The Geometry of Admissible Transport III

Admissible Yang–Mills Fibrations and Thin Haag–Kastler Reconstruction

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Abstract. We formulate the continuation problem that lies between finite physical recognition and thin algebraic quantum field theory in the Wilson–tube Yang–Mills construction. The source recognition theory proves that every compact connected simple global form admits a genuine detector packet of at most two irreducibles, a finite sector-complete enlargement, and an isolated Casimir electric seed (Recognition Theorem 6.2, Corollary 6.3, Lemma 7.1, and Proposition 8.1 of [1]); magnetic physicalization, cofinal compatibility, and realization of the prescribed relative and coefficient-band data remain separately certified physical obligations.

Starting from a complete recognized source certificate, we identify the finite physical datum that must survive refinement and define certified admissible continuation by preservation or controlled transport of its sector markings, gauge and spacetime descent data, selected physical carrier, complete-ground projection, coercive form, Ward structure, cohomological coordinates, and provenance. On Cat-valued admissible branches these continuations form a pseudofunctor over the refinement category and hence a Grothendieck fibration. The represented carrier is naturally module-valued: admissible physical fibers arise as ranges of projections in a Grassmannian of self-dual Hilbert W^*-modules, while represented descent may remain strict, corridorwise, coefficient-line-banded, or correspondence-valued.

After a package-preserving reduction from finite physical demands to finite Čech systems, incision-aware Cauchy descent yields isotony, Einstein causality, selected time-slice reconstruction, regional dynamics, and observable Ward compatibility. An independent comparison–Mosco passage preserves the complete ground and a uniform off-ground coercive floor. Their assembly produces a thin observable Yang–Mills Haag–Kastler net without requiring global trivialization of the richer represented realization.

The theorem is general for every compact connected simple global form satisfying the stated physical realization and continuation hypotheses. The canonical selected $SU(3)$ branches provide the presently certified end-to-end witness, while the $SU(2)$ construction independently exhibits nontrivial coefficient-band realization with an unchanged finite analytic shadow. The original thick-to-thin Paper IV is recovered as a specialized branch of the present framework rather than superseded by it.

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1 Introduction

The thick-to-thin Yang–Mills reconstruction can be written too weakly as

$$\mathcal{A}_{\varepsilon}^{\mathrm{YM}}(O) \longrightarrow \mathcal{A}_{0}^{\mathrm{YM}}(O).$$

At that level the problem appears to be one of convergence of local observable algebras. The actual construction carries substantially more information. A finite physical realization contains the global form of the gauge group, a genuine detector packet and its physically realized sector markings, the selected physical carrier, the complete-ground projection, coercive form data, incision and seam geometry, Cauchy structure, Ward data, relative and coefficient-line information, and the provenance of the realized branch. The local observable algebra is only one coordinate of this object.

The question addressed here is not whether the finite realization exists, but how a recognized physical realization is allowed to move. A family of realization fibers is not yet a fibration. The missing datum is certified continuation.

We package the required finite datum as

$$\mathfrak{C}_{\lambda}^{\mathrm{YM}} = \Big(\Sigma_{\lambda},\, \mathfrak{G}_{\lambda}^{\mathrm{YM}},\, \mathbb{S}_{\lambda}^{\Sigma,\mathrm{YM}},\, P_{\lambda}^{K},\, P_{\Omega,\lambda},\, \mathfrak{m}_{\lambda}^{\mathrm{YM}},\, \mathfrak{H}_{\lambda}^{\mathrm{YM}},\, L_{\lambda}^{\mathrm{prov}}\Big).$$

The physical detector sectors Σ_{λ}, the relative refinement class $r_D \in H^2(\mathrm{Fib}(Q_D))$, and the coefficient-band class $b_D \in H^3(BB_D; U(1))$ are distinct data. Likewise the Gauss projection, selected physical carrier, and complete-ground projection remain distinct unless a separate theorem identifies them.

Once certified continuations have been constructed and shown coherent, the admissibility assignment becomes a Cat-valued pseudofunctor

$$\mathrm{Adm}^{\mathrm{YM}} : \mathcal{R}_{\mathrm{adm}}^{op} \to \mathbf{Cat}$$

on the appropriate branch. Its Grothendieck construction gives the admissible Yang–Mills fibration.

The represented realization introduces a second distinction. The natural local carrier is a self-dual Hilbert W^*-module, and the selected physical object is the range of an admissibly varying projection,

$$P_{\lambda}^{K} \in \mathrm{Gr}_{\mathrm{adm}}(H_{\lambda}).$$

Thus the primitive analytic geometry is Grassmannian-valued; the carrier bundle is its range realization.

Two independent survival mechanisms complete the thin passage. Incision-aware Cauchy descent controls the regional algebraic/dynamical structure, while comparison–Mosco transport controls the closed physical form and complete ground. Their assembly yields

$$\mathcal{A}_{0}^{\mathrm{YM}} : \Big(\mathrm{RegLoc}_{\mathrm{inc}}^{0,\mathrm{adm}}, \perp_{\mathrm{inc}}\Big) \to W^*\mathrm{Alg}_n.$$

The theorem is general in G_{Γ}, but its hypotheses mark a domain of applicability rather than an unconditional realization theorem for every compact-simple group.

This formulation does not supersede the original five-paper Wilson–tube construction. Appendix C records the coverage of the original thick-to-thin Paper IV as a specialization of the present admissible-transport framework.

2 Compact-Simple Recognition at the Source Boundary

Theorem 2.1 (Compact-Simple Entry). *Let G_{Γ} be a compact connected simple global form. Then there exists a finite genuine detector packet Π_{Γ}, the packet may be chosen with at most two irreducible representations, there exists a finite sector-complete enlargement $\Pi_{\Gamma}^{\mathrm{sec}}$, and every such sector-complete packet admits a finite isolated Casimir electric seed with positive excitation floor and identity marked Gram operator. Physical magnetic realization, a positive finite coercive frustum, cofinal compatibility, seam survival, and realization of a prescribed sectorial type remain additional hypotheses.*

The statement combines Recognition Theorem 6.2 and Corollary 6.3 with Lemma 7.1 and Proposition 8.1 of [1]; its separation of universal eligibility from higher physical realization is summarized in Recognition Theorem 15.3.

Definition 2.2 (Recognized Yang–Mills Source)**.** A recognized Yang–Mills source is a tuple

$$\mathcal{C}_{\mathrm{src}} = (G_\Gamma, \Pi_\Gamma^{\mathrm{sec}}, \mathcal{J}, q, v, M, \kappa, (x_D)_{D\in\mathcal{J}})$$

satisfying

$$\mathrm{RecCert}_{\mathcal{J}}(G_\Gamma; \Pi_\Gamma^{\mathrm{sec}}, q, v, M, \kappa).$$

This is Recognition Definition 16.1; the principal sufficient assembly theorem is Recognition Theorem 16.2 in [1].

3 Finite Physical Cargo and Certified Continuation

Recognition tells us that a source realization exists. It does not yet tell us how that realization moves. The distinction is already visible upstream: the finite recognition fiber is an inverse image and carries no Grothendieck lifting property merely by virtue of being called a fiber. The present paper begins by adding the continuation structure that finite recognition deliberately leaves open.

