The Geometry of Admissible Transport II

Constraint Projection, Frustum Geometry, and Klauder Physicalization

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Authorial reconstruction draft – August 9, 2026

Abstract

A finite Yang–Mills reconstruction carries more than one constraint burden, and imposing the constraints separately obscures the carrier on which they must coexist. We replace the packet by a single positive quadratic constraint operator K_ε, and use its finite spectral projections $E_{K_\varepsilon}([0,\delta^2])$ to obtain simultaneous constrained carriers. The spectral width δ is not the geometric thickness ε; the two become related only when the finite carrier is tested against the physical reserve required by coercivity, seam attachment, complete-ground marking, Ward compatibility, and admissible comparison.

On each source-certified finite spectral packet, feasibility is self-referential: admitting a carrier changes the physical profile by which that same carrier is judged. In the finite setting the fixed-point mechanism reduces to order. If a certified seed exists, the feasible cumulative projections possess a unique maximal element $P^K_{\varepsilon,*}$. Adjoining its complete-ground marking, buffered support pair, incision, physical profile, finite Ward profile, and transport provenance gives a positive-thickness physical frustum $\mathfrak{F}^{\mathrm{phys}}_{\varepsilon,*}$.

The Ward layer is kept typed throughout. Allowed Gauss–Ward coercive mixing, Ward-induced carrier leakage, and failure of a passage to intertwine the Ward variation are distinct. For composable Ward-marked passages,

$$\mathfrak{W}_{ST} = S\mathfrak{W}_T + \mathfrak{W}_S T.$$

On the certified band-coefficient subsystem the resulting frusta admit a minimal finite reconstruction realization compatible with local descent and admissible scale refinement. A final comparison problem is deliberately left outside the paper. The relative refinement coordinate and the full finite-reconstruction defect ledger remain separately meaningful without a certified canonical cross-description. This is not taken to obstruct the physical continuation; it marks the effective boundary of the representational language used here and is exported to interface theory.

Research notes – not peer reviewed. Trust, but verify. Always.

Contents

1 Introduction

Upstream interface. Throughout this paper, *The Geometry of Admissible Transport 0* is treated as a frozen upstream interface contract. Its finite reconstruction cargo is consumed only through the canonical export registry. In particular, whenever finite Hamiltonian or coercive data pass through a comparison or refinement arrow, the Hamiltonian descent datum is the indivisible package

$$\mathsf{HamDesc}_D = \big(\mathsf{Desc}_D^{\mathrm{el}}, \mathsf{Desc}_D^{\mathrm{mag}}, \mathsf{Desc}_D^{\mathrm{sb}}, \mathsf{BC}_D^H, \boldsymbol{\varepsilon}_D^H\big), \qquad \boldsymbol{\varepsilon}_D^H = (\varepsilon_D^{\mathrm{UV}}, \varepsilon_D^{\mathrm{IR}}, \varepsilon_D^{\mathrm{mix}}).$$

The Beck–Chevalley cells and residual budgets are fields of this transported object, not optional annotations. All conclusions below are finite-level unless a stronger conclusion level is explicitly invoked.

The difficulty is not that the finite Yang–Mills realization carries many constraints. The difficulty is that the constraints are not naturally independent. Gauss conditions, Ward compatibility, representation-theoretic multiplicities, support restrictions, and the eventual complete-ground sector live on different parts of the finite construction, yet the physical carrier must sustain them simultaneously. The naive formulation therefore forgets the object that matters. Solving the constraints separately and intersecting the resulting kernels may recover an exact common zero space, but it does not organize the finite representation blocks, the physical reserve needed to attach neighboring regions, or the finite room required before the thin limit is taken. What is missing is one positive object whose spectrum can carry the simultaneous burden. That is the role of the quadratic constraint. Let

Φ α, ε α∈ A ε

be the finite constraint packet at positive reconstruction thickness ε > 0. On a common dense form domain define $\sum$

αβ

k ε [ψ] = G ε Φ α, ε ψ, Φβ, ε ψ , G ε > 0,

α,β

and let K ε be the positive self-adjoint operator associated with its closure. On the exact branch,

ker K ε = ker Φ α, ε ,

α∈ A ε

but the exact kernel is not yet the finite object we need. The useful structure is the ordered spectral family

$$PK$$

$$\varepsilon, \delta = E\ K\ \varepsilon\ ([0, \delta]).$$

Two finite scales now appear, and they must not be collapsed:

$$\delta \neq \varepsilon.$$

The parameter δ measures spectral resolution in the simultaneous constraint problem. The parameter ε

measures positive geometric thickness in the Wilson/support reconstruction. One retains finite room in constraint space; the other retains finite room for localization, seams, common comparison domains, and transport. Their relation is earned only when a spectral carrier is tested against the physical geometry in which it must continue. That test is not external to the carrier. Once a cumulative spectral projection P is admitted, the physical Hamiltonian, mixed coercive channels, target-complement reserves, seam burdens, complete-ground marking, and Ward data must be recomputed on that carrier. The candidate therefore participates in the criterion by which it is judged:

$$P \longrightarrow \text{physical profile on } P \longrightarrow \text{finite reserve} \longrightarrow \text{admissibility of } P.$$

This is the fixed-point mechanism underlying the paper. On a source-certified finite packet, however, no infinite-dimensional fixed-point theorem is needed. Feasibility may be tested directly on the finite ordered family

$$P0 < P1 < \cdots < P\ N\ .$$

If

$$J\ \varepsilon = \ j : P\ j \text{ satisfies the complete finite physical certificate}$$

is nonempty, then

$$PK$$

$$\varepsilon,\ast = P\max J\ \varepsilon$$

is the unique maximal simultaneous constrained carrier supported by its own physical consequences. The spectral problem still does not know where the carrier can be compared. Positive-thickness reconstruction therefore forces a second object:

$$O\ \varepsilon- \Subset O\ +\varepsilon\ .$$

The inner region records physical retention. The outer region records the collar in which restriction, seam attachment, and comparison remain controlled. Their interface is marked by the incision

$$\Sigma\varepsilon \, .$$

After adjoining the complete-ground projection, physical profile, finite Ward profile, and transport selector, one obtains phys

$$F\varepsilon,\ast = O\ \varepsilon- \, , \ O\ +\varepsilon \, , \ \Sigma\ \varepsilon \ ; \ P\ K$$

$$\varepsilon,\ast \, , \ P\ \Omega;\varepsilon,\ast \, , \ m\ \varepsilon,\ast \, , \ W\varepsilon \, , \ S\ \varepsilon \, .$$

