}}^{(2)})F^W, \quad G = T^*T, \quad F^{\text{phys}} = TG^{-1/2}. \quad (\text{VIII.28})$$

**Theorem VIII.4** (Independent  $SU(2)$  strict-control witness). *The system (VIII.26) has a rank-one complete ground projection and satisfies*

$$\begin{aligned} E_0^{(2)} &= 0.9180581777722170525\dots, \\ G &= 0.9999999997154206590\dots, \\ (F^{\text{phys}})^* H_{\text{mag}} F^{\text{phys}} &= 1.0595842731643929577\dots, \\ \|K^{(2)}\| &= 0.0519985588623950596\dots \end{aligned} \quad (\text{VIII.29})$$

*The mixed block is nonzero, and the resolved comparison gives*

$$H^{(2)} \geq E_0^{(2)} I + 3.093219423836803087\dots (I - P_{\text{vac}}^{(2)}). \quad (\text{VIII.30})$$

*For every tensor depth  $n$ , the anchored family has*

$$P_{\text{vac},n}^{(2)} = (P_{\text{vac}}^{(2)})^{\otimes n}, \quad G_n = GI, \quad \|K_n^{(2)}\| = \|K^{(2)}\|, \quad (\text{VIII.31})$$

*with one common positive coercive profile. The canonical complete-ground-line bordisms compose exactly with identity reassociator, so*

$$[\Omega^{(2)}]_{\text{band}} = 0, \quad [\widehat{\Omega}^{(2)}]_{\text{cs,ctrl}} = 0. \quad (\text{VIII.32})$$

*The fundamental Wilson channel is center odd, but the higher class remains trivial. Strictness is preserved under genuinely neutral stabilization.*

The proof and interval certificate are summarized in Appendix H; the complete calculation and reproducibility ledger are included in the accompanying supplement. The witness is depth-cofinal at one fixed positive-thickness stratum. It does not vary tube thickness, representation cutoff, geometry, or boundary data cofinally.

## E. What the realizations verify

The two realizations show that several component mechanisms are genuinely inhabited, but they do not inhabit the entire front-end hypothesis package. Their verified roles and boundaries are summarized as follows.

- **Local observable system.** The  $SU(3)$  branch gives a causal Weyl net and a local-normal  $W^*$ -net. The  $SU(2)$  control does not construct a spacetime net.
- **Time-slice and exhaustion.** These are proved for the selected  $SU(3)$  incised region category and are not asserted for the fixed-stratum  $SU(2)$  depth family.
- **Finite physicalization.** The  $SU(3)$  assembly uses the explicitly local Friedrichs branch and the inherited positive renormalized threshold. The  $SU(2)$  seed is computed and certified end to end, including nonzero mixed leakage.
- **Complete-ground transport.** The  $SU(3)$  implementation is coefficient-line valued; the  $SU(2)$  transport is the exact tensor-depth ground-line correspondence.
- **Center structure.** The  $SU(3)$  branch carries triality-sensitive linked flux. The  $SU(2)$  control has a center-odd fundamental channel.
- **Higher class.** For the linked-flux branches,  $[\Omega_m^{\text{fib}}] = mu_3 \neq 0$  for  $m = 1, 2$ , and the total controlled coordinates are  $(m, 0, r)$ . For the strict control,  $[\hat{\Omega}^{(2)}]_{\text{cs,ctrl}} = 0$ .
- **Observable composition.** The  $SU(3)$  adjoint maps compose strictly despite the twisted implementation; the  $SU(2)$  system is strict at both levels.
- **Positive threshold.** The  $SU(3)$  canonical comparison Hamiltonian has threshold  $\sqrt{\kappa_{\text{cof}}}$ . The  $SU(2)$  family has uniform fixed-stratum depth bounds.

- **Continuum claim.** Neither realization is identified with the final thin Yang–Mills theory.

These examples do not remove the remaining thin-reconstruction obligations. In particular, they do not establish that the canonical  $SU(3)$  comparison field equals the final thin Yang–Mills observable net, that the  $SU(2)$  fixed-stratum family extends to a full multiparameter cofinal branch, or that either positive threshold is a relativistic Yang–Mills mass gap.

## IX. CONCLUSION

This article proves a general conditional finite positive-thickness realization architecture and closes its selected source-free  $SU(3)$  front end. The full manuscript remains Outcome B because the model-specific downstream H4–H19 synthesis is still conditional:

**The selected positive-thickness front end is inhabited; the downstream H4–H19 realization remains to be closed in this lineage.**

The physical branch, obstruction-carrier, and front-end finite-thickness realization subproblems now have Outcome A at their declared scope.

The stationary front end is fully inhabited on the selected retained  $SU(3)$  dyadic system. P1 supplies the ordinary pre-incision time-zero regional packet; P2 supplies the protected stationary Cauchy anchor; P3 places the packet on one common finite-dimensional core and verifies DN0–DN8; P4 proves effective anchored descent on objects, one-morphisms, and two-morphisms; and P5 constructs fixed-scale decorated two-stacks and a rooted correspondence-valued normal lax section. That stationary branch is realizable on every selected region and therefore has

$$M_{\text{reg}, SU(3), m}^{\text{stat}} = M, \quad \Sigma_{\text{obs}, SU(3), m}^{\text{stat}} = \emptyset.$$

This conclusion is retained exactly. It is not the linked-flux physical branch.

The nonstationary problem is now formulated independently as a rooted physical lifting stack on ordinary Minkowski testing regions. Its objects retain the physical Wilson quotient, complete-ground and complementary corners, one common core, propagation-adapted Cauchy evolution, Gauss graph forms, Ward insertions and domains, linked center-flux data, fixed-thickness analytic certificates, roots, markings, selected scale correspondences, and every

regional-scale–Cauchy mixed cell. Restriction stability makes its regular locus open and its carrier closed. The prescribed-incision witness embeds fully faithfully at fixed scale on every ordinary region compactly contained away from the straight worldsheet and forms a correspondence-valued lax transformation across selected scales.

The decisive branch correction is source-freeness. The ambient candidate stack can permit primitive collar or worldsheet currents; the selected vacuum Yang–Mills substack cannot. It also carries one cofinal transverse concentration-exclusion certificate. Fixed-scale smoothness is insufficient, because smooth transverse currents can concentrate weakly to a delta measure while retaining a nonzero meridional phase. Under one common superlinear Orlicz bound, vanishing singular modulus, and vanishing joint Gauss–Ward–Cauchy–scale–mixed remainder, the compressed meridional defect disappears on shrinking transverse disks.

A hypothetical source-free lift through the prescribed worldsheet would then produce a P4–P5 compatible point relaxation. Such a relaxation is impossible: in Sector II it would produce the forbidden scale-natural strictifying primitive, while in Sector III it would carry  $I_3(\Omega_m) = \omega^m \neq 1$  into the flat point channel. Therefore

$$\boxed{M_{\text{reg}}^{\text{YM,vac}} = M \setminus \Sigma_{\text{pres}}, \quad \Sigma_{\text{obs}}^{\text{YM,vac}} = \Sigma_{\text{pres}} = \{(t, 0, 0, z)\}.$$

This is the required persistent-obstruction and dynamical-incision theorem for the selected source-free physical branch. It does not identify stationary and physical dynamics and does not derive a carrier from a nontrivial cocycle alone. The frustum/linked-flux witness, source-free restriction, concentration-exclusion estimate, and retained gerbe coherence together prove that the obstruction cannot be removed by admissible point relaxation and identify its failure carrier. Only afterward is

$$M_{\text{inc}}^{\text{YM,vac}} := M \setminus \Sigma_{\text{obs}}^{\text{YM,vac}}$$

formed. Thus the incision is not a cutoff, regulator, imposed boundary condition, or device introduced to manufacture coercivity. Its role is constructive: the derived complement supplies the selected ambient spacetime realization in which the finite positive-thickness Yang–Mills packets and their coherent regional, Cauchy, Čech, and scale comparisons exist.

Carrier equality promotes the explicit detector  $\delta(t, x, y, z) = (x, y)$  and the existing tubular and finite-atlas data from a prescribed witness to the derived selected-vacuum carrier. Thus

H1 and H2 are inhabited at that scope. The independent  $SU(2)$  model remains a strict fixed-stratum/depth control, while the linked-flux  $SU(3)$  computation remains the physical Sector-III witness.

The selected physical H3 theorem now places every finite regional-chart- Cauchy-scale demand in one endpoint-normalized ledger at the actual selected positive thicknesses. Every comparison composite and both paths of every mixed cell are represented by their common geometric endpoint, so exact coherence introduces no artificial repeated displacement cost. Compactness and the H1-H2 protected geometry give positive residual obstruction, chart, container, Cauchy-slab, temporal, and spacelike margins. Thus H3.1-H3.10 are closed.

The resulting protected physical Cauchy diamonds satisfy CIR0-CIR5. Every finite family of certificates, inclusions, chart comparisons, protected Cauchy comparisons, background maps, and mixed cells has one common finite refinement at its actual selected thickness. For each chosen finite admissible protected cover, the certificate preorders over every nonempty intersection and face inclusion are directed and weakly contractible. Localization at vertical refinements therefore recovers the rooted or marked Čech nerve of that cover, compatibly with its face and degeneracy maps. No assertion is made for every open cover, for saturation of an ambient Grothendieck topology, or for equality with all arbitrary regular opens. On the same selected nerve, the source-free lifting stack and retained positive-thickness  $SU(3)$  realization are identified by mutually inverse pseudofunctors with unit, counit, and triangle modifications. Observable CP4 holds after descent for the compatible-family  $W^*$ -algebra, while the represented  $\mu_3$  gerbe remains nonneutral.

The inherited Paper-I/Paper-II complete-ground certificate is now a cartesian simplicial decoration on that nerve. P4 extraction and reconstruction, P5 fixed-scale descent and stackification, and every RID unit, counit, triangle, face, degeneracy, selected-Cauchy, finite-scale, and mixed cell preserve the projection, declared core, shifted preform, and endpoint profile. Hence the retained preform satisfies

$$q_{p,j;O}^{\text{prov}}[\Psi] \geq \kappa_{\text{cof}} \|(I - P_{\Omega,p,j;O})\Psi\|^2, \quad \kappa_{\text{cof}} > 337/1000,$$

on the declared profile-covered positive-thickness family. General controlled scale seams use independently certified endpoints; no target lower bound is manufactured from a one-sided

defect. This closes the Paper-III front end.

