The Geometry of Admissible Transport 0

Finite Cohomological and Coercive Recognition for Semisimple Gauge Systems

Finite source certificates for downstream reconstruction

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Abstract. This paper establishes the finite source-recognition layer of the Geometry of Admissible Transport program for every compact connected semisimple global form $G = (G_1 \times \cdots \times G_N)/\Gamma$. It proves the existence of a finite genuine sector-complete detector packet and a universal weighted Casimir seed, while keeping physical realization as a separate certified problem. Four recognition coordinates remain distinct: the filling obstruction, the relative mapping-fiber class in degree two, the internal coefficient-band associator in degree three, and the electric/coercive coordinate. A certified tensor selector transgresses the realized degree-three class along a marked mixed loop and divides the selected tensor data into three realized sectors without identifying the two native cohomology complexes. A separately calibrated regional–refinement surface also defines the mixed phase $\mathbf{\Pi}_{\square}$; when comparator validity passes, a nontrivial phase obstructs finite admissible decoupling. The converse is not claimed. Analytically, the quadratic Casimir supplies the universal electric seed, while the exact ultraviolet–infrared mixed block, complete-ground projection, independent target complement, and seam reserves remain explicit. Compact typed good loci and coherent quantitative extension then produce compatible families with one persistent positive finite floor. Distinct gauge and spacetime Čech directions carry the Gauss–Ward and finite AQFT probe interfaces. The resulting certificate exports typed inputs for later reconstruction without choosing a unique GNS realization or claiming a continuum net, Grassmannian Fredholm fiber, Klauder projector, invariant relativistic spectrum, or independent Yang–Mills mass-gap theorem.

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1 Introduction

1.1 The source problem before transport

Admissible transport begins only after there is something admissible to transport. In a gauge theory this preliminary point is easy to blur. A Lie group supplies representations; a finite Hamiltonian supplies eigenvalues; a Cech nerve supplies cochains; and a Wilson loop supplies a gauge-invariant formal observable. These ingredients originate in the gauge, Wilson-loop, and Hamiltonian lattice formulations [17, 16, 7]; none of them, by itself, is a physically typed source for a reconstruction theorem.

The source must survive several interfaces at once. The representations must be genuine for the chosen global form rather than merely for its simply connected cover. The selected detector sectors must remain nonzero after physical reduction. A formal refinement mismatch must admit a correction in the declared admissible complex. The resulting coefficient lines must carry their actual reassociator. Regional localization, detector readout, Gauss reduction, and Ward identities must be exposed rather than compressed into a single finite matrix package. Finally, the physical Hamiltonian must retain a positive lower bound on the complement of its *complete* ground space, including every one-sided seam and ultraviolet–infrared mixed block used in the comparison.

This paper organizes those demands into one finite recognition architecture. Its theorem-bearing object is not a group name, a cocycle, or a gap number in isolation. It is a typed finite proposal together with a physically admissible realization, its local interface, its two native cohomological coordinates, its selected band loop, its calibrated mixed surface, its exact ground projection, and its coercive ledger.

This is the role of GAT 0 in the series. Its finite source objects and dependency boundaries consolidate the architecture developed across [8, 12, 11, 10, 13, 9]. Later papers study transport, Fredholm and Grassmannian realization, cofinal and continuum reconstruction, operator-algebraic completion, and invariant spectral consequences. GAT 0 first determines the finite cargo and the exact hypotheses under which that cargo is ready to be consumed.

1.2 Why the gauge group is not yet a certificate

Let

$$\widetilde{G} = \prod_{\nu=1}^{N} G_\nu, \qquad G = \widetilde{G}/\Gamma, \qquad \Gamma \subset Z(\widetilde{G}),$$

where each G_ν is compact, simply connected, and simple. The factors may belong to any classical or exceptional Lie family. Representation theory determines the genuine highest weights and residual central characters, but it does not determine:

1. whether the corresponding detector sectors survive Gauss reduction and physical compression with nonzero rank;
2. whether the magnetic and regional blocks exist on the required thick carrier;
3. whether a refinement mismatch can be filled inside the physically allowed correction complex;
4. whether the relative lift and coefficient-band reassociator are simultaneously realized;
5. whether independently assembled endpoint complements control every one-sided seam;
6. whether one positive coercive margin survives the ultraviolet–infrared mixture; or
7. whether the finite packages extend as one compatible family over a demand-complete refinement system.

The first algebraic burden can nevertheless be discharged uniformly. Every compact connected semisimple global form admits a finite faithful unitary representation. After decomposition into irreducibles and finite enlargement by one genuine irreducible in every residual central degree, one obtains a sector-complete packet whose lifted common kernel is exactly Γ. This is the semisimple detector-packet theorem (theorem 13.3). It proves algebraic availability, not physical survival.

The same factorization supplies a universal electric coordinate. For positive normalizations a_ν and factor Casimirs $C_{2,\nu}$, set

$$C_2^G(\lambda) = \sum_{\nu=1}^{N} a_\nu C_{2,\nu}(\lambda_\nu), \qquad c_G = \min_\nu a_\nu c_\nu > 0,$$

where c_ν is the least nonzero Casimir eigenvalue of G_ν. Every nontrivial genuine quotient label satisfies $C_2^G(\lambda) \geq c_G$. The Casimir is therefore the universal electric coercive agent. The underlying compact semisimple representation theory is standard [4, 3]. The Casimir bound is not a surrogate for the magnetic block, mixed corner, exact ground projection, or seam reserve.

1.3 Four native coordinates and the tensor selector

Fix a finite demand diagram D and a realization-complete proposal R_D. The recognition problem has four native coordinates:

$$o_D^{\mathrm{fill}}, \qquad r_D \in H^2(\mathrm{Fib}(Q_D)), \qquad b_D \in H^3(B\mathcal{B}_D; U(1)), \qquad (C_D^{\mathrm{el}}, P_{\mathrm{vac},D}, \kappa_D).$$

The filling obstruction o_D^{fill} asks whether the raw refinement mismatch reaches the allowed physical correction theory. A realized point must pass this test, so $o_D^{\mathrm{fill}} = 0$. Once a filling is chosen, the mapping-fiber class r_D records *which* relative lift is realized; it need not vanish. Only after the coefficient lines of the physical package have been constructed does the internal class b_D ask whether their comparison maps reassociate strictly. The electric/coercive coordinate asks which positive analytic structure survives on the complement of the exact complete-ground space. None of these questions answers another.

The internal H^3 class must nevertheless participate in tensor selection. Merely writing (r_D, b_D) does not tell a selected product which reassociator branch it follows. The missing object is a marked mixed band loop obtained from the two orders of the gauge and regional–refinement operations. A finite selector-comparison certificate makes this loop functorial under admissible changes of filling and band chart. Loop transgression then gives

$$H^3(B\mathcal{B}_D; U(1)) \longrightarrow H^2(F_D^{\mathrm{sel}}; U(1)), \qquad b_D \longmapsto s_D^{\mathsf{S}}.$$

This produces the frozen three-sector ledger:

sector	$\widehat{\kappa}_D^{\mathrm{cs}}$	b_D
I	0	0
II	$\neq 0$	0
III	arbitrary	$\neq 0$.

Sector III is further divided into a silent subtype $s_D^{\mathsf{S}} = 0$ and a detected subtype $s_D^{\mathsf{S}} \neq 0$. This is the precise sense in which the selected degree-two data are divided by the realized degree-three sector. The native H^2 and H^3 complexes are not identified or quotiented.

1.4 Why a mixed square needs calibration

The two Cech directions already produce mixed squares. A noncommuting square, however, is not automatically interaction. It may measure a mismatched background, support, representative, quotient marking, boundary condition, or structural semidirect action. An interaction-sensitive comparison must first subtract what the two marginals would produce independently.

For matched gauge and regional–refinement marginals, the paper constructs an independent surface using only the marginal arrows, the declared direct, semidirect, or twisted action, and the native band coherence. The actual mixed comparator is excluded from this construction. Contractibility of the calibration fiber requires more than rewriting confluence: the structural relations must form a complete homotopy basis and the normal form must carry no residual central isotropy. When those hypotheses hold, the independent surface is canonical up to admissible relative bordism.

The actual surface is then doubled against the oppositely oriented calibrated surface. Realized banded 2-transport evaluates the closed double as

$$\mathbf{\Pi}_{\Box} = t_{\mathbb{I}_{\Box}}^{-1} t_{\mathbb{A}_{\Box}} \in U(1).$$

The comparator-validity certificate requires identical typed boundary, support, background, provenance, independently certified complement, and coefficient band. Under that certificate,

$$\mathbf{\Pi}_{\Box} \neq 1 \quad \Longrightarrow \quad \text{finite admissible decoupling fails.}$$

This is the mixed-interaction obstruction (theorem 20.6). It is deliberately one-sided. Trivial calibrated holonomy does not exclude a stable, cofinal, higher-descent, or represented interaction shadow.

1.5 Casimir coercivity and the ultraviolet–infrared mixture

Let $P_{\mathrm{vac},D}$ be the exact complete-ground projection and suppose the nonvacuum carrier has a typed orthogonal decomposition

$$(I - P_{\mathrm{vac},D})\mathcal{H}_D = \mathcal{H}_D^{\mathrm{UV}} \oplus \mathcal{H}_D^{\mathrm{IR}}.$$

The ultraviolet block receives its universal positive seed from the weighted Casimir. The infrared block contains the verified magnetic and regional remainder. If the centered Hamiltonian satisfies

$$\langle \xi \oplus \eta, (H_D - E_{0,D})(\xi \oplus \eta)\rangle \geq u_D\|\xi\|^2 + v_D\|\eta\|^2 - 2m_D\|\xi\|\,\|\eta\|,$$

then diagonal positivity is not enough. The mixed block must satisfy the strict determinant condition $m_D^2 < u_D v_D$. The exact smaller eigenvalue of the comparison matrix is

$$\kappa_D^{\mathrm{UV/IR}} = \frac{u_D + v_D - \sqrt{(u_D - v_D)^2 + 4m_D^2}}{2} > 0,$$

and gives

$$H_D \geq E_{0,D}I + \kappa_D^{\mathrm{UV/IR}}(I - P_{\mathrm{vac},D}).$$

Every independent target complement, determinant reserve, and seam deduction used in the full Hamiltonian must be included before the final minimum is taken. The resulting number is a finite coercive certificate. It is not yet a continuum spectral gap.

1.6 From one finite point to a compatible family

Pointwise positive floors do not produce persistence. A persistent finite floor belongs to a compatible family over a demand-complete directed system $\mathcal{J}$. The family theorem assumes a physically realized seed, compact Hausdorff bounded typed good loci, coherent continuous full restriction, quantitative full extension, one invariant cutoff, nonnegative seam reserves at a common target floor, and demand completeness. Compact inverse-limit selection then gives a compatible family $(\mathfrak{C}_D^{\mathrm{GAT0}})_{D\in\mathcal{J}}$ with

$$H_D \geq E_{0,D}I + \kappa(I - P_{\mathrm{vac},D}) \qquad (D \in \mathcal{J})$$

for one $\kappa > 0$.

At fixed group, packet, type, profile, cutoff, floor, selector, and calibration, compactness also yields an exact alternative: either a compatible family exists or some finite collection of demands has no compatible joint realization. The latter excludes only those fixed data. Excluding a group would require exhaustion of every genuine sector-complete packet, magnetic realization, sectorial type, profile, cutoff, selector, calibration, and allowed finite package.

When a Cech subsystem is used, it must be related to the full demand system by a proved cofinal comparison preserving the complete package. The word “Cech” is not itself a cofinality theorem.

1.7 Ward and AQFT seeds on the two nerve directions

Gauge/admissibility descent and spacetime localization are recorded by two typed Cech directions. Their bisimplicial bookkeeping object exposes mixed faces without identifying the directions. The finite Ward seed occupies the gauge direction and records the Gauss domain or averaging projector, the full physical kernel/range, observable preservation, locality after reduction, boundary matching, finite variations, algebra covariance, ground-state Ward identities, and restriction/readout compatibility.

The finite AQFT seed occupies the spacetime direction. It records the region poset, supported local algebras, isotony, separation and locality status, finite generation, dynamics/covariance, complete-ground state, detector readout, Gauss–Ward slot, and still-unfilled transport slot. Every entry has status

$$\{\text{proved},\ \text{conditional},\ \text{minimal},\ \text{open},\ \text{not posed}\}.$$

These seeds are substantive because they prevent physical reduction, localization, readout, and transport from being silently identified. They are not a completed Ward–Takahashi hierarchy or Haag–Kastler net. A downstream theorem may consume the seed only when its named requirement profile passes.

1.8 Main results

The principal results can be read as a recognition ladder.

1. **Semisimple detector and Casimir theorem.** Every compact connected semisimple central quotient admits a finite genuine sector-complete detector packet with exact lifted kernel Γ and a positive weighted Casimir seed. This discharges algebraic and electric eligibility for all classical and exceptional factors.
2. **Finite physical/cohomological recognition.** A well-typed proposal with a full admissible physical lift, realized native H^2 and H^3 data, finite local interface, exact complete ground, and positive block/complement/seam estimates determines a finite recognition certificate.
3. **Banded tensor-selector theorem.** The selector-comparison certificate constructs a marked loop canonical up to the permitted admissible homotopies. Transgression of the realized associator class produces the selector class and the three-sector ledger.
4. **Calibrated mixed-interaction obstruction.** Under nonanticipatory independent calibration, complete boundary matching, and realized banded 2-transport, nontrivial $\mathbf{\Pi}_{\square}$ obstructs finite admissible decoupling.
5. **UV/IR coercive assembly.** The weighted Casimir supplies the universal UV seed, while the strict determinant condition controls mixing with the IR block. The exact smaller eigenvalue gives the finite coercive remainder.
6. **Compatible-family recognition.** Compact good loci and quantitative full extension produce a compatible source family with one selection cutoff and one persistent positive finite floor. Empty inverse limit at fixed data has a finite incompatible-demand witness.
7. **No-loss typed export.** The compatible family exports as a contravariant pseudofunctor

$$\mathsf{RecIn}_{\mathcal{J}} : \mathcal{J}^{\mathrm{op}} \longrightarrow \mathbf{RecIn}^{\mathrm{fin}},$$

   preserving the native cohomology, selector, interaction, projection, coercivity, Ward, AQFT, and defect ledgers without choosing a common GNS realization.

The exact endpoint is therefore

$$\boxed{(\mathsf{RecIn}_{\mathcal{J}}, \{\mathsf{Pass}_{\mathsf{Req}}\}_{\mathsf{Req}})\,.}$$

The first coordinate is the full typed finite cargo. The second records which named downstream theorem profiles have actually passed.

1.9 Witness boundary

The universal statements of this paper are algebraic, electric, and conditional-recognition statements. Higher physical realization is witnessed at the following explicit scopes.

1. The $SU(2)$ construction provides paired strict and nontrivially banded finite branches with the same physical Hilbert data, Hamiltonian, complete ground, magnetic data, mixed-block profile, and positive coercive floor, but distinct internal H^3 sectors.
2. For the selected low-packet linear $SU(3)$ collar, the filling obstruction vanishes when the admissible correction fibers are the full invariant block spaces. For the marked fusion-frame comparison actually present, the relative class is zero. Triality does not force a nontrivial relative H^2 class. The calibrated-interaction construction separately constructs an enriched detected Sector-III and mixed-interaction witness, conditional on realization of its additional regional–refinement cycle and comparison cells.
3. The abstract $SU(n)$ center-band calculation gives a nontrivial internal H^3 class whose selected transgression can be silent. An enriched gauge–regional–refinement cocycle gives a detected Sector-III witness, and a clock–shift square gives an explicit calibrated interaction phase. These are finite local witnesses, not universal claims about every Wilson–tube realization of $SU(n)$.