Ward structure enters here in three different ways and is kept separated accordingly. The Gauss–Ward block controls one allowed analytic coupling. Ward stability asks whether the variation remains inside the selected physical carrier. Ward transport covariance asks whether a restriction, detector map, or comparison intertwines the Ward action. These are different defects: W

$$\epsilon\ GW \neq L\delta, \ \varepsilon \neq WT \, .$$

The distinction becomes essential under composition, where

$$WST = SWT + WS\ T \, .$$

The final finite question is whether these physically selected frusta populate the general finite admissible reconstruction system without forcing scale evolution to become invertible. On the certified subsystem they do. Local descent is groupoidal where it must be; scale evolution remains a bordism/refinement correspondence and need not be an equivalence. The resulting architecture is

$$\text{quadratic constraint} \longrightarrow \text{finite spectral carrier} \longrightarrow \text{self-admissible maximum}$$

$$\longrightarrow \text{support frustum} \longrightarrow \text{Ward-marked physical datum} \longrightarrow \text{minimal finite reconstruction}$$

A separate relative refinement coordinate is also available in the source construction. We do not force it into the full finite-reconstruction defect ledger without an actual chain-level comparison. The absence of that cross-description is not interpreted as failure of the physical continuation. It is the point at which this descriptive language reaches its natural boundary. Several firewalls are therefore maintained throughout:

$$P\ G \neq P\ K \neq P\Omega \, ,$$

$$\text{mixing} \neq \text{leakage} \neq \text{twisting},$$

$$\text{transport} \neq \text{migration} \neq \text{residual escape},$$

and

$$\text{H-gap} \not\Rightarrow \text{M 2 -gap}.$$

The paper is a recognition and mechanism paper. It does not rewrite the frozen five-paper Wilson-tube Yang–Mills construction. Its task is narrower: to expose why the finite constrained carrier used downstream is legitimate, how it is selected, what geometry is required to transport it, and where the present representational language stops.

2 Simultaneous Quadratic Constraint Physicalization

2.1 Constraint packet and common form domain

Let H εkin be the finite kinematical Hilbert space or finite reconstruction carrier at thickness ε. Let

$$\Phi\ \alpha, \varepsilon\ \ \alpha\in A\ \varepsilon$$

be the declared finite quantum constraint packet. We assume a common dense form domain

$$D\varepsilon \subset D\ (\Phi\ \alpha, \varepsilon\ )$$

$$\alpha\in A\ \varepsilon$$

on which all mixed quadratic pairings are defined. Let

$$\alpha\beta$$

$$G\varepsilon = G\varepsilon\ \alpha,\beta\in A\ \varepsilon$$

be a positive-definite Hermitian constraint metric. Define $\sum$

$$\alpha\beta$$

$$k\ \varepsilon\ [\psi] = G\ \varepsilon\ \Phi\ \alpha, \varepsilon\ \psi, \Phi\beta, \varepsilon\ \psi\ ,\ \psi \in D\varepsilon\ .$$

$$\alpha,\beta\in A\ \varepsilon$$

Assume throughout that k ε is closable and write again k ε for its closure.

Definition 2.1 (Simultaneous quadratic constraint operator). The positive self-adjoint operator associated

with the closed form k_ε is denoted

K_ε.

When the $\Phi_{\alpha,\varepsilon}$ are bounded on the finite carrier one may write formally

$$K_\varepsilon = \sum_{\alpha,\beta} G_{\varepsilon\,\alpha\beta}\, \Phi^*_{\alpha,\varepsilon} \Phi_{\beta,\varepsilon},$$

but the form definition is primary.

Lemma 2.2 (Positivity estimate). There exists $c_\varepsilon > 0$ such that

$$k_\varepsilon[\psi] \geq c_\varepsilon \sum_{\alpha\in A_\varepsilon} \|\Phi_{\alpha,\varepsilon}\psi\|^2$$

for every $\psi \in D_\varepsilon$.

Proof. Since G_ε is positive definite on the finite coefficient space indexed by A_ε, its smallest eigenvalue

$$c_\varepsilon = \lambda_{\min}(G_\varepsilon)$$

is strictly positive. Apply $G_\varepsilon \geq c_\varepsilon I$ to the coefficient vector

$$(\Phi_{\alpha,\varepsilon}\psi)_{\alpha\in A_\varepsilon}.$$

□

Theorem 2.3 (Exact simultaneous physicalization). On the exact branch,

$$\ker K_\varepsilon = \bigcap_{\alpha\in A_\varepsilon} \ker \Phi_{\alpha,\varepsilon}.$$

Proof. For a nonnegative closed quadratic form, the zero-form space is the kernel of its representing positive self-adjoint operator. The preceding positivity estimate then shows that k ε [ψ] = 0 if and only if all Φ α, ε ψ

vanish. □

Remark 2.4. This theorem does not identify the simultaneous constraint projection with the Gauss projection

or the complete-ground projection.

2.2 Finite Klauder bands

For δ ≥ 0, define

2

PK

ε, δ = E K ε [0, δ] .

For δ > 0, the range

K K kin

H ε, δ = P ε, δ H ε

is the finite simultaneous constraint carrier.

Proposition 2.5 (Spectral concentration estimate). For every ψ ∈ D (k ε) and every δ > 0,

2 –2

‖ (1 – P K

ε, δ)ψ‖ ≤ δ k ε [ψ].

Proof. Let µ ψ be the spectral measure of K ε associated with ψ. Then

∫ ∫

k ε [ψ] = λ dµ ψ (λ) ≥ δ dµ ψ (λ),

[0,∞) (δ 2 ,∞)

which gives the result. □

2.3 Representation-block decomposition

Suppose

$$H\ \varepsilon\mathrm{kin} = H\ \varepsilon,\lambda$$

$$\lambda\in\Lambda\ \varepsilon$$

is a finite representation decomposition and the closed form k ε reduces this decomposition. Then

$$K\varepsilon = K\ \varepsilon,\lambda\ .$$

$$\lambda\in\Lambda\ \varepsilon$$

Proposition 2.6 (Blockwise simultaneous physicalization). Under the preceding reducing hypothesis,

$$PK$$

$$\varepsilon,\ \delta = E\ K\ \varepsilon,\lambda\ (\ [0,\ \delta 2\ ]).$$

$$\lambda\in\Lambda\ \varepsilon$$

Proof. This is the spectral calculus for direct sums of self-adjoint operators. □

Remark 2.7. The proposition does not identify different representation dimensions or require them to agree.

Every block is subjected to the same simultaneous spectral criterion.