3.1 Transport-ready physical cargo

Definition 3.1 (Finite Physical Yang–Mills Cargo)**.** Let λ denote a recognized positive-thickness physical stage. The associated transport-ready cargo is

$$\mathfrak{C}_\lambda^{\mathrm{YM}} = \left(\Sigma_\lambda, \mathfrak{G}_\lambda^{\mathrm{YM}}, \mathbb{S}_\lambda^{\Sigma,\mathrm{YM}}, P_\lambda^K, P_{\Omega,\lambda}, \mathfrak{m}_\lambda^{\mathrm{YM}}, \mathfrak{H}_\lambda^{\mathrm{YM}}, L_\lambda^{\mathrm{prov}}\right).$$

No new finite physical datum is introduced by this definition. It packages source-certified data according to what downstream continuation must preserve.

The coordinates have different jobs. The set Σ_λ records physically realized detector sectors. The datum $\mathfrak{G}_\lambda^{\mathrm{YM}}$ carries finite gauge/admissibility descent. The spacetime coordinate $\mathbb{S}_\lambda^{\Sigma,\mathrm{YM}}$ carries the incision, regional localization, Cauchy markings, and physical separation data. The projection P_λ^K selects the admissible physical carrier, whereas $P_{\Omega,\lambda}$ selects the complete ground inside that carrier. The closed physical form is $\mathfrak{m}_\lambda^{\mathrm{YM}}$. The symbol $\mathfrak{H}_\lambda^{\mathrm{YM}}$ retains the separately typed relative, coefficient, comparison, and higher data that remain in scope, and $L_\lambda^{\mathrm{prov}}$ records branch and transport provenance.

3.2 Projection firewall

The physical construction contains several projections with different roles. Throughout,

$$P_\lambda^G \neq P_\lambda^K \neq P_{\Omega,\lambda}$$

unless a separate theorem identifies two of them on a declared branch. The relevant nesting is

$$\mathrm{Ran}\, P_{\Omega,\lambda} \subseteq \mathrm{Ran}\, P_\lambda^K.$$

A vector can therefore be physically admissible without belonging to the complete ground.

3.3 Why the physical algebra is not enough

Let $\mathcal{A}_\lambda^{\mathrm{YM}}(O)$ denote the source-certified physical regional observable algebra. It is not defined merely by a compression $P\mathcal{F}(O)P$. Compression is instead used to expose a possible locality defect after physical reduction.

Lemma 3.2 (Compression-Mediation Identity)**.** *Let $P = P^* = P^2$ and let A_L, A_R commute. Then*

$$[PA_LP, PA_RP] = PA_R(I-P)A_LP - PA_L(I-P)A_RP.$$

Proof. Insert $P = I - (I-P)$ between the commuting operators and subtract. □

The identity shows that geometric support separation before reduction is not by itself enough. Physical reduction can communicate through the complement of the selected carrier. The finite locality certificate must therefore control the relevant mediation channels, seam data, and physical support geometry. This is one reason the regional algebra cannot be the complete transport object.

3.4 Source fingerprint and transport obligations

The source recognition theory already identifies the preservation burden for every downstream interface: global-form fidelity, detector and sector fidelity, complete-ground fidelity, coercive fidelity, separately typed cohomological fidelity, refinement fidelity, cofinal fidelity, and provenance fidelity. The current cargo is designed precisely so that these obligations have a place to live.

This yields the guiding rule of continuation:

transport by comparison, never by notation.

Two stages that use the same symbols G, H, P_Ω, κ, or a sector label have not thereby been identified. An actual continuation map must carry the required data and certify their compatibility.

Definition 3.3 (Certified Admissible Continuation)**.** Let $a : \lambda \to \mu$ be an admitted refinement arrow. A certified continuation along a is a branch-appropriate comparison

$$\mathfrak{T}_a : \mathfrak{C}^{\mathrm{YM}}_\mu \rightsquigarrow \mathfrak{C}^{\mathrm{YM}}_\lambda$$

that assigns every in-scope transport coordinate one of the statuses preserved, controlled defect, externalized, or outside scope. At minimum it controls global-form and detector identity, physical sector markings, gauge/admissibility descent, incision-aware spacetime descent, Cauchy markings, P^K, P_Ω, $\mathfrak{m}$, Ward structure, the retained relative and coefficient data at their own types, and provenance.

For a typed structure X, write the transport defect schematically as

$$\Delta_a(X) = T_a X_\mu - X_\lambda T_a$$

whenever both terms are defined in the same comparison space. For composable certified passages $\lambda \xrightarrow{a} \mu \xrightarrow{b} \nu$, one has

$$\Delta_{ba}(X) = T_a \Delta_b(X) + \Delta_a(X) T_b,$$

with the branch-appropriate compositor inserted when the represented passage is weakly coherent. The point is not the elementary identity itself, but that the defect remains typed under composition.

At a single stage there is now a recognized physical cargo; across admitted refinement there is certified continuation. The next section asks whether those continuations assemble coherently enough to produce a genuine continuation geometry.

4 The Admissible Yang–Mills Fibration

The finite recognition theory uses *fiber* first in the weak sense of an inverse image. We have now added the missing ingredient: certified continuation between recognized physical realizations. This section identifies the branch on which that continuation assembles into a genuine fibration.

4.1 The admissibility pseudofunctor

Let $\mathcal{R}_{\mathrm{adm}}$ denote the admitted refinement category for the selected reconstruction branch. For each stage λ, let $\mathrm{Adm}^{\mathrm{YM}}(\lambda)$ be the category of recognized physical Yang–Mills realizations at that stage. For an admitted refinement arrow $a : \lambda \to \mu$, certified continuation supplies contravariant reindexing

$$a^* : \mathrm{Adm}^{\mathrm{YM}}(\mu) \longrightarrow \mathrm{Adm}^{\mathrm{YM}}(\lambda).$$

For composable arrows $\lambda \xrightarrow{a} \mu \xrightarrow{b} \nu$, assume the source-certified comparison data supply an invertible compositor

$$\chi_{a,b} : a^* b^* \Longrightarrow (ba)^*$$

and unit comparisons satisfying the usual pentagon and unit laws.

Proposition 4.1 (Admissibility Pseudofunctor). *On any Cat-valued admissible branch, the assignment*

$$\mathrm{Adm}^{\mathrm{YM}} : \mathcal{R}_{\mathrm{adm}}^{op} \longrightarrow \mathbf{Cat}$$

is a pseudofunctor.

Proof. The object assignment is the finite admissibility category at each refinement stage. Certified continuation supplies the reindexing functors. The source-certified compositors and unit comparisons provide the pseudofunctor comparison cells, and their coherence gives the pentagon and unit identities. □

The proof is formally short because the physical work occurred before the proposition. The nonformal claim is that the admitted Yang–Mills continuation theory has already certified the arrows for which these reindexings exist.

4.2 Grothendieck construction

Let

$$\mathcal{E}^{\mathrm{YM}} = \int_{\mathcal{R}_{\mathrm{adm}}} \mathrm{Adm}^{\mathrm{YM}}$$

be the Grothendieck construction. An object is a pair (λ, X) with $X \in \mathrm{Adm}^{\mathrm{YM}}(\lambda)$. A morphism $(\lambda, X) \to (\mu, Y)$ lying over $a : \lambda \to \mu$ is represented by a fiber morphism $X \to a^*Y$.