Its terminal output is the coherent directed system

$$\left\{ \mathfrak{D}_{\text{YM}, \mathfrak{d}}^{\text{thick}, \text{fin}} \right\}_{\mathfrak{d} \in \mathfrak{F}_{\text{adm}}},$$

represented on the recovered incised spacetime and equipped with the declared refinement-compatible comparison structure. Intrinsic data determine refinement-compatible equivalence classes in this system; presentation choices are identified under common refinement; and margin, defect, and concentration-exclusion quantities remain control data certifying admissibility. No object at thickness zero is part of this Paper-III endpoint.

The downstream H4–H19 mathematics remains stable as a conditional theorem. No later collar charge, Ward character, entropy surgery, closed form, comparison operator, or spectral threshold is used to manufacture the front-end habitat. CP4 remains confined to the ordinary observable shadow and does not erase represented twisting. The invariant ground datum remains a projection or corner, never a preferred global vector.

Paper IV receives the thick-to-thin obligations: thin-tail margin uniformity, admissible thin descent, expectation-algebra and Mosco comparison, Orlicz and entropy tightness, cofinal independence, survival of locality and time-slice, and construction of the physical Haag–Kastler net and thin closed form<sup>38</sup>. Paper V receives no-leakage, covariance and invariant-mass spectral closure, and threshold transfer<sup>39</sup>. No zero-thickness net, Poincare spectrum, confinement theorem, scattering theorem, or Yang–Mills mass gap is claimed here<sup>40</sup>.

**Scope boundary.** Paper III ends with a coherent directed system of intrinsic finite positive-thickness realizations on the derived incised spacetime. The system retains finite-Čech compatibility, branch type, complete-ground provenance, Gauss–Ward data, entropy surgery, closed forms, and all declared control ledgers. Paper IV alone constructs the thin net and thin closed form, and it must carry each finite diagram through the limiting process before any passage to a larger window or cofinal tail.

## **SUPPLEMENTARY MATERIAL**

See the supplementary material for the strict-SU(2) certificate ledger, controlled-SU(3) and repair documentation, explicit finite obstruction and defect checks, machine-readable verification scripts and outputs, and the supporting certificate ledgers for the finite-dimensional witnesses used in this article.

## **ACKNOWLEDGMENTS**

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## **USE OF GENERATIVE ARTIFICIAL INTELLIGENCE**

The author used OpenAI’s ChatGPT, including GPT-4.5 through GPT-5.6, as an interactive research and adversarial editorial tool during the development of this manuscript and the broader five-paper series. Its use included assistance with organizing mathematical material, testing formulations and proof strategies, improving exposition, checking notation and cross-references, preparing and reviewing LaTeX source, and supporting computational and document-audit workflows.

The conceptual framework, ontologies, definitions, conjectures, identification of the principal obstructions, load-bearing components of the proofs, and overall proof strategies originated with and remain solely those of the author.

All mathematical claims, definitions, theorems, proofs, computations, interpretations, citations, and final wording were reviewed and approved by the author, who assumes full responsibility for the content of the manuscript. No generative artificial-intelligence system is credited as an author.

## AUTHOR DECLARATIONS

### Conflict of Interest

The author has no conflicts to disclose.

### Author Contributions

Jeffrey S Little: Conceptualization; Formal analysis; Investigation; Methodology; Writing—original draft; Writing—review and editing.

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## DATA AVAILABILITY

All finite-dimensional numerical values and certification ledgers used in this article are contained in the article and its supplementary material. The submission package includes the full strict-SU(2) certificate ledger, the controlled-SU(3) and two-repair supplements, and machine-readable scripts that reproduce the reported  $SU(2)$  seed constants and exhaustively check the cyclic  $\mathbb{Z}_3$  pentagon and detector. The scripts are independent cross-checks; the rigorous enclosures and exact arguments are stated in the manuscript and supplements.

## Appendix A: Conventions and branch architecture

This appendix records the conventions needed to read the branch-sensitive statements without importing stronger conclusions than the hypotheses justify.

## 1. Operator-algebraic maps

All observable algebras are von Neumann algebras. A morphism in  $\mathbf{W}^*\mathbf{Alg}_n$  is a normal unital  $*$ -homomorphism. Normality means ultraweak continuity and is equivalent to the existence of a bounded predual map. It does not imply injectivity. When  $T : \mathcal{M}_0 \rightarrow \mathcal{M}_1$  is normal, the state transport is contravariant:

$$T_* : \mathcal{M}_{1,*} \rightarrow \mathcal{M}_{0,*}, \quad T_*(\omega) = \omega \circ T. \quad (\text{A.1})$$

For composable maps  $T_a$  and  $T_b$ ,

$$(T_b T_a)_* = T_{a,*} T_{b,*}. \quad (\text{A.2})$$

This order is used in every scale-bordism cocycle formula.

A normal spatial isomorphism is stronger than a normal homomorphism. A coefficient-line unitary

$$U : \mathcal{H}_0 \widehat{\otimes} L \rightarrow \mathcal{H}_1 \quad (\text{A.3})$$

implements a faithful algebra map because unitary conjugation is injective. In the weak branch,  $U$  is replaced by a normal  $W^*$ -correspondence or an ultraweakly closed intertwiner space; no equality of spectra, multiplicities, or kernels follows.

## 2. Graph triples and conjugate duals

A graph triple is

$$\mathcal{V} \xrightarrow{j} \mathcal{H} \xrightarrow{j^*} \mathcal{V}^*, \quad (\text{A.4})$$

where  $\mathcal{V}$  is a Hilbert space continuously and densely embedded in the pivot Hilbert space  $\mathcal{H}$ .

A bounded Hermitian graph form is represented by

$$\mathbb{G} : \mathcal{V} \rightarrow \mathcal{V}^*. \quad (\text{A.5})$$

If  $J : \mathcal{V}_0 \rightarrow \mathcal{V}_1$  is a graph unitary, its conjugate-dual inverse is

$$J^{-*} = (J^{-1})^* : \mathcal{V}_0^* \rightarrow \mathcal{V}_1^*. \quad (\text{A.6})$$

Graph covariance is

$$\mathbb{G}_1 J = J^{-*} \mathbb{G}_0, \quad (\text{A.7})$$

equivalently

$$g_1[J\Psi, J\Phi] = g_0[\Psi, \Phi]. \quad (\text{A.8})$$

A bounded map  $\mathbb{G} : \mathcal{V} \rightarrow \mathcal{V}^*$  need not take values in  $j^* \mathcal{H}$ , so it need not define a Hilbert-space operator. A densely defined symmetric pivot realization is closable. A closed symmetric operator need not be self-adjoint. A self-adjoint operator has a spectral measure and an exponentiated unitary group, but the existence of that group does not by itself prove that it implements the intended gauge action on the observable algebra.

### 3. Complete ground sectors

The invariant ground datum is a projection  $P_\Omega$ , not a vector. Its normal-state face is

$$\mathfrak{S}_\Omega = \{\rho \geq 0 : \text{Tr } \rho = 1, \rho = P_\Omega \rho P_\Omega\}. \quad (\text{A.9})$$

Statements such as Gauss annihilation, Ward invariance, and entropy surgery are formulated for the entire range of  $P_\Omega$  or for all states in  $\mathfrak{S}_\Omega$ . No cyclicity, separating property, Reeh–Schlieder property, or one-dimensionality is assumed.

### 4. Banded classes and effective descent

A coefficient-line descent class is an equivalence class of line objects, line multiplications, and scalar tensorators. Its vanishing means that coherent units can be chosen in the lines. This is called coefficient strictification. It is distinct from effective descent, which asks for an actual global object in the target category. Thus

$$[\widehat{\Omega}]_{\text{cs,ctrl}} = 0 \not\Rightarrow \text{a global Hilbert representation.} \quad (\text{A.10})$$

Similarly, the boundary-character class  $[\chi^\partial]_{\text{cs}}$  is independent of both  $[\widehat{\Omega}]_{\text{cs,ord}}$  and  $[\widehat{\Omega}]_{\text{cs,ctrl}}$ . Vacuum coefficient strictification does not automatically provide global collar-charge zero-points.

The notation  $[\Omega]_{\text{band}}$  is intrinsic and degree-free. A Čech class in degree two is written only after a constant abelian band and a good cover are selected. When central reduction fails, the correct object is correspondence-valued; a scalar phase is not imposed.

## 5. Exact, controlled, and weak branches

The article uses the following branch table.

**Exact branch.:** The algebraic transport is a normal homomorphism, injective only when separately proved; the represented transport is coefficient-line unitary; the analytic conclusion is equality of the stated forms, states, seams, or operators on their specified domains.

**Controlled branch.:** The algebraic or represented transport carries a finite-stage defect; the analytic conclusion is an explicit graph or predual estimate whose modulus decays along the selected cofinal family.

**Weak branch.:** The transport is correspondence-valued; the conclusion is a bimodule or relation-valued intertwining statement and not unitary equivalence.

An exact algebraic face does not force an exact analytic face. A controlled defect tending to zero does not change a topological obstruction class. A weak correspondence does not imply unitary equivalence.

## 6. Five independent axes

For bookkeeping, a completed thick object has branch signature

$$\mathfrak{b} = (b_{\text{alg}}, b_{\text{rep}}, b_{\text{an}}, b_{\text{op}}, b_{\text{sec}}). \quad (\text{A.11})$$

The entries record:

- whether algebra transport is strict, pseudofunctorial, or correspondence-valued;
- whether representation transport is unitary-line or weak correspondence transport;

- whether analytic transport is exact or controlled;
- whether only graph forms, closed operators, self-adjoint operators, or exponentiated groups are available;
- whether the chart–scale system lies in Sector I, controlled Sector II, or Sector III.

No component determines another. In particular,

$$\begin{aligned}
&\text{normal algebra map} \not\Rightarrow \text{injective map}, \\
&[\widehat{\Omega}]_{\text{cs,ctrl}} = 0 \not\Rightarrow \text{effective Hilbert descent}, \\
&\text{state surgery} \not\Rightarrow \text{graph-space surgery}, \\
&\bar{q}_r \geq \kappa_*(I - P_{\Omega,r}) \not\Rightarrow \text{a thin positive threshold}.
\end{aligned} \tag{A.12}$$

These non-implications are part of the theorem architecture, not merely cautionary remarks.

## Appendix B: Controlled scale cohomology and abstract sector witnesses

This appendix separates three statements that must not be conflated: the ordinary chart–scale descent class, the controlled class relevant to uniformly local refinement, and the physical fixed-thickness  $SU(3)$  class derived in Appendix I. The rooted scale nerve is contractible in unrestricted ordinary cohomology; controlled Sector II records the absence of a strictification selected from uniformly local, scale-natural shell data.