2.4 The physical Hamiltonian comes afterward

Once a simultaneous physical carrier has been chosen, let phys

$$H\ \varepsilon,\ \delta$$

denote the physical Hamiltonian on H ε, K . When its complete ground spectral island is defined, set δ

$$P\Omega;\varepsilon,\ \delta = E\ H\ \mathrm{phys}\ E\ 0;\varepsilon,\ \delta\ \ .$$

$$\varepsilon,\ \delta$$

Then

PΩ;ε, δ ≤ P K

ε, δ .

Proposition 2.8 (Complete-ground support reconstruction). For every orthogonal projection P,

P= s(ρ),

ρ∈F(P)

for any support-saturating family of normal states supported in P.

Proof. Every s(ρ) ≤ P. Support saturation implies that the orthogonal complement in P of their join is

zero. □

Exact-ground interface convention. The symbol $P_{\Omega;\varepsilon,\delta}$ denotes the Paper-II complete-ground marking on the selected Klauder carrier. When the corresponding physical Hamiltonian is consumed as a GAT 0 certified finite Hamiltonian H_D, this marking is identified with the inherited exact complete-ground projection $P_{\mathrm{vac,D}}$ only after

$$H^{\mathrm{phys}}_{\varepsilon,\delta} - E_{0;\varepsilon,\delta} I \geq 0, \qquad \ker\Big(H^{\mathrm{phys}}_{\varepsilon,\delta} - E_{0;\varepsilon,\delta} I\Big) = \mathrm{Ran}\, P_{\Omega;\varepsilon,\delta}.$$

Thus $P_{\Omega;\varepsilon,\delta} = P_{\mathrm{vac,D}}$ is a typed, conditional identification. In all cases, $P_{\Omega;\varepsilon,\delta} \leq P^{K}_{\varepsilon,\delta}$, and no identification of the complete-ground projection with the Klauder constraint projection is implied.

2.5 Finite Ward profile after physicalization

Gauss and Ward data enter the finite physicalization in different roles. The Gauss law belongs naturally to the constraint-selection problem that determines the physical carrier. The Ward structure is broader: after physicalization it records the compatibility of the regional algebra, the finite ground state, restriction maps, and detector/readout maps with the declared finite gauge action.

Accordingly, the Ward datum is not identified here with a single additional kernel condition inside K ε .

Definition 2.9 (Finite Ward profile). On a selected finite physical carrier, the Ward profile is the typed

package

Wε = U ε , δ ε , ωvac

ε , WStat ε ,

where U ε records the declared finite gauge action, δ ε its induced variation on regional generators, ωvac

ε the complete-ground state/face used in the Ward identity, and WStat ε the collection of verified Ward statements for regional restriction and detector/readout operations.

The present paper uses the Ward profile in three distinct ways:

$$\text{constraint compatibility} \neq \text{coercive Gauss–Ward channel}$$

$$\neq \text{transported Ward identity.}$$

The Gauss–Ward block in Section 3 is the analytic/coercive shadow of this broader finite interface. In particular, a completely positive detector compression does not by itself preserve the Ward profile; an intertwining relation, invariant detector range, or an explicitly recorded Ward defect is required.

Hypothesis 2.10 (Ward-compatible physicalization). The selected physical carrier is stable under the

declared finite gauge action, and every restriction, detector/readout, and admitted comparison used in the finite reconstruction either preserves the corresponding Ward identity or carries an explicitly typed defect measuring its failure.

Proposition 2.11 (Finite Ward persistence on the certified subsystem). Under Ward-compatible physicaliza-

tion, the finite Ward profile is a marked component of the physical datum and is preserved under all local restriction and comparison maps on the certified subsystem, up to the explicitly recorded Ward defects.

Proof. The claim is a typing statement. Gauge stability keeps the Ward variation inside the selected physical

carrier. By hypothesis, each restriction, detector/readout, and comparison map either intertwines the declared finite gauge action or supplies a defect in the prescribed Ward slot. Thus the Ward profile remains defined after physicalization and is transported together with the physical datum. □

2.6 Ward stability, transport covariance, and defect composition

The finite Ward profile carries two distinct failure modes which must not be conflated.

Definition 2.12 (Ward-induced carrier leakage). For the selected Klauder carrier, define

W K K

Lδ, ε = (1 – P ε,∗) δ ε P ε,∗ .

The condition W

$$L\delta, \varepsilon = 0$$

means that the infinitesimal Ward variation remains inside the selected physical carrier.

Definition 2.13 (Ward intertwining defect). Let

$$T : x \dashrightarrow y$$

be an admitted restriction, detector/readout map, or physical comparison between finite Ward-marked objects. Define

$$WT = T \delta x - \delta y T .$$

Then

$$WT = 0$$

means that T intertwines the declared Ward variation exactly.

Thus

$$\text{Ward stability} \neq \text{Ward transport covariance.}$$

The first is internal to one selected carrier. The second compares the Ward structures of two finite objects.

Lemma 2.14 (Ward defect composition). Let

$$T\ S$$

$x\dashrightarrow y \dashrightarrow z$

be composable admitted maps between finite Ward-marked objects. Then

$$WST = SWT + WS\ T .$$

Proof. By definition,

$$WST = ST \delta x - \delta z ST$$

$$= S(T \delta x - \delta y T) + (S\delta y - \delta z S)T$$

$$= SWT + WS\ T .$$

□

Corollary 2.15 (Exact Ward compatibility composes). If

$$WT = 0 \text{ and } WS = 0,$$

then

$$WST = 0.$$

Corollary 2.16 (Finite Ward defect estimate). If the relevant operator norms are defined and

$$\|WT\| \leq \eta T , \|WS\| \leq \eta S ,$$

then

$$\|WST\| \leq \|S\|\eta T + \eta S \|T\|.$$

The three Ward-related quantities appearing in this paper therefore have different meanings:

$$\epsilon GW = \text{allowed analytic Gauss–Ward mixing},$$

W

$$L\delta, \varepsilon = \text{Ward variation escaping the selected carrier},$$

$$WT = \text{failure of a passage to intertwine Ward structure}.$$

They are not combined into a single generic Ward defect. For a detector/readout map

$$E : Aphys \longrightarrow Adet ,$$

the corresponding readout defect is

$$W E = E \circ \delta phys - \delta det \circ E.$$

Complete positivity of E alone does not imply that this defect vanishes.

Remark 2.17. This finite statement does not establish continuum Gauss law, BRST cohomology, the full

Ward–Takahashi hierarchy, anomaly cancellation, or gauge covariance of the completed Haag–Kastler net.