Theorem 4.2 (Admissible Yang–Mills Fibration). *The projection*

$$\pi_{\mathrm{Adm}}^{\mathrm{YM}} : \int_{\mathcal{R}_{\mathrm{adm}}} \mathrm{Adm}^{\mathrm{YM}} \longrightarrow \mathcal{R}_{\mathrm{adm}}$$

is a cloven Grothendieck fibration. For every admitted arrow $a : \lambda \to \mu$ and $Y \in \mathrm{Adm}^{\mathrm{YM}}(\mu)$, a chosen cartesian lift is represented by

$$(a, \mathrm{id}_{a^*Y}) : (\lambda, a^*Y) \longrightarrow (\mu, Y).$$

Proof. This is the standard Grothendieck construction for a contravariant pseudofunctor. Pseudofunctoriality identifies pullback along a composite with iterated pullback up to the prescribed compositor, and the resulting universal factorization is exactly the cartesian lifting property. The chosen lifts are coherent under composition by the unitors and compositors. □

The physical certification of the reindexing and the formal Grothendieck construction are different steps. The first is the Yang–Mills content; the second packages it.

4.3 Fibred Yoneda recognition

Lemma 4.3 (Fibred Yoneda Recognition). *For every $\lambda \in \mathcal{R}_{\mathrm{adm}}$, evaluation at the identity induces an equivalence*

$$\mathrm{Cart}_{/\mathcal{R}_{\mathrm{adm}}}\left(\mathcal{R}_{\mathrm{adm}}/\lambda, \mathcal{E}^{\mathrm{YM}}\right) \simeq \mathrm{Adm}^{\mathrm{YM}}(\lambda).$$

Equivalently,

$$\mathrm{PsNat}\left(\mathcal{R}_{\mathrm{adm}}(-,\lambda), \mathrm{Adm}^{\mathrm{YM}}\right) \simeq \mathrm{Adm}^{\mathrm{YM}}(\lambda).$$

Proof. Given $X \in \mathrm{Adm}^{\mathrm{YM}}(\lambda)$, assign to $a : \mu \to \lambda$ the reindexed object a^*X. Pseudofunctorial compositors provide the comparison on triangles and define a cartesian functor from the representable fibration. Conversely, a cartesian functor from the representable fibration is determined, up to canonical equivalence, by the image of id_λ. The two constructions are quasi-inverse. □

Thus one admissible realization determines its representable cartesian continuation profile. The Yang–Mills content is not the fibred Yoneda equivalence but the construction of the admissibility pseudofunctor to which it applies.

4.4 Physical sectors and represented branches

Physical detector sectors remain internal coordinates of the realized object. Ordinary refinement is not a sector transition. Likewise, the cohomological sectorial type $v_D = (r_D, b_D)$ is a different layer again.

The fibration just constructed lives at the admissibility level. Represented descent may be globally strict, corridorwise strict, coefficient-line-banded, or correspondence-valued. Those branch targets need not themselves be one ordinary Cat-valued Grothendieck fibration. This separation becomes load-bearing in the represented AQFT realization.

5 Module-Valued and Grassmannian AQFT Realization

The admissibility fibration records how recognized Yang–Mills objects continue. It does not yet specify the analytic geometry of the carrier being continued. That carrier is not most naturally a primitive Hilbert bundle. The vacuum realization is module-valued, and the varying physical carriers arise as ranges of admissibly varying projections.

5.1 Ambient module and admissible Grassmannian

Let H_λ be the self-dual Hilbert W^*-module associated with the represented vacuum realization at stage λ, over a von Neumann algebra A_λ, and set

$$B_\lambda = \mathrm{End}^*_{A_\lambda}(H_\lambda).$$

Define

$$\mathrm{Gr}_{\mathrm{adm}}(H_\lambda) = \left\{ P \in B_\lambda : \begin{array}{c} P = P^* = P^2, \\ PH_\lambda \text{ satisfies the declared physical admissibility conditions} \end{array} \right\}.$$

The selected physical carrier determines

$$P^K_\lambda \in \mathrm{Gr}_{\mathrm{adm}}(H_\lambda), \qquad H^{\mathrm{phys}}_\lambda = P^K_\lambda H_\lambda,$$

with

$$P_{\Omega,\lambda} H_\lambda \subseteq P^K_\lambda H_\lambda \subseteq H_\lambda.$$

Thus P^K and P_Ω occupy different levels of the projection geometry.

The primitive varying-carrier datum is the Grassmannian-valued projection profile $\lambda \mapsto P^K_\lambda$; the module/Hilbert carrier bundle is its range realization. Coherent Grassmannian transport therefore does not amount to global Hilbert-bundle trivialization.

5.2 Branch-typed represented object

Write

$$\mathbf{A}^{\mathrm{YM,rep}}_\lambda = \left(\mathbf{A}^{\mathrm{YM,obs}}_\lambda ; \mathfrak{E}^{\mathrm{YM}}_\lambda \right),$$

where the enhancement retains the module carrier, Grassmannian projection, line-assisted comparison maps or correspondences, compositors and reassociators, represented Ward implementers, complete-ground projections, and branch/coefficient provenance required by the selected branch.

The four represented target types used here are

$$\begin{aligned} \mathrm{RD1} &: \ \text{global strict represented descent,} \\ \mathrm{RD2} &: \ \text{corridorwise strict descent with coherent seams,} \\ \mathrm{RD3} &: \ \text{central coefficient-line-banded descent,} \\ \mathrm{RD4} &: \ \text{correspondence-valued descent.} \end{aligned}$$

These are represented branch types, not physical detector sectors.

5.3 Observable projection

Define

$$\Pi^{\mathrm{YM}}_{\mathrm{obs}} : \mathbf{A}^{\mathrm{YM,rep}}_{\lambda} \longrightarrow \mathbf{A}^{\mathrm{YM,obs}}_{\lambda}$$

by forgetting the represented enhancement. The observable arrow is an explicit coordinate of the represented transport certificate; it is not manufactured from an arbitrary correspondence.

The simplest local witness of information loss is a line-assisted comparison $J : H\widehat{\otimes}L \to K$. For $z \in U(1)$,

$$\mathrm{Ad}_{zJ} = \mathrm{Ad}_J\,.$$

The observable action can therefore forget represented phase data without proving that those data are trivial in the represented descent problem.

The source prototype is Paper-IV master block H3, with common-refinement comparison supplied on the canonical branch by Paper-IV Theorem 4.1 and the end-to-end closure theorem, Theorem 5.1 [2]. The present theorem separates that represented target from the Cat-valued admissibility fibration.

Theorem 5.1 (Branch-Typed Represented AQFT Realization)**.** *Assume the admissible Yang–Mills continuation system of the preceding section and a branch-supported represented realization satisfying the declared comparison, Grassmannian-carrier, Cauchy, Ward, and provenance compatibilities. Then the observable Yang–Mills realization defines a cartesian AQFT realization over the admissibility fibration, while the represented enhancement descends in the target appropriate to RD1–RD4. No global Hilbert-space trivialization is required.*

This theorem is deliberately branch typed. The reconstruction does not require the global unitary equivalence characteristic of the interaction-picture setting addressed by Haag-type no-interaction results. Likewise, $\Pi^{\mathrm{YM}}_{\mathrm{obs}}$ is an AQFT forgetful projection and carries no claim about distinguishability under physical interrogation.

6 Cauchy-Marked Incised Descent

The represented realization now has the correct carrier geometry. What remains is to identify the finite spacetime structure that survives the thin passage. The source-side order is essential:

$$\text{finite physical demands} \longrightarrow \text{certified cofinal Čech subsystem} \longrightarrow \text{descent.}$$

A Čech nerve is legitimate only after the selected finite subsystem is shown to be directed, test complete, and capable of carrying the complete typed package under restriction.