### 1. Cyclic carry cocycles

Let  $N \geq 2$ ,  $\zeta_N = e^{2\pi i/N}$ , and let  $\tilde{a} \in \{0, \dots, N-1\}$  denote the preferred representative of  $a \in \mathbb{Z}_N$ . Define

$$\epsilon_N(a, b) = \frac{\tilde{a} + \tilde{b} - \widetilde{a+b}}{N} \in \{0, 1\}. \tag{B.1}$$

The carry identity is

$$\epsilon_N(a, b) + \epsilon_N(a+b, c) = \epsilon_N(b, c) + \epsilon_N(a, b+c). \tag{B.2}$$

For  $m \in \mathbb{Z}_N$ , put

$$\Omega_m(a, b, c) = \zeta_N^{\widetilde{ma\epsilon_N(b,c)}}. \tag{B.3}$$

**Proposition B.1** (Cyclic pentagon and detector). *The cochain  $\Omega_m$  is normalized and satisfies  $\delta\Omega_m = 1$ . Moreover,*

$$I_N(\Omega) = \prod_{b=0}^{N-1} \Omega(1, b, 1) \quad (\text{B.4})$$

*is invariant under normalized two-coboundaries, and*

$$I_N(\Omega_m) = \zeta_N^m. \quad (\text{B.5})$$

*Thus  $\Omega_m$  is nontrivial in  $H^3(B\mathbb{Z}_N; U(1))$  for  $m \neq 0$ .*

*Proof.* Equation (B.2) cancels the exponent in  $\delta\Omega_m$ . The factors in  $I_N(\delta\lambda)$  telescope for every normalized two-cochain  $\lambda$ . In (B.3),  $\epsilon_N(b, 1)$  is nonzero only for  $b = N - 1$ .  $\square$

## 2. Ordinary root contraction

Let  $\mathcal{R}$  have objects  $r_n$ ,  $n \geq 0$ , and a unique arrow  $a_{n;k} : r_n \rightarrow r_{n+k}$  for every  $k \geq 0$ . The object  $r_0$  is initial. An ordinary normalized two-cochain is a family  $\lambda_n(k, \ell)$ , and

$$(\delta_{\mathcal{R}}\lambda)_n(k, \ell, p) = \frac{\lambda_{n+k}(\ell, p)\lambda_n(k, \ell + p)}{\lambda_n(k + \ell, p)\lambda_n(k, \ell)}. \quad (\text{B.6})$$

**Theorem B.2** (Ordinary cone contraction). *For constant  $U(1)$  coefficients,  $H^q(B\mathcal{R}; U(1)) = 0$  for  $q > 0$ . For the depth-independent cyclic scale cocycle*

$$\Omega_r^{\text{sc}}(k, \ell, p) = \zeta_N^{\tilde{r}k\epsilon_N(\ell, p)}, \quad (\text{B.7})$$

*the explicit primitive*

$$\lambda_{r;n}^{\text{root}}(k, \ell) = \Omega_r^{\text{sc}}(n, k, \ell) \quad (\text{B.8})$$

*satisfies  $\delta_{\mathcal{R}}\lambda_r^{\text{root}} = \Omega_r^{\text{sc}}$ .*

*Proof.* The initial object supplies an extra degeneracy of the nerve. Applying the three-cocycle identity to the cone arrow  $r_0 \rightarrow r_n$  gives the explicit formula. Direct substitution of (B.8) into (B.6) yields (B.7).  $\square$

The primitive is not stationary. For  $r \neq 0$ ,

$$\frac{\lambda_{r;n+1}^{\text{root}}(N-1, 1)}{\lambda_{r;n}^{\text{root}}(N-1, 1)} = \zeta_N^r. \quad (\text{B.9})$$

It remembers the absolute depth even when adjacent renormalized shell packages are isomorphic.

### 3. Uniformly scale-controlled cochains

A decorated scale simplex records a finite shell neighborhood of each vertex, all comparisons among its vertices, the complete-ground projections, coefficient lines, multiplication maps, and the declared analytic controls. A normalized  $p$ -cochain is *uniformly scale controlled* when it is determined by a single finite-radius functional of this decorated simplex, depends only on reduced band grades, is natural under control-preserving isomorphisms, and is uniformly Lipschitz in the shell metric. Face restriction is nonexpansive, so the ordinary coboundary preserves these conditions.

**Proposition B.3** (Controlled cochain complex). *The uniformly scale-controlled normalized cochains form a subcomplex*

$$(C_{\text{sc,ctrl}}^\bullet(\mathfrak{D}; U(1)), \delta) \subseteq (C_{\text{ord}}^\bullet(N_\bullet \mathcal{R}; U(1)), \delta). \quad (\text{B.10})$$

*On an exact renormalized dilation orbit, naturality forces depth stationarity and band preservation reduces the complex canonically to the normalized cochain complex of  $\mathbb{Z}_N$ .*

*Proof.* Every term in the coboundary is the value of the original cochain on a face of one decorated simplex. Finite radius, naturality, normalization, and band preservation are inherited by face restriction, and the Lipschitz constants add. Exact shell translation identifies all source depths; reduced-grade dependence then gives the normalized  $\mathbb{Z}_N$  complex.  $\square$

**Theorem B.4** (Controlled cyclic nonvanishing and sharp defect). *On the exact dilation orbit,*

$$[\Omega_r^{\text{sc}}]_{\text{sc,ctrl}} = ru_N \in H^3(B\mathbb{Z}_N; U(1)). \quad (\text{B.11})$$

*For  $N = 3$  and  $r = 1, 2$ , every normalized two-cochain satisfies*

$$\max_{b=1,2} d_{U(1)}(\Omega_r^{\text{sc}}(1, b, 1), (\delta\lambda)(1, b, 1)) \geq \frac{\pi}{3}, \quad (\text{B.12})$$

*and the bound is sharp.*

*Proof.* Proposition B.3 reduces the equation to the normalized group complex. Proposition B.1 excludes a primitive. For the quantitative bound, the product of the two marked residuals is  $\omega^r$ ; the geodesic triangle inequality gives  $2 \max d \geq 2\pi/3$ . Choosing equal residual phases gives equality.  $\square$

#### 4. Abstract strictly refining controlled Sector II

Assign thickness  $e(r_n) = 2^{-n}$  and grade  $|a_{n;k}| = k \pmod{N}$ . Every nonidentity arrow strictly decreases thickness; every fixed-thickness chart category is strict. Give the scale coefficient lines reassociator

$$\alpha_{c,b,a} = \Omega_m(|c|, |b|, |a|)I. \quad (\text{B.13})$$

**Theorem B.5** (Abstract strictly refining controlled Sector-II witness). *For  $m \neq 0$ , every fixed-thickness class is zero, the unrestricted ordinary scale class is zero by Theorem B.2, and the controlled total class is nonzero by Theorem B.4. The system is therefore controlled Sector II. It uses no nonidentity equal-thickness scale endomorphism.*

This theorem establishes categorical nonemptiness only. It does not derive the coefficient system from a Yang–Mills Hamiltonian. The physical  $SU(3)$  scale corridor and the fixed-thickness linked-flux class are separate constructions in Section VIII and Appendix I.

### Appendix C: Closed forms, complete kernels, and leakage

#### 1. Closure of the coercive preform

Let  $\mathcal{H}$  be a Hilbert space,  $P$  an orthogonal projection, and  $q^{\text{pre}}$  a densely defined nonnegative closable form on a core  $\mathcal{C}$ . Assume

$$P\mathcal{C} \subseteq \mathcal{C}, \quad q^{\text{pre}}[P\psi, \phi] = 0, \quad (\text{C.1})$$

and

$$q^{\text{pre}}[\psi] \geq \kappa \|(I - P)\psi\|^2, \quad \kappa > 0. \quad (\text{C.2})$$

Let  $\bar{q}$  be its closure, with graph norm

$$\|\psi\|_q^2 = \|\psi\|^2 + \bar{q}[\psi]. \quad (\text{C.3})$$

**Lemma C.1** (Graph continuity of the ground projection). *The projection  $P$  extends to a contraction on  $D(\bar{q})$  in the graph norm, and*

$$\bar{q}[P\psi] = 0, \quad \bar{q}[\psi] = \bar{q}[(I - P)\psi]. \quad (\text{C.4})$$

*Proof.* Choose  $\psi_n \in \mathcal{C}$  converging to  $\psi$  in the graph norm. By (C.1),

$$q^{\text{pre}}[P(\psi_n - \psi_m)] = 0, \quad (\text{C.5})$$

so  $P\psi_n$  is Cauchy in the graph norm exactly when it is Cauchy in  $\mathcal{H}$ . It converges to  $P\psi$ . Closure gives  $P\psi \in D(\bar{q})$  and  $\bar{q}[P\psi] = 0$ . Polarization of (C.1) and passage to the closure eliminate all cross terms, giving (C.4).  $\square$

**Theorem C.2** (Closed coercive decomposition). *Under (C.1)–(C.2),*

$$D(\bar{q}) = P\mathcal{H} \hat{\oplus} (I - P)D(\bar{q}), \quad (\text{C.6})$$

$$\bar{q}[\psi] \geq \kappa \|(I - P)\psi\|^2, \quad (\text{C.7})$$

and

$$\ker \bar{q} = P\mathcal{H}. \quad (\text{C.8})$$

If  $K$  is the positive self-adjoint operator associated with  $\bar{q}$ , then

$$K = 0_{P\mathcal{H}} \oplus K^\perp, \quad K^\perp \geq \kappa I, \quad (\text{C.9})$$

so

$$E_K(\{0\}) = P, \quad \text{spec}(K) \subseteq \{0\} \cup [\kappa, \infty). \quad (\text{C.10})$$

*Proof.* The domain decomposition follows from Lemma C.1. Let  $\psi_n \in \mathcal{C}$  converge to  $\psi$  in the graph norm. Then  $(I - P)\psi_n \rightarrow (I - P)\psi$  in  $\mathcal{H}$ , and lower semicontinuity gives

$$\bar{q}[\psi] = \lim_n q^{\text{pre}}[\psi_n] \geq \kappa \lim_n \|(I - P)\psi_n\|^2, \quad (\text{C.11})$$

which is (C.7). If  $\bar{q}[\psi] = 0$ , the bound gives  $(I - P)\psi = 0$ , so  $\psi \in P\mathcal{H}$ ; the reverse inclusion follows from Lemma C.1. The representation theorem for closed nonnegative forms gives  $K$ . The form vanishes on  $P\mathcal{H}$  and is bounded below by  $\kappa$  on the orthogonal complement, giving (C.9) and (C.10).  $\square$

If  $U : \mathcal{H}_0 \rightarrow \mathcal{H}_1$  is unitary, carries  $P_0$  to  $P_1$ , maps a core for  $q_0^{\text{pre}}$  to a core for  $q_1^{\text{pre}}$ , and preserves the preform, then it extends to a graph unitary and

$$K_1 = UK_0U^*. \quad (\text{C.12})$$

This applies to exact chart and scale transports. For a controlled graph comparison  $V_a^q$ , the defect (VI.42) is bounded on the product graph space and composes by (VI.43). A small such defect is weaker than norm-resolvent convergence and does not imply convergence of spectral projections.