3 Coupled Physical Coercivity and the Complete Finite Reserve Certificate

3.1 Physical mixing is not leakage

Assume

$$q\ GW\ [u, v] \geq \gamma G\ \|u\|\ 2 + \gamma W\ \|v\|\ 2 - 2\epsilon\ GW\ \|u\|\ \|v\|$$

with γG , $\gamma W > 0$. Let

$$\gamma G\ -\epsilon\ GW$$

$$MGW = .$$

$$-\epsilon\ GW\ \gamma W$$

Proposition 3.1 (Gauss–Ward block coercivity). If

$$\epsilon\ GW < \gamma\ G\ \gamma W\ ,$$

then

$$q\ GW\ [u, v] \geq \kappa\ GW\ \|u\|\ 2 + \|v\|\ 2\ ,$$

where $\sqrt{}\ \gamma\ G + \gamma W - 2$

$$(\gamma G - \gamma W\ )\ 2 + 4\epsilon\ GW$$

$$\kappa\ GW = > 0.$$

Proof. The determinant and trace of MGW are positive, hence MGW > 0. The displayed constant is its

smaller eigenvalue. □

Thus

$$\text{mixing} \neq \text{leakage}.$$

3.2 Operator-valued block coercivity

Let A –L ∗ K = K1 ⊕ K2 , M= . –L B

Assume $A \geq a\ 0\ Q\ A$ on the reduced first channel and define

–1 ∗ S B| A = B – L Ared L .

Proposition 3.2 (Reduced Schur-complement coercivity). If

$$S B| A \geq b\ 0\ Q\ B$$

on the declared reduced second channel with b 0 > 0, then the coupled form is coercive on the reduced excitation space.

Proof. Complete the square:

2

$$x\ x\ –1\ ∗$$

$$,M = A1/2\ x – Ared\ L\ y + \langle y, S B| A y\rangle.$$

y y □

> **Coercivity firewall.** The local Gauss–Ward and reduced Schur-complement estimates above realize individual finite coercive channels. They do not replace the complete GAT 0 finite coercive certificate. Invoking that certificate additionally requires the exact complete-ground projection, the declared UV and IR diagonal bounds, the literal mixed corner, an independent target-complement reserve, seam reserves, and the determinant condition.

3.3 Carrier leakage and ground migration

Let

$$Ua : Hxkin \to Hykin .$$

Definition 3.3 (Carrier leakage).

$$Lacar = (1 – P\ K\ K$$

$$y\ )U\ a\ P\ x .$$

Definition 3.4 (Ground-support migration).

$$\mathcal{M}a\Omega = (P\ K$$

$$y – P\Omega;y\ )U\ a\ P\Omega;x .$$

The latter remains inside the destination physical carrier, so

$$\text{ground migration} \neq \text{carrier leakage.}$$

3.4 Internal and seam reserve

Let a index the required physical two-block decompositions. For each a, let

u a (P) > 0, v a (P) > 0, m a (P) $\geq$ 0,

and set

Δa (P) = u a (P)v a (P) – m a (P) 2 .

For a seam or extension e, write

c e (P), b e (P), ne (P), ηe (P)

for the source-certified capacity, burden, target-complement capacity, and mixed seam coupling. Define

R κ (e; P) = c e (P) – b e (P), ne (P), [c e (P) – b e (P)]ne (P) – ηe (P) 2 .

Imported reserve provenance. When the candidate carrier is evaluated as a consumer of GAT 0, the target-complement and seam quantities used below are carrier-recomputed realizations of the inherited finite coercive ledger. Their positivity is checked on the candidate P, while their type and provenance remain upstream.

For bookkeeping, write

$$\mathsf{Cert}^{II}_{\varepsilon}(P) = \mathsf{Sel}^{II}_{\varepsilon}(P) \wedge \mathsf{Cert}^{0}_{D}\big|_{P}.$$

Here $\mathsf{Sel}^{II}_{\varepsilon}(P)$ contains the Paper-II selection requirements: membership in the declared cumulative Klauder packet, recomputation of the physical profile on P, and actual action of the declared comparison and selector data on that carrier. The factor $\mathsf{Cert}^{0}_{D}\big|_{P}$ denotes the carrier-recomputed GAT 0 finite certificate, including exact complete ground, UV reserve, IR reserve, literal mixed corner, independent target complement, seam reserve, and determinant condition. The substantive clauses of Definition 3.5 remain unchanged.

Definition 3.5 (Complete finite reserve certificate). A cumulative Klauder spectral carrier P at thickness ε

satisfies the complete finite reserve certificate if:

(i) P belongs to the declared finite spectral packet of K ε ;

(ii) every required physical block satisfies its diagonal and determinant reserve conditions; (iii) every declared target complement retains the prescribed positive capacity; (iv) every required seam reserve vector lies in its declared admissible cone;

(v) the physical Hamiltonian on PH εkin possesses the declared complete-ground marking PΩ;ε, P ;

(vi) the physical profile m ε, P lies in the source-certified bounded coercive class;

(vii) every finite comparison satisfies the declared carrier-preservation condition; (viii) all comparison maps retain their declared transport selector and act on the carriers for which they are defined.

No cohomological strictness requirement is included here.

3.5 Feasible carriers and diagnostic reserves

Let

$$0 \leq \lambda 0 < \lambda 1 < \cdots < \lambda N$$

be the distinct thresholds in the finite packet and

$$P_j = E_{K_\varepsilon}([0, \lambda_j]).$$

Definition 3.6 (Feasible index set).

$$J \varepsilon = j \in 0, \ldots, N \ : P j \text{ satisfies the complete finite reserve certificate} .$$

Feasibility is primitive; no monotonicity in j is assumed. For quantitative diagnostics one may define, when appropriate, a feasible threshold set

$$\mathcal{B}\varepsilon (P) = s \geq \lambda(P) : \text{the s-spectral enlargement satisfies the complete certificate}$$

and its supremum

$$B \varepsilon (P) = \sup \mathcal{B}\varepsilon (P).$$

Only when suitable interval/closure properties are established do we define

$$D \varepsilon (P) = B \varepsilon (P) - \lambda(P)$$

as a self-consistency diagnostic.

4 Maximal Self-Admissible Klauder Carriers

4.1 Seed nonemptiness

Hypothesis 4.1 (Certified feasible Klauder seed). There exists at least one cumulative spectral carrier P_{j_0} in the declared finite packet for which

$$\mathsf{Cert}_{\varepsilon}^{II}(P_{j_0}) = 1.$$

The GAT 0 interface supplies the finite coercive ingredients entering this certificate, but does not by itself assert that an arbitrary Paper-II Klauder packet contains such a carrier.