6.1 Two descent directions

The finite physical package carries two distinct descent problems. The gauge nerve

$$\check{N}^{g}_{\bullet}$$

controls compatibility of local gauge presentations and coefficient data. The spacetime nerve

$$\check{N}^{\mathrm{sp}}_{\bullet}$$

controls localization, regional inclusions, Cauchy development, incision geometry, and the physical separation relation. They act on the same specimen but compare different things.

Because both directions act on the same realization, a mixed square may carry a comparison cell $\Xi_{ij;\alpha\beta}$. Such a cell compares two paths; it is not automatically a cocycle, a cohomology class, or the internal band class b_D.

6.2 Cauchy marking and incision-aware orthogonality

Cauchy data belong to the spacetime nerve. For an admitted Cauchy pair $S \hookrightarrow O$, the finite package carries extension and return maps

$$\mathsf{Ext}_{\lambda} : \mathcal{A}^{\mathrm{YM}}_{\lambda}(S) \to \mathcal{A}^{\mathrm{YM}}_{\lambda}(O), \qquad \mathsf{Res}_{\lambda} : \mathcal{A}^{\mathrm{YM}}_{\lambda}(O) \to \mathcal{A}^{\mathrm{YM}}_{\lambda}(S).$$

The compositions $\mathsf{Res}_\lambda\mathsf{Ext}_\lambda$ and $\mathsf{Ext}_\lambda\mathsf{Res}_\lambda$ test return and reconstruction, respectively; they are not the same condition.

Let $\mathrm{RegLoc}_{\mathrm{inc}}^{\lambda,\mathrm{adm}}$ denote the admitted incised region category. Its orthogonality relation $\perp_{\mathrm{inc}}$ is the thin continuation of the finite physical-separation relation and includes the supported collars, seam neighborhoods, Gauss channels, and declared mediation regions. It is therefore stronger than disjointness of thin cores.

6.3 Locality and comparison defects

Regional inclusion is transported independently of refinement. On a common comparison carrier, the inclusion defect is the difference between the transferred larger-region inclusion and the transferred smaller-region observable. Vanishing of this defect yields thin isotony.

For physically separated regions, the finite Gauss/seam locality certificate controls the commutator of transferred bounded physical observables. The following elementary lemma transfers that control to the comparison realization.

Lemma 6.1 (Strong Stability of Bounded Commutators)**.** *Let $A_{\lambda,i}$ and $B_{\lambda,i}$ be uniformly bounded families with $A_{\lambda,i} - B_{\lambda,i} \to 0$ strongly for $i = 1, 2$. If $[B_{\lambda,1}, B_{\lambda,2}] \to 0$ strongly, then*

$$[A_{\lambda,1}, A_{\lambda,2}] \to 0$$

strongly.

Proof. Write $A_{\lambda,i} = B_{\lambda,i} + E_{\lambda,i}$. Expanding the commutator gives the finite commutator plus three terms containing at least one $E_{\lambda,i}$. Uniform boundedness and strong convergence of the errors force those terms to zero strongly. □

The conclusion is stated on bounded observable elements. Unbounded finite field generators remain confined to their declared common invariant domains.

6.4 Cauchy, dynamics, and Ward transport

After transfer to the common comparison system, define return and time-slice defects by

$$\mathfrak{C}_\lambda^{\mathrm{ret}} = \widehat{\mathsf{Res}}_\lambda\widehat{\mathsf{Ext}}_\lambda - I, \qquad \mathfrak{C}_\lambda^{\mathrm{ts}} = \widehat{\mathsf{Ext}}_\lambda\widehat{\mathsf{Res}}_\lambda - I.$$

Vanishing of both defects gives mutually inverse limiting Cauchy core maps. Where the limiting maps extend normally, one obtains the selected von Neumann time-slice isomorphism.

Regional Cauchy dynamics carries its own comparison defect. Normality, group composition, and continuity in time are inherited only at the scope where the corresponding source hypotheses are certified. Regional Cauchy dynamics is not yet a full Poincaré spectral realization.

Ward structure has two compatibility directions: internal spacetime propagation and cross-refinement transport. For unbounded derivations the theorem additionally requires a common invariant core and closability; no such assumption is hidden in the statement.

The source prototype is Paper-IV master block H2. On the canonical branch, its observable naturality, Cauchy dynamics, and time-slice compatibility are part of Proposition 4.2 and Theorem 5.1 of [2]. The formulation below isolates the spacetime-nerve mechanism from the other Paper-IV master blocks.

Theorem 6.2 (Cauchy-Marked Incised Descent Interface)**.** *Let a recognized Yang–Mills source be transported along a certified cofinal branch, and suppose a package-preserving Čech reduction has been established. Assume the finite incised spacetime descent data carry coherent regional inclusions, incision-aware physical separation with Gauss/seam mediation control, Cauchy extension and return maps, admitted regional Cauchy dynamics, and Ward propagation together with cross-refinement compatibility. If the corresponding comparison defects vanish in the declared bounded/core topology, then the limiting observable core inherits isotony, Einstein causality for $\perp_{\mathrm{inc}}$, selected incised time-slice reconstruction, regional Cauchy dynamics, and observable Ward compatibility. Where the source-certified maps admit normal extension, these structures extend to the regional von Neumann algebras.*

Proof. Regional inclusion compatibility gives isotony. Incision-aware physical separation and the finite Gauss/seam locality certificate give asymptotically vanishing commutators for physical representatives; the bounded commutator lemma transfers this to the comparison realization. Vanishing return and time-slice defects give mutually inverse limiting Cauchy core maps, and normal extension gives the regional von Neumann time-slice statement. The dynamical comparison defect gives regional Cauchy evolution, while the internal and cross-refinement Ward defects give the limiting observable Ward action on the declared core. □

The five thin AQFT properties are therefore different markings of one finite physical spacetime descent object rather than independent endpoint assumptions.

7 Comparison–Mosco Transport and the Complete Ground

The spacetime side of the thin passage does not by itself control the complete ground. Because the physical carriers vary with refinement, a preferred vacuum vector is too representation dependent to serve as the primitive datum. The stable analytic object is the pair

$$(\mathfrak{m}_\lambda^{\mathrm{YM}}, P_{\Omega,\lambda}).$$

Let $\mathcal{H}_n^K$ be the selected physical carriers and $\mathcal{H}_0^K$ the thin comparison carrier. The comparison maps $J_n : \mathcal{H}_n^K \to \mathcal{H}_0^K$ are analytic comparison maps; they need not be global unitary trivializations of the represented descent system.

The main text uses the Phase III.A.2 numbering:

M1. quasi-unitary comparison and domain control;

M2. relative form defect;

M3. comparison-weak compactness;

M4. complete-ground projection transport;

M5. uniform off-ground coercivity;

M6. inclusion-compatible recovery and monotone exhaustion.

Appendix B records the source lock and the collision with the later Paper-IV M-package.