## 2. A leakage counterexample

Let

$$\mathcal{H} = \ell^2(\mathbb{N}) \tag{C.13}$$

and let  $P_n$  be the projection onto

$$\text{span}\{e_1, \dots, e_n\}. \tag{C.14}$$

Define

$$q_n[\psi] = \|(I - P_n)\psi\|^2. \tag{C.15}$$

Each form is closed and satisfies

$$q_n[\psi] \geq \|(I - P_n)\psi\|^2, \quad \ker q_n = P_n \mathcal{H}, \tag{C.16}$$

with the uniform constant  $\kappa_* = 1$ .

However,

$$P_n \longrightarrow I \quad \text{strongly}, \tag{C.17}$$

and therefore

$$q_n[\psi] \longrightarrow 0 \tag{C.18}$$

for every  $\psi \in \mathcal{H}$ . The pointwise and Mosco limit is the zero form, whose kernel is all of  $\mathcal{H}$ . Thus a uniform endpoint estimate does not prevent the complete ground sectors from absorbing every limiting direction.

**Proposition C.3** (No automatic threshold transfer). *The family (C.15) proves*

$$\left( q_n \geq \kappa_*(I - P_n) \text{ for all } n \right) \not\Rightarrow \left( q_\infty \geq \kappa_*(I - P_\infty) \right) \tag{C.19}$$

*for any nontrivial limiting ground complement. Uniform positive-thickness coercivity alone is therefore insufficient for a thin spectral threshold.*

### 3. A sufficient transfer pattern

Suppose the forms  $q_n$  are realized in a common comparison Hilbert space, the projections satisfy

$$P_n \rightarrow P_\infty \quad \text{strongly,} \quad (\text{C.20})$$

and a candidate closed form  $q_\infty$  has a strong recovery property: for every  $\psi \in D(q_\infty)$  there are  $\psi_n \rightarrow \psi$  with

$$q_n[\psi_n] \longrightarrow q_\infty[\psi]. \quad (\text{C.21})$$

Then the endpoint bounds imply

$$\begin{aligned} q_\infty[\psi] &= \lim_n q_n[\psi_n] \\ &\geq \kappa_* \lim_n \|(I - P_n)\psi_n\|^2 \\ &= \kappa_* \|(I - P_\infty)\psi\|^2. \end{aligned} \quad (\text{C.22})$$

The recovery or upper-identification statement is essential. The usual Mosco lower-bound condition alone has the opposite logical direction: it bounds  $q_\infty$  above by a liminf of approximating energies and cannot by itself produce (C.22).

In the Yang–Mills program, (C.20) is the ground-sector non-leakage input and (C.21) is the limiting energy-identification input. Neither is proved in the present article.

## Appendix D: Normal compatible-family descent and unbounded invariance

This appendix collects two technical mechanisms used repeatedly in the main text: normal equalizer descent for bounded observables and the passage from graph-form invariance to strong commutation for self-adjoint constraints.

### 1. Compatible families as a normal equalizer

Let  $\mathcal{G}$  be a small groupoid and let

$$\mathfrak{M} : \mathcal{G} \longrightarrow \mathbf{W}^* \mathbf{Alg}_n \quad (\text{D.1})$$

be a functor such that every arrow is represented by a normal  $*$ -isomorphism. Write  $\mathcal{M}_p = \mathfrak{M}(p)$  and  $\Phi_\nu = \mathfrak{M}(\nu)$ . The bounded product

$$\prod_{p \in \text{Ob } \mathcal{G}}^{\ell^\infty} \mathcal{M}_p \quad (\text{D.2})$$

is a von Neumann algebra with coordinatewise operations and predual  $\ell^1$ -sum; see Refs. <sup>41,42</sup> for the standard operator-algebraic background.

For every arrow  $\nu : p \rightarrow q$ , define the normal linear difference map

$$D_\nu((X_r)_r) = X_q - \Phi_\nu(X_p). \quad (\text{D.3})$$

The compatible-family algebra is

$$\text{Desc}(\mathfrak{M}) = \bigcap_{\nu \in \text{Mor } \mathcal{G}} \ker D_\nu. \quad (\text{D.4})$$

**Proposition D.1** (Normal equalizer). *The algebra  $\text{Desc}(\mathfrak{M})$  is a von Neumann subalgebra of (D.2).*

*Proof.* Each  $D_\nu$  is ultraweakly continuous because coordinate evaluation and  $\Phi_\nu$  are normal. Hence  $\ker D_\nu$  is ultraweakly closed. The intersection (D.4) is therefore ultraweakly closed. Compatibility is preserved by adjoints and products because the  $\Phi_\nu$  are  $*$ -homomorphisms. The compatible families contain the unit, so they form a von Neumann subalgebra.  $\square$

Fix an object  $p$ . Its isotropy group acts normally on  $\mathcal{M}_p$ . Let

$$\mathcal{M}_p^{\text{Iso}(p)} = \{X \in \mathcal{M}_p : \Phi_\gamma(X) = X \text{ for all } \gamma : p \rightarrow p\}. \quad (\text{D.5})$$

**Theorem D.2** (Evaluation and isotropy). *If  $\mathcal{G}$  is connected, coordinate evaluation is a normal  $*$ -isomorphism*

$$\text{ev}_p : \text{Desc}(\mathfrak{M}) \xrightarrow{\cong} \mathcal{M}_p^{\text{Iso}(p)}. \quad (\text{D.6})$$

*In particular, evaluation is onto  $\mathcal{M}_p$  if and only if the observable isotropy acts trivially.*

*Proof.* A compatible family has  $X_p$  fixed by every loop at  $p$ , proving that evaluation lands in (D.5). It is injective because connectedness lets one transport  $X_p$  to every object. Conversely, choose for each  $q$  an arrow  $\nu_q : p \rightarrow q$  and set  $X_q = \Phi_{\nu_q}(X_p)$ . If  $\nu'_q$  is another choice, then

$(\nu'_q)^{-1}\nu_q$  is an isotropy loop at  $p$ , so the fixed-point condition gives the same  $X_q$ . The resulting family is compatible and evaluates to  $X$ . Normality of evaluation and its inverse follows from normality of the component maps.  $\square$

A one-object groupoid with isotropy group  $H$  illustrates the necessity of the condition: the descended algebra is  $\mathcal{M}^H$ , not  $\mathcal{M}$ . In the Wilson-tube construction, observable isotropy loops agree with the identity on the ultraweakly generating physical core; normality then forces them to be the identity on the bicommutant.

## 2. Descent of natural transformations

Let  $R : \mathcal{G}_1 \rightarrow \mathcal{G}_0$  be a functor and let

$$J_p : \mathfrak{M}_0(Rp) \longrightarrow \mathfrak{M}_1(p) \quad (\text{D.7})$$

be normal unital  $*$ -homomorphisms satisfying

$$\Phi_\nu^{(1)} J_p = J_q \Phi_{R\nu}^{(0)} \quad (\text{D.8})$$

for every  $\nu : p \rightarrow q$ . Define

$$\text{Desc}(J, R)((X_r)_r) = (J_p(X_{Rp}))_p. \quad (\text{D.9})$$

**Proposition D.3** (Normal descent of a natural transformation). *Equation (D.9) defines a normal unital  $*$ -homomorphism between the compatible-family algebras. It is injective or invertible only when the component family has the corresponding property and the comparison functor sees the relevant connected components.*

*Proof.* Naturality (D.8) proves that the image family is compatible. Coordinatewise normality gives normality on the product and hence on the equalizer. The final statement follows by evaluating in chosen presentations and applying Theorem D.2; normality alone contains no injectivity information.  $\square$

Regional inclusions, strict background maps, and homomorphic scale maps are all instances of Proposition D.3. It also proves composition: two natural transformations descend to the composite transformation because their component maps compose pointwise.

### 3. Normal extension of commutator estimates

Let  $\mathfrak{A}_0 \subseteq \mathcal{M}_0$  and  $\mathfrak{A}_1 \subseteq \mathcal{M}_1$  be unital  $*$ -subalgebras that are strongly dense in their bicommutants inside a common representation. Suppose

$$\|[A, B]\| \leq \eta \|A\| \|B\| \quad (\text{D.10})$$

for all  $A \in \mathfrak{A}_0$ ,  $B \in \mathfrak{A}_1$ .

**Proposition D.4** (Kaplansky extension). *The same estimate holds for all  $X \in \mathcal{M}_0$  and  $Y \in \mathcal{M}_1$ :*

$$\|[X, Y]\| \leq \eta \|X\| \|Y\|. \quad (\text{D.11})$$

*Proof.* By Kaplansky density, choose nets  $A_i \rightarrow X$  and  $B_j \rightarrow Y$  strongly- $*$  with  $\|A_i\| \leq \|X\|$  and  $\|B_j\| \leq \|Y\|$ . For fixed  $j$ , bounded strong- $*$  convergence gives  $[A_i, B_j] \rightarrow [X, B_j]$  strongly. The operator norm is lower semicontinuous under strong convergence on bounded sets, so

$$\|[X, B_j]\| \leq \eta \|X\| \|B_j\|. \quad (\text{D.12})$$

Taking the second bounded strong- $*$  limit gives (D.11).  $\square$

### 4. Graph invariance and spectral strong commutation

Let  $a$  be a densely defined closed semibounded form on  $\mathcal{H}$  with associated self-adjoint operator  $A$ . Let  $B \in B(\mathcal{H})$  satisfy

$$BD(a) \subseteq D(a), \quad a[B\psi, \phi] = a[\psi, B^*\phi] \quad (\text{D.13})$$

for all  $\psi, \phi \in D(a)$ .

**Lemma D.5** (Resolvent commutation). *For every  $\lambda$  below the lower bound of  $A$ ,*

$$B(A - \lambda)^{-1} = (A - \lambda)^{-1}B. \quad (\text{D.14})$$

*Proof.* For  $f \in \mathcal{H}$ , let  $u = (A - \lambda)^{-1}f$ . The vector  $u \in D(a)$  is characterized by

$$a[u, v] - \lambda \langle u, v \rangle = \langle f, v \rangle \quad (\text{D.15})$$

for every  $v \in D(a)$ . Apply (D.13) to obtain

$$a[Bu, v] - \lambda \langle Bu, v \rangle = \langle Bf, v \rangle. \quad (\text{D.16})$$

Uniqueness of the variational solution gives  $Bu = (A - \lambda)^{-1}Bf$ .  $\square$

The functional calculus then gives commutation with every bounded Borel function of  $A$ , in particular with all spectral projections. Hence

$$\{A\}'_{\text{str}} = \{B \in B(\mathcal{H}) : BE_A(\Delta) = E_A(\Delta)B \text{ for all Borel } \Delta\} \quad (\text{D.17})$$

is a von Neumann algebra.