Remark 4.2. The exact imported theorem supplying this seed must match the same certificate used here. A

theorem proving only internal coercivity is insufficient if seam or target-complement data are also required.

Theorem 4.3 (Maximal self-admissible Klauder carrier). Assume the certified finite seed hypothesis. Then

there exists a unique maximal cumulative spectral carrier satisfying the complete finite reserve certificate. Explicitly,

$$j_{\varepsilon}^{*} = \max J_{\varepsilon}$$

is well defined, and

$$P_{\varepsilon,*}^{K} = P_{j_{\varepsilon}^{*}}$$

is the unique maximal self-admissible Klauder carrier in the declared finite spectral packet.

Proof. The set J_ε is nonempty and finite. Hence it has a largest element. Since the P_j form a chain, the

corresponding projection is the unique maximal feasible carrier. □

4.2 Derived Klauder windows

ε,∗ = P j∗ and j ∗ < N, define

If P K

spec

$$I_{\varepsilon,*}^{\text{spec}} = [\lambda_{j_*}, \lambda_{j_*+1}).$$

If the quantitative reserve analysis supplies a smaller feasible threshold set, define

K spec

$$I^K_{\varepsilon,*} = I^{\text{spec}}_{\varepsilon,*} \cap \mathcal{B}_\varepsilon(P^K_{\varepsilon,*}).$$

Corollary 4.4 (Admissible Klauder window). For every

$$\delta^2 \in I^K_{\varepsilon,*},$$

PK K

ε, δ = P ε,∗ ,

and the corresponding finite constrained carrier satisfies the complete physical certificate.

4.3 Saturation and capacity cliffs

The maximal feasible carrier is spectrally saturated when its feasible threshold set reaches no new spectral block. A stronger capacity-cliff configuration occurs when the old carrier's reserve reaches the next threshold, but the enlarged carrier fails the complete certificate after its profile is recomputed. This is a conditional phenomenon, not a general alternative forced by maximality.

4.4 Complete-ground marking on the selected carrier

Let phys phys

H ε,∗ = H ε, P K

ε,∗

and, when the complete ground spectral island is defined,

PΩ;ε,∗ = E H phys E 0;ε,∗ .

ε,∗

Then

PΩ;ε,∗ ≤ P K

ε,∗ .

Selected-carrier exact-ground closure. When $H^{\text{phys}}_{\varepsilon,*}$ is evaluated as a GAT 0 certified finite Hamiltonian, the marking $P_{\Omega;\varepsilon,*}$ is required to satisfy the exact-ground condition of EXP-0.014 before it is identified with $P_{\text{vac,D}}$. The inclusion

$$P_{\Omega;\varepsilon,*} \le P^K_{\varepsilon,*}$$

does not identify the complete-ground projection with the Klauder constraint projection.

5 Support Corners, Persistent Incision, and the Physical Frustum

5.1 Buffered support pairs and incision

Assume a nested support pair

$$O\ \varepsilon- \Subset O\ +\varepsilon\ .$$

Definition 5.1 (Support corner). A support corner at thickness ε is a nested pair

sup

$$\mathfrak{C}\ \varepsilon = (O\ \varepsilon-\ ,\ O\ +\varepsilon\ )$$

satisfying the declared finite localization and common-domain hypotheses.

Definition 5.2 (Incision). An incision datum

$$\Sigma\varepsilon$$

is the declared support/interface marking associated with the support corner and retained by the positive- thickness reconstruction.

Thus

$$\text{incision} \neq \text{twisting.}$$

Remark 5.3 (Maximal realizability). The discovery analysis suggests that the incision may arise as the

boundary of a maximal support region on which the finite physical realization remains admissible. This is retained as a candidate mechanism, not used as a theorem unless separately sourced.

5.2 Common comparison domains and selectors

cmp Let D ε denote the declared common comparison core associated with the support corner. Let Sel be the set of transport selectors. For an admitted comparison T, write S (T) ∈ Sel.

We preserve the distinction

$$N\ G \neq N\ M\ .$$

Hypothesis 5.4 (Selector-compatible physicalization). For every selector s ∈ Sel, physical compression and

the required closures define maps within the same selector fiber.

5.3 Physical frustum

Definition 5.5 (Physical frustum). The physical frustum at thickness ε is

phys

$$F\varepsilon,* = O\ \varepsilon- , O\ +\varepsilon , \Sigma\ \varepsilon ; P\ K$$

$$\varepsilon,* , P\ \Omega;\varepsilon,* , m\ \varepsilon,* , W\varepsilon , S\ \varepsilon .$$

Definition 5.6 (Geometric frustum).

geom

$$F\varepsilon,* = O\ \varepsilon- , O\ +\varepsilon , \Sigma\ \varepsilon ; P\ K$$

$$\varepsilon,* , P\ \Omega;\varepsilon,* , m\ \varepsilon,* .$$

Definition 5.7 (Categorical frustum system). The categorical frustum system consists of the geometric frusta

together with their admitted typed comparison maps and coherence cells.

> **Frustum provenance.** The physical frustum is a Paper-II construction built from imported GAT 0 cargo; it is not itself a GAT 0 export. The inherited payload includes the exact-ground lineage when recognized, the finite coercive profile, comparison-domain data, selector provenance, and any active $\mathsf{HamDesc}_D$. The new Paper-II data include the maximal Klauder carrier, incision, finite Ward profile, and self-admissibility structure.
>
> **Incision firewall.** Incision compatibility is support-geometric. It neither implies nor is implied by coefficient strictness, Hamiltonian descent, or vanishing of any Ward defect.

5.4 Incision-compatible transport

Let a : x → y be an admitted comparison with support component a # .

Definition 5.8 (Incision compatibility). The comparison a is incision-compatible if

$$a\ \#\ (\Sigma\ x\ ) \sim adm\ \Sigma\ y\ .$$

Proposition 5.9 (Incision persistence under composition). Assume a : x → y and b : y → z are incision-

compatible admitted comparisons and that ∼adm is transitive and respected by support composition. Then

b ◦ a is incision-compatible.