Theorem 7.1 (Thin Complete-Ground Identification)**.** *Under M1–M6,*

$$\mathfrak{m}_n^{\mathrm{YM}} \xrightarrow{\mathrm{M},J} \mathfrak{m}_0^{\mathrm{YM}},$$

and

$$\mathfrak{m}_0^{\mathrm{YM}}[\Psi] \geq \kappa_* \|(I - P_{\Omega,0})\Psi\|^2, \qquad \ker \mathfrak{m}_0^{\mathrm{YM}} = \operatorname{Ran} P_{\Omega,0}.$$

Proof. Comparison-weak compactness and the relative form control give the Mosco lower bound. Inclusion-compatible recovery gives the Mosco upper bound. Complete-ground transport identifies the limiting ground/off-ground splitting, and the uniform finite estimate passes to the limit to give

$$\mathfrak{m}_0^{\mathrm{YM}}[\Psi] \geq \kappa_* \|(I - P_{\Omega,0})\Psi\|^2.$$

Hence every zero-form vector lies in $\operatorname{Ran} P_{\Omega,0}$. Conversely, ground-compatible recovery transports finite ground-null vectors to the limiting ground and gives $\operatorname{Ran} P_{\Omega,0} \subseteq \ker \mathfrak{m}_0^{\mathrm{YM}}$. The two inclusions give the exact kernel identity. □

The associated closed-form theorem gives the comparison resolvent and semigroup conclusions at the imported scope. The present paper does not require a separate Riesz argument to establish the thin complete ground.

Two firewalls remain. First, P_0^K and $P_{\Omega,0}$ remain different projection levels. Second, the positive form estimate is a Hamiltonian/form threshold and not yet an invariant relativistic mass threshold.

8 Admissible Thin Yang–Mills Reconstruction and Observable Globalization

The two limiting problems have now been solved at their proper levels. The main theorem is therefore an assembly theorem.

Theorem 8.1 (Admissible Thin Yang–Mills Reconstruction)**.** *Let $G_\Gamma = \widetilde{G}/\Gamma$ be a compact connected simple global form. Assume a complete recognized source certificate, physicalization into a transport-ready family, certified continuation defining the admissibility pseudofunctor on the selected Cat-valued branch, a branch-appropriate represented AQFT realization with admissible Grassmannian carrier transport, a package-preserving finite Čech reduction, the hypotheses of Theorem 6.2, and the comparison–Mosco hypotheses M1–M6 of Paper-IV Criterion 3.3 [2].*

Then there exists a thin observable Yang–Mills net

$$\mathcal{A}_0^{\mathrm{YM}} : \left(\mathrm{RegLoc}_{\mathrm{inc}}^{0,\mathrm{adm}}, \perp_{\mathrm{inc}}\right) \longrightarrow W^*\mathrm{Alg}_n$$

with isotony, Einstein causality for $\perp_{\mathrm{inc}}$, selected incised time-slice, regional Cauchy dynamics, observable Ward compatibility, and complete-ground structure

$$\mathfrak{m}_0^{\mathrm{YM}}[\Psi] \geq \kappa_* \|(I - P_{\Omega,0})\Psi\|^2, \qquad \ker \mathfrak{m}_0^{\mathrm{YM}} = \mathrm{Ran}\, P_{\Omega,0}.$$

The represented realization remains in the target appropriate to RD1–RD4 and need not admit one global Hilbert-space trivialization.

Proof. The recognized source certificate supplies the finite physical data and source fingerprint. Certified continuation and the Grothendieck theorem produce the admissibility fibration on the Cat-valued admissibility layer. The branch-typed realization carries the objects into the represented AQFT architecture while retaining the module/Grassmannian carrier data required by the chosen branch. The Cauchy-marked descent theorem gives the thin regional observable structure, and the comparison–Mosco theorem gives the limiting closed physical form, the uniform off-ground estimate, and the exact complete-ground kernel. Compatibility of the regional inclusion data yields the asserted orthogonal Haag–Kastler functor. No step requires strictification of the complete represented enhancement. □

The theorem is general in G_Γ but conditional at the physical realization and continuation interfaces. Universal compact-simple recognition removes algebraic detectability and electric nonemptiness as family-level obstructions; the remaining conditions define the theorem's domain of applicability rather than a family-level exclusion theorem.

8.1 Observable globalization and represented nontriviality

Where the admitted incised region category supports the corresponding directed inductive construction, the regional observable net determines the quasi-local observable algebra

$$\mathcal{A}_0^{\mathrm{YM,qloc}} = \varinjlim_{O} \mathcal{A}_0^{\mathrm{YM}}(O)$$

in the source-certified category. "Global" here means global at the observable AQFT coordinate.

Proposition 8.2 (Observable Strictness Does Not Force Represented Strictness)**.** *Strict composition of the observable coordinate imposes no vanishing condition on represented enhancement data omitted by $\Pi_{\mathrm{obs}}^{\mathrm{YM}}$ unless a separate comparison theorem relates those data to observable composition.*

Proof. The observable projection retains only the observable coordinate. Strict composition in its image constrains the retained observable arrows. A condition on an omitted represented coordinate requires an additional comparison theorem. □

Thus strict observable AQFT does not imply global represented unitary equivalence, and a global observable ground-state datum does not imply global trivialization of the richer module-valued vacuum realization.

The mechanism can now be read backward: the thin observable theory succeeds because the finite construction transports more structure than the observable endpoint ultimately displays. In this sense, the continuation is faithful enough before the observable projection becomes forgetful; this is an interpretive statement, not a claim of categorical faithfulness.

9 Recognition, Witnesses, and the Current Realization Boundary

The main reconstruction theorem is conditional in exactly the place where the physics becomes group dependent. The current status has three complementary levels: universal compact-simple algebraic/electric entry, an explicit $SU(2)$ coefficient-band witness, and the canonical $SU(3)$ end-to-end thin reconstruction witness.

Corollary 9.1 (Universal Compact-Simple Entry)**.** *Every compact connected simple global form satisfies the algebraic and electric entry hypotheses of the main reconstruction theorem. This is precisely the universal part of Recognition Theorem 15.3(i)–(ii) [1].*

The remaining source-side burden begins with magnetic physicalization and continues through nonzero physical ranks, complete-ground and coercive control, realization of the prescribed sectorial type, quantitative seam survival, coherent finite-demand extension, and one uniform selection cutoff. Failure to verify one such route is not a group-level nonexistence theorem; positive realization requires one certified branch, while nonrealizability would require exhaustion of all admissible branches.

9.1 $SU(2)$ coefficient-band witness

The simply connected $SU(2)$ construction has a different role from the selected $SU(3)$ branch. It exhibits two physically realized coefficient laws with the same finite analytic shadow through fixed-stratum tensor depth:

$$(0,0), \qquad (0,[\Omega_1]_{\mathrm{band}}),$$

with $[\Omega_1]_{\mathrm{band}} \neq 0 \in H^3(B\mathbb{Z}_2; U(1))$.

Corollary 9.2 ($SU(2)$ Band-Neutrality Witness)**.** *By Recognition Theorem 13.3 and Proposition 13.4 of [1], the paired simply connected $SU(2)$ fixed-stratum depth branches realize the types*

$$(0,0), \qquad (0,[\Omega_1]_{\mathrm{band}})$$

with identical finite analytic shadows and the same uniform finite coercive profile. Hence nontrivial represented band class does not by itself force a change of the finite analytic coercive data in this explicit construction.

The family is not cofinal in the full multiparameter geometric scale category. It is therefore not a thin-reconstruction theorem for $SU(2)$.