**Theorem D.6** (Strong commutation from a core). *Let  $\mathfrak{A}$  be a represented unital  $*$ -algebra that preserves  $D(a)$  and satisfies (D.13). Then*

$$\mathfrak{A}'' \subseteq \{A\}'_{\text{str}}. \quad (\text{D.18})$$

*Proof.* Lemma D.5 places every  $B \in \mathfrak{A}$  in the spectral strong commutant. Since (D.17) is a von Neumann algebra, it contains the bicommutant generated by  $\mathfrak{A}$ .  $\square$

This is the argument used for the Gauss generators. It is stronger and safer than trying to extend a formal commutator equality by ultraweak density.

## 5. Commutant isotropy does not preserve graph domains

Let  $\mathcal{H} = \ell^2(\mathbb{Z})$ , let  $\mathcal{M} = \mathbb{C}I$ , and let

$$\mathcal{V} = \left\{ x = (x_n) : \sum_{n \in \mathbb{Z}} 2^{2|n|} |x_n|^2 < \infty \right\}. \quad (\text{D.19})$$

Every unitary lies in  $\mathcal{M}'$ . Choose a permutation  $\sigma$  of  $\mathbb{Z}$  that sends an infinite sequence  $n_k \rightarrow \infty$  to  $\sigma(n_k) = k$ . The induced permutation unitary  $Z$  can map a vector concentrated on the  $n_k$  with weighted-square summability into one whose coefficients at  $k$  fail the weighted condition, or conversely. Hence

$$Z \in \mathcal{M}' \quad \text{but} \quad Z\mathcal{V} \neq \mathcal{V}. \quad (\text{D.20})$$

This is why the selected Cauchy and overlap commutant defects must satisfy explicit domain and graph-form isotropy conditions before unbounded path independence is asserted.

## Appendix E: Finite controlled geometry and causal propagation

This appendix records the finite-stage geometric facts used in the support and covariance arguments. The statements are local to compact test diagrams and do not assert a global resolution of every possible incision.

### 1. Finite adapted atlases

Let  $\Sigma \subset M$  be a closed controlled stratified subset. Fix a finite family consisting of relatively compact admissible regions, compact support pairs, collar footprints, and local Cauchy slabs. Local finiteness of the stratification and compactness imply that only finitely many strata meet the selected family. Choose tubular neighborhoods for those strata and shrink the regions so that all relevant closures remain inside the control charts.

**Proposition E.1** (Finite adapted refinement). *Every finite geometric test family admits a finite precompact atlas  $\{U_\alpha\}$  with the following properties:*

1. *every selected closure and causal shadow is covered by finitely many  $U_\alpha$ ;*
2. *chart transitions preserve the selected tubular and collar controls;*
3. *every selected region, support carrier, and Cauchy slice has positive distance from the unwanted chart and incision boundaries;*
4. *any two such finite atlases possess a common adapted refinement;*
5. *a strict controlled background embedding transports the atlas to an adapted atlas of its image.*

*Proof.* Cover each compact set by finitely many adapted control charts and take the finite union. Precompact shrinking gives positive buffer distances. Intersections of two finite adapted atlases, followed by precompact shrinking, give a common refinement. Strict background embeddings preserve the control structures by definition, so transported charts remain adapted. Distances are understood intrinsically in the transported image; an embedding need not preserve all ambient-target geodesic distances.  $\square$

The control charts can be resolved along the finitely many incident strata to manifolds with corners

$$\beta_\alpha : \widehat{U}_\alpha \longrightarrow \widetilde{U}_\alpha \quad (\text{E.1})$$

with incision front faces  $\mathfrak{F}_{S,\alpha}$ . Adapted restrictions and chart transitions lift and preserve the incidence structure. This geometric resolution does not imply that the principal gauge bundle extends across the front face; that is a separate bundle datum.

## 2. Margins and thickening thresholds

Let  $h$  be a control Riemannian metric on a compact stage. If  $\mathcal{O}^{+\delta}$  is produced by a controlled thickening profile with Lipschitz constant  $C_{\mathcal{O}}$ , then

$$d_h(\overline{\mathcal{O}^{+\delta}}, \Sigma) \geq d_h(\overline{\mathcal{O}}, \Sigma) - C_{\mathcal{O}}\delta. \quad (\text{E.2})$$

For two compact regions define a continuous spacelike-margin functional  $\mu_U(\mathcal{O}_1, \mathcal{O}_2) > 0$  on a common globally hyperbolic neighborhood. Controlled thickening gives

$$\mu_U(\mathcal{O}_1^{+\delta_1}, \mathcal{O}_2^{+\delta_2}) \geq \mu_U(\mathcal{O}_1, \mathcal{O}_2) - C_1\delta_1 - C_2\delta_2. \quad (\text{E.3})$$

Thus a finite family has a single positive threshold  $\delta_{\mathfrak{F}}$  preserving chart containment, incision clearance, and all selected causal-separation relations. This proves the finite-stage exact-locality threshold (IV.22).

## 3. Causal shadows

Let  $N$  be globally hyperbolic with Cauchy slices  $\Sigma_s$ . For compact  $K \subseteq \Sigma_{s_1}$ , the shadow (IV.5) is compact because the causal diamond between compact sets and Cauchy slices is compact. It is monotone and preserves finite unions. If  $s_1 < s_2 < s_3$ , every causal curve from  $K$  to  $\Sigma_{s_3}$  meets  $\Sigma_{s_2}$ , while concatenation gives the reverse inclusion. Hence (IV.6) holds.

Assume the finite-stage shadow maps are Hausdorff-Lipschitz:

$$d_H(\text{Sh}_{s_2, s_1}(K), \text{Sh}_{s_2, s_1}(K')) \leq \Lambda d_H(K, K'). \quad (\text{E.4})$$

Then

$$\text{Sh}_{s_2, s_1}(K^{+\varepsilon}) \subseteq \text{Sh}_{s_2, s_1}(K)^{+\Lambda\varepsilon}, \quad (\text{E.5})$$

which is (IV.7). In a standard constant-time Minkowski frame,  $\Lambda = 1$ .

A strict controlled background map  $\psi : N \rightarrow N'$  is causally compatible on its image. Therefore

$$\psi(\text{Sh}_{s_2, s_1}^N(K)) = \text{Sh}_{\psi s_2, \psi s_1}^{\psi N}(\psi K), \quad (\text{E.6})$$

and similarly for domains of dependence and causal hulls. These identities establish the geometric faces of the background–scale–Cauchy coherence diagram.

#### 4. From generator propagation to regional locality

Suppose each elementary Wilson label  $w$  on  $\Sigma_{s_1}$  is assigned a propagated label  $\mathcal{P}_{s_2, s_1} w$  satisfying:

1. the propagated carrier lies in the causal shadow of the original thick carrier;
2. physical relations, adjoints, and products are preserved;
3. reverse propagation is inverse on the physical quotient;
4. propagation composes under subdivision of the Cauchy path.

Then the formal assignment descends to the algebraic isomorphism (IV.8), and support exactness (IV.9) follows by induction on Wilson words and linearity. If two propagated thick carriers are causally disjoint, each pair of elementary factors commutes; repeated use of the Leibniz identity for commutators proves commutation of the full words and polynomials. Proposition D.4 then extends the result to the regional von Neumann closures.

In the quasilocal branch, choose localized approximants satisfying (IV.20). Expanding

$$[A, B] = [A - A_0, B - B_0] + [A - A_0, B_0] + [A_0, B - B_0] \quad (\text{E.7})$$

and using  $\|[X, Y]\| \leq 2\|X\|\|Y\|$  gives (IV.21). The exact and quasilocal branches are therefore logically distinct: asymptotic leakage control does not imply exact positive-thickness commutation unless the leakage vanishes.

## Appendix F: Sectorial gluing of Gauss kernels and collar Ward data

This appendix records the gluing statements that distinguish the strict and twisted gauge sectors. The central point is that complete-ground annihilation is first a chartwise kernel condition; a global operator equation is available only after the represented and unbounded systems descend effectively.

### 1. Chart covariance of graph constraints

Let  $\xi_\beta$  be a compactly supported interior parameter on a chart overlap and let

$$\xi_\alpha = \mathbb{C}_{\alpha\beta}^g \xi_\beta \quad (\text{F.1})$$

be its transport by the chart diffeomorphism and gauge transition function. The overlap unitary restricts to a graph unitary

$$\mathbb{U}_{\alpha\beta,r}^{V,G} : \mathcal{V}_{\beta,r}^G \hat{\otimes} L_{\alpha\beta,r} \longrightarrow \mathcal{V}_{\alpha,r}^G. \quad (\text{F.2})$$

Graph covariance is

$$\mathbb{G}_{\alpha,r}(\xi_\alpha) \mathbb{U}_{\alpha\beta,r}^{V,G} = (\mathbb{U}_{\alpha\beta,r}^{V,G})^{-*} (\mathbb{G}_{\beta,r}(\xi_\beta) \hat{\otimes} I). \quad (\text{F.3})$$

In a closed operator branch, (F.3) implies equality of the transported domains and conjugacy of the closures. In a self-adjoint branch it implies conjugacy of the spectral measures.

Suppose the complete-ground graph projection satisfies

$$\mathbb{G}_{\alpha,r}(\xi_\alpha) P_{\Omega,\alpha,r}^{V,G} = 0. \quad (\text{F.4})$$

Then (F.3) and (V.6) give

$$\mathbb{U}_{\alpha\beta,r}^{V,G} (\ker \mathbb{G}_{\beta,r}(\xi_\beta) \hat{\otimes} L_{\alpha\beta,r}) = \ker \mathbb{G}_{\alpha,r}(\xi_\alpha). \quad (\text{F.5})$$

In particular, the complete-ground subspaces form a coefficient-line-banded kernel object.

**Proposition F.1** (Sectorial ground annihilation). *At a fixed thickness, the family*

$$\mathcal{H}_{\Omega,\alpha,r} = P_{\Omega,\alpha,r} \mathcal{H}_{\alpha,r} \quad (\text{F.6})$$

is a Gauss-null descent subobject. If  $[\Omega_{e(r)}]_{\text{band}} = 0$  and effective Hilbert and Gauss-domain descent hold, it gives a global projection  $P_{\Omega,r}$  and global constraints satisfying

$$G_r(\xi)P_{\Omega,r} = 0. \quad (\text{F.7})$$

If the fixed-thickness class is nonzero, the chartwise family (F.6) is the invariant conclusion and (F.7) is not asserted.