Proof. Apply compatibility twice and use transitivity. □

5.5 Carrier leakage

For an admitted comparison T_a, define

$$T_a^{\text{phys}} = P_{y,*}^K \, T_a \, P_{x,*}^K$$

and

$$\mathcal{L}_a^{\text{car}} = (1 - P_{y,*}^K) \, T_a \, P_{x,*}^K.$$

Proposition 5.10 (Zero carrier-leakage criterion).

$$\mathcal{L}_a^{\text{car}} = 0$$

if and only if

$$T_a \operatorname{Ran} P_{x,*}^K \subseteq \operatorname{Ran} P_{y,*}^K.$$

Proof. Immediate from the orthogonal decomposition defined by $P_{y,*}^K$. □

Carrier/Hamiltonian firewall. Zero carrier leakage states only that the selected source carrier is carried into the selected target carrier:

$$\mathcal{L}_a^{\text{car}} = 0 \quad \not\Rightarrow \quad \text{Hamiltonian descent compatibility.}$$

Whenever the same comparison transports the physical Hamiltonian or its coercive form, the complete $\mathsf{HamDesc}_D$ datum must additionally be carried.

Theorem 5.11 (Physical frustum realization). Fix a positive-thickness finite fiber ε. Assume:

(i) the maximal self-admissible Klauder carrier exists; (ii) the corresponding complete-ground marking is defined; (iii) the physical profile satisfies the imported positive-frustum estimate; (iv) the source localization provides a buffered support pair with common comparison core; (v) an incision datum is defined; (vi) the declared comparisons are selector-compatible and have zero inadmissible carrier leakage at the required stage.

(vii) whenever an admitted comparison transports physical Hamiltonian or coercive-form data, it carries the full inherited $\mathsf{HamDesc}_D$ package, including BC_D^H and ε_D^H.

Then $\mathfrak{F}_{\varepsilon,*}^{\text{phys}}$ is a well-defined positive-thickness physical frustum.

6 Finite Physical Realization and Admissible Reconstruction

6.1 The realization problem

phys Let Frfin denote the class of finite physical frusta and Recadm fin the general finite admissible reconstruction structure. The realization is factored as

phys Ecmp phys Erec

Frfin –→ Diagfin –→ Recadm

fin .

The first map enriches a frustum by its local descent data; the second verifies the ambient/admissible coefficient typing, defect ledger, and refinement compatibility.

> **Pass-through integrity.** The realization map E_{rec} does not forget active upstream packages. Whenever present, the finite reconstruction payload retains
>
> $$P_{\mathrm{vac,D}}, \qquad \mathsf{HamDesc}_D, \qquad \mathsf{BC}_D^H, \qquad \varepsilon_D^H,$$
>
> together with typed defect, Ward, selector, incision, and provenance data. Hamiltonian descent occupies its own declared reconstruction slot and is not absorbed into a generic defect coordinate.

6.2 Local reconstruction population

Within a fixed finite stage, let

x i = local physical/Hilbert realization on chart i,

gi j = unitary local transition,

c i j k : gi j g j k ==⇒ gik ,

and a i j kl the coefficient-line reassociation datum. Thus the local finite diagram is groupoidal where the reconstruction requires invertibility.

6.3 Ambient and admissible coefficients

Let Z amb σ

denote the ambient central coefficient corrections and

Z adm amb

σ ⊆ Zσ

the physically admitted subgroup.

Proposition 6.1 (Admissible coefficient subsystem). Assume each physical admissibility condition defining

Z adm σ is stable under the coefficient operation and inversion, and restriction maps preserve those conditions. Then Z adm

σ ≤ Zσ

amb

for every simplex σ, restrictions preserve admissible coefficients, and

L σ = Z amb adm

σ /Z σ

is well defined.

6.4 Typed defect realization

The finite reconstruction defect ledger is written schematically as

L def = (u, e, τ, ω; κ, l, r).

The source defects include analytic non-flatness, refinement mismatch with admissible filling, coefficient-band reassociation, seam reserve, and provenance data. They are not identified indiscriminately.

Theorem 6.2 (Typed defect realization). Suppose every source physical defect is supplied with a declared

simplicial degree, ambient coefficient object, admissible coefficient subgroup where appropriate, restriction law, and physical provenance. Suppose further that the corresponding finite-reconstruction slot has the same source/target type. Then there is a componentwise assignment into L def preserving degree, coefficient type, restriction, and provenance. No degree-changing identification occurs without an explicit connecting, transgression, suspension, or mapping-fiber mechanism.

6.5 Scale bordisms as refinement morphisms

Let

a : ε → ε′

be an admitted scale arrow. The frozen construction supplies a scale correspondence

⊗ L a −→ H ε ′ .

Ja : H ε b

This map need not be invertible. It is not a local gluing arrow gi j . Instead, it acts at the refinement level, transporting the complete finite diagram and its markings.

Theorem 6.3 (Typed scale-bordism refinement). Let $a : \varepsilon \to \varepsilon'$ be an admitted scale arrow with scale correspondence

$$J_a : \mathcal{H}_\varepsilon \widehat{\otimes} L_a \longrightarrow \mathcal{H}_{\varepsilon'}.$$

Assume that J_a satisfies:

(i) object compatibility;

(ii) local-gluing compatibility;

(iii) compositor compatibility up to the declared admissible correction;

(iv) associator compatibility up to the declared higher correction;

(v) preservation of $Z_\sigma^{\mathrm{adm}} \subseteq Z_\sigma^{\mathrm{amb}}$;

(vi) preservation of the typed finite defect ledger and its provenance;

(vii) preservation or explicitly controlled transport of the finite Ward profile;

(viii) preservation of the transport selector;

(ix) preservation of the Klauder-carrier and complete-ground markings in their declared typed slots;

(x) whenever physical Hamiltonian/form data are active, transport of the complete package

$$\mathsf{HamDesc}_D = \left(\mathsf{Desc}_D^{\mathrm{el}}, \mathsf{Desc}_D^{\mathrm{mag}}, \mathsf{Desc}_D^{\mathrm{sb}}, \mathsf{BC}_D^H, \boldsymbol{\varepsilon}_D^H\right), \qquad \boldsymbol{\varepsilon}_D^H = (\varepsilon_D^{\mathrm{UV}}, \varepsilon_D^{\mathrm{IR}}, \varepsilon_D^{\mathrm{mix}}),$$

or an explicit typed record of the corresponding adjusted coercive data.

Then J_a induces an admissible refinement morphism of the corresponding *finite reconstruction data*.

> **Level firewall.** This theorem is finite-level. A composable collection of such arrows yields a persistent finite family only after the separate compatible-family hypotheses of GAT 0 (EXP-0.026) are discharged. No continuum, represented, invariant-mass, or physical-Hamiltonian identification follows from this refinement theorem alone.