9.2 Canonical $SU(3)$ witness

Recognition Theorem 14.4 of [1] supplies the selected simply connected $SU(3)$ source packet carrying the three triality sectors simultaneously,

$$K_{\triangle}^{SU(3)} = \mathbb{C}\Omega \oplus S_0 \oplus S_1 \oplus S_2, \qquad \Sigma_{SU(3)} = \{0,1,2\}.$$

Its positive-thickness source family has a uniform retained coercive floor and a preserved meridional detector for the selected nontrivial triality branches $m = 1, 2$. The downstream canonical reconstruction discharges the complete continuation, Cauchy, represented-descent, comparison–Mosco, and thin observable hypotheses; see Paper-IV Theorem 5.1 and Corollary 5.2 of [2].

Theorem 9.3 (Canonical $SU(3)$ End-to-End Witness)**.** *For each selected branch $m \in \{1,2\}$, the canonical dyadic physical $SU(3)$ reconstruction satisfies the hypotheses of the main reconstruction theorem. Consequently there exists a thin observable incised Haag–Kastler net together with a chart–corridor-banded represented realization, retained discrete branch detector, and preserved complete-ground structure.*

This is the strongest currently certified end-to-end witness in the present paper. The observable endpoint is global in the quasi-local AQFT sense, while the represented endpoint remains global only in its branch-appropriate chart–corridor-banded sense.

9.3 Current realization boundary

For a new compact connected simple global form the algebraic search is already finished. The remaining constructive program is to provide a sector-complete magnetic physicalization, realize the required relative and coefficient-band data, obtain quantitative extension and uniform compact selection, justify the finite-demand/Čech comparison, preserve the correct global form, and verify the continuation, Cauchy, and comparison–Mosco hypotheses of the main theorem.

The theorem therefore supplies a domain of applicability rather than a pathology ledger. A realization outside that domain is not ruled out; the present theorem simply does not certify its thin continuation. Recognition Theorem 16.6 and the group-level quantifier of Warning 15.4 [1] are the source-side counterparts of this boundary.

10 Literature Interface

The present construction meets several established mathematical frameworks, but none of them is used as a substitute for the physical continuation argument.

Locally covariant and orthogonal AQFT. The thin observable endpoint is naturally compared with the Brunetti–Fredenhagen–Verch formulation of locally covariant quantum field theory, in which a QFT is organized functorially over admissible spacetime embeddings [3]. The orthogonality relation

$$\perp_{\mathrm{inc}}$$

is also naturally placed beside the orthogonal-category formulation of algebraic quantum field theory developed by Benini, Schenkel, and Woike [5]. The present orthogonality relation is not introduced abstractly: it is inherited from the finite physical separation certificate, including the incision, collars, seams, and Gauss-mediated support conditions.

Fibered and higher AQFT. Benini and Schenkel formulate quantum field theories on categories fibered in groupoids over spacetime in order to organize additional geometric structures such as bundles, connections, and spin structures [4]. The admissible Yang–Mills fibration is analogous in form but different in role: its base is the admitted refinement geometry and its fibers are physically certified realizations. Homotopy and higher AQFT frameworks show more generally that strict observable structures may coexist with coherent higher data [6]; the represented enhancement used here is not identified with a specific homotopy-AQFT model category unless such an identification is separately constructed.

Gluing. Recent AQFT gluing results emphasize that the global theory is not generally recovered by gluing only the underlying local algebras; the AQFT structures themselves must be glued [7]. The present reconstruction pushes this distinction one level earlier: before the thin AQFT exists, the finite Yang–Mills realization already carries gauge, Cauchy, seam, Ward, spectral, and provenance data that must survive the limiting passage.

Fibred categories and Yoneda. The passage from a Cat-valued pseudofunctor to its Grothendieck fibration and the fibred Yoneda lemma are standard categorical tools; we follow the formulation reviewed by Streicher [8]. The Yang–Mills content lies in proving that the certified refinement maps actually define the required pseudofunctor.

W^*-modules and correspondences. Self-dual Hilbert modules over von Neumann algebras go back to the foundational work of Paschke and are closely related to concrete von Neumann modules and correspondences [9, 10, 11]. W^*-correspondences provide a standard language for normal bimodule transport between von Neumann algebras [12]. In the present reconstruction these structures support the module-valued vacuum carrier and the admissible Grassmannian range geometry; the construction-specific reason that these are the correct Yang–Mills carriers is imported from the frozen vacuum/realization papers rather than from the general module literature.

Varying-carrier form convergence. The comparison–Mosco package used in Section 7 belongs to the broader analytic theory of energy forms and operators on varying Hilbert spaces, where quasi-unitary comparison can imply generalized norm-resolvent, semigroup, spectral-projection, and spectral convergence [13]. The present theorem nevertheless uses the exact M1–M6 package imported from the frozen Yang–Mills reconstruction.

Haag. Haag-type no-interaction theorems are relevant only at the narrow interface stated in Section 5: they concern strong global unitary identifications characteristic of the interaction-picture setting. The present construction does not require that identification [14, 15]. No stronger claim about Haag's theorem is used.

The admissible-fibration formulation arose independently from the reconstruction problem developed in the Wilson–tube series. The comparisons above locate the resulting structure within a wider AQFT and operator-algebraic landscape; they do not supply the physical certification on which the main theorem depends.

A Source and Notation Concordance

This appendix fixes notation imported from the finite recognition, physicalization, Grassmannian-carrier, represented-AQFT, and comparison–Mosco stages. Its purpose is to prevent distinct source objects from being collapsed by shorthand.

A.1 Global form, detector sectors, and sectorial type

Let

$$G_\Gamma = \widetilde{G}/\Gamma, \qquad \Gamma \le Z(\widetilde{G}),$$

be a compact connected simple global form. The physical detector-sector set is denoted Σ_D, with a typical label $\chi \in \Sigma_D$. No universal cardinality is assumed. For the selected simply connected $SU(3)$ witness, $\Sigma_{SU(3)} = \{0, 1, 2\}$.

The cohomological sectorial type is

$$v_D = (r_D, b_D),$$

with

$$r_D = [\Delta\zeta_D, \tau_D]_{\mathrm{ref/adm}} \in H^2(\mathrm{Fib}(Q_D)), \qquad b_D = [\Omega_D]_{\mathrm{band}} \in H^3(BB_D; U(1)).$$

The two classes live in different complexes. No equality, implication, or transgression between them is assumed without an explicit map; this is the source firewall of Recognition Theorem 5.4 [1].

A.2 Recognized source certificate

The source-side recognition datum is

$$\mathcal{C}_{\mathrm{src}} = \left(G_\Gamma, \Pi_\Gamma^{\mathrm{sec}}, \mathcal{J}, q, v, M, \kappa, (x_D)_{D\in\mathcal{J}}\right),$$

certified by

$$\mathrm{RecCert}_{\mathcal{J}}(G_\Gamma; \Pi_\Gamma^{\mathrm{sec}}, q, v, M, \kappa).$$

The source clauses RC1–RC7 certify genuine algebraic detection, physical finite realization, realized sectorial type, full compatibility, quantitative seam survival, uniform selection, and demand completeness.

A.3 Finite demands and two Cech directions

The native source index system is a finite-demand system inside the full geometric scale category. A finite Cech subsystem may replace it only after directedness, test completeness, and preservation of the full typed package are proved; this is exactly the scope of Recognition Proposition 10.6 in [1]. The gauge and spacetime nerves are denoted

$$\check{N}^{g}_{\bullet}, \qquad \check{N}^{\mathrm{sp}}_{\bullet},$$

and remain distinct. Cauchy data are markings of the spacetime nerve rather than a third Cech direction.