*Proof.* Equation (F.5) proves the descent-subobject statement. A coefficient trivialization converts the overlap correspondences to an ordinary unitary cocycle, but the global operator equation still requires that the Hilbert objects and unbounded domains descend effectively. Under those additional hypotheses the local kernel equations glue to (F.7).  $\square$

Across a scale arrow  $a : r_0 \rightarrow r_1$ , exact graph covariance is

$$\mathbb{G}_{\alpha,r_1}(\mathbb{R}_a^g \xi) \mathbb{V}_{\alpha;a}^{V,G} = (\mathbb{V}_{\alpha;a}^{V,G})^{-*}(\mathbb{G}_{\alpha,r_0}(\xi) \hat{\otimes} I). \quad (\text{F.8})$$

It transports the complete Gauss-null ground subspaces exactly. In the controlled branch define the graph defect by the difference of the two sides of (F.8). For composable arrows it satisfies the same transported-sum law as (VI.43); therefore finite glued chains inherit the edgewise decay modulus.

## 2. Collar quotient and splitting independence

Let the exact sequence (VI.2) be fixed. For  $\lambda \in \mathfrak{g}_\partial$ , choose any admissible lift  $\xi$ . If  $\xi'$  is another lift, then  $\xi' - \xi \in \mathfrak{g}_{\text{int}}$ . Hence the quotient class (VI.19) is independent of the lift.

A splitting  $s^\partial$  gives the representative

$$\mathbb{Q}_s^\partial(\lambda) = \mathbb{G}^{\text{adm}}(s^\partial \lambda). \quad (\text{F.9})$$

If  $\tilde{s}^\partial = s^\partial + \tau$ , where  $\tau : \mathfrak{g}_\partial \rightarrow \mathfrak{g}_{\text{int}}$ , then

$$\mathbb{Q}_s^\partial(\lambda) = \mathbb{Q}_s^\partial(\lambda) + \mathbb{G}^{\text{int}}(\tau \lambda). \quad (\text{F.10})$$

On  $\mathcal{K}^{G,\text{int}}$ , the second term vanishes, proving that the reduced charge form is intrinsic.

The charge may transform across a chart overlap by an affine scalar term as in (VI.23). On a triple overlap, consistency gives

$$\chi_{\alpha\gamma}^{\text{ch}}(\lambda) = \chi_{\alpha\beta}^{\text{ch}}(\lambda) + \chi_{\beta\gamma}^{\text{ch}}(\lambda) \quad (\text{F.11})$$

with the parameter transported to the appropriate chart. Scale characters satisfy an analogous composition law, and chart and scale characters obey a mixed square. Changing local charge zero-points by scalar linear functionals changes these characters by a total coboundary. Their total class  $[\chi^\partial]_{\text{cs}}$  is therefore intrinsic.

A continuous infinitesimal character  $\chi : \mathfrak{g}_\partial \rightarrow i\mathbb{R}$  satisfies

$$\chi([\lambda, \mu]) = 0. \quad (\text{F.12})$$

It factors through the continuous abelianization. In particular, if the boundary algebra is topologically perfect, every such infinitesimal character vanishes. Nontrivial affine data must then arise from a global or discrete quotient, a germ-level structure, or a central extension rather than from an arbitrary infinitesimal semisimple gauge algebra.

### 3. Ward action versus charge zero-points

Let  $Q_\alpha^\partial$  and  $Q_\beta^\partial$  be self-adjoint representatives related by (VI.23). Define the adjoint derivation

$$\mathcal{D}_\lambda^\alpha(A) = i[Q_\alpha^\partial(\lambda), A]. \quad (\text{F.13})$$

The scalar shift commutes with  $A$ , so

$$\mathcal{D}_\lambda^\alpha \circ \text{Ad}_{U_{\alpha\beta}} = \text{Ad}_{U_{\alpha\beta}} \circ \mathcal{D}_\lambda^\beta. \quad (\text{F.14})$$

Thus the Ward derivation descends even when the local charge zero-points do not.

If the current defect (VI.24) is scalar,

$$\mathfrak{a}^\partial(\lambda, \mu) = c^\partial(\lambda, \mu)I, \quad (\text{F.15})$$

then

$$[\mathcal{D}_\lambda, \mathcal{D}_\mu] + \mathcal{D}_{[\lambda, \mu]} = 0 \quad (\text{F.16})$$

although the charge representation is projective. A scalar current cocycle therefore changes the implementation but not the observable Ward algebra.

**Theorem F.2** (Charge descent is stronger than Ward descent). *A global family of collar-charge operators requires triviality of the boundary-character class together with effective unbounded charge descent. The adjoint Ward derivations require neither a choice of global scalar zero-point nor the vanishing of a scalar current cocycle. They still require effective descent of the observable, Hilbert, and insertion-domain systems.*

#### 4. Ground-state Ward identities

Let  $\rho$  be a normal state supported in  $P_{\Omega, \alpha, r}$ . From (VI.15) and the interior Ward identity,

$$\rho(\pi(\delta_\xi A)) = 0. \quad (\text{F.17})$$

This holds for every state in the complete ground face, not only for a vector state. On chart overlaps the pushforward of  $\rho$  by  $\mathbb{U}_{\alpha\beta, r}$  satisfies the corresponding identity in the target chart. Therefore the complete-ground Ward functionals form a sectorial descent family. In Sector I, after effective descent, they become ordinary global Ward identities; in Sector III they remain coefficient-line banded.

#### Appendix G: Thin-reconstruction criteria

The present theorem ends with a complete positive-thickness diagram. This appendix records the additional criteria that a subsequent reconstruction must verify before any zero-thickness object can be claimed. The criteria are stated on each fixed finite background–scale–chart–Cauchy test stage. They are not consequences of Theorems III.17 and III.18 unless explicitly indicated.

##### TR1: Cofinal corridor control.:

The selected scale-bordism arrows must admit a cofinal subsystem on which all chart, support, representation, and analytic corridor choices are simultaneously comparable. The arrow labels must be retained; endpoint thickness pairs are insufficient in controlled Sector II.

**TR2: Observable compactness.:**

Regional unit-ball elements on the selected Wilson cores must have convergent representatives in a common normal comparison carrier. The limit must be independent of the chosen presentation and compatible with regional inclusions.

**TR3: Normal-state tightness.:**

Finite-entropy and complete-ground normal states must be tight under the contravariant predual scale maps. Limit points must remain normal on the candidate regional algebras.

**TR4: Sectorially typed representation compactness.:**

In Sector I one may seek ordinary Hilbert comparison carriers. In controlled Sector II the carrier must retain the controlled scale twisting, and in Sector III it must remain coefficient-line or correspondence valued. Compactness is not allowed to strictify the sector by fiat.

**TR5: Graph-domain recovery.:**

The Gauss and closed-form graph spaces must possess a common limiting core or an equivalent recovery mechanism. Convergence of bounded observables does not control unbounded domains.

**TR6: Entropy lower semicontinuity and continuity control.:**

Relative entropy must be lower semicontinuous on the chosen tight state sets. To pass the surgery-bordism balance law through a controlled defect, one also needs a continuity modulus converting predual distance into entropy control on those sets.

**TR7: Seam convergence.:**

State seams, represented seams, and vector-form seams must converge and remain compatible with glued scale composition. The limiting seam must be independent of parenthesization and of the selected finite corridor decomposition.

**TR8: Defect convergence.:**

The locality, Gauss, Ward, surgery, and closed-form defects must vanish uniformly on each fixed finite test stage. The cocycle formulas then guarantee vanishing on every fixed finite glued chain.

**TR9: Gauge and Ward stability.:**

Gauge-parameter transport, insertion domains, complete-ground kernels, collar quotient classes, and Ward derivations must remain stable under the comparison maps. A limiting Ward action does not automatically supply global limiting charge zero-points.

**TR10: Causal and background stability.:**

The limiting regional assignment must preserve the causal-development and background-covariance squares. Exact thin locality and a time-slice theorem require separate proofs after the regional algebras are constructed.

**TR11: Coercive liminf readiness.:**

The objectwise estimate already gives

$$\liminf_r \bar{q}_r[\Psi_r] \geq \kappa_* \liminf_r \|(I - P_{\Omega,r})\Psi_r\|^2. \quad (\text{G.1})$$

This is a readiness inequality, not a bound for a limiting form.

**TR12: Cofinal independence.:**

Two cofinal extraction systems and two choices of comparison carriers must possess a common refinement on which the resulting candidate thin objects are canonically equivalent in the appropriate sector.

Two additional threshold-transfer conditions are indispensable:

TT1 :  $P_{\Omega,r}$  is tight and converges to a candidate  $P_{\Omega,\infty}$ ,

TT2 : the limiting form admits recovery sequences or an equivalent upper identification. (G.2)

Under TR11, TT1, and TT2, the argument in Appendix C yields

$$q_\infty[\Psi] \geq \kappa_* \|(I - P_{\Omega,\infty})\Psi\|^2. \quad (\text{G.3})$$

Without TT1 and TT2, the leakage example (C.15) applies.

**Theorem G.1** (Typed thin-readiness statement). *Assume TR1–TR12 on every finite test stage. Then the positive-thickness diagram has compatible observable, state, graph-domain, entropy, seam, defect, and cofinal-comparison witnesses for formulating a sector-preserving*

*zero-thickness reconstruction. The statement does not construct the thin algebra, representation, Gauss/Ward system, entropy surgery, or closed form. A positive limiting threshold additionally requires TT1 and TT2.*

The division of labor is therefore precise. The present article constructs the thick diagram and proves its internal coherence. A later reconstruction must build the thin objects from the readiness witnesses. A subsequent non-leakage theorem must establish TT1 and TT2 strongly enough to preserve the positive threshold. None of those later conclusions is encoded into the definition of readiness.

## Appendix H: Certified $SU(2)$ strict-control realization

This appendix records the proof data behind Theorem VIII.4. Its purpose is not to supply a second continuum construction. It verifies nonemptiness of the strict finite/depth interface after the complete interacting-ground physicalization and the scale-bordism requirements are imposed simultaneously.