6.6 Minimal finite physical realization

Theorem 6.4 (Minimal finite physical realization). Under the certified local lifting, band-coefficient,

support/frustum, and scale-transport hypotheses, the selected physical frustum defines a minimal finite reconstruction datum retaining

$$x, g, c, a, Z^{\mathrm{amb}}, Z^{\mathrm{adm}},$$

the band defect/coherence data,

$$P^K,\ P_\Omega,\ \Sigma,\ \mathfrak{m},\ \mathcal{W},\ \mathsf{S},$$

and, whenever Hamiltonian data are active, the full $\mathsf{HamDesc}_D$ package with its BC cells and residual budgets, together with the corresponding finite provenance data. This realization is compatible with admissible refinement on the certified subsystem.

6.7 Representational boundary

The independent relative refinement coordinate

$$[\Delta\ \zeta\ ,\ \tau]\ \mathrm{ref/adm}$$

is not canonically identified here with the complete finite reconstruction defect ledger.

Remark 6.5 (Representational boundary). The relative refinement coordinate and the finite reconstruction

defect ledger encode distinct views of the same finite physical passage. The present construction does not require, and does not assume, a canonical chain-level identification between these representations. Failure to supply such an identification is not interpreted as failure of the physical reconstruction. It marks the boundary of the finite reconstruction language used in this paper.

Open Problem 6.6 (Relative reconstruction crosswalk). Construct, if it exists, a canonical chain-level

comparison fin

$$[\Delta\zeta, \tau]_{\mathrm{ref/adm}} \longrightarrow L^{\mathrm{fin}}_{\mathrm{def}},$$

or prove that the two descriptions arise naturally as distinct projections of a richer interface datum. No such arrow is assumed in this paper.

The general interface-theoretic principle suggested by this boundary is exported rather than proved here.

7 Mechanism and Structural Consequences

7.1 Why the quadratic constraint is the correct primitive

The family of constraints

$$\Phi\ \alpha,\ \varepsilon$$

is difficult not because there are many equations but because solving them separately destroys the geometry by which they are meant to coexist. The quadratic form

$$k\varepsilon$$

places their simultaneous burden into one positive object. Thus quadratic aggregation converts compatibility into spectral geometry.

7.2 Why finite spectral width is useful

The spectral width δ and geometric thickness ε are distinct but structurally parallel:

finite spectral resolution + finite geometric resolution = a finite object on which comparison is possible

7.3 Why the carrier must test itself

The carrier changes the physical profile on which its own admissibility is evaluated:

$$P \longrightarrow \text{physical profile on } P \longrightarrow \text{reserve supported by } P \longrightarrow \text{admissibility of } P.$$

This is the self-consistency mechanism underlying the maximality theorem. The mechanism is fixed-point-like; the proof is finite order.

7.4 Why the frustum is forced

Physical localization and physical comparison require different support scales:

$$O_{\varepsilon^-} \Subset O_{+\varepsilon}.$$

The frustum is the positive-thickness datum in which physical support and comparison support coexist.

7.5 Local descent and scale evolution

The source audit yields a structural distinction:

local descent may be invertible while admissible scale evolution is not.

Within a fixed finite stage, g_{ij}, c_{ijk}, a_{ijkl} belong to local descent. Across scales, J_a is a bordism/refinement correspondence. Conflating these roles creates a category error.

7.6 Local rigidity without global strictness

Each finite fiber may possess a sharply selected physical carrier and a positive coercive profile without requiring literal identification of carriers across refinement. Thus the present construction exhibits a local form of local analytic rigidity does not require global categorical strictness. This is not promoted here to a universal theorem.

> **Conclusion-level convention.** The certified conclusions of this paper are finite-level (F) unless a stronger level is explicitly stated. Persistent (P), continuum (C), represented (R), and physical (Ph) conclusions require separate hypotheses and are not obtained by silent promotion from the finite frustum construction.

8 Relation to Existing Structures and Scope

8.1 Klauder projection-operator quantization

The closest external antecedent is the projection-operator method for constrained quantization. The feature used here is the replacement of a family of quantum constraints by a single positive quadratic object and the spectral selection of a finite physical carrier.

Bibliography placeholder: exact Klauder citation to be locked in final source audit.

8.2 Master-constraint adjacency

There is a broader family of constructions combining several constraints into one positive operator. The present paper retains the terminology simultaneous quadratic constraint operator to avoid importing stronger conceptual associations not needed here.

8.3 Kato-type spectral transport

Kato-type transport is relevant only after the physical carrier has been selected and is allowed to vary. Projection drift is not itself leakage.

Bibliography placeholder: exact Kato/spectral-subspace transport citation to be locked.

8.4 Finite reconstruction

The finite reconstruction theory begins after the physical frustum has been constructed. Paper II proves physical existence and legitimacy; the general reconstruction theory supplies coherent continuation on the certified subsystem.

8.5 Ward boundary

The finite Ward profile is retained as part of the positive-thickness physical datum. The paper does not promote this finite compatibility to a continuum Ward–Takahashi theorem, BRST statement, anomaly-cancellation result, or gauge-covariance theorem for the completed Haag–Kastler net. Those require separate downstream arguments.

8.6 AQFT boundary

The positive-thickness physical frusta are not yet a Haag–Kastler net. Thick-to-thin local quantum reconstruction belongs to the next paper.

8.7 Higher descent boundary

Incision is support geometry, not coefficient twisting. A strict analytic physical carrier does not imply a globally strict represented coefficient-line lift.

8.8 Vacuum boundary

The selected Klauder projection and complete-ground projection remain distinct:

$$P^{K}_{\varepsilon,*} \neq P_{\Omega;\varepsilon,*}$$

unless separately proved. Paper II constructs a legitimate carrier for the complete-ground object; it does not prove full vacuum lineage.

8.9 Invariant spectral boundary

A finite Hamiltonian/coercive threshold does not imply an invariant mass threshold:

$$H\text{-gap} \not\Rightarrow M^2\text{-gap}.$$

8.10 Five-paper freeze policy

The five-paper Wilson-tube Yang–Mills sequence remains

FROZEN — PRIMARY CONSTRUCTION.

The present paper is explanatory and recognitional. It does not rewrite or supersede the primary construction. Likewise, GAT 0 is treated here as a frozen upstream interface contract: Paper II consumes its exported finite recognition and refinement data but neither rewrites nor strengthens the GAT 0 theorem envelope.

9 Conclusion and Provenance Ledger

The finite constrained problem begins with a collection of quantum constraints and ends with a positive- thickness physical object that can be transported through the certified minimal finite reconstruction system:

$$\Phi^{\text{phys}}_{\alpha,\varepsilon} \longrightarrow K_\varepsilon \longrightarrow P^K_{K_{\varepsilon,\delta}} \longrightarrow P_{\varepsilon,*} \longrightarrow F_{\varepsilon,*}.$$

The selected carrier is the unique maximal carrier in the declared finite packet satisfying the complete finite physical certificate. The physical frustum adds the support corner, incision, complete-ground marking, profile, and provenance needed for positive-thickness transport.