A.4 Projection notation

Three projection levels are permanently distinguished:

$$\begin{aligned} P^{G}_{\lambda} &= \text{Gauss/constraint projection}, \\ P^{K}_{\lambda} &= \text{selected physical carrier}, \\ P_{\Omega,\lambda} &= \text{complete-ground projection}. \end{aligned}$$

The standing firewall is

$$P^{G}_{\lambda} \neq P^{K}_{\lambda} \neq P_{\Omega,\lambda}$$

unless a separate theorem identifies two of them on a declared branch. The relevant nesting is

$$\operatorname{Ran} P_{\Omega,\lambda} \subseteq \operatorname{Ran} P^{K}_{\lambda}.$$

A.5 Physical algebra and transport-ready cargo

The regional observable algebra $\mathcal{A}^{\mathrm{YM}}_{\lambda}(O)$ is the source-certified physical algebra and is not defined merely as a compression $P\mathcal{F}(O)P$. The present paper packages the continuation data as

$$\mathfrak{C}^{\mathrm{YM}}_{\lambda} = \left(\Sigma_{\lambda}, \mathfrak{G}^{\mathrm{YM}}_{\lambda}, \mathbb{S}^{\Sigma,\mathrm{YM}}_{\lambda}, P^{K}_{\lambda}, P_{\Omega,\lambda}, \mathfrak{m}^{\mathrm{YM}}_{\lambda}, \mathfrak{H}^{\mathrm{YM}}_{\lambda}, L^{\mathrm{prov}}_{\lambda}\right).$$

No new finite physical datum is introduced by this packaging.

A.6 Admissibility fibration and represented branch

At stage λ, the admitted realization category is $\mathrm{Adm}^{\mathrm{YM}}(\lambda)$. On a Cat-valued branch, admitted refinement gives a pseudofunctor

$$\mathrm{Adm}^{\mathrm{YM}} : \mathcal{R}^{op}_{\mathrm{adm}} \to \mathbf{Cat}$$

and hence the Grothendieck fibration

$$\pi^{\mathrm{YM}}_{\mathrm{Adm}} : \int \mathrm{Adm}^{\mathrm{YM}} \to \mathcal{R}_{\mathrm{adm}}.$$

Represented descent is separately typed by RD1–RD4 and must not be identified with the admissibility fibration.

A.7 Self-dual module and Grassmannian carrier

The represented vacuum carrier is a self-dual Hilbert W^{*}-module H_{λ} over A_{λ}, with endomorphism algebra

$$B_{\lambda} = \mathrm{End}^{*}_{A_{\lambda}}(H_{\lambda}).$$

The primitive varying-carrier datum is the admissible projection profile

$$P^{K}_{\lambda} \in \mathrm{Gr}_{\mathrm{adm}}(H_{\lambda}),$$

with physical range $P^{K}_{\lambda}H_{\lambda}$. An ordinary Hilbert bundle is therefore a range realization of the Grassmannian-valued family, not the primitive datum.

A.8 Permanent typing rules

$$\chi \neq r_D \neq b_D \neq \mathrm{RD}j, \qquad P^G \neq P^K \neq P_\Omega, \qquad \check{N}^g_\bullet \neq \check{N}^{\mathrm{sp}}_\bullet.$$

Further distinctions used throughout are:

$$\begin{aligned}
\text{regional inclusion} &\neq \text{refinement},\\
\text{refinement} &\neq \text{physical sector transition},\\
\text{Grassmannian transport} &\neq \text{Hilbert-bundle trivialization},\\
\text{observable strictness} &\not\Rightarrow \text{represented strictness},\\
\text{complete-ground persistence} &\neq \text{vacuum lineage},\\
\text{Hamiltonian/form threshold} &\neq \text{invariant-mass threshold}.
\end{aligned}$$

B Comparison–Mosco Hypotheses and Source Lock

The present manuscript uses the Phase III.A.2 primitive comparison–Mosco numbering of Paper-IV Criterion 3.3 and Theorem 3.4 in [2]. A later Paper-IV registry repartitions the same broad analytic burden under another M1–M6 enumeration; the two numberings are not interchangeable.

B.1 Moving carriers

Let $\mathcal{H}_n^K$ be the selected finite physical carriers and $\mathcal{H}_0^K$ the thin comparison carrier. Let q_n be closed nonnegative forms with complete-ground projections $P_{\Omega,n}$. In the main-text notation,

$$q_n \leftrightarrow \mathfrak{m}_n^{\mathrm{YM}}, \qquad q_0 \leftrightarrow \mathfrak{m}_0^{\mathrm{YM}}.$$

B.2 M1–M6

M1 — Quasi-unitary comparison and domain control. There are uniformly controlled comparison maps

$$J_n : \mathcal{H}_n^K \to \mathcal{H}_0^K$$

that place the moving form domains into a common or graph-dense comparison geometry.

M2 — Relative form defect. There is a defect $\theta_n \to 0$ controlling the difference between transported finite forms and the limiting form in the common graph norm.

M3 — Comparison-weak compactness. Every graph-bounded moving family has a comparison-weakly convergent subnet or, in the sequential formulation, a convergent subsequence at the certified scope.

M4 — Complete-ground projection transport. The complete-ground projections are exactly intertwined or converge strongly in the comparison geometry:

$$J_n P_{\Omega,n} J_n^{-1} \longrightarrow P_{\Omega,0}.$$

M5 — Uniform off-ground coercivity. There exists one $\kappa_* > 0$ such that on a cofinal tail

$$q_n[u] \geq \kappa_* \|(I - P_{\Omega,n})u\|^2.$$

M6 — Inclusion-compatible recovery and monotone exhaustion. Every limiting form vector admits recovery vectors compatible with regional inclusions and the selected exhaustion, with comparison-strong convergence and the source-certified limsup control.

Theorem B.1 (Imported Comparison–Mosco Transfer). *Assume M1–M6. Then the transported forms converge in the comparison–Mosco sense to a unique closed nonnegative form q_0,*

$$q_0[u] \geq \kappa_* \|(I - P_{\Omega,0})u\|^2, \qquad \ker q_0 = \operatorname{Ran} P_{\Omega,0},$$

and the associated resolvents and heat semigroups converge strongly in the comparison sense. Under the stronger uniform relative-form estimate, one obtains norm-resolvent convergence.

The lower-bound, exact kernel, and resolvent statements are conclusions, not additional members of M1–M6.

C Recovery of the Original Paper-IV Thick-to-Thin Reconstruction

The present paper reorganizes the thick-to-thin passage through recognition, admissible continuation, fibred realization, Cauchy-marked descent, and comparison–Mosco transport. It does not replace or weaken the original Paper IV. The load-bearing source anchors are Phase III.A.2 Theorems 2.6–2.7, 3.1, 3.4, 3.7–3.10, 4.1, Proposition 4.2, Corollary 4.5, and Theorem 5.1 of [2]. Every theorem-level conclusion of original Paper IV used in the canonical $SU(3)$ thick-to-thin reconstruction is recovered at its original scope as a specialization of the present framework.