### 1. Fusion reconstruction and complete ground projection

The  $SU(2)$  character fusion rule

$$\chi_{1/2}\chi_j = \chi_{j-1/2} + \chi_{j+1/2}, \quad \chi_{-1/2} = 0, \quad (\text{H.1})$$

shows that compression of multiplication by  $\chi_{1/2}$  to (VIII.25) is

$$X = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}. \quad (\text{H.2})$$

With electric normalization  $4j(j+1)$ ,

$$H_{\text{el}} = \text{diag}(0, 3, 8, 15), \quad H_{\text{mag}} = I - \frac{1}{2}X, \quad (\text{H.3})$$

which gives (VIII.26). Its scaled characteristic polynomial is

$$p(\lambda) = 16\lambda^4 - 480\lambda^3 + 4356\lambda^2 - 12932\lambda + 8561. \quad (\text{H.4})$$

The polynomial is square free, and exact Sturm counting isolates one simple root in each of four disjoint intervals. In particular, the lowest root is

$$E_0^{(2)} = 0.9180581777722170525783499839449488 \dots \quad (\text{H.5})$$

and the complete projection

$$P_{\text{vac}}^{(2)} = 1_{\{E_0^{(2)}\}}(H^{(2)}) \quad (\text{H.6})$$

has rank one. A numerical eigenvector may be used to evaluate intervals, but it is not part of the invariant datum.

## 2. Physicalization and the nonzero mixed block

Let  $F^W(1) = \phi^W$  with (VIII.27). Since  $F^W$  is normalized,

$$G = 1 - |\langle g_+, \phi^W \rangle|^2 = 0.9999999997154206590229225986252481 \dots, \quad (\text{H.7})$$

where  $g_+$  is any normalized representative of the complete ground line. Therefore  $F^{\text{phys}}$  in (VIII.28) is well defined and satisfies

$$(F^{\text{phys}})^* F^{\text{phys}} = 1, \quad P_{\text{vac}}^{(2)} F^{\text{phys}} = 0. \quad (\text{H.8})$$

The magnetic scalar must be recomputed after this reduction:

$$B_{\text{mag}}^{\text{phys}} = (F^{\text{phys}})^* H_{\text{mag}} F^{\text{phys}} = 1.059584273164392957736871096703039 \dots \quad (\text{H.9})$$

Set

$$P_{\text{ch}} = F^{\text{phys}}(F^{\text{phys}})^*, \quad P_{\text{res}} = 0, \quad P_{\text{UV}} = I - P_{\text{vac}}^{(2)} - P_{\text{ch}}, \quad (\text{H.10})$$

and  $H^\flat = H^{(2)} - E_0^{(2)}I$ . The interval certificate gives

$$\begin{aligned} P_{\text{ch}} H^\flat P_{\text{ch}} &= 3.114097481076100331337464231810777 \dots P_{\text{ch}}, \\ P_{\text{UV}} H^\flat P_{\text{UV}} &\geq 8.096308674897366858801704823980880 \dots P_{\text{UV}}. \end{aligned} \quad (\text{H.11})$$

The exact mixed block

$$K^{(2)} = P_{\text{UV}} H^{\text{b}} P_{\text{ch}} \quad (\text{H.12})$$

is nonzero and satisfies the norm in (VIII.29). Taking

$$c_{\text{ch}} = 3.093770498, \quad c_{\text{UV}} = 8, \quad \varepsilon_{\text{mix}} = 0.052, \quad (\text{H.13})$$

gives the exact determinant margin

$$c_{\text{ch}} c_{\text{UV}} - \varepsilon_{\text{mix}}^2 = 24.747459984 > 0. \quad (\text{H.14})$$

The smaller eigenvalue of the scalar comparison matrix yields (VIII.30). This proof uses the channel, ultraviolet, and leakage decomposition; it is not replaced by direct diagonalization of  $H^{(2)}$ .

### 3. Tensor depth and exact bordism transport

For  $n \geq 1$ , let

$$\mathcal{H}_n^{(2)} = (\mathcal{K}^{(2)})^{\otimes n}, \quad H_n^{(2)} = \sum_{r=1}^n I^{\otimes(r-1)} \otimes H^{(2)} \otimes I^{\otimes(n-r)}. \quad (\text{H.15})$$

Strong commutation of the tensor summands gives

$$E_{0,n}^{(2)} = n E_0^{(2)}, \quad P_{\text{vac},n}^{(2)} = (P_{\text{vac}}^{(2)})^{\otimes n}. \quad (\text{H.16})$$

Let  $V = \text{Ran } P_{\text{vac}}^{(2)}$  and define the phase-free anchored coefficient space

$$\mathcal{E}_n = \bigoplus_{r=1}^n V^{\otimes(r-1)} \otimes \mathbb{C} \otimes V^{\otimes(n-r)}. \quad (\text{H.17})$$

After removal of the joint ground line, distinct shell channels are orthogonal, hence  $G_n = G I_{\mathcal{E}_n}$ .

The global mixed operator is an orthogonal direct sum of copies of the seed block:

$$K_n^{(2)} = \bigoplus_{r=1}^n K^{(2,r)}, \quad \|K_n^{(2)}\| = \|K^{(2)}\|. \quad (\text{H.18})$$

No channel vector is sent to a multiple-excitation sector. The common Paper-II profile gives

$$H_n^{(2)} - E_{0,n}^{(2)} I \geq 0.24975(I - P_{\text{vac},n}^{(2)}), \quad (\text{H.19})$$

while the tensor-sum argument gives the stronger bound with the seed constant in (VIII.30).

For a batch insertion of  $k$  ground shells, canonical tensor reassociation defines

$$J_{n;k}^{(2)} : \mathcal{H}_n^{(2)} \widehat{\otimes} V^{\widehat{\otimes} k} \longrightarrow \mathcal{H}_{n+k}^{(2)}. \quad (\text{H.20})$$

Its restriction to complete ground lines is unitary and has overlap one. The old coefficient embedding transports the shifted Hamiltonian, ground projection, raw and physicalized channels, Gram operator, centered magnetic compression, and mixed block in the exact pullback directions. For all  $k, \ell \geq 0$ ,

$$J_{n+k;\ell}^{(2)} \left( J_{n;k}^{(2)} \widehat{\otimes} I \right) = J_{n;k+\ell}^{(2)} \quad (\text{H.21})$$

after canonical tensor identification. The coefficient reassociator is therefore the identity.

#### 4. Center marking and neutral stabilization

The center unitary

$$Z\chi_j = (-1)^{2j}\chi_j \quad (\text{H.22})$$

satisfies  $ZXZ^{-1} = -X$ . The two background fibers

$$H^{[\epsilon]} = H_{\text{el}} + I - \frac{(-1)^\epsilon}{2}X, \quad \epsilon \in \mathbb{Z}_2, \quad (\text{H.23})$$

obey  $ZH^{[\epsilon]}Z^{-1} = H^{[\epsilon+1]}$ . The physicalization and the depth bordisms are covariant across these fibers. A  $k$ -shell fundamental marking has parity  $k \bmod 2$ , but (H.21) still gives identity reassociation. This proves that a physical center grade does not itself force a higher obstruction.

If the system is tensored with a genuinely neutral factor having trivial gauge and center action, grade zero, strict transport, and identity reassociator, the stabilized reassociator is the product of two identity cocycles. Thus strictness is preserved. If the neutral factor has excitations, its gap may lower the ultraviolet constant, but it cannot alter the zero cohomology class.

- **Complete ground datum:** rank-one spectral projection at  $E_0^{(2)} = 0.918058177772\dots$
- **Gram scalar:**  $G = 0.999999999715\dots$

- **Physical magnetic recompression:**  $1.059584273164\dots$
- **Mixed norm:**  $\|K^{(2)}\| = 0.0519985588624\dots \neq 0$ .
- **Resolved seed constant:**  $3.093219423837\dots$
- **Depth propagation:**  $G_n = GI$  and  $\|K_n^{(2)}\| = \|K^{(2)}\|$ .
- **Strict classes:**  $[\Omega^{(2)}]_{\text{band}} = [\widehat{\Omega}^{(2)}]_{\text{cs,ctrl}} = 0$ .
- **Scope:** fixed-stratum depth exhaustion only.

## Appendix I: Local Friedrichs propagation and fixed-thickness linked flux

This appendix records the two realization theorems used in Section VIII. The first is restricted to a local zero-order internal term; a spectral lower bound alone is not a propagation theorem. The second derives the  $SU(3)$  fixed-thickness cocycle from meridional geometry and the center action on a selected complete-ground Wilson/collar line.

### 1. The local incised-space operator

Let

$$X_{\text{inc}} = \mathbb{R}^3 \setminus \ell, \quad \ell = \{(0, 0, z) : z \in \mathbb{R}\}, \quad (\text{I.1})$$

and let  $E_m \rightarrow X_{\text{inc}}$  be a finite-rank Hermitian bundle with smooth flat unitary connection  $\nabla_m$  and positive meridional holonomy  $\omega^m I$ . For the causal canonical branch, take

$$A_m = \eta \nabla_m^* \nabla_m + M_{V_m}, \quad \eta > 0, \quad (\text{I.2})$$

where

$$V_m \in C^\infty(X_{\text{inc}}; \text{End}_{\text{Herm}} E_m), \quad V_m(x) \geq -C_V I. \quad (\text{I.3})$$

Thus  $M_{V_m}$  is pointwise and commutes with scalar cutoffs. An abstract nonlocal  $h_{\text{int}}$  is excluded from the causal statement.

For  $\rho > 0$ , put

$$X_\rho = \{x : \text{dist}(x, \ell) > \rho\} \quad (\text{I.4})$$

and define the Dirichlet form

$$\mathfrak{a}_{m,\rho}[u, v] = \eta \langle \nabla_m u, \nabla_m v \rangle + \langle V_m u, v \rangle \quad (\text{I.5})$$

on

$$Q_{m,\rho} = H_0^1(X_\rho; E_m) \cap D(M_{V_m,+}^{1/2}). \quad (\text{I.6})$$

The shifted form  $\mathfrak{a}_{m,\rho} + (C_V + 1)\|\cdot\|^2$  is closed. Its Friedrichs operator is denoted  $A_{m,\rho}$ . The global minimal form  $\mathfrak{a}_m$  is the closure of  $C_c^\infty(X_{\text{inc}}; E_m)$  in the analogous shifted form norm.

**Proposition I.1** (Uniform collar positivity). *Assume*

$$\mathfrak{a}_m[v] \geq \kappa_{\text{cof}} \|v\|^2, \quad v \in Q_m, \quad (\text{I.7})$$

with  $\kappa_{\text{cof}} > 0$ . Extension by zero  $J_\rho : L^2(X_\rho; E_m) \rightarrow L^2(X_{\text{inc}}; E_m)$  satisfies

$$J_\rho Q_{m,\rho} \subseteq Q_m, \quad \mathfrak{a}_m[J_\rho u, J_\rho v] = \mathfrak{a}_{m,\rho}[u, v]. \quad (\text{I.8})$$

Consequently,

$$A_{m,\rho} \geq \kappa_{\text{cof}} I \quad (\text{I.9})$$

uniformly in  $\rho$ .