No other Lie family or nontrivial quotient is claimed physically realized beyond the level recorded in the final certificate ledger.

1.10 What is not proved here

The following are outside the theorem scope of GAT 0:

1. a canonical comparison between arbitrary admissible realization fibers;
2. a unique Hilbert-space or GNS realization of all sectorial carriers;
3. strict rather than twisted or sectional duality;
4. a completed net over all bounded spacetime regions, Poincare covariance, spectrum condition, Haag duality, split property, or Reeh–Schlieder theorem;
5. the Grassmannian Fredholm and determinant-line realization reserved for the next paper;
6. Klauder's window projection for the square of the Gauss constraints;
7. Mosco–Riesz passage to a continuum Hamiltonian;
8. identification of a reconstructed continuum form operator with the physical Stone generator of time translations;
9. an invariant relativistic mass spectrum or independent Yang–Mills mass gap theorem;
10. cofinal, represented, or spectral persistence of the finite interaction shadow; or
11. a complete characterization of interaction by $\mathbf{\Pi}_\square$.

These are not omissions hidden by notation. They are explicit downstream requirement profiles. GAT 0 supplies the finite cargo and the typed comparison slots on which those theorems must act. In particular, the finite coercive floor proved here is not the continuum Yang–Mills mass-gap conclusion posed in [5].

1.11 Organization

Sections 2–4 define finite demands, formal proposals, thick constructors, physical admissibility, and realization fibers. Sections 5–8 construct the two Cech directions, mixed squares, physical package, regional algebras, and finite Gauss–Ward/AQFT interfaces. Sections 9–12 separate the relative H^2 lift from the internal H^3 band and construct the tensor selector. Sections 13–17 develop semisimple detector packets, Casimir-controlled UV/IR coercivity, capacity and burden, comparison neutrality, and compatible families. Sections 18–23 construct the calibrated interaction shadow and record the $SU(2)$, $SU(3)$, and $SU(n)$ witnesses and the Lie-family ledger. Sections 24–29 assemble the unified certificate and export it to the downstream reconstruction architecture.

2 Authority, imported data, and the typed source interface

The purpose of this opening construction is to make the source of admissible transport into a mathematical object. A formal refinement proposal specifies a finite question; a thick constructor supplies candidate answers; a physical predicate cuts out the admissible answers; and cohomological and analytic certificates classify what has actually survived. These stages are related, but no stage is a synonym for another.

2.1 Semisimple global forms and finite demands

Throughout the paper,

$$\widetilde{G} = \prod_{\nu=1}^{N} G_\nu, \qquad G = \widetilde{G}/\Gamma, \qquad \Gamma \subset Z(\widetilde{G}), \tag{1}$$

where each G_ν is compact, connected, simply connected, and simple. Thus G is a compact connected semisimple global form, including products and central quotients of the classical and exceptional families. A representation of G is called *genuine* when its lift to $\widetilde{G}$ is trivial on Γ. Algebraic availability of a genuine representation is not called physical realization.

Let $\mathcal{R}_{\mathrm{geom}}$ be the directed geometric demand category. Its objects include finite demands

$$\beta = (n, \varepsilon, \vartheta, \Lambda, b), \tag{2}$$

whose entries record tensor or regional depth, tube thickness, geometric regularity, representation cutoff, and boundary or seam data. The arrows are declared refinements or incidence maps. At this point an arrow asks for a comparison; it does not assert that a physical comparison exists.

Write $\mathcal{I}_{\mathrm{fin}}$ for the directed collection of finite subdiagrams $D \subset \mathcal{R}_{\mathrm{geom}}$. It is *demand complete* when every finite collection of objects and arrows of $\mathcal{R}_{\mathrm{geom}}$ occurs in some $D \in \mathcal{I}_{\mathrm{fin}}$. Varying only n at a fixed $\sigma_0 = (\varepsilon_0, \vartheta_0, \Lambda_0, b_0)$ is cofinal only in the corresponding depth subcategory; it is not automatically cofinal in $\mathcal{R}_{\mathrm{geom}}$.

2.2 The five-stage boundary

For a finite demand shape D, let $\mathfrak{R}_D^{\mathrm{form}}$ denote the typed class of formal proposals, $\mathfrak{T}_D(R_D)$ the candidate thick realizations over a proposal, and $\mathfrak{A}_D^{\mathrm{phys}}$ the class of physically admissible finite packages. The source architecture is

$$\boxed{\begin{array}{c}\text{formal proposal} \longrightarrow \text{thick candidate} \longrightarrow \text{physical realization} \\ \longrightarrow \text{classified/coercive realization} \longrightarrow \text{compatible reconstruction input}\end{array}} \tag{3}$$

The first arrow is a constructor. The second is the physical admissibility test. The third attaches the native cohomological and analytic coordinates. The fourth requires compatible-family and typed-comparison theorems developed later in the paper. None of these arrows is supplied by naming the gauge group.

Warning 2.1 (Source and downstream authority)**.** GAT 0 constructs finite recognition cargo. It does not infer a continuum net, a preferred representation, a GNS completion, a Grassmannian or Fredholm fiber, a Klauder constraint-window projection, or an invariant mass spectrum. Those objects may consume the certificates produced here, but they do not occur in the definition of a finite source.

2.3 Typed arrow registry

The following arrow types are fixed for the remainder of the paper.

Formal refinement arrow $a : D \to D'$. An arrow of $\mathcal{R}_{\mathrm{geom}}$ or of a finite demand diagram. It contains no physical operator.

Constructor operation $c : y \rightsquigarrow y'$. A thickening, collar extension, finite gluing, detector insertion, or block addition used while attempting to populate one realization fiber.

Internal localization arrow $\iota^{x_D}_{O_2 O_1}$. For $O_1 \preccurlyeq O_2$ inside one fixed physical package x_D, this is the inclusion of supported local algebras. It is neither refinement nor cross-fiber transport.

Gauge-presentation comparison $T_{ij;\alpha}$. A marked comparison of two gauge presentations on the same admitted regional support O_α.

Regional restriction $\rho_{\alpha\beta;i}$. A support-changing map inside one fixed realized package and one fixed gauge presentation.

Mixed comparison cell. The cell $\Xi_{ij;\alpha\beta}$ compares the two typed composites $\rho_{\alpha\beta;j} T_{ij;\alpha}$ and $T_{ij;\beta}\rho_{\alpha\beta;i}$.

Detector readout $\mathcal{C}_D$. A compression from the localization carrier to a selected finite detector packet. It is completely positive in the bounded setting but need not be multiplicative.

Cross-fiber comparison c_a. A later map, correspondence, or bimodule between recognized packages over different proposals. Its existence and validity must be separately certified; it is never inferred from a, ι, or algebraic similarity.

Proposition 2.2 (No arrow collapse)**.** *The source and target data of the preceding arrow types are different. Consequently none of the following implications is formal:*

$$a \Rightarrow \mathsf{c}_a, \qquad \iota \Rightarrow \mathsf{c}_a, \qquad \mathcal{C}_D \textit{ multiplicative}, \qquad \Xi = 1 \Rightarrow \textit{global transport.}$$

Each implication requires its own construction theorem.

Proof. A formal refinement arrow lives in the demand category; an internal inclusion lives in one region poset and one represented physical package; a detector readout changes carrier; a mixed cell compares two composites on one rectangle; and a cross-fiber comparison changes at least one proposal or physical lift. Their domains and codomains therefore prevent identification without additional typed data. $\square$

2.4 Equivalence and status conventions

We use different symbols because different invariants must survive different changes:

$R_D \cong_{\mathrm{form}} R'_D$	type-preserving equivalence of formal proposals,
$x_D \sim_{\mathrm{adm}} x'_D$	declared finite admissible equivalence,
$U \sim_{\mathrm{u}} U'$	unitary equivalence on a fixed typed carrier,
$\gamma \simeq_{\mathrm{adm}} \gamma'$	admissible homotopy of selected band loops,
$\Sigma \sim^{\mathrm{adm}}_{\mathrm{bord}} \Sigma'$	admissible bordism of calibrated comparison surfaces.

No Morita, homotopy, bordism, or unitary equivalence is silently included in $\sim_{\mathrm{adm}}$.

Every partial interface carries a status in

$$\mathsf{Stat} = \{\mathsf{proved}, \mathsf{conditional}, \mathsf{minimal}, \mathsf{open}, \mathsf{not\ posed}\}. \tag{4}$$

A status declaration must name the carrier, support convention, hypotheses, and theorem or witness on which it rests.

3 Finite demands, formal proposals, and thick constructors

3.1 Formal proposals

Definition 3.1 (Finite refinement proposal)**.** A finite refinement proposal of shape D is a tuple

$$R_D = (D, G_D, \Pi_D, \Sigma_D, T_D^{\mathrm{geom}}, L_D^{\mathrm{coef}}, F_D^{\mathrm{req}}, S_D^{\mathrm{req}}, M_D) \in \mathfrak{R}_D^{\mathrm{form}}, \tag{5}$$

where:

1. G_D is a semisimple global form as in equation (1);
2. Π_D is a finite candidate detector packet with declared sector set Σ_D;
3. T_D^{geom} contains the requested thickening, incidence, regional, and collar data;
4. L_D^{coef} contains the formal coefficient lines, band labels, and grading data;
5. F_D^{req} lists the required fillings and comparison cells;
6. S_D^{req} contains seams, boundaries, orientations, and markings; and
7. M_D records the provenance, common support, background, tolerance, orientation, and validity conditions required to interpret every comparison.

Definition 3.2 (Well-typed and realization-complete)**.** A proposal is *well typed* when every arrow has declared source and target; every representation is genuine or explicitly marked unverified; every coefficient line and filling has a declared base, boundary, and coefficient system; every seam and comparison records its validity conditions; and no physical operator, ground projection, or positive constant is included merely by notation. It is *realization complete* when it contains enough formal information to run every clause of the physical predicate in definition 4.1.

Realization completeness means that the question can be asked. It does not mean the realization fiber is nonempty.

Definition 3.3 (Formal equivalence)**.** Two proposals of the same shape are formally equivalent when type-preserving identifications preserve the semisimple global form, genuine detector and sector labels, geometric incidence, coefficient systems, requested fillings, seams, markings, and validity metadata. The equivalence cannot use a successful filling, a physical Hamiltonian, or a coercive estimate.

3.2 Thick Wilson constructors

A thin Wilson loop records holonomy but does not provide transverse electric or magnetic support, collars, detector regions, seam variables, a common operator domain, or a complete ground projection. A loop γ is therefore replaced at positive thickness by a finite tubular carrier $T_\varepsilon(\gamma)$ with the incidence and boundary data prescribed by R_D.

Definition 3.4 (Thick Wilson candidate)**.** A thick Wilson candidate over R_D is a tuple

$$y_D = (T_D, \widetilde{\Pi}_D, \widetilde{\Sigma}_D, \mathcal{H}_D^{\mathrm{cand}}, \Gamma_D^{\mathrm{cand}}, E_D^{\mathrm{cand}}, M_D^{\mathrm{cand}}, H_D^{\mathrm{cand}}, P_D^{\mathrm{cand}}, B_D^{\mathrm{cand}}, L_D^{\mathrm{cand}}, F_D^{\mathrm{cand}}, S_D^{\mathrm{cand}}) \tag{6}$$

whose formal shadow is R_D. The notation "candidate" asserts no physical admissibility, exact-ground, or coercivity statement. The class of such candidates is denoted $\mathfrak{T}_D(R_D)$.

Candidate constructors may thicken, extend collars, glue finitely many carriers, insert detector channels, assign coefficient lines, and add electric, magnetic, residual, ultraviolet, boundary, and mixed blocks. Every operation must preserve the formal shadow and record its domain of validity.

Warning 3.5 (Constructor is not transport)**.** A constructor operates during an attempt to populate one inverse image over R_D. Transport compares already recognized physical packages. The former does not create the latter.

For $y_D \in \mathfrak{T}_D(R_D)$, let $\mathcal{A}_D^{\mathrm{cand}}(y_D)$ be the algebraic span of its supported thick Wilson observables and declared insertions on the candidate domain. Closure, adjoints, positivity, and constraint preservation remain tests rather than assumptions.

3.3 Finite demands before Čech specialization

Fix a threshold profile q and a compatible sectorial type v. For a finite shape D, let

$$X_D^{\mathrm{good},v}(q) \tag{7}$$

be the space of admissible-equivalence classes of full typed good diagrams. An object of this locus retains endpoint operators, regional carriers, physical channels, seams, bordisms, relative fillings, coefficient lines, reassociators, exact ground projections, vacuum comparisons, coercive reserves, and all validity metadata. It is not an equivalence class of endpoint Hamiltonians alone.

For $D \subset D'$, a full restriction map, when it has been constructed, is written

$$\rho_{D',D} : X_{D'}^{\mathrm{good},v}(q) \longrightarrow X_{D}^{\mathrm{good},v}(q).$$

Proposition 3.6 (Source-native testing principle)**.** *Recognition over $\mathcal{R}_{\mathrm{geom}}$ must first be tested on the demand-complete system $\mathcal{I}_{\mathrm{fin}}$. A chosen Čech subsystem may replace it only after a cofinal comparison proves both:*

1. *every finite geometric demand is represented in the subsystem; and*
2. *restriction preserves the complete typed package in equation (7).*

In particular, agreement of local endpoint Hamiltonians is insufficient.

Proof. The recognition predicates depend on coefficient lines, fillings, reassociators, seams, ground projections, target complements, and validity metadata. A subsystem that forgets any of these coordinates cannot detect failure of the corresponding clause. Test-completeness and preservation of the full package are therefore necessary before inverse limits over the two indexing systems can be compared. □

4 Physical admissibility and finite realization fibers

4.1 The physical predicate

Definition 4.1 (Finite physical realization predicate)**.** A candidate $y_D \in \mathfrak{T}_D(R_D)$ is physically admissible when the following clauses hold.

(PR1) *Genuine global-form realization.* The semisimple global form is fixed and every selected detector representation is genuine, with the declared lifted common kernel.

(PR2) *Sector survival.* Every sector required by R_D is represented with nonzero physical rank. Algebraic central grading is not substituted for physical rank.

(PR3) *Carrier realization.* The tubes, collars, boundaries, regions, orientations, incidence data, and common supports exist on the stated finite domains.

(PR4) *Full operator realization.* The declared physical Hamiltonian H_D is defined on its controlled finite domain and contains every electric, magnetic, residual, ultraviolet, boundary, and mixed block used in the proof.

(PR5) *Exact complete ground.* $P_{\mathrm{vac},D}$ is the exact complete-ground projection of the realized package, not a blockwise or provisional substitute.

(PR6) *Coefficient and filling realization.* The coefficient band, coefficient lines, relative fillings, and finite comparison cells exist in their declared complexes and satisfy their boundary conditions.

(PR7) *Gauss–Ward realization.* The finite constraint operators or physical projector are declared; regional observables preserve the physical space; post-reduction locality is audited; and every Ward identity used by the package is stated with its domain.

(PR8) *Seam and marking realization.* Boundary fluxes, collars, seam operators, orientations, quotient markings, and provenance conditions required by the source interface are satisfied.

(PR9) *Fiberwise algebraic admissibility.* Supported observables and insertions close under the declared products and adjoints on a common invariant domain while preserving sectors and seams.

(PR10) *Analytic certification.* Every positivity, domain, mixed-corner, and coercive claim used at this stage is proved for the complete physical package.

The ten-clause presentation refines, without strengthening, the eight-clause predicates of the source manuscripts: Gauss–Ward, seams, markings, algebraic closure, and analytic certification are separated so that their later status fields cannot be conflated.