9.1 Claim-status ledger

Result	Provenance	Level	Status
Simultaneous quadratic constraint construction	Paper II	F	CERTIFIED
Exact common-kernel identity	Paper II	F	CERTIFIED
Finite Klauder spectral carrier	Paper II	F	CERTIFIED
Representation-block accommodation	Paper II	F	CERTIFIED
Finite Ward profile	Paper II	F	CERTIFIED ON DECLARED SUBSYSTEM
Ward defect composition	Paper II	F	CERTIFIED
Finite coercive ingredients	GAT 0	F	IMPORTED
Complete finite coercive certificate	GAT 0 + carrier recomputation	F	IMPORTED/REALIZED
Feasible Klauder seed	GAT 0 + Paper II	F	CONDITIONAL
Maximal self-admissible carrier	Paper II	F	CERTIFIED CONDITIONAL
Exact complete-ground marking	GAT 0 + Paper-II realization	F	CONDITIONAL ON EXACT-GROUND TEST
Physical frustum	Paper II	F	CERTIFIED CONDITIONAL
Typed finite reconstruction	GAT 0 + Paper II	F	CERTIFIED ON DECLARED SUBSYSTEM
Hamiltonian descent	GAT 0	F	IMPORTED PASSTHROUGH
Persistent compatible family	GAT 0 + Paper-II data	P	REQUIRES SEPARATE COMPATIBILITY
Thin AQFT reconstruction	downstream	C/R	OPEN
Vacuum lineage	downstream	R/Ph	OPEN
Invariant spectral transfer	downstream	Ph	OPEN
Relative reconstruction crosswalk	interface theory	—	OPEN

9.2 Frozen GAT 0 import ledger

The paper-level import envelope is

$\mathsf{Imports}_{II\leftarrow 0} = \{\mathsf{EXP}_{0.011}, \mathsf{EXP}_{0.012}, \mathsf{EXP}_{0.014}, \mathsf{EXP}_{0.015}, \mathsf{EXP}_{0.016}, \mathsf{EXP}_{0.017}, \mathsf{EXP}_{0.022}, \mathsf{EXP}_{0.023}, \mathsf{EXP}_{0.024}, \mathsf{EXP}_{0.025}, \mathsf{EX}$

The principal uses are:

- EXP-0.011: plaquette-local physical Hamiltonian seed where the Paper-II physical Hamiltonian consumes the typed electric/magnetic/seam finite seed;
- EXP-0.012: atomic Hamiltonian descent certificate $\mathsf{HamDesc}_D$;
- EXP-0.014: exact complete-ground projection $P_{\mathrm{vac,D}}$;
- EXP-0.015–0.016: semisimple Casimir seed and exact UV–IR finite coercive assembly;
- EXP-0.017: persistent finite spectral certificate, only when its compatible-family hypotheses are separately met;
- EXP-0.022–0.025: finite reconstruction input, typed comparison, finite reconstruction bicategory, and no-loss export;
- EXP-0.026: persistent compatible-family export;
- EXP-0.027–0.028: downstream requirement profiles and target-relative readiness gate.

There is no invented GAT 0 export corresponding to the former `IMP-BORD`: the concrete scale-bordism realization is Paper-II data acting through the typed reconstruction-comparison interface. Likewise, there is no `IMP-INV` import establishing invariant spectral transfer; that conclusion remains downstream.

Requirement-profile disposition. Paper II advances finite entries of $\mathsf{Req}_{\mathrm{Kl}}$, $\mathsf{Req}_{\mathrm{Ward}}$, and $\mathsf{Req}_{G\text{-phys}}$. No entire profile is declared passed merely because some finite entries are discharged. The pass predicate of EXP-0.027 remains equal to one only when every required entry and dependency of the target profile has been discharged.

9.3 Downstream handoffs

Define the Paper-II finite export package

$$\boxed{\mathsf{PhysFr}^{II}_{\varepsilon} = \left(\mathfrak{F}^{\mathrm{phys}}_{\varepsilon,*}, \mathsf{HamDesc}_{D(\varepsilon)}, L^{\mathrm{fin}}_{\mathrm{def},\varepsilon}, \mathsf{Prov}_{\varepsilon}\right).}$$

The frustum component already retains $P^{K}_{\varepsilon,*}$, $P_{\Omega;\varepsilon,*}$, Σ_ε, $\mathfrak{m}_{\varepsilon,*}$, $\mathcal{W}_\varepsilon$, and S_ε.

Paper III receives

$$\{\mathsf{PhysFr}^{II}_{\varepsilon}\}_{\varepsilon>0}$$

as finite positive-thickness input, not as a continuum net.

Paper V receives the exact-ground-lineage subpackage

$$\left(P_{\Omega;\varepsilon,*}, \Sigma_\varepsilon, \mathsf{S}_\varepsilon, \mathsf{HamDesc}_{D(\varepsilon)}, \mathsf{Prov}_\varepsilon\right),$$

with no automatic vacuum-lineage claim. The full relative reconstruction crosswalk is not required for either handoff.

9.4 Final perspective

The finite constrained carrier was available because three mechanisms aligned. First, the representation-theoretic constraint packet could be assembled into one positive quadratic spectral object. Second, the finite physical estimates selected which spectral carriers could sustain themselves together with the interfaces required for continuation. Third, positive geometric thickness retained enough support geometry for those carriers to be compared before the thin limit. Thus: simultaneous constraint selection + finite reserve + buffered localization = transportable finite physical carrier

The present paper reaches a natural stopping point. The physical carrier has been selected, its positive- thickness geometry has been identified, and its minimal admissible reconstruction has been established on the certified subsystem. What fails beyond this point is not physical continuation but the expectation that every faithful description of that continuation must admit a canonical translation into every other. The subsequent vacuum and continuum problems remain physical problems; the general comparison of their representational shadows belongs elsewhere.

> The finite reserve and refinement compatibility invoked here are inherited from the frozen GAT 0 interface; the simultaneous spectral selection, self-admissible maximality, Ward-marked frustum, and resulting positive-thickness physical realization are the new content of Paper II.
>
> $\boxed{\text{Paper II terminates at a certified finite positive-thickness physical reconstruction.}}$
>
> Persistent compatibility may be inherited only when its separate hypotheses are satisfied. Thin, continuum, represented, invariant-spectral, and physical continuum conclusions are not claimed here.

Research notes — not peer reviewed. Trust, but verify. Always.