*Proof.* Approximate  $u \in H_0^1(X_\rho; E_m) \cap D(V_{m,+}^{1/2})$  in the collar form norm by compactly supported smooth sections. Their zero extensions are Cauchy in the global form norm and the integrals agree. Apply (I.7) to the zero extension.  $\square$

In a local unitary frame,  $\nabla_m^* \nabla_m = -\Delta + \text{lower-order terms}$ . Hence

$$\sigma_{\text{prin}}(\partial_t^2 + A_{m,\rho}) = (-\tau^2 + \eta|\xi|^2)I, \quad (\text{I.10})$$

and the characteristic speed is at most  $\sqrt{\eta}$ .

## 2. Fixed-collar and collar-limit propagation

Let  $d_\rho$  be the intrinsic distance on  $X_\rho$  and  $K \subset X_\rho$  closed. Choose  $\vartheta \in W^{1,\infty}(\mathbb{R})$  with  $\vartheta = 0$  on  $(-\infty, 0]$ ,  $\vartheta = 1$  on  $[1, \infty)$ , and  $\vartheta' \geq 0$ , and set

$$\chi_{\delta,t}(x) = \vartheta\left(\frac{d_\rho(x, K) - \sqrt{\eta}t}{\delta}\right). \quad (\text{I.11})$$

For a smooth solution  $u_{tt} + A_{m,\rho}u = f$ , use the shifted energy density

$$e[u] = \frac{1}{2} \left( |u_t|^2 + \eta |\nabla_m u|^2 + \langle (V_m + C_V + 1)u, u \rangle \right) \quad (\text{I.12})$$

and flux  $j[u] = -\eta \operatorname{Re} \langle u_t, \nabla_m u \rangle$ . Since  $|j| \leq \sqrt{\eta} e$ , the weighted exterior energy

$$E_{\delta,\rho}(t) = \int_{X_\rho} \chi_{\delta,t}^2 e[u(t)] \quad (\text{I.13})$$

satisfies

$$\frac{d}{dt} E_{\delta,\rho}(t) \leq (C_V + 2) E_{\delta,\rho}(t) + \frac{1}{2} \int_{X_\rho} \chi_{\delta,t}^2 |f(t)|^2. \quad (\text{I.14})$$

The cutoff flux is nonpositive, and the Dirichlet trace gives no collar-boundary source. The constants are independent of  $\rho$  and  $\delta$ .

**Theorem I.2** (Fixed-collar finite propagation). *If the initial data are supported in  $K$  and the forcing is absent outside their forward cone, then*

$$\operatorname{supp} u(t) \subseteq \{x : d_\rho(x, K) \leq \sqrt{\eta}|t - t_0|\}. \quad (\text{I.15})$$

Equivalently,  $d_\rho(E, F) > \sqrt{\eta}|t|$  implies

$$1_F \cos(tA_{m,\rho}^{1/2})1_E = 0, \quad 1_F A_{m,\rho}^{-1/2} \sin(tA_{m,\rho}^{1/2})1_E = 0. \quad (\text{I.16})$$

*Proof.* The weighted exterior energy is initially zero. Gronwall's inequality and (I.14) keep it zero. Let  $\delta \downarrow 0$  and use Fatou's lemma. General finite-energy data follow by approximation. The operator statements are the position- and velocity-data cases.  $\square$

For  $\rho_n \downarrow 0$ , the zero-extended form domains increase. Every compactly supported smooth section of  $X_{\text{inc}}$  belongs to all sufficiently small collar domains, and the global domain is the closure of their union. The liminf and recovery conditions therefore give Mosco convergence of the shifted forms<sup>35,43,44</sup>. The associated operators converge in embedded strong resolvent sense. Proposition I.1 removes a zero-frequency singularity; the first-order wave generators then converge by Trotter–Kato, uniformly on compact time intervals.

**Theorem I.3** (Incised Friedrichs/collar finite propagation). *The global finite-energy Cauchy problem for  $\partial_t^2 + A_m$  is well posed, and its retarded, advanced, and causal propagators satisfy*

$$\operatorname{supp}(E_m f) \subseteq J_{\sqrt{\eta}}(\operatorname{supp} f). \quad (\text{I.17})$$

The result is independent of the collar sequence. On every relatively compact collar-Cauchy admissible region  $N$ , the Green sequence

$$0 \longrightarrow C_c^\infty(N; E_m) \xrightarrow{P_m} C_c^\infty(N; E_m) \xrightarrow{E_m} \text{Sol}_{\text{sc}}(N) \longrightarrow 0 \quad (\text{I.18})$$

is exact. Consequently the quotient symplectic functor is causal, extension by zero along causally compatible inclusions is injective, and a neighborhood of a Cauchy section generates the same Weyl algebra.

*Proof.* Approximate data and sources by collar data with supports in arbitrarily small intrinsic neighborhoods. The collar solutions vanish outside the enlarged cones by Theorem I.2; compact-time strong convergence and bounded multiplication by characteristic functions preserve vanishing, and the neighborhood enlargement is then removed. Exactness follows from Cauchy uniqueness, the retarded/advanced identities, and the standard Cauchy cutoff  $P_m(\chi u) = [P_m, \chi]u$ , whose support remains local because  $V_m$  is pointwise. This is the direct local mechanism underlying the comparison results; the related Laplace-type and Green-hyperbolic frameworks are discussed in Refs. <sup>45–50</sup>.  $\square$

### 3. Fixed-thickness linked-flux geometry

Fix one positive thickness. The universal angular cover is

$$\widetilde{X}_{\text{inc}} = (0, \infty) \times \mathbb{R}_{\widetilde{\theta}} \times \mathbb{R}, \quad T(r, \widetilde{\theta}, z) = (r, \widetilde{\theta} + 2\pi, z). \quad (\text{I.19})$$

For  $m \in \mathbb{Z}_3$ , define

$$E_m = (\widetilde{X}_{\text{inc}} \times \mathbb{C}^3) / ((T\widetilde{x}, v) \sim (\widetilde{x}, \omega^m v)). \quad (\text{I.20})$$

The induced flat connection has positive meridional holonomy  $\omega^m I_3$ .

Choose three angular chart objects and strict lifted corridors  $\Gamma_n$  advancing the lifted angle by  $2\pi n/3$ . Then

$$\Gamma_n \Gamma_k = \Gamma_{n+k}, \quad \Gamma_3 = T. \quad (\text{I.21})$$

Transporting the three chart objects to one selected base chart by complete-ground comparisons gives a coefficient-preserving skeleton with underlying one-groupoid  $B\mathbb{Z}_3$ ; the deck

transformation remains the coefficient carry  $C_m$ . For the preferred lifts  $\tilde{\alpha} \in \{0, 1, 2\}$ ,

$$\tilde{U}_{m,\alpha}\tilde{U}_{m,\beta} = C_m^{\epsilon_3(\alpha,\beta)}\tilde{U}_{m,\alpha+\beta}. \quad (\text{I.22})$$

The canonical endpoint package contains a declared rank-one, center-natural line

$$L_{m,\alpha} \subset \text{Hom}_{\text{Gauss},\Omega,\text{cen}}(V_{j,m}^\Omega \hat{\otimes} \mathcal{C}_\alpha, V_{j+\alpha,m}^\Omega), \quad \dim L_{m,\alpha} = 1, \quad (\text{I.23})$$

where  $\mathcal{C}_\alpha$  has triality  $\alpha$ :

$$\rho_\alpha(\omega I_3) = \omega^{\tilde{\alpha}} I. \quad (\text{I.24})$$

Rank one is selected endpoint data, not a multiplicity-one theorem for the full intertwiner space.

**Proposition I.4** (Meridional carry–triality commutation). *For  $\ell_\alpha \in L_{m,\alpha}$ , center-background naturality gives*

$$C_{m,j+\alpha}^\Omega \ell_\alpha = \ell_\alpha(C_{m,j}^\Omega \hat{\otimes} \rho_\alpha(\omega^m I_3)). \quad (\text{I.25})$$

Hence the induced action on the line is

$$C_m \cdot \ell_\alpha = \omega^{m\tilde{\alpha}} \ell_\alpha. \quad (\text{I.26})$$

Moving one carry line  $K_m$  through  $L_{m,\alpha}$  is therefore the ordinary tensor flip followed by this conjugation action:

$$\ell \hat{\otimes} k \mapsto \omega^{m\tilde{\alpha}} k \hat{\otimes} \ell. \quad (\text{I.27})$$

The pairwise products, with all carry lines placed to the left, are

$$\mu_{m;\alpha,\beta}^{\text{fib}} : L_{m,\alpha} \hat{\otimes} L_{m,\beta} \longrightarrow K_m^{\epsilon_3(\alpha,\beta)} \hat{\otimes} L_{m,\alpha+\beta}. \quad (\text{I.28})$$

**Theorem I.5** (Physical fixed-thickness linked-flux class). *The two parenthesized products of three fixed-thickness coefficient lines differ by*

$$\Omega_m^{\text{fib}}(\alpha, \beta, \gamma) = \omega^{m\tilde{\alpha}\epsilon_3(\beta,\gamma)}. \quad (\text{I.29})$$

The marked triple and detector are

$$\Omega_m^{\text{fib}}(1, 2, 1) = \omega^m, \quad I_3(\Omega_m^{\text{fib}}) = \prod_{b=0}^2 \Omega_m^{\text{fib}}(1, b, 1) = \omega^m. \quad (\text{I.30})$$

Thus

$$[\Omega_m^{\text{fib}}] = mu_3 \in H^3(B\mathbb{Z}_3; U(1)), \quad (\text{I.31})$$

and the  $m = 1, 2$  branches are Sector III at every selected positive thickness. The class survives complete-ground compression, center-natural rephasing, and genuinely neutral stabilization; the induced observable adjoint maps compose strictly.

*Proof.* The left-associated product produces  $K_m^{\epsilon_3(\alpha, \beta) + \epsilon_3(\alpha + \beta, \gamma)}$ . The right-associated product first produces  $K_m^{\epsilon_3(\beta, \gamma)}$  to the right of  $L_{m, \alpha}$ . Proposition I.4 moves it left and supplies the scalar in (I.29); the carry identity makes the remaining powers equal. The detector is invariant under normalized coboundaries by Proposition B.1. Scalars on external coefficient lines cancel under adjoint action, and neutral factors contribute the identity cocycle.  $\square$

The fixed-thickness class is generated by meridional geometry, not by scale transport. It is logically independent of the controlled scale class of Appendix B.

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