Let $\mathfrak{A}_D^{\text{phys}}$ be the class of candidates satisfying (PR1)–(PR10). The forgetful map

$$\pi_D : \mathfrak{A}_D^{\text{phys}} \longrightarrow \mathfrak{R}_D^{\text{form}}$$

retains the formal shadow.

Definition 4.2 (Admissible realization fiber)**.** For $R_D \in \mathfrak{R}_D^{\text{form}}$, set

$$\operatorname{AdmFib}_D(R_D) := \pi_D^{-1}(R_D). \tag{8}$$

The word "fiber" means only this inverse image. No local triviality, Grothendieck-fibration structure, lifting axiom, stack condition, or global transport is assumed.

Definition 4.3 (Finite physical realization)**.** A realization-complete proposal is physically realized when $\operatorname{AdmFib}_D(R_D) \neq \varnothing$. A chosen $x_D \in \operatorname{AdmFib}_D(R_D)$ is a finite physical realization.

4.2 Failure and nonemptiness firewalls

A failed candidate records the first unresolved or violated PR clause together with enough typed data to reproduce the failure. One failed candidate does not prove that the fiber is empty.

Proposition 4.4 (Empty-fiber firewall)**.** *The following statements are distinct:*

1. *a particular $y_D \in \mathfrak{T}_D(R_D)$ fails a PR clause;*
2. *a selected sector contains no realized point;*
3. $\operatorname{AdmFib}_D(R_D) = \varnothing$*;*
4. *no proposal for the same gauge group is physically realizable.*

The implication from (1) to (3) requires either exhaustion of the declared candidate class or an obstruction theorem uniform on that class. No theorem of this paper infers (4) from (3).

4.3 The universal finite defect ledger

For a declared sector $[b]$, let $X_{D,G,[b]}^{\text{cand}}$ be the typed candidate configuration class. A finite defect ledger is

$$\mathbf{d}_{D,G}^{[b]}(x) = (d_{\text{type}}, d_{\text{det}}, d_{\text{fill}}, d_G, d_{\text{seam}}, d_W, d_{\text{coh}}, d_{\text{pos}}, d_{\text{vac}}, d_{\text{alg}})(x), \tag{9}$$

with nonnegative coordinates testing the correspondingly named PR clauses. For positive weights w_j, put

$$q_{D,G}^{[b]}(x) = \sum_j w_j d_j(x).$$

Definition 4.5 (Complete defect ledger)**.** The ledger is complete on the declared candidate class when

$$q_{D,G}^{[b]}(x) = 0 \quad \Longleftrightarrow \quad x \text{ determines a point of } \operatorname{AdmFib}_{D,[b]}(R_D).$$

Completeness is a theorem burden; it is not implied by nonnegativity.

Fix a declared candidate equivalence $\sim_D$ which preserves every PR status, and suppose the quotient

$$\mathscr{X}_{D,G,[b]} := X_{D,G,[b]}^{\text{cand}} / \sim_D$$

is a set. A candidate class is *status complete* when every PR clause descends to a well-defined truth-valued predicate on this quotient. No algorithm deciding those predicates is assumed. For $j = 1, \ldots, 10$ define

$$\delta_j(\xi) = \begin{cases} 0, & \text{the } j\text{th PR clause holds on } \xi, \\ 1, & \text{the } j\text{th PR clause fails on } \xi, \end{cases} \qquad q_{D,G}^{[b],\text{can}}(\xi) := \sum_{j=1}^{10} w_j \delta_j(\xi), \quad w_j > 0. \tag{10}$$

Proposition 4.6 (Canonical complete ledger and closed encoding)**.** *On every status-complete set-sized candidate quotient, the binary ledger equation (10) is complete. Moreover, on*

$$\mathcal{H}^{\mathrm{can}}_{D,G,[b]} := \ell^2(\mathscr{X}_{D,G,[b]}),$$

the diagonal multiplication operator

$$\mathbb{H}^{[b],\mathrm{can}}_{D,G} e_\xi = q^{[b],\mathrm{can}}_{D,G}(\xi) e_\xi \tag{11}$$

is bounded, nonnegative, and self-adjoint. Its quadratic form

$$Q^{[b],\mathrm{can}}_{D,G}(\psi) = \sum_{\xi \in \mathscr{X}_{D,G,[b]}} q^{[b],\mathrm{can}}_{D,G}(\xi) |\psi(\xi)|^2 \tag{12}$$

is everywhere defined and closed, and the encoding $\iota_{D,G,[b]}(x) = e_{[x]}$ is faithful modulo $\sim_D$. Finally,

$$\ker \mathbb{H}^{[b],\mathrm{can}}_{D,G} \cong \ell^2\Big(\mathrm{AdmFib}_{D,[b]}(R_D)/\!\sim_D\Big)\,. \tag{13}$$

Proof. All weights are positive, so the canonical defect vanishes exactly when every PR clause holds. This is precisely physical admissibility and proves ledger completeness. Since $0 \le q^{[b],\mathrm{can}}_{D,G} \le \sum_j w_j$, diagonal multiplication is a bounded nonnegative self-adjoint operator on the stated ℓ^2 space; its quadratic form is therefore everywhere defined and closed. Distinct $\sim_D$-classes give distinct canonical basis vectors, proving faithfulness. The kernel is the closed span of exactly those basis vectors whose candidate classes have zero total defect, which gives equation (13). □

Set $\mathbb{H}^{\mathrm{can}}_{D,G} = \bigoplus_{[b]} \mathbb{H}^{[b],\mathrm{can}}_{D,G}$.

Theorem 4.7 (Finite spectral recognition reduction)**.** *For a status-complete set-sized candidate quotient, the canonical recognition operator satisfies equation (13). Consequently*

$$\mathrm{AdmFib}_D(R_D) \neq \varnothing \quad \Longleftrightarrow \quad 0 \in \sigma_p(\mathbb{H}^{\mathrm{can}}_{D,G}).$$

If the declared candidate quotient is nonempty, then

$$\mathrm{AdmFib}_D(R_D) = \varnothing \quad \Longleftrightarrow \quad \inf \sigma(\mathbb{H}^{\mathrm{can}}_{D,G}) \ge \min_j w_j > 0. \tag{14}$$

Proof. The kernel identity is proposition 4.6. A nonzero kernel is equivalent to the presence of at least one zero-defect basis class, which is equivalent to a realized point. If there is no realized point, every candidate fails at least one PR clause and hence has canonical defect at least $\min_j w_j$. Conversely a positive lower bound excludes a zero-defect basis class. □

Warning 4.8 (Recognition spectrum is not physical energy)**.** $\mathbb{H}^{\mathrm{can}}_{D,G}$ measures certified realization failure by a diagonal, possibly nonseparable and nonlocal enumeration of candidate classes. It need not be computationally efficient or geometrically natural. The Yang–Mills Hamiltonian H_D measures energy on an already realized package. A spectral gap above $\ker \mathbb{H}^{\mathrm{can}}_{D,G}$ is not a physical mass gap and does not imply the complete-ground estimate for H_D without a separately proved bridge.

4.4 The finite certificate at this stage

Definition 4.9 (Finite physical–cohomological–coercive certificate)**.** A finite certificate over R_D is

$$\mathsf{Cert}_D = (R_D, x_D, o^{\mathrm{fill}}_D, r_D, b_D, P_{\mathrm{vac},D}, \kappa_D, \mathsf{Stat}_D), \tag{15}$$

where $x_D \in \mathrm{AdmFib}_D(R_D)$, o^{fill}_D is the filling obstruction, $r_D \in H^2(\mathrm{Fib}(Q_D))$ is the realized relative class, $b_D \in H^3(B\mathcal{B}_D; U(1))$ is the realized coefficient-band class, and

$$H_D \ge E_{0,D} I + \kappa_D (I - P_{\mathrm{vac},D}), \qquad \kappa_D > 0,$$

holds on the certified finite carrier. The status ledger names which local, Ward, AQFT, comparison, and reconstruction slots have actually been proved.

The four native recognition coordinates remain separate. Their precise relation through the tensor selector is constructed in definition 11.1.

5 The two typed Čech directions

There are two kinds of locality in the finite source. Gauge/admissibility descent asks whether different local presentations describe one admissible gauge object. Spacetime localization asks where the observables of an already realized object live. Refinement changes the demand itself and therefore acts externally to both directions.

5.1 Gauge/admissibility nerve

Definition 5.1 (Finite gauge-presentation cover)**.** For a realization-complete proposal R_D, a gauge-presentation cover is a finite family $\mathcal{U}_D^g = \{U_i^g\}_{i \in I_g}$ together with a hereditary compatibility predicate. A tuple $(i_0, \ldots, i_q)$ is compatible when the detector markings, coefficient lines, local lift data, relative fillings, Gauss data, and validity metadata have a common admitted domain.

The corresponding Čech nerve is

$$\check{N}_q^g(R_D) = \{(i_0, \ldots, i_q) : (i_0, \ldots, i_q) \text{ is gauge compatible}\}.$$

Face and degeneracy maps delete and repeat indices. Pairwise simplices may carry mismatch and filling data; triple and higher simplices may carry comparison and reassociation data. The nerve itself contains none of those physical data until they are attached.

5.2 Spacetime-localization nerve

Definition 5.2 (Finite spacetime cover)**.** For a candidate or realization x_D with admitted finite region poset $\mathcal{O}_{D,x_D}$, a spacetime cover is a finite family $\mathcal{U}_{D,x_D}^{\mathrm{sp}} = \{O_\alpha\}_{\alpha \in I_{\mathrm{sp}}}$. A tuple is compatible when its supported intersection is an admitted region, collar, seam, mediation region, or declared empty intersection. Compatibility is hereditary.

Its nerve is

$$\check{N}_p^{\mathrm{sp}}(D, x_D) = \{(\alpha_0, \ldots, \alpha_p) : (\alpha_0, \ldots, \alpha_p) \text{ is spacetime compatible}\}.$$

Unlike the formal gauge cover, this nerve can depend on the physical lift: different points of the same formal fiber may admit different regional systems.

5.3 The compatible two-nerve carrier

Definition 5.3 (Rectangular compatibility)**.** A gauge simplex $\mathbf{i}$ and a spacetime simplex $\boldsymbol{\alpha}$ are rectangularly compatible when every gauge presentation, regional restriction, detector marking, coefficient line, filling, seam, operator domain, and background required by the pair is simultaneously meaningful on the common support. The condition is hereditary under all faces and degeneracies.

Definition 5.4 (Finite gauge–spacetime index)**.** Set

$$\mathcal{N}_{p,q}(D, G, x_D) = \{(\boldsymbol{\alpha}, \mathbf{i}) : \boldsymbol{\alpha} \in \check{N}_p^{\mathrm{sp}},\ \mathbf{i} \in \check{N}_q^g,\ (\boldsymbol{\alpha}, \mathbf{i}) \text{ rectangularly compatible}\}. \tag{16}$$

Horizontal simplicial operators act only on $\boldsymbol{\alpha}$ and vertical operators only on $\mathbf{i}$.

Proposition 5.5 (Underlying two-nerve construction)**.** *Hereditary gauge, spacetime, and rectangular compatibility make $\mathcal{N}_{\bullet,\bullet}(D, G, x_D)$ a finite bisimplicial set. Its horizontal and vertical simplicial operators commute strictly on indices.*

Proof. The hereditary hypotheses keep every face and degeneracy inside the declared compatible subsets. Horizontal and vertical operations alter different tuples, so they commute on the indexing set. □

Warning 5.6 (The "double Čech nerve" is not a strict physical double)**.** The phrase refers to the two compatible gauge/admissibility and spacetime directions. Strict commutation of their combinatorial operators does not imply strict commutation of the physical comparisons carried over them. Refinement remains an external demand direction. No total differential or mixed obstruction is defined until the physical rectangle and cube laws are proved.

5.4 Mixed cells

At a $(1,1)$-bisimplex, write $T_{ij;\alpha}$ for the gauge comparison on O_α and $\rho_{\alpha\beta;i}$ for regional restriction in presentation i. A mixed comparison cell is typed as

$$\Xi_{ij;\alpha\beta} : \rho_{\alpha\beta;j} T_{ij;\alpha} \Longrightarrow T_{ij;\beta} \rho_{\alpha\beta;i}. \tag{17}$$

Higher bisimplices ask for compatibility of these cells with the gauge and spacetime compositors.

Definition 5.7 (Bigraded finite realization carrier)**.** A bigraded carrier is a family $\mathcal{C}_D^{p,q}$ over $\mathcal{N}_{p,q}$ with declared gauge and spacetime comparison operators

$$\delta_g : \mathcal{C}_D^{p,q} \to \mathcal{C}_D^{p,q+1}, \qquad \delta_{\mathrm{sp}} : \mathcal{C}_D^{p,q} \to \mathcal{C}_D^{p+1,q},$$

and mixed cells as in equation (17). The notation does not assert a strict double complex.

A doubly indexed vacuum datum is a family $V_{i,\alpha}$ of local physical or zero-defect subspaces with the gauge comparisons, regional restrictions, and mixed cells. It is not a global trivialization and does not select a unique GNS realization.

6 Mixed squares, seams, and Gauss reduction

6.1 Coordinatewise mixed compatibility

A mixed square is audited coordinate by coordinate. Its ledger includes Casimir, Gauss, seam, boundary flux, filling, coefficient-band, magnetic, ground, regional-algebra, and detector-readout coordinates. For a coordinate q, define the route defect

$$m^q_{ij;\alpha\beta} = \rho^q_{alpha\beta;j} T^q_{ij;\alpha} - T^q_{ij;\beta} \rho^q_{\alpha\beta;i} \tag{18}$$

whenever subtraction is meaningful on a common linear target. Intrinsically, one instead retains the cell Ξ^q and asks whether it is the identity in the declared comparison groupoid.

Warning 6.1 (Mixed coherence is not yet interaction)**.** A nonidentity Ξ or nonzero m^q may be caused by support, background, quotient marking, representative choice, boundary data, or a structural semidirect action. It becomes the interaction-sensitive phase $\mathbf{\Pi}_\square$ only after the independent calibration and doubled-surface theorem of section 19.

6.2 The strict Casimir coordinate

For the semisimple group in equation (1), let

$$C_D^{\mathrm{el}} = \sum_{\nu=1}^{N} a_\nu C_{2,\nu}, \qquad a_\nu > 0,$$

on a finite genuine marked packet, with the vacuum line assigned eigenvalue zero. A gauge comparison is *Casimir marked* when it preserves factor labels, representation labels, and the vacuum line. A regional map is *Casimir local* when it is an isometry on the electric form domain and intertwines the factor action.

Proposition 6.2 (Strict Casimir descent)**.** *If the vertical maps are genuine Casimir-marked unitary intertwiners, the horizontal maps are Casimir-local isometries, and both routes agree on the marked Casimir spectral projections, then*

$$\Xi^{C_2}_{ij;\alpha\beta} = 1.$$

Any nontrivial mixed defect on that rectangle lies in another coordinate.

Proof. The Casimir is central in each factor's enveloping algebra and acts by a scalar on every marked irreducible. Both families of maps therefore preserve its spectral form. Agreement on the marked spectral projections identifies the two route maps on that coordinate. □

This strict coordinate is the finite descent shadow of the universal Casimir coercive agent. It does not yet identify the complete physical vacuum or control the UV/IR mixed block.

6.3 Gauss compression

Let $P^G_{i,\alpha}$ be the finite physical projector at a vertex. When the finite Haar average is available,

$$P^G_{i,\alpha} = \int_G U_{i,\alpha}(g)\, dg.$$

For a candidate route R, its physical compression has the typed form $P^G_{mathrmt} R P^G_{mathrms}$.

Definition 6.3 (Gauss mixed defect)**.** The post-Gauss defect of a mixed square is

$$\begin{aligned} m^G_{ij;\alpha\beta} &= P^G_{j,\beta} \rho_{\alpha\beta;j} T_{ij;\alpha} P^G_{i,\alpha} \\ &\quad - P^G_{j,\beta} T_{ij;\beta} \rho_{\alpha\beta;i} P^G_{i,\alpha}. \end{aligned}$$

Vanishing is physically meaningful only when both routes preserve the declared physical source and target spaces.

Lemma 6.4 (Gauss-compression commutator defect)**.** *Let $P = P^G$ and let A_L, A_R commute on the extended carrier. Then*

$$[PA_LP, PA_RP] = PA_R(I - P)A_LP - PA_L(I - P)A_RP. \tag{19}$$

Thus pre-constraint commutation descends only when the two off-physical excursions cancel or vanish.

Proof. Insert $I = P + (I - P)$ between the two operators in each compressed product and use $[A_L, A_R] = 0$. □

Theorem 6.5 (Strict post-Gauss square)**.** *Suppose the uncompressed mixed square is strict, the gauge comparisons intertwine the four Gauss projectors, and the regional maps carry physical source ranges into physical target ranges. Then $m^G_{ij;\alpha\beta} = 0$ and the two routes define the same map between the physical regional spaces.*

Proof. Projector naturality moves the source and target projectors through their adjacent gauge comparisons. Physical-range preservation moves them through the regional maps. The remaining middle composites agree by strictness of the uncompressed square. □

For a tensor factorization, gauge invariance of the spectator vacuum is a sufficient condition for physical-range preservation. Boundary Gauss terms, edge modes, and collar constraints must nevertheless be included explicitly; they are not covered by a bare tensor product.

6.4 Seams and boundary flux

For $O_1 \preccurlyeq O_2$, let $S^{(i)}_{O_2 O_1}$ be the finite seam operator in presentation i. It contains all declared boundary-flux matching, collar constraints, and mediation variables.

Definition 6.6 (Boundary-flux defect)**.** For a regional embedding $I^{(i)}_{21}$, define $B^{(i)}_{21}$ by

$$S^{(i)}_{O_2} I^{(i)}_{21} - I^{(i)}_{21} S^{(i)}_{O_1} = B^{(i)}_{21}. \tag{20}$$

Strict seam compatibility is the special case $B^{(i)}_{21} = 0$.

Theorem 6.7 (Gauge-covariant seam reduction)**.** *Assume the extended mixed square is strict, the gauge comparisons intertwine the seam operators, and all relevant common domains are preserved. Then*

$$B_{21}^{(j)} T_{ij;O_1} = T_{ij;O_2} B_{21}^{(i)}. \tag{21}$$

In particular, if the regional maps are seam compatible, the seam coordinate of the mixed cell is strict.

Proof. Expand both boundary defects, use naturality of the two seam operators, and replace one regional–gauge composite by the other using the strict mixed square. □

A nonzero B_{21} is not automatically inconsistent. It is local interface data that must enter the admissible filling complex. Only that complex can decide whether it can be absorbed coherently. Hence

$$B_{21} \longrightarrow \text{relative filling datum} \longrightarrow o_D^{\text{fill}} \longrightarrow r_D$$

is a sequence of constructions, not a chain of equalities.

7 The finite physical package: localization and readout

7.1 One package, two carrier roles

Definition 7.1 (Finite Yang–Mills physical package)**.** For $x_D \in \mathrm{AdmFib}_D(R_D)$, the physical package is

$$\mathcal{P}_D = (T_D, \mathcal{H}_D^{\text{loc}}, K_D^{\text{det}}, J_D, \Pi_D, \Sigma_D, H_D^{\text{el}}, M_D, H_D, P_{\text{vac},D}, \mathcal{O}_{D,x_D}, \mathcal{A}_D^{\text{loc}}, P_D^G, \mathsf{GW}_D, S_D, F_D). \tag{22}$$

Here $\mathcal{H}_D^{\text{loc}}$ supports regional operations and their commutators; K_D^{det} is the finite marked detector/coercive packet; $J_D : K_D^{\text{det}} \hookrightarrow \mathcal{H}_D^{\text{loc}}$ is an isometry; and $P_D^{\text{det}} = J_D J_D^*$ is its range projection.

The two spaces may coincide, but a proper inclusion is allowed and is often necessary. In particular, recognition of a few gauge sectors may require far less room than nontrivial regional localization.

Definition 7.2 (Detector readout)**.** For a bounded regional operator A, set

$$\mathcal{C}_D(A) = J_D^* A J_D.$$

Its multiplicative defect is

$$\mathfrak{m}_D(A, B) = \mathcal{C}_D(AB) - \mathcal{C}_D(A)\mathcal{C}_D(B) = J_D^* A(I - P_D^{\text{det}})BJ_D. \tag{23}$$

Proposition 7.3 (Readout multiplicativity criterion)**.** *If P_D^{det} reduces a unital $*$-algebra $\mathcal{B} \subset B(\mathcal{H}_D^{\text{loc}})$, then $\mathcal{C}_D|_{\mathcal{B}}$ is a unital $*$-homomorphism. Without reduction, equation (23) is the exact defect.*

Proof. Insert $P_D^{\text{det}} + (I - P_D^{\text{det}})$ between A and B. Reduction kills the off-packet term and also preserves adjoints. □

Theorem 7.4 (Localization–detector separation)**.** *Suppose the regional algebra is represented on $\mathcal{H}_D^{\text{loc}}$, the detector packet recognizes the declared gauge sectors, the relation between the localization and detector ground projections is exact, and the detector Hamiltonian satisfies the certified finite coercive estimate. Then:*

1. *regional locality may be posed and proved on $\mathcal{H}_D^{\text{loc}}$;*
2. *marked gauge recognition and finite coercivity may be certified on K_D^{det};*
3. *failure of compressed detector observables to commute does not by itself disprove uncompressed regional locality; and*

4. *compressed commutation does not prove locality without support and Gauss–seam certificates.*

Proof. The first two claims are the distinct typed roles of the two carriers. The third follows because off-packet excursions in equation (23) can create compressed commutator terms. The fourth follows because a matrix compression contains no information about full thick support, mediation regions, or constraint reduction. □

7.2 Realization defect versus physical Hamiltonian

The package contains both the recognition operator $\mathbb{H}_{D,G}$ from theorem 4.7 and the physical Hamiltonian H_D. Their kernels can coincide only after a bridge theorem. Likewise, the exact complete-ground projection of H_D cannot be replaced by the kernel projection of one electric or detector block.

Proposition 7.5 (Finite energy firewall)**.** *Suppose*

$$H_D \geq E_{0,D} I + \kappa_D (I - P_{\mathrm{vac},D}), \qquad \kappa_D > 0.$$

Then κ_D is a finite physical excitation floor for the realized package. It is not, without compatible-family, continuum, symmetry, and invariant spectral transfer theorems, a relativistic mass gap.

8 Regional algebras, the Gauss–Ward seed, and the finite AQFT seed

8.1 Fiberwise regional algebras

Definition 8.1 (Finite admissible region system)**.** For $x_D \in \mathrm{AdmFib}_D(R_D)$, let $\mathcal{O}_{D,x_D}$ consist of the tubes, collars, supported intersections, mediation regions, and finite unions admitted by the carrier, seam, domain, and marking data. Write $O_1 \preccurlyeq O_2$ for admissible supported inclusion and $O_1 \perp_{x_D} O_2$ only when full thick supports, collars, seam neighborhoods, and declared coupling regions are separated.

Definition 8.2 (Fiberwise local algebra)**.** For $O \in \mathcal{O}_{D,x_D}$, let $\mathcal{A}^{\mathrm{loc}}_{D,x_D}(O)$ be the unital $*$-algebra generated on the common invariant domain by the admissible thick Wilson observables, electric and magnetic insertions, and seam-compatible supported operators in O. An inclusion gives

$$\iota^{x_D}_{O_2 O_1} : \mathcal{A}^{\mathrm{loc}}_{D,x_D}(O_1) \hookrightarrow \mathcal{A}^{\mathrm{loc}}_{D,x_D}(O_2).$$

A finite local-algebra certificate records domain control, injective isotony, inclusion coherence, admissible locality, proper-cover generation, and exact vacuum/Hamiltonian compatibility. A closure stronger than the algebraic $*$-closure must be named and proved.

Theorem 8.3 (Fiberwise separated-support commutation)**.** *Let O_L, O_R be nonempty proper thick regions with a declared mediation region O_M. Assume:*

1. *left and right generator families have support contained in O_L and O_R with no undeclared tail through O_M;*
2. *all generators preserve a common controlled domain;*
3. *either an edge-mode factorization, a direct calculation of the generator commutators, or a split certificate with conditional expectations embeds the two families into commuting algebras;*
4. *every cross-region seam or constraint term is absent, central on the package, or assigned to the mediation algebra; and*
5. *physical Gauss reduction preserves the vanishing commutators.*

Then

$$[\mathcal{A}_L, \mathcal{A}_R] = 0$$

on the declared common domain, or in the bounded represented algebra when that closure has been certified.

Proof. In the factorized case the generator families act on different factors; in the direct case their commutators vanish by hypothesis; and in the split case they lie in commuting represented algebras. Products remain commuting by the Leibniz rule. The mediation and constraint hypotheses prevent seam assignment or physical projection from recreating a cross-region term. □

Disjoint thin cores do not satisfy the theorem by themselves. Their thick collars can overlap, electric insertions can differentiate shared variables, and Gauss reduction can identify boundary fluxes.

8.2 Finite Gauss–Ward seed on the gauge direction

Definition 8.4 (Finite Gauss certificate)**.** A finite Gauss certificate consists of an extended localization carrier $\mathcal{H}_D^{\mathrm{ext}}$, a finite variation space $\mathfrak{g}_D^{\mathrm{fin}}$, constraint operators $G_D(\xi)$ or an averaging projector P_D^G, and a physical space satisfying

$$\mathcal{H}_D^{\mathrm{phys}} = \bigcap_{\xi\in\mathfrak{g}_D^{\mathrm{fin}}} \ker G_D(\xi) = \operatorname{Ran} P_D^G.$$

It records constraint typing, physical-space identification, observable preservation, locality after reduction, and boundary/seam matching, each with its status.

The eventual implementation of Gauss constraints by Klauder's spectral projection for a window of the squared constraints is not assumed here. The finite certificate records the constraint tuple and physical projector data that such a downstream construction must realize.

Definition 8.5 (Finite Ward profile)**.** A finite Ward profile is

$$\mathsf{W}_D = (U_D, \delta_D, \omega_D^{\mathrm{vac}}, maths f W Stat_D),$$

recording: the finite gauge transformations or variations on regional generators; algebra covariance or invariance; the ground-state identities $\omega_D^{\mathrm{vac}}(\delta_\xi A) = 0$ on the certified class; and compatibility or explicit defects under regional restriction and detector readout.

Definition 8.6 (Ward seed over the two-nerve carrier)**.** A finite Ward seed is a family

$$\mathsf{Ward}_{i,\alpha} = (P_{i,\alpha}^G, U_{i,\alpha}, \delta_{i,\alpha}, \omega_{i,\alpha}^{\mathrm{vac}}, \mathsf{WStat}_{i,\alpha})$$

over the vertices of $\mathcal{N}_{\bullet,\bullet}$, together with:

1. gauge-direction comparison cells preserving or measuring the Ward data;
2. spacetime-direction restriction cells preserving or measuring the Ward data; and
3. mixed modifications relating the two orders on each admitted rectangle.

Every equality is interpreted only on the common support and domain recorded by rectangular compatibility.

Proposition 8.7 (Ward-seed well-posedness)**.** *A certified family in definition 8.6 makes finite gauge invariance, constraint preservation, ground-state Ward identities, and their gauge/regional compatibility well typed on the two Čech directions. It does not prove a continuum Ward–Takahashi hierarchy, BRST cohomology, or anomaly cancellation.*

8.3 Finite AQFT seed on the spacetime direction

Definition 8.8 (Finite AQFT probe seed)**.** The finite AQFT seed over x_D is

$$\mathsf{AQFT}_D^{\mathrm{fin}} = (\mathcal{O}_{D,x_D}, \mathcal{A}_{D,x_D}^{\mathrm{loc}}, \iota^{x_D}, \perp_{x_D}, \mathsf{Stat}_D, \mathsf{Dyn}_D, \mathsf{Vac}_D, \mathsf{Read}_D, \mathsf{GWSlot}_D, \mathsf{TrSlot}_D). \tag{24}$$

The entries record regional indexing and isotony; locality and generation status; finite dynamics and symmetry; the exact ground projection and the finite state data; detector readout and its defect; the Gauss–Ward slot; and the unfilled cross-fiber transport slot.

The seed lives horizontally over the spacetime nerve and retains its vertical gauge markings. Thus Ward and AQFT information inhabit one compatible two-directional carrier without being collapsed into one differential.

Definition 8.9 (Representation-family slot)**.** The vacuum entry of equation (24) carries a typed family $\mathfrak{Rep}_D$ of admissible state/representation realizations together with their statuses and comparison requirements. No unique GNS realization is selected. Strict duality, twisted or sectional duality, and other admitted realization types remain distinct members or branches until a downstream theorem decides among them.

Theorem 8.10 (Well-posed finite Ward–AQFT interface)**.** *Assume a finite local-algebra certificate, a localization–detector package, a finite Gauss certificate, a Ward seed, and a finite AQFT probe seed on a common rectangularly compatible two-nerve carrier. Then the finite questions of isotony, thick-support locality, proper-cover generation, constraint preservation, ground-state Ward compatibility, finite dynamics, detector readout, and future admissible-transport preservation are mathematically well posed and separately status-bearing.*

Proof. The regional poset and algebra assignment type isotony, locality, and generation. The constraint projector and Ward profile type physical reduction and gauge identities. The Hamiltonian and exact ground projection type finite dynamics and state questions. The detector embedding types readout and its off-packet defect. Rectangular compatibility gives common domains for the gauge, regional, and mixed comparisons. Finally, the transport slot records what a later cross-fiber arrow must preserve without postulating such an arrow. □

Warning 8.11 (Finite seed firewall)**.** Theorem 8.10 is not a Haag–Kastler reconstruction theorem. It proves neither a net over all bounded spacetime regions nor Poincaré covariance, the spectrum condition, uniqueness or cyclicity of a vacuum, Haag duality, split nuclearity, local normality, a unique GNS realization, or continuum completion. It ensures that none of these later questions is erased at the finite source boundary.

8.4 Finite-source and local-interface handoff

Sections 2–8 have now produced the typed finite object on which the two native cohomologies act. The gauge/admissibility and spacetime-localization nerves are separate, the mixed square retains its seam and Gauss defects, the localization and detector carriers are distinct, and Ward/AQFT data are present as status-bearing seeds. The cohomological-selector construction can therefore construct the relative H^2 lift, the internal H^3 associator, and the tensor selector without changing the source object or confusing a mixed coherence defect with interaction.

9 The refinement–admissibility obstruction and the relative H^2 lift

The first cohomological question occurs before reassociation. A formal refinement proposal determines a raw mismatch; the physical interface permits only certain corrections. One must first ask whether the raw mismatch reaches the allowed correction theory. Only after the answer is affirmative can one ask which relative lift has been selected.

9.1 Strict defect coefficients inside lax physical descent

Let $\check{N}^g_\bullet(R_D)$ be the gauge/admissibility nerve of section 5. Assign finite additive coefficient objects $\mathcal{A}_D(U_I)$ and $\mathcal{B}^{\mathrm{adm}}_D(U_I)$ to every admitted overlap U_I. The first records raw refinement mismatch coordinates; the second records corrections permitted by the same support, seam, marking, Gauss–Ward, and validity conditions used by the physical predicate.

Definition 9.1 (Strict defect coefficient system)**.** A strict defect coefficient system consists of presheaves $\mathcal{A}_D$ and $\mathcal{B}^{\mathrm{adm}}_D$ whose restriction maps compose strictly, together with a natural transformation

$$Q_D : \mathcal{A}_D \Longrightarrow \mathcal{B}_D^{\mathrm{adm}}.$$

The physical gauge comparisons themselves may remain lax. Strictness is required only of the additive coordinates that record their defects.

Write

$$A_D^q = \check{C}^q(\check{N}^g; \mathcal{A}_D), \qquad B_D^q = \check{C}^q(\check{N}^g; \mathcal{B}_D^{\mathrm{adm}})$$

with alternating differentials d_A, d_B.

Theorem 9.2 (Ordinary-complex criterion)**.** *For a strict defect coefficient system,*

$$d_A^2 = 0, \qquad d_B^2 = 0, \qquad Q_D d_A = d_B Q_D.$$

The shifted mapping-fiber complex, in the standard homological-algebraic mapping-cone convention [1, 15],

$$\mathrm{Fib}(Q_D)^k = A_D^k \oplus B_D^{k-1}, \qquad D_{Q_D}(\alpha, \beta) = (d_A\alpha, Q_D\alpha - d_B\beta) \tag{25}$$

satisfies $D_{Q_D}^2 = 0$.

Proof. Strictly composable restrictions give the usual cancellation in the alternating Čech differential. Naturality of Q_D gives commutation with each face restriction and hence with their alternating sums. Substitution in equation (25) gives $D_{Q_D}^2 = 0$. □

Warning 9.3 (Hypercohomology trigger)**.** If the coefficient restrictions themselves compose only up to nontrivial cells, or the correction object is nonabelian so that the additive differential is undefined, theorem 9.2 does not apply. One must construct a total complex, crossed-module cocycle theory, nonabelian model, or hypercohomology object. The notation $H^2(\mathrm{Fib}(Q_D))$ is not used until its differential and equivalence relation have been supplied.

Thus physical laxity can be recorded by an ordinary complex when its defect coordinates restrict strictly. Only laxity of the coefficient carrier forces the cohomology theory itself to change.

9.2 Filling obstruction and relative lift

Let

$$\Delta\zeta_D \in A_D^2, \qquad d_A \Delta\zeta_D = 0,$$

be the raw mismatch of the fixed proposal. Because Q_D is a cochain map, $Q_D\Delta\zeta_D$ is closed.

Definition 9.4 (Filling obstruction)**.** The finite filling obstruction is

$$o_D^{\mathrm{fill}} := [Q_D\Delta\zeta_D] \in H^2(B_D^\bullet). \tag{26}$$

Theorem 9.5 (Finite filling-existence criterion)**.** *Assume $B_D^\bullet$ faithfully represents every admissible correction allowed by the declared physical interface. Then an admissible filling*

$$\tau_D \in B_D^1, \qquad Q_D\Delta\zeta_D = d_B\tau_D,$$

exists if and only if $o_D^{\mathrm{fill}} = 0$.

Proof. The class $[Q_D\Delta\zeta_D]$ vanishes exactly when its closed representative is a d_B-coboundary. Faithfulness identifies such degree-one cochains with the physically permitted fillings. □

When a filling exists,

$$D_{Q_D}(\Delta\zeta_D, \tau_D) = 0$$

and hence the selected pair determines a second, logically different datum.

Definition 9.6 (Relative refinement–admissibility lift)**.** For a chosen admissible filling, set

$$r_D = [\Delta\zeta_D, \tau_D]_{\mathrm{ref/adm}} \in H^2(\mathrm{Fib}(Q_D)). \tag{27}$$

Warning 9.7 (Existence is not triviality)**.** The equality $o_D^{\mathrm{fill}} = 0$ means that at least one filling exists. It does not imply $r_D = 0$. Nor does it produce detector rank, operator domains, positivity, an exact ground projection, or coercivity.

The distinction is visible in the exact triangle

$$\mathrm{Fib}(Q_D) \longrightarrow A_D^\bullet \xrightarrow{Q_D} B_D^\bullet$$

and the segment

$$H^1(B_D^\bullet) \longrightarrow H^2(\mathrm{Fib}(Q_D)) \xrightarrow{p_*} H^2(A_D^\bullet) \xrightarrow{Q_{D*}} H^2(B_D^\bullet). \tag{28}$$

A lift of $[\Delta\zeta_D]$ exists precisely when its image is the zero filling obstruction. When lifts exist, their ambiguity is controlled by the preceding degree-one term modulo the identifications induced from the A_D side.

9.3 Representative invariance and the lift torsor

Admissible changes of representative have the form

$$\Delta\zeta'_D = Delta\zeta_D + d_A\lambda, \qquad \tau'_D = \tau_D + Q_D\lambda - d_B\mu. \tag{29}$$

Proposition 9.8 (Representative invariance)**.** *The transformation in equation (29) leaves r_D unchanged and replaces $Q_D\Delta\zeta_D$ by a cohomologous cocycle. Hence o_D^{fill} is also unchanged.*

Proof. The change of the pair is the mapping-fiber coboundary $D_{Q_D}(\lambda, \mu)$. □

Proposition 9.9 (Lift-torsor structure)**.** *Fix $\Delta\zeta_D$ with $o_D^{\mathrm{fill}} = 0$. The set of relative lift classes is a torsor under the quotient of $H^1(B_D^\bullet)$ determined by equation (28) and the declared admissible equivalence.*

Proof. Two fillings differ by a closed degree-one correction. Replacing that difference by a d_B-coboundary gives the same mapping-fiber class. The remaining ambiguity is exactly the preceding term of the long exact sequence, modulo the image of the A_D-side equivalence. □

9.4 Seam completeness and restriction stability

Let $\mathfrak{b}_{21}$ denote the local boundary-flux defect from definition 6.6. A seam-complete filling model contains a restriction-compatible encoding

$$\jmath_D : \{\text{admitted local seam defects}\} \longrightarrow B_D^1$$

that loses no support, boundary, orientation, marking, or validity condition. Local seam defects are coordinates of τ_D; they are not themselves degree-two classes.

Proposition 9.10 (Gauge and restriction stability)**.** *Suppose a gauge change or typed restriction induces cochain maps f_A, f_B with $f_B Q_D = Q_{D'} f_A$. Then it carries o_D^{fill} to the corresponding image obstruction and carries r_D to the corresponding mapping-fiber class. In particular, vanishing, nonvanishing, and the lift-torsor action are invariant under certified changes of presentation.*

Proof. The commuting square induces a cochain map of mapping fibers. Cohomology therefore carries both the image class and the relative pair functorially. □

10 The realized coefficient band and the internal H^3 associator

The degree-three class is defined only after a physical point $x_D \in \mathrm{AdmFib}_D(R_D)$ exists. It records how the realized coefficient-line comparisons reassociate. It neither decides whether a filling exists nor transports the package to another refinement fiber.

10.1 The coefficient band and its pentagon

Let $\mathcal{L}_a$ be the invertible coefficient line carried by an admitted arrow a of the finite physical diagram. The invertible lines and their band-local isomorphisms form a pointed central Picard groupoid $\mathcal{B}_D$. For three composable arrows, the two bracketings are compared by

$$\alpha^{\Omega}_{c,b,a} : (\mathcal{L}_c \otimes \mathcal{L}_b) \otimes \mathcal{L}_a \Longrightarrow \mathcal{L}_c \otimes (\mathcal{L}_b \otimes \mathcal{L}_a), \qquad \alpha^{\Omega}_{c,b,a} = \Omega_D(|c|, |b|, |a|)I. \tag{30}$$

The pentagon for four composable arrows is exactly the normalized cocycle law

$$\delta\Omega_D = 1. \tag{31}$$

Definition 10.1 (Internal coefficient-band class)**.** The realized reassociator determines

$$b_D = [\Omega_D]_{\mathrm{band}} \in H^3(B\mathcal{B}_D; U(1)), \tag{32}$$

where cochains and coboundaries are restricted to the permitted band-local equivalence theory of the physical package.

Thus the H^3 obstruction is literally the associator obstruction: it is the class of the scalar or central cells needed to compare the two bracketings, and its cocycle condition is the pentagon.

Definition 10.2 (Finite coherence type)**.** A realized package is *strictifiable* when $b_D = 0$ and *intrinsically twisted* when $b_D \neq 0$ in the declared band-local equivalence theory.

Theorem 10.3 (Strictification criterion)**.** *Assume the coefficient band represents every permitted change of reassociator and loses no marking or validity condition relevant to strictification. Then*

$$b_D = 0 \quad \Longleftrightarrow \quad \begin{array}{c}\textit{the realized finite comparison system is strictifiable}\\ \textit{within the declared equivalence}\end{array}$$

Proof. If $b_D = 0$, then $\Omega_D = \delta\eta$ for a permitted normalized band 2-cochain η. Modifying the tensorators by η^{-1} removes the associator. Conversely, a permitted strictification supplies such a cochain, so Ω_D is a band coboundary. □

Proposition 10.4 (Band representative invariance)**.** *Replacing Ω_D by $(\delta\eta)\Omega_D$ leaves b_D and the strictifiable/twisted classification unchanged.*

10.2 The two-cohomology firewall

Theorem 10.5 (Finite cohomological firewall)**.** *The three cohomological statements*

$$o_D^{\mathrm{fill}} \in H^2(B_D^{\bullet}), \qquad r_D \in H^2(\mathrm{Fib}(Q_D)), \qquad b_D \in H^3(B\mathcal{B}_D; U(1))$$

have different cochains, coboundaries, equivalences, and native questions. No equality, implication, quotient, or transgression among them is formal. Every relation requires an explicitly constructed comparison map.

Proof. The first class is the image of a mismatch in the admissible correction complex. The second is a cocycle in the shifted mapping fiber of that image map. The third is a normalized central cocycle of the realized coefficient band. Their definitions provide no common cochain complex or comparison map. □

The selector in section 11 respects this firewall. It does not identify the native H^2 and H^3 groups; it constructs a marked loop and transgresses b_D to a new selected degree-two class.

10.3 Semisimple center markings

For the semisimple global form $G = \widetilde{G}/\Gamma$, the residual center is $Z(G) = Z(\widetilde{G})/\Gamma$. Genuine irreducibles provide characters of this residual finite abelian group. A center-sensitive coefficient band is a constructed abelian band together with a realized map from those surviving characters into its marking system.

Proposition 10.6 (Associator cocycle from comparison data)**.** *Suppose triple gauge-comparison cells are faithfully represented in a strict central coefficient band and the two reassociations agree on every admitted quadruple overlap. Then their coefficients form a normalized degree-three cocycle and determine the class b_D of definition 10.1.*

Proof. On a quadruple overlap the two composites of triple cells are the two sides of the pentagon. Their ratio is the alternating coboundary $\delta\Omega_D$. Equality of the physical composites therefore gives equation (31). □

For $SU(n)$ the center marking is cyclic of order n; for the spin families it may be binary, cyclic of order four, or have two independent binary coordinates; E_6 and E_7 supply ternary and binary markings. A band built only from central characters is trivial for E_8, F_4, G_2. This does not exclude a separately constructed noncentral higher-coherence band.

Warning 10.7 (Algebraic capacity is not realization)**.** The presence of residual center characters or a nonzero ambient $H^3(B\mathcal{B}_D; U(1))$ shows only that a sector is algebraically available. It does not prove that the Wilson–tube constructor produces a physical package carrying that class.

11 The three realized sectors and the banded tensor selector

The internal H^3 class must act on the selected tensor calculus if it is to do more than decorate the recognition ledger. The necessary bridge is not a map from the native mapping-fiber complex to the band complex. It is a marked loop produced by the two orders of the gauge and regional–refinement operations. Transgression along that loop gives the degree-two coherence seen by the selected product.

11.1 Selected fillings and the two mixed routes

Fix a selected filling

$$x = (\Delta\zeta_D, \tau_D, \mathfrak{m}_D), \qquad d_A\Delta\zeta_D = 0, \qquad Q_D\Delta\zeta_D = d_B\tau_D,$$

where $\mathfrak{m}_D$ contains the orientation, common support, provenance, background, factor and quotient labels, ground and Gauss projections, Gauss–Ward data, and selector markings.

Let w^G be the gauge leg and w^M the marked regional–refinement leg. The two routes are

$$L_x = w^M_{b;ij} w^G_{ab;i}, \qquad R_x = w^G_{ab;j} w^M_{a;ij}. \tag{33}$$

They must have the same typed source and target on the selected support. Only their invertible images in the coefficient band are denoted $\overline{L}_x, \overline{R}_x$; no underlying noninvertible physical correspondence is inverted.

Let $\mathsf{Fill}^{\mathrm{sel}}_{D,2}$ be the finite 2-groupoid whose objects are selected fillings, whose invertible 1-arrows are representative changes

$$\sigma = (\lambda, \mu) : x \to x', \qquad \Delta\zeta'_D = \Delta\zeta_D + d_A\lambda, \quad \tau'_D = \tau_D + Q_D\lambda - d_B\mu, \tag{34}$$

and whose 2-arrows are changes-of-change. Localizing by the latter gives $\mathsf{Fill}^{\mathrm{sel}}_D$ and

$$F^{\mathrm{sel}}_D = B\mathsf{Fill}^{\mathrm{sel}}_D.$$

11.2 The selector-comparison certificate

Definition 11.1 (Selector-comparison certificate)**.** The predicate $\mathsf{SelComp}_D = 1$ means:

(SC1) $\overline{L}_x$ and $\overline{R}_x$ are invertible band arrows with the same typed endpoints for every selected filling x.

(SC2) Every representative change $\sigma : x \to x'$ carries invertible band 2-cells

$$\Lambda^L_\sigma : \overline{L}_{x'} u_s(\sigma) \Longrightarrow u_t(\sigma)\overline{L}_x, \qquad \Lambda^R_\sigma : \overline{R}_{x'} u_s(\sigma) \Longrightarrow u_t(\sigma)\overline{R}_x,$$

preserving all data in $\mathfrak{m}_D$.

(SC3) The identity and composition laws hold with the native band compositors; changes of parenthesization are compared by α^Ω.

(SC4) Every change-of-change gives commuting modifications between the corresponding L- and R-cells.

(SC5) Typed restriction preserves these cells, their modifications, the selected support, and the distinction between gauge and regional–refinement legs.

This certificate is finite. It is additional realized data and does not follow merely from $Q_D d_A = d_B Q_D$.

Theorem 11.2 (Mixed boundary realization)**.** *If $\mathsf{SelComp}_D = 1$, the assignment*

$$x \longmapsto \gamma_x = \overline{R}_x^{-1}\overline{L}_x \tag{35}$$

extends canonically to a functor

$$\partial^{\mathrm{band}}_D : \mathsf{Fill}^{\mathrm{sel}}_D \longrightarrow \Lambda\mathcal{B}_D,$$

where $\Lambda\mathcal{B}_D$ is the inertia groupoid of band loops. On a representative change σ, the arrow is the spherical pasting

$$C_\sigma = (\Lambda^R_\sigma)^{-1} * \Lambda^L_\sigma.$$

The functor is natural under every typed restriction satisfying (SC5).

Proof. By (SC1), equation (35) is a well-typed object of the inertia groupoid. Paste the inverse R-comparison to the L-comparison. The common target identification cancels with opposite orientation, leaving the source conjugacy

$$\gamma_{x'} \Longrightarrow u_s(\sigma)\gamma_x u_s(\sigma)^{-1}.$$

The identity and composition laws follow from (SC3); the only parenthesization ambiguity agrees by the pentagon $\delta\Omega_D = 1$. Clause (SC4) makes the construction independent of a change-of-change, so it descends to the localized groupoid. Clause (SC5) gives naturality. ☐

Passing to classifying spaces gives

$$\ell_D := B(\partial^{\mathrm{band}}_D) : F^{\mathrm{sel}}_D \longrightarrow B\Lambda\mathcal{B}_D \simeq L(B\mathcal{B}_D). \tag{36}$$

Lemma 11.3 (Canonical selected band loop)**.** *Relative to the marked support, orientation, band, and comparison cells in $\mathsf{SelComp}_D$, the map ℓ_D is canonical up to admissible free homotopy. It is unchanged by permitted relative-representative changes, band-chart changes, subdivision, and cyclic change of marked base vertex. Orientation reversal gives the inverse loop.*

Proof. The filling equation survives equation (34) because $Q_D d_A = d_B Q_D$ and $d_B^2 = 0$. Theorem 11.2 supplies conjugacy in the inertia groupoid and functoriality under composition. A band-chart change gives a natural isomorphism of loop functors and hence a homotopy of their classifying-space maps. Subdivision inserts identity or inverse factors, and cyclic basepoint change is conjugacy. Finally, $(\overline{R}^{-1}\overline{L})^{-1} = \overline{L}^{-1}\overline{R}$. ☐

The canonicity is certificate-relative. Changing the selected support, orientation, band, or comparison cells defines a different selector problem.

11.3 Transgression and the selector class

Put $X_D = B\mathcal{B}_D$ and let

$$\mathrm{ev} : S^1 \times LX_D \longrightarrow X_D$$

be evaluation. Loop transgression is the slant product

$$\mathrm{Tr}_{S^1}(c) = \mathrm{ev}^* \, c/[S^1].$$

It lowers degree by one and commutes with the coboundary.

Definition 11.4 (Banded tensor-selector class)**.** For the realized associator class b_D, define

$$s_D^{\mathsf{S}} := \ell_D^* \, \mathrm{Tr}_{S^1}(b_D) \in H^2(F_D^{\mathrm{sel}}; U(1)). \tag{37}$$

Theorem 11.5 (Finite banded tensor-selector theorem)**.** *Assume x_D is physically realized, its coefficient band carries the class b_D, and $\mathsf{SelComp}_D = 1$. Then the class s_D^{S} is independent of:*

1. *the normalized cocycle representative of b_D;*
2. *the admissible representative of the selected filling;*
3. *the permitted band-local chart; and*
4. *subdivision and cyclic change of the selected loop's marked base vertex.*

For every typed restriction preserving the complete selector datum,

$$i^* s_{D'}^{\mathsf{S}} = s_D^{\mathsf{S}}.$$

The target is a new selected degree-two cohomology group. It is not the native class $r_D \in H^2(\mathrm{Fib}(Q_D))$ without an additional comparison theorem.

Proof. Since slant product is a cochain map of degree -1,

$$\delta \, \mathrm{Tr}_{S^1}(\Omega_D) = \mathrm{Tr}_{S^1}(\delta\Omega_D) = 1.$$

Thus the transgressed representative is a degree-two cocycle. Replacing Ω_D by $(\delta\eta)\Omega_D$ changes it by the coboundary of $\mathrm{Tr}_{S^1}(\eta)$. By lemma 11.3, changes of filling and chart alter ℓ_D only by admissible homotopy, and homotopic pullbacks agree in cohomology. Subdivision and cyclic basepoint changes are included in the same lemma. Naturality follows from naturality of evaluation, slant product, and the restriction-compatible loop functor. □

Corollary 11.6 (Transgression respects admissible equivalence)**.** *Admissibly homotopic selected loops have the same transgressed selector class. Band-cobordant loop families have equal boundary selector classes whenever the realized band cocycle extends over the bordism. Thus selector evaluation depends only on the certified admissible-homotopy class and on the realized band class.*

Proof. The first statement is homotopy invariance of cohomological pullback. For the second, apply the cochain Stokes formula to the extending cocycle; its coboundary vanishes, so the two oriented boundary evaluations agree. □

11.4 The general realized-sector theorem

Let $\widehat{\kappa}_D^{\mathrm{cs}}$ denote the selected scale/refinement coordinate. When a certified realization comparison with the native mapping fiber has been supplied, it is represented by r_D; until then, that identification is not made.

Theorem 11.7 (Realization of the three finite sectors)**.** *Let Cert_D be a finite physical certificate with realized band and $\mathsf{SelComp}_D = 1$. Exactly one of the following conditions holds:*

$$\begin{aligned} \mathrm{I}_D: &\quad \widehat{\kappa}_D^{\mathrm{cs}} = 0, & b_D = 0;\\ \mathrm{II}_D: &\quad \widehat{\kappa}_D^{\mathrm{cs}} \neq 0, & b_D = 0;\\ \mathrm{III}_D: &\quad \widehat{\kappa}_D^{\mathrm{cs}} \text{ arbitrary}, & b_D \neq 0. \end{aligned}$$

Conversely, sector $K \in \{\mathrm{I}, \mathrm{II}, \mathrm{III}\}$ is realized on D if and only if there exists a physical certificate satisfying the corresponding displayed condition and the selector-comparison certificate. Inside Sector III there is a canonical detection split

$$\mathrm{III}_{\mathrm{silent}} : s_D^{\mathsf{S}} = 0, \qquad \mathrm{III}_{\mathrm{det}} : s_D^{\mathsf{S}} \neq 0.$$

The split does not create a fourth sector.

Proof. Vanishing or nonvanishing of b_D first separates the strict-band branch from Sector III. On the strict-band branch, vanishing or nonvanishing of the selected scale/refinement coordinate separates I from II. These cases are disjoint and exhaustive. The converse is precisely nonemptiness of the corresponding stratum of the certified realization fiber. Because transgression is a homomorphism, $b_D = 0$ implies $s_D^{\mathsf{S}} = 0$. For $b_D \neq 0$, its image may vanish because neither transgression nor pullback need be injective; this yields the two Sector-III subtypes. □

This theorem is the exact sense in which the selected H^2 data are *divided by* the realized H^3 sector. The division is a sectoring of the selected tensor calculus by the associator and its transgression. It is not a quotient of the native complexes and does not erase r_D.

Corollary 11.8 (Coherent selected fusion)**.** *Assume the selected supports are closed under the permitted finite tensor products and define*

$$X \widehat{\boxtimes}_{\mathrm{adm}} Y := \mathsf{S}(X \boxtimes Y)$$

whenever the selected support is nonzero. The realized sector packages form a coherent partial semigroup. Its associativity cell is the restriction of α^{Ω}, and fourfold products satisfy the pentagon. It is strictly associative after a permitted band-local change exactly on the strictifiable branch $b_D = 0$.

Proof. Closure keeps all displayed products in the selected realization category. The two triple products are related by α^{Ω}, and the two composites on four factors agree because $\delta\Omega_D = 1$. The strictification claim is theorem 10.3. No common Hilbert-space or GNS realization is used. □

12 Fusion and collar realization of the cohomological coordinates

The selector theorem is intrinsic, but explicit witnesses are evaluated in fusion blocks. The associator determines which comparison channels belong together; the filling obstruction is then tested inside those channels. In this precise order, H^3 organizes the H^2 calculation without replacing it.

12.1 Semisimple fusion blocks

Let Π_D be a finite genuine packet for the semisimple global form G. For a marked triple $\Lambda = (\lambda_1, \lambda_2, \lambda_3)$, set

$$\mathcal{M}_\Lambda^G := \mathrm{Hom}_G(V_{\lambda_1} \otimes V_{\lambda_2}, V_{\lambda_3}^*), \qquad N_\Lambda^G = \dim \mathcal{M}_\Lambda^G. \tag{38}$$

The triple is admitted only if this space is nonzero and its residual central characters satisfy the neutral vertex condition

$$\chi_{\lambda_1}\chi_{\lambda_2}\chi_{\lambda_3} = 1.$$

After a normalized fusion selector and a realized coherence sector $[b]$ are chosen, the finite boundary carrier decomposes as

$$E_{\partial,D} = \bigoplus_{\Lambda,[b]} \mathcal{M}^G_\Lambda \otimes \mathcal{R}_{\Lambda,[b]}, \tag{39}$$

where $\mathcal{R}_{\Lambda,[b]}$ is the finite collar/radial factor. The physical reassociator acts on the multiplicity blocks; it is not inferred from the standard associativity of $\mathrm{Rep}(G)$.

Proposition 12.1 (Multiplicity-one scalarization)**.** *If $N^G_\Lambda = 1$, every G-equivariant seam coordinate on the intertwiner factor is scalar after the fusion selector is normalized. The remaining calculation lies entirely in the collar/radial and overlap-incidence factors.*

Proof. The endomorphism algebra of a one-dimensional intertwiner multiplicity space is $\mathbb{C}$. □

Assume the seam extraction and defect coefficient complex preserve equation (39):

$$B^\bullet_D = \bigoplus_{\Lambda,[b]} B^\bullet_{D,\Lambda,[b]}, \qquad Q_D = \bigoplus_{\Lambda,[b]} Q_{D,\Lambda,[b]}.$$

Theorem 12.2 (Semisimple fusion–associator reduction)**.** *Assume the finite packet is genuine for G, the realized boundary carrier has the decomposition equation (39), the coefficient band and its comparison cells preserve the block system, Q_D is a block-preserving cochain map, and the coefficient complex is the displayed finite direct sum. Then*

$$o^{\mathrm{fill}}_D = 0 \iff [Q_{D,\Lambda,[b]}\Delta\zeta_{D,\Lambda,[b]}] = 0$$

for every realized fusion/coherence block. In finite bases this is equivalent to the rank test

$$\operatorname{rank} M_{1,\Lambda,[b]} = \operatorname{rank}[M_{1,\Lambda,[b]} \mid b_{D,\Lambda,[b]}]$$

on every required block.

Proof. Finite-dimensional representations of a compact semisimple group are completely reducible, including after imposing genuineness for the central quotient. The hypotheses therefore give a finite direct-sum decomposition of cochain complexes and of the image cocycle. Cohomology commutes with finite direct sums, so the total class vanishes exactly when every block class vanishes. The displayed rank equality is the finite-dimensional solvability criterion in each block. □

This theorem applies factorwise to products, componentwise to noncyclic center markings, and without center labels to trivial-center factors. Higher multiplicity produces a finite matrix problem rather than a scalar one.

12.2 Balanced fillings

When a block obstruction vanishes, its filling space is affine:

$$\mathcal{S}_{\Lambda,[b]} = \{t \in B^1_{D,\Lambda,[b]} : d_B t = b_{D,\Lambda,[b]}\} = \tau_{\Lambda,[b]} + \ker d_B.$$

With the Hermitian structure inherited from an orthonormal boundary basis, define the balanced filling by

$$\tau^{\mathrm{bal}}_{D,\Lambda,[b]} = d_B^*(d_B d_B^*)^{-1} b_{D,\Lambda,[b]} = d_B^\dagger b_{D,\Lambda,[b]} \tag{40}$$

on $(\ker d_B)^\perp$.

Proposition 12.3 (Balanced representative)**.** *The filling in equation (40) is the unique minimum-norm solution orthogonal to $\ker d_B$. Every other solution is $\tau^{\mathrm{bal}} + z$ with $z \in \ker d_B$. Its mapping-fiber class need not vanish.*

Proof. Decompose the finite domain orthogonally as $\ker d_B \oplus (\ker d_B)^\perp$. The restriction of d_B to the second summand is bijective onto its range. The Moore–Penrose solution is its unique inverse image orthogonal to the kernel. □

12.3 The selected triangular $SU(3)$ status

For the low triangular packet, triality conservation and the familiar fusion rules produce multiplicity-one singlet and fundamental/antifundamental channels, together with the finite multiplicity-two adjoint cubic block. In the declared linear vertex/collar model the admissible correction spaces are the full invariant block spaces: support, orientation, triality, $SU(3)$-equivariance, and fusion selection are already built into the coefficient fibers. Gauss and finite Ward compatibility are therefore intrinsic to the invariant seam corrections.

Theorem 12.4 (Low-packet triangular filling theorem)**.** *For the selected linear triangular $SU(3)$ collar model, assume no additional external filling restriction beyond the invariant block coefficient fibers. Then the filling differential is surjective on every multiplicity-one block and on the multiplicity-two adjoint block. Hence*

$$o_{\triangle}^{\mathrm{fill}} = 0.$$

The balanced filling exists blockwise and determines a well-defined relative class

$$r_{\triangle}^{\mathrm{bal}} = [\Delta\zeta_{\triangle}, \tau_{\triangle}^{\mathrm{bal}}] \in H^2(\mathrm{Fib}(Q_{\triangle})).$$

Proof. The scalar triangular incidence map is surjective on each multiplicity-one block. Tensoring that finite map with the identity preserves surjectivity on the adjoint multiplicity block. The total obstruction is the direct sum of the block obstructions, so it vanishes. The Moore–Penrose construction then gives the balanced representative. □

Warning 12.5 (What the triangular theorem does not say)**.** Vanishing of the low-packet filling obstruction does not imply that the balanced relative class is zero, does not construct a twisted H^3 branch, and does not prove the same result after nonlinear, quantized, or additional edge-mode restrictions are imposed.

For the comparison data actually present in the source witness, the gauge presentation changes are pure marked changes of orthonormal fusion frame. Writing $T_{ij} = F_i F_j^{-1}$ gives $T_{ij}T_{jk} = T_{ik}$.

Theorem 12.6 (Current-data $SU(3)$ relative branch)**.** *For the gauge-presentation comparison system determined solely by the certified marked fusion frames,*

$$\Delta\zeta_{\triangle}^{\mathrm{frame}} = 0, \qquad \tau_{\triangle}^{\mathrm{bal}} = 0, \qquad r_{\triangle}^{\mathrm{frame}} = 0.$$

Thus triality and the low-packet collar filling do not force a nontrivial relative H^2 class. A nonzero class requires additional realized pairwise physical comparison factors whose triple residual survives in the declared coefficient system.

Proof. Pure frame changes satisfy the strict cocycle identity. Their raw mismatch and seam-extraction image vanish, and the balanced filling of zero is zero. The zero mapping-fiber pair represents the trivial class. □

An enriched $SU(3)$ branch may still carry a nontrivial realized coefficient band and hence Sector III. Such a branch must be constructed from additional physical comparison cells and their quadruple coherence; it cannot be inferred from triality or ordinary change of fusion basis.

12.4 Cohomological-selector handoff

Sections 9–12 have now supplied the missing verb between the two cohomologies. The native H^2 class still records the realized relative lift. The realized H^3 class is the associator obstruction. The selector-comparison certificate constructs a canonical band loop, and transgression along that loop produces the degree-two selector class that divides the selected tensor calculus into the three realized sectors. The semisimple-coercive construction can therefore attach the general semisimple detector and Casimir/coercive theory without changing the sector definitions.

13 Semisimple global forms and genuine detector packets

The cohomological selector of Sections 9–12 classifies a realized finite proposal, but it does not by itself guarantee that the proposal can see every residual central degree of the gauge group. That is a representation-theoretic question. For a general compact connected semisimple global form, the right input is not one preferred representation but a finite genuine packet whose joint kernel remembers the global quotient and whose central characters test every residual degree.

Write

$$\widetilde{G} = \prod_{\nu=1}^{N} G_\nu, \qquad G = \widetilde{G}/\Gamma, \qquad \Gamma \subset Z(\widetilde{G}), \tag{41}$$

where the G_ν are compact, simply connected, and simple. They may belong to any of the classical families A, B, C, D or the exceptional families G_2, F_4, E_6, E_7, E_8. Define the residual central band and its degree group by

$$B_\Gamma := Z(\widetilde{G})/\Gamma, \qquad \widehat{B}_\Gamma = \{\chi \in \widehat{Z(\widetilde{G})} : \chi|_\Gamma = 1\}. \tag{42}$$

Thus $\widehat{B}_\Gamma$ is precisely the group of central characters that descend to genuine representations of G.

Definition 13.1 (Genuine sector-complete detector packet)**.** A finite family $\Pi_\Gamma^{\mathrm{sec}}$ of finite-dimensional unitary representations is a genuine sector-complete detector packet for G when:

1. every member is a representation of G, rather than a merely projective representation of G;
2. after pullback to $\widetilde{G}$, the common kernel of the packet is exactly Γ;
3. its central degrees generate $\widehat{B}_\Gamma$; and
4. it contains a nontrivial irreducible object in every degree $\beta \in \widehat{B}_\Gamma$, including a nontrivial object in the neutral degree.

The last clause matters. A trivial central character does not mean a trivial representation; adjoint-type excitations live in the neutral residual degree.

Lemma 13.2 (Representation of every residual degree)**.** *For every $\beta \in \widehat{B}_\Gamma$ there exists an irreducible representation V_β of G with central character β. When $\beta = 1$, V_1 may be chosen nontrivial.*

Proof. Choose a maximal torus of $\widetilde{G}$. Irreducible representations of G are indexed by the dominant weights in the intermediate weight lattice whose central characters are trivial on Γ. Restriction of that lattice to $Z(\widetilde{G})/\Gamma$ is onto its character group. Hence β has a weight representative. Adding a sufficiently dominant root lattice element preserves the central character and produces a dominant weight. Its highest-weight module descends to G and has degree β. For $\beta = 1$, any nontrivial simple-factor summand of the adjoint representation is center-trivial and descends. □

Theorem 13.3 (Finite semisimple detector-packet theorem)**.** *Every compact connected semisimple global form $G = \widetilde{G}/\Gamma$ admits a finite genuine sector-complete detector packet.*

Proof. The compact Lie group G has a faithful finite-dimensional unitary representation U. Decompose U into finitely many irreducible constituents. Their pullbacks to $\widetilde{G}$ have common kernel exactly Γ. For each of the finitely many degrees $\beta \in \widehat{B}_\Gamma$, adjoin one V_β from lemma 13.2. The resulting finite packet is genuine, retains common lifted kernel Γ, and contains a nontrivial irreducible object in every residual degree. Its degrees therefore generate $\widehat{B}_\Gamma$. □

Corollary 13.4 (Finite tensor-word enlargement)**.** *If a finite genuine packet has central degrees generating $\widehat{B}_\Gamma$, it admits a finite enlargement by tensor words, contragredients, and an adjoint summand such that every residual degree occurs in a nontrivial irreducible summand.*

Proof. Express each degree as a product of generator degrees and inverses, form the corresponding tensor word, and select a nonzero irreducible summand. The center acts on every such summand by the prescribed product character. Add an adjoint summand for the neutral degree if necessary. Finiteness of $\widehat{B}_\Gamma$ makes the enlargement finite. □

13.1 From algebraic availability to physical rank

For every residual degree β, let $P_{\beta,D}$ denote the realized detector-sector projection inside the excitation carrier of a finite package. The algebraic theorem supplies the labels; the physical certificate must record

$$P_{\beta,D} \neq 0 \qquad (\beta \in \widehat{B}_\Gamma) \tag{43}$$

for every sector declared by the proposal.

Warning 13.5 (Algebraic packet versus realized sector)**.** Theorem 13.3 does not show that the marked sectors survive Gauss reduction, magnetic physicalization, seam matching, local Ward constraints, or detector compression. A zero $P_{\beta,D}$ is a physical realization failure, even when V_β exists algebraically.

The finite GAT 0 detector ledger therefore contains both the marked packet $\Pi_\Gamma^{\mathrm{sec}}$ and the ranks of all realized projections. This is the precise sense in which the theorem covers general semisimple global forms without pretending that representation theory alone solves the physical existence problem.

14 Casimir-driven ultraviolet–infrared coercivity

The universal coercive input is the quadratic Casimir. It supplies an electric floor on every nontrivial representation, including neutral adjoint-type sectors. It does not, by itself, provide a magnetic floor, identify the complete ground space, or control the mixed ultraviolet–infrared corner. Those quantities must remain visible in the operator estimate.

14.1 The plaquette-local physical Hamiltonian seed

The recognition operator of proposition 4.6 is not physical energy; see warning 4.8. A physical finite Hamiltonian enters only after a realized gauge carrier, Gauss projection, couplings, and local cellulation have been supplied. The standard electric–magnetic bridge is the Hamiltonian lattice construction of Wilson and Kogut–Susskind [16, 7].

Let E_D and Plq_D be the oriented edge and plaquette sets of a realized finite cellulation. Put $\mathcal{H}_D^{\mathrm{kin}} = L^2(G^{\mathsf{E}_D})$, let P_D^G be the Gauss-averaging projection, and choose a gauge-stable finite Peter–Weyl detector cutoff P_D^Λ. For each edge e, let $C_{2,e}^G$ be the positive factorwise-weighted Laplace–Casimir of equation (50) acting in the e variable. For each plaquette p, let $U_{\partial p}$ be its ordered boundary holonomy and choose a genuine finite-dimensional unitary detector representation ρ_p of the declared global form G. Define the bounded multiplication operator

$$V_{p,\rho_p} := I - \frac{1}{\dim \rho_p} \operatorname{Re} \chi_{\rho_p}(U_{\partial p}). \tag{44}$$

Definition 14.1 (Plaquette-local physical Hamiltonian seed)**.** Let $\alpha_{e,D}, \beta_{p,D} > 0$, and let $B_D^{\mathrm{sb}} \geq 0$ be a self-adjoint gauge-invariant seam/boundary operator on the finite detector carrier. The associated physical Hamiltonian seed is

$$\boxed{H_D^{\mathrm{plaq}} := P_D^G P_D^\Lambda \left(\sum_{e \in \mathsf{E}_D} \alpha_{e,D} C_{2,e}^G + \sum_{p \in \mathsf{Plq}_D} \beta_{p,D} V_{p,\rho_p} + B_D^{\mathrm{sb}} \right) P_D^\Lambda P_D^G.} \tag{45}$$

The coefficients carry the lattice scale and coupling normalization. For a uniform spatial lattice one recovers, up to the selected Casimir and character conventions, the familiar electric coefficient proportional to g^2/a and magnetic coefficient proportional to $1/(g^2 a)$.

Proposition 14.2 (Typed electric–magnetic Hamiltonian interface)**.** *On the finite-dimensional carrier* $\mathcal{H}_D^{\mathrm{phys},\Lambda} := \mathrm{Ran}(P_D^G P_D^\Lambda)$*, the operator in* equation (45) *is self-adjoint, gauge invariant, and nonnegative. Its edge-Casimir sum is the universal electric/UV coercive seed, while its plaquette-character sum is the magnetic curvature contribution. Given an exact ground projection and a typed splitting* equation (53)*, verified lower bounds for the two diagonal compressions of* $H_D^{\mathrm{plaq}} - E_{0,D}$ *supply* u_D *and* v_D*, and a verified norm bound for its off-diagonal compression supplies* m_D*. Consequently* equation (55)*, rather than Casimir positivity alone, is the certificate that promotes this seed to the complete coercive bound* equation (56)*.*

Proof. For a unitary representation, $|\chi_{\rho_p}(g)| \le \dim \rho_p$; hence $0 \le V_{p,\rho_p} \le 2I$. The positive Laplace–Casimir is nonnegative, as is B_D^{sb}. Edge Casimirs are invariant under the left–right gauge action, and $U_{\partial p}$ transforms by conjugation at the plaquette base point, so its character is gauge invariant. The gauge-stable cutoff commutes with Gauss averaging. Compression to its finite-dimensional range therefore gives the asserted nonnegative self-adjoint physical operator. The final statements are precisely the diagonal and off-diagonal block compressions of that operator with respect to equation (53); the norm estimate on the mixed block gives equation (54). □

Definition 14.3 (Hamiltonian descent certificate on the two-nerve)**.** Over every vertex $(\alpha, i) \in \mathcal{N}_{0,0}(D, G, x_D)$, choose a plaquette-local seed $H_{D;i,\alpha}^{\mathrm{plaq}}$ as in definition 14.1. A Hamiltonian descent certificate is

$$\mathsf{HamDesc}_D = (\mathsf{Desc}_D^{\mathrm{el}}, \mathsf{Desc}_D^{\mathrm{mag}}, \mathsf{Desc}_D^{\mathrm{sb}}, \mathsf{BC}_D^H, \boldsymbol{\varepsilon}_D^H), \qquad \boldsymbol{\varepsilon}_D^H = (\varepsilon_D^{\mathrm{UV}}, \varepsilon_D^{\mathrm{IR}}, \varepsilon_D^{\mathrm{mix}}), \tag{46}$$

with the following typed content.

(HD1) Gauge comparisons $T_{ij;\alpha}$ intertwine the Gauss projections, cutoffs, edge Casimirs, and plaquette-character operators.

(HD2) For every regional isometry $\rho_{\alpha\beta;i}$, the pullback defect

$$\Delta_{\alpha\beta;i}^H := \rho_{\alpha\beta;i}^* H_{D;i,\beta}^{\mathrm{plaq}} \rho_{\alpha\beta;i} - H_{D;i,\alpha}^{\mathrm{plaq}} \tag{47}$$

is supported on the declared cut-edge, cut-plaquette, collar, and seam carrier and is represented in B_D^{sb}.

(HD3) On every $(1,1)$-bisimplex, the two routes in equation (17) intertwine the Hamiltonian up to a declared Hamiltonian Beck–Chevalley residual $\Delta_{ij;\alpha\beta}^H$. Gauge and spacetime compositors carry these residuals coherently over higher bisimplices.

(HD4) After passage to the exact excitation carrier and its typed UV/IR splitting, the total descended residual $\mathcal{E}_D^H$ satisfies

$$\begin{aligned} \|P_D^{\mathrm{UV}} \mathcal{E}_D^H P_D^{\mathrm{UV}}\| &\le \varepsilon_D^{\mathrm{UV}}, \\ \|P_D^{\mathrm{IR}} \mathcal{E}_D^H P_D^{\mathrm{IR}}\| &\le \varepsilon_D^{\mathrm{IR}}, \\ \|P_D^{\mathrm{UV}} \mathcal{E}_D^H P_D^{\mathrm{IR}}\| &\le \varepsilon_D^{\mathrm{mix}}. \end{aligned} \tag{48}$$

Here P_D^{UV} and P_D^{IR} are the projections associated with equation (53); no global GNS realization is selected.

Proposition 14.4 (Plaquette Hamiltonian descent on the gauge–spacetime nerve)**.** *Suppose the local cellulations and detector representations have matched provenance on rectangularly compatible charts. Then the edge-Casimir and uncut plaquette-character terms of* $H_{D;i,\alpha}^{\mathrm{plaq}}$ *descend strictly in the gauge direction. In the spacetime direction, every failure of strict restriction is carried by the regional defect* equation (47)*. Consequently the family defines a Hamiltonian descent certificate whenever the cut terms are exhausted by the declared seam/boundary ledger, the mixed cells satisfy their higher coherence laws, and the three bounds* equation (48) *are finite.*

Proof. Bi-invariance of the Laplace–Casimir gives strict gauge intertwining of the electric term. A plaquette boundary holonomy changes by conjugation under a gauge transition, and its character is conjugation invariant; hence the magnetic term also intertwines. Regional restriction preserves every edge and plaquette wholly supported in the smaller chart. The remaining terms are exactly those meeting the cut, collar, or seam and therefore give equation (47). Rectangular compatibility makes the two composites in equation (17) comparable on one carrier. The declared Beck–Chevalley cell and its higher coherence laws then supply the mixed descent datum. Finite detector compression makes every residual bounded, and the three block estimates give the stated quantitative certificate. □

When $b_D = 0$ and the comparison cells strictify, this is ordinary operator descent. For nontrivial b_D, the higher Hamiltonian comparisons are equivariant under the certified reassociator and define a coherent local family, not a single preferred global Hamiltonian or GNS realization. A nonzero $\Delta^H_{ij;\alpha\beta}$ is only a Hamiltonian descent residual; by warning 6.1, it is not the calibrated interaction shadow $\mathbf{\Pi}_\square$ without the validity and bordism data of Sections 18–20.

Warning 14.5 (Link/plaquette typing firewall)**.** The Casimir in equation (45) lives on edges: it is the electric kinetic operator. The direct plaquette operator is the holonomy character equation (44). A dual cellulation may index an edge-electric label by a corresponding dual plaquette, but that typed duality arrow must be recorded; it does not turn the Casimir into the magnetic plaquette potential. Nor does H_D^{plaq} identify its exact ground space or prove a persistent spectral gap without the later ground, IR, mixed-corner, and transport certificates.

14.2 Exact complete-ground certification

Let H_D be a self-adjoint finite Hamiltonian with ground energy $E_{0,D}$.

Definition 14.6 (Exact complete-ground projection)**.** An orthogonal projection $P_{\mathrm{vac,D}}$ is the exact complete-ground projection when

$$H_D - E_{0,D} I \geq 0, \qquad \ker(H_D - E_{0,D} I) = \operatorname{Ran} P_{\mathrm{vac,D}}. \tag{49}$$

Set $Q_D^{\mathrm{exc}} := I - P_{\mathrm{vac,D}}$.

Every coercive conclusion below is an estimate on $Q_D^{\mathrm{exc}}\mathcal{H}_D$. A selected vacuum vector, a perturbative vacuum candidate, or one sectorwise ground state cannot replace the kernel equality in equation (49).

14.3 The universal semisimple Casimir seed

We use the conventional compact semisimple normalization of the quadratic Casimir and its scalar action on irreducible representations [4, 3].

Choose positive normalizations $a_\nu > 0$ and let $C_{2,\nu}$ be the positive quadratic Casimir of G_ν. On an irreducible product label $\lambda = (\lambda_1, \ldots, \lambda_N)$ put

$$C_2^G(\lambda) := \sum_{\nu=1}^{N} a_\nu C_{2,\nu}(\lambda_\nu). \tag{50}$$

Let $c_\nu > 0$ be the least nonzero Casimir eigenvalue of the factor G_ν in the chosen normalization, and define

$$c_G := \min_{1 \leq \nu \leq N} a_\nu c_\nu > 0. \tag{51}$$

Proposition 14.7 (Universal electric Casimir floor)**.** *Every nontrivial irreducible representation of G satisfies*

$$C_2^G(\lambda) \geq c_G.$$

If an electric block has coefficient $g_{\mathrm{el}}^2 > 0$ and its kernel is exactly the declared electric vacuum carrier, then

$$H_D^{\mathrm{el}} \geq g_{\mathrm{el}}^2 c_G I \tag{52}$$

on the electric nonvacuum complement.

Proof. A nontrivial product label has at least one nontrivial component λ_ν. Positivity of all Casimir terms yields

$$C_2^G(\lambda) \geq a_\nu C_{2,\nu}(\lambda_\nu) \geq a_\nu c_\nu \geq c_G.$$

Passing to the quotient by Γ only restricts the permitted labels and cannot lower the bound. Multiplication by g_{el}^2 proves equation (52). ☐

14.4 The literal ultraviolet–infrared block

Suppose the exact excitation carrier has a typed orthogonal decomposition

$$Q_D^{\mathrm{exc}}\mathcal{H}_D = \mathcal{H}_D^{\mathrm{UV}} \oplus \mathcal{H}_D^{\mathrm{IR}}. \tag{53}$$

Neither summand is required to reduce H_D. For $\xi \in \mathcal{H}_D^{\mathrm{UV}}$ and $\eta \in \mathcal{H}_D^{\mathrm{IR}}$, retain the mixed corner in the form estimate

$$\begin{aligned} &\langle \xi \oplus \eta, (H_D - E_{0,D})(\xi \oplus \eta)\rangle \\ &\qquad \geq u_D\|\xi\|^2 + v_D\|\eta\|^2 - 2m_D\|\xi\|\,\|\eta\|. \end{aligned} \tag{54}$$

Here u_D is the verified UV remainder, v_D the independently verified complete-IR floor, and m_D a norm bound for the exact mixed corner. When the Casimir supplies the UV input, u_D is no larger than $g_{\mathrm{el}}^2 c_G$ after all proved UV and seam deductions.

Theorem 14.8 (Operator UV/IR coercive assembly)**.** *Assume definition 14.6 and equation (53) and equation (54), with*

$$u_D > 0, \qquad v_D > 0, \qquad m_D^2 < u_D v_D. \tag{55}$$

Then

$$H_D \geq E_{0,D}I + \kappa_D^{\mathrm{UV/IR}}(I - P_{\mathrm{vac,D}}), \tag{56}$$

where

$$\boxed{\kappa_D^{\mathrm{UV/IR}} = \frac{u_D + v_D - \sqrt{(u_D - v_D)^2 + 4m_D^2}}{2} > 0.} \tag{57}$$

Proof. The right side of equation (54) is the quadratic form of

$$C_D = \begin{pmatrix} u_D & -m_D \\ -m_D & v_D \end{pmatrix}$$

on $(\|\xi\|, \|\eta\|)$. The matrix is positive definite because its first principal minor and determinant are positive. Its smaller eigenvalue is equation (57). Therefore the form is bounded below by $\kappa_D^{\mathrm{UV/IR}}(\|\xi\|^2 + \|\eta\|^2)$, which is exactly equation (56) on the full excitation complement. ☐

Corollary 14.9 (Two-Čech descent-to-coercivity transfer)**.** *Assume definition 14.3 and the exact-ground and UV/IR hypotheses of theorem 14.8. Suppose a coherently descended reference form has verified block data (u_D^0, v_D^0, m_D^0), and the passage to the realized Hamiltonian has total residual $\mathcal{E}_D^H$ satisfying equation (48). Set*

$$\boxed{u_D = u_D^0 - \varepsilon_D^{\mathrm{UV}}, \qquad v_D = v_D^0 - \varepsilon_D^{\mathrm{IR}}, \qquad m_D = m_D^0 + \varepsilon_D^{\mathrm{mix}}.} \tag{58}$$

If

$$u_D^0 > \varepsilon_D^{\mathrm{UV}}, \qquad v_D^0 > \varepsilon_D^{\mathrm{IR}}, \qquad (m_D^0 + \varepsilon_D^{\mathrm{mix}})^2 < (u_D^0 - \varepsilon_D^{\mathrm{UV}})(v_D^0 - \varepsilon_D^{\mathrm{IR}}), \tag{59}$$

then the realized plaquette Hamiltonian obeys equation (56) with the values in equation (58). In particular, the Casimir floor may enter u_D^0 through proposition 14.7; a positive v_D^0 must still be proved from the complete magnetic/IR and seam package.

Proof. On each diagonal block, a self-adjoint residual lowers a verified form bound by at most its operator norm. On the off-diagonal block, the triangle inequality raises the mixed-corner norm by at most $\varepsilon_D^{\mathrm{mix}}$. Thus equation (54) holds with the adjusted triple equation (58). Conditions equation (59) are exactly equation (55) for that triple, so theorem 14.8 applies. □

Warning 14.10 (The mixed block cannot be suppressed)**.** Positive diagonal numbers u_D and v_D do not imply coercivity when the UV and IR summands fail to reduce the Hamiltonian. The exact threshold is the determinant condition $m_D^2 < u_D v_D$. Replacing the coupled block by the slogan "UV plus IR" discards the mechanism that can close the floor.

Definition 14.11 (Coercive cone)**.** For $\kappa \geq 0$, define

$$\mathcal{C}(\kappa) := \{(u, v, m) \in [0, \infty)^3 : u \geq \kappa, v \geq \kappa, m^2 \leq (u - \kappa)(v - \kappa)\}. \tag{60}$$

Proposition 14.12 (Cone characterization)**.** *For $u, v, m \geq 0$, the following are equivalent:*

$$\lambda_{\min}\begin{pmatrix} u & -m \\ -m & v \end{pmatrix} \geq \kappa, \qquad \begin{pmatrix} u & -m \\ -m & v \end{pmatrix} \geq \kappa I, \qquad (u, v, m) \in \mathcal{C}(\kappa).$$

Proof. Subtract κI. A Hermitian 2×2 matrix is positive semidefinite exactly when both diagonal entries and its determinant are nonnegative. □

Corollary 14.13 (Uniform UV/IR persistence criterion)**.** *Suppose a compatible finite-demand family satisfies*

$$u_D \geq u_* > 0, \qquad v_D \geq v_* > 0, \qquad m_D \leq m^*, \qquad (m^*)^2 < u_* v_*$$

for every D, and its restriction arrows preserve the exact complete-ground packages. Then every member has the common lower bound

$$\kappa_* = \frac{u_* + v_* - \sqrt{(u_* - v_*)^2 + 4(m^*)^2}}{2} > 0. \tag{61}$$

Proof. Replace u_D, v_D, m_D in the form bound by the weaker uniform bounds and apply theorem 14.8. Exact ground preservation keeps the estimated carrier equal to the complete excitation complement at every demand. □

If H_D has compact resolvent, equation (56) implies $\operatorname{spec}(H_D) \cap (E_{0,D}, E_{0,D} + \kappa_D^{\mathrm{UV/IR}}) = \varnothing$. This is a finite spectral gap of the certified package. It is not yet an invariant relativistic mass gap.

15 Capacity, burden, and legal finite surgery

The phrase "capacity exceeds burden" becomes mathematical only after both terms are attached to certified operators. Capacity is remaining distance inside the coercive cone. Burden is a proved loss caused by a comparison, extension, or seam. The invariant cohomological selection cost is a third quantity and is not automatically either of the first two.

15.1 Coercive slack and the seam ledger

For $s = (u, v, m)$ define

$$C(s) = \begin{pmatrix} u & -m \\ -m & v \end{pmatrix}, \qquad \mathrm{Cap}_\kappa(s) := \lambda_{\min}(C(s)) - \kappa. \tag{62}$$

Thus $\mathrm{Cap}_\kappa(s) \geq 0$ exactly when $s \in \mathcal{C}(\kappa)$. For a finite diagram, its verified capacity is the minimum over all declared surviving sector blocks and all endpoint packages. It is capacity relative to the verified scalar comparison blocks, not necessarily the exact spectral excess of the Hamiltonian.

Let $J : \mathcal{H} \to \mathcal{H}'$ be a non-surjective refinement isometry that preserves the complete ground carrier. In one target sector write

$$(I - P'_{\mathrm{vac}})\mathcal{H}' = \mathcal{I} \oplus \mathcal{N}, \tag{63}$$

where $\mathcal{I}$ is the inherited image and $\mathcal{N}$ is the new target complement.

Lemma 15.1 (Endpoint capacity–burden criterion)**.** *Fix a target floor $\kappa \geq 0$. Suppose the centered target form $\widehat{H}'$ satisfies*

$$\widehat{H}'|_{\mathcal{I}} \geq (\kappa + c - b)I_{\mathcal{I}}, \qquad \widehat{H}'|_{\mathcal{N}} \geq (\kappa + n)I_{\mathcal{N}},$$

with $c, b, n \geq 0$, and its exact seam corner obeys

$$\|P_{\mathcal{I}}\widehat{H}'P_{\mathcal{N}}\| \leq \eta.$$

If

$$c \geq b, \qquad \eta^2 \leq (c - b)n, \tag{64}$$

then $\widehat{H}' \geq \kappa(I - P'_{\mathrm{vac}})$ on that sector.

Proof. For $x \in \mathcal{I}$ and $y \in \mathcal{N}$, the form of $\widehat{H}' - \kappa I$ is bounded below by

$$(c - b)\|x\|^2 + n\|y\|^2 - 2\eta\|x\|\,\|y\|.$$

The comparison matrix $\left(\begin{smallmatrix} c-b & -\eta \\ -\eta & n \end{smallmatrix}\right)$ is positive semidefinite exactly under equation (64). ☐

Here c is inherited capacity, b is verified image degradation, n is independent target-complement capacity, and η is the mixed seam burden. The new complement is not controlled by the source estimate. Even a perfect source intertwiner therefore needs an endpoint theorem for $\mathcal{N}$.

Definition 15.2 (Seam reserve vector)**.** For a seam-sector e, define

$$\mathcal{R}_\kappa(e) := \big(c - b,\ n,\ (c - b)n - \eta^2\big). \tag{65}$$

It is feasible when all three entries are nonnegative.

The vector keeps diagonal exhaustion and determinant exhaustion separate. A single informal scalar burden would lose this geometry.

15.2 Representative-independent selection cost

Equip the finite mapping-fiber cochain spaces of section 9 with fixed inner products. For $r_D = [\Delta\zeta_D, \tau_D] \in H^2(\mathrm{Fib}(Q_D))$, set

$$\|r_D\|_{\mathrm{rel,D}} := \inf\{\|(\Delta\zeta_D, \tau_D) + D_{Q_D}(\lambda, \mu)\|_D : (\lambda, \mu) \text{ admissible}\}. \tag{66}$$

Proposition 15.3 (Relative quotient-norm invariance)**.** *The quantity in equation (66) depends only on r_D. In finite dimension the infimum is attained by orthogonal projection to the complement of the coboundary subspace.*

Proof. Changing representatives translates the affine set of candidates by the same coboundary subspace. That subspace is closed in finite dimension, so it has a nearest-point projection. ☐

Let $c_{\mathrm{band}}(b_D^{\mathrm{int}})$ be a declared invariant cost of the realized H^3 band class. With fixed positive weights, one may form

$$\mathrm{Curv}_{\mathrm{inv}}(D, x) = \sum_{(b,a),\nu} w_\nu \|\mathfrak{D}_{b,a}^{(\nu)}\|^2 + w_{\mathrm{rel}}\|r_D\|^2_{\mathrm{rel,D}} + w_{\mathrm{band}}c_{\mathrm{band}}(b_D^{\mathrm{int}}), \tag{67}$$

and the normalized density

$$\mathcal{S}_D^{\mathrm{inv}}(x) := \frac{\mathrm{Curv}_{\mathrm{inv}}(D, x)}{\mathrm{Vol}_{\mathrm{WT}}(D)}. \tag{68}$$

A cutoff $\mathcal{S}_D^{\mathrm{inv}}(x) \le M$ selects a bounded good locus.

Warning 15.4 (Selection cost is not coercive loss)**.** There is no general identity $\kappa_{\mathrm{new}} = \kappa_{\mathrm{old}} - \mathrm{Curv}_{\mathrm{inv}}$. The relative class or band cost becomes an analytic burden only after a separate theorem maps it to a Hamiltonian comparison defect. Nor is the normalized density automatically monotone under restriction.

Proposition 15.5 (Relative-burden descent criterion)**.** *Let Z_D^2 and B_D^2 be the relative cocycles and coboundaries. Suppose a linear map*

$$T_D : Z_D^2 \longrightarrow \mathcal{B}(\mathcal{H}_D^{\mathrm{IR}}, \mathcal{H}_D^{\mathrm{UV}})$$

describes a contribution to the mixed analytic corner and satisfies $T_D(B_D^2) = 0$. Then it descends to a bounded map

$$\overline{T}_D : H^2(\mathrm{Fib}(Q_D)) \longrightarrow \mathcal{B}(\mathcal{H}_D^{\mathrm{IR}}, \mathcal{H}_D^{\mathrm{UV}})$$

and

$$\|\overline{T}_D(r_D)\| \le L_D \|r_D\|_{\mathrm{rel,D}}, \qquad L_D = \|\overline{T}_D\|.$$

Consequently, a verified decomposition $K_D = K_{\mathrm{ref,D}} + \overline{T}_D(r_D) + K_{\mathrm{col,D}}$ gives

$$\|K_D\| \le \|K_{\mathrm{ref,D}}\| + L_D \|r_D\|_{\mathrm{rel,D}} + \|K_{\mathrm{col,D}}\|.$$

Proof. Vanishing on coboundaries makes T_D constant on cohomology classes. The quotient is finite-dimensional, hence the descended map is bounded. The displayed estimates follow from the quotient norm and the triangle inequality. □

This proposition states the required bridge; it does not manufacture it. Without such a coboundary-invariant analytic map, an H^2 class is a typed cohomological coordinate, not a spectral penalty.

15.3 Legal finite surgery

Definition 15.6 (Spectrally legal finite surgery)**.** A surgery between two certified finite packages is spectrally legal on a declared sector when it preserves the topological marking and the complete physical predicate, maps and reflects the declared defect kernel, and carries explicit two-sided form and transverse-norm comparison constants. A physical coercivity transfer additionally requires a bridge intertwining the defect kernel projector with the exact complete-ground projector.

Proposition 15.7 (Finite surgery transfer)**.** *Suppose a source defect form satisfies $\mathcal{Q}_D \ge \lambda_D(I - P_D^{\mathrm{def}})$ and a legal surgery S obeys*

$$\mathcal{Q}_{D'}(S\Psi) \ge c_S \mathcal{Q}_D(\Psi), \qquad \|(I - P_{D'}^{\mathrm{def}})S\Psi\|^2 \le C_S \|(I - P_D^{\mathrm{def}})\Psi\|^2.$$

Then on the transported subspace

$$\mathcal{Q}_{D'} \ge \frac{c_S}{C_S} \lambda_D (I - P_{D'}^{\mathrm{def}}).$$

If source and target also carry explicit physical coercivity bridges, a positive source Hamiltonian floor transfers with the product of the surgery and bridge constants. Exact isometric intertwining preserves the floor.

Proof. Apply the target-to-source form comparison, the source lower bound, and the transverse norm comparison. Kernel reflection prevents a positive-defect direction from becoming a new zero-defect mode. The physical statement is the same chain of inequalities with the source and target bridge estimates inserted at its ends. □

Warning 15.8 (Finite surgery firewall)**.** Preservation of the recognized finite object does not automatically preserve a Hamiltonian lower bound. The coercive bridge is an independent hypothesis. Nothing in proposition 15.7 constructs cross-fiber transport or a continuum limit.

16 Comparison-cell and coefficient-band analytic neutrality

The two cohomological coordinates can change while the finite analytic shadow stays fixed, but only under typed covariance hypotheses. There are three comparison levels which must not be collapsed. For a scale arrow $a : s(a) \to t(a)$, the invariant physical correspondence may be one-sided,

$$J_a : \mathcal{H}_{s(a)} \widehat{\otimes} L_a \longrightarrow \mathcal{H}_{t(a)}, \tag{69}$$

with no inverse assumed. Its vacuum compression has a polar partial isometry. Separately, two finite realizations of a common composite may be related by genuine unitary comparison two-cells. Only the latter cells, and unitary polar factors on their supported vacuum carriers, enter the neutrality theorems.

16.1 Profile-unitary equivalence

Definition 16.1 (Finite analytic shadow)**.** The finite analytic shadow $\mathfrak{A}_D(x)$ is the unitary-equivalence class of all objectwise complete physical packages, sectorwise UV/IR comparison triples, exact mixed corners, ground projections, vacuum-comparison singular values, analytic refinement-defect norms, seam reserve vectors, and other operators or norms explicitly named by the fixed threshold profile. It forgets the cohomology classes and their separately declared selection costs.

Call two full finite diagrams profile-unitarily equivalent when endpoint, marked coefficient, and coefficient-line unitaries conjugate every declared Hamiltonian, complete-ground projection, Wilson map, sector projection, mixed corner, vacuum compression, and analytic comparison cell, with the source and target unitary of each typed arrow in the correct position.

Classical eigenprojection and singular-value perturbation estimates provide nearby quantitative comparison tools [6, 2, 14]. The next theorem uses the stronger exact hypothesis