C.1 Master-block crosswalk

$H0$	recognized source and transport-ready finite physical cargo
$H1$	observable comparison, normal envelopes, and thin observable globalization
$H2$	incision-aware locality, Cauchy dynamics, and selected time-slice
$H3$	admissibility fibration, branch-typed represented descent, and ground transport
$H4$	retained entropy/state-surgery coordinate, invoked branchwise under the original hypotheses
$H5$	Gauss–Ward, collar, boundary, and graph-domain transport
$H6$	comparison–Mosco, TT1–TT2, and exact complete-ground kernel
$H7$	detector fidelity and sector conservativity
$H8$	admissible common-refinement and reconstruction-choice independence (Theorems 2.6 and 4.1 of [2])

Theorem C.1 (Recovery of Original Paper IV). *Let $\mathcal{D}_{\mathrm{IV}}$ be a positive-thickness Yang–Mills datum satisfying the original Paper-IV master hypotheses H0–H8. Then $\mathcal{D}_{\mathrm{IV}}$ determines a recognized transport-ready datum satisfying the corresponding hypotheses of the present reconstruction theorem. The present framework therefore yields the same ordinary thin observable Haag–Kastler system and complete-ground form structure as original Paper IV. Where the original branch additionally carries entropy/state-surgery, Gauss–Ward, sector-detector, and reconstruction-choice data, those coordinates are retained and recover the corresponding original Paper-IV outputs under their original hypotheses. For the canonical $m = 1, 2$ branches, the source conclusion is Paper-IV Theorem 5.1; admissible reconstruction-choice independence is Theorem 4.1 [2].*

Entropy-surgery block. The H4 entropy/state-surgery package is covered by retention and branchwise application rather than being redefined as ordinary AQFT structure. This is deliberate:

$$\text{state bordism} \neq \text{ordinary observable AQFT}.$$

Non-supersession. The original five-paper sequence remains the primary construction. The present paper identifies the general continuation geometry in which the Paper-IV mechanism lives. Thus coverage does not mean supersession.

D Branch-Typed Represented Descent and Grassmannian Carrier Geometry

The admissibility fibration lives at the Cat-valued physical-continuation layer. Represented descent is richer and is typed by the target needed to carry the actual continuation data.

D.1 Local descent versus scale transport

Local represented transition maps may be invertible where the local reconstruction requires groupoidal descent. Across scales, however, the comparison may be a line-assisted map

$$J_a : H_\varepsilon \widehat{\otimes} L_a \longrightarrow H_{\varepsilon'}$$

and need not be invertible. Hence local gluing and scale transport are distinct.

D.2 Carrier leakage and ground migration

For a selected physical transport T_a, define the carrier leakage

$$\mathcal{L}_a^{\mathrm{car}} = (I - P_y^K) T_a P_x^K.$$

Zero carrier leakage expresses preservation of the selected physical range and does not require surjectivity. Ground migration is separately measured inside the destination physical carrier and is not the same defect.

D.3 Represented branch types

$$\begin{aligned}
\mathrm{RD1}: &\quad \text{global strict represented descent,}\\
\mathrm{RD2}: &\quad \text{corridorwise strict descent with coherent seams,}\\
\mathrm{RD3}: &\quad \text{coefficient-line-banded descent,}\\
\mathrm{RD4}: &\quad \text{correspondence-valued descent.}
\end{aligned}$$

These are represented target types, not physical detector sectors and not cohomological classes. The canonical selected $SU(3)$ branch lies in RD3.

Theorem D.1 (Branch-Typed Represented Coverage)**.** *Let $\pi^{\mathrm{YM}}_{\mathrm{Adm}} : \mathcal{E}^{\mathrm{YM}} \to \mathcal{R}_{\mathrm{adm}}$ be the admissibility fibration. Assume each certified physical continuation admits a represented lift carrying the selected physical projection, complete-ground marking, coefficient/provenance data, and declared local descent structure. Then the represented lift is interpreted in the branch RD1–RD4 appropriate to its actual comparison structure, while the ordinary observable coordinate may still descend through $\Pi^{\mathrm{YM}}_{\mathrm{obs}}$ whenever the observable arrows satisfy the Haag–Kastler comparison hypotheses.*

E Dependency Graph, Theorem Status, and Domain of Applicability

This appendix records proof provenance and applicability. The published theorem typography does not distinguish imported from new results; the distinction is maintained in the source ledger.

E.1 Status conventions

Status	Meaning
CERTIFIED	proved here or standard after all physical hypotheses are verified
IMPORTED	substantive theorem proved in a frozen upstream source
CONDITIONAL	proved under a declared physical, analytic, or branch hypothesis
EXPORTED	retained here but theorem deliberately deferred downstream
OUTSIDE SCOPE	not a proof obligation of the present manuscript

E.2 Theorem-status ledger

ID	Result	Status
T2.1	Compact-Simple Entry	IMPORTED
D3.1	Transport-ready finite physical cargo	NEW RECOGNITION DEFINITION
L3.2	Compression-Mediation Identity	CERTIFIED / DIRECT
D3.3	Certified admissible continuation	NEW DEFINITION
P4.1	Admissibility pseudofunctor	CERTIFIED GIVEN SOURCE COHERENCE
T4.2	Admissible Yang–Mills fibration	CERTIFIED / STANDARD
L4.3	Fibred Yoneda recognition	CERTIFIED / STANDARD
T5.1	Branch-typed represented AQFT realization	CONDITIONAL / CROSSWALK
T6.2	Cauchy-marked incised descent interface	CERTIFIED GIVEN COMPARISON HYPOTHESES
T7.7	Thin complete-ground identification	CERTIFIED / IMPORTED ANALYTIC CORE
T8.1	Admissible thin Yang–Mills reconstruction	NEW ASSEMBLY THEOREM
T9.3	Canonical SU(3) end-to-end witness	IMPORTED APPLICATION + PRESENT ASSEMBLY
TC.1	Recovery of original Paper IV	NEW COVERAGE THEOREM
TD.1	Branch-typed represented coverage	NEW ORGANIZATIONAL THEOREM

E.3 Domain of applicability

The hypotheses of the main reconstruction theorem define its certified domain of applicability. A realization outside this domain is not thereby ruled out; it means only that the present theorem does not certify its continuation to the thin observable endpoint.

Interface	Applicability requirement
A1	detector packet survives the physical realization at the required sector scope
A2	a physically admissible magnetic realization exists
A3	finite carrier, complete ground, and coercive certificate satisfy source conditions
A4	prescribed relative and coefficient-band data are physically realized at their own types
A5	finite-demand system admits the required quantitative extensions
A6	seam and one-sided target-complement control survive with the required reserve
A7	certified continuation preserves every fingerprint coordinate required downstream
A8	admitted continuation maps satisfy the coherence required by the selected admissibility branch
A9	represented realization is interpreted in a target capable of carrying its actual descent type
A10	finite-demand family admits the claimed package-preserving Čech reduction
A11	regional locality, Cauchy, dynamical, and Ward comparison hypotheses hold
A12	comparison–Mosco hypotheses M1–M6 hold
A13	the regional system has the directedness needed when quasi-local globalization is invoked

Schematically,

$$X \in \mathfrak{D}_{\mathrm{Thm}\,8.1} \Longrightarrow \text{certified thin reconstruction.}$$

The converse is neither claimed nor required. A branch outside the applicability domain may still exist physically; it is simply not certified by Theorem 8.1 as stated.

E.4 Export ledger

The full module-valued vacuum-lineage problem is exported downstream. The invariant mass bridge is exported to the invariant spectral-transfer paper. No comparison map between $r_D \in H^2(\mathrm{Fib}(Q_D))$ and $b_D \in H^3(BB_D; U(1))$

is supplied here. Haag duality is outside scope. The observable projection is not interpreted as a measurement quotient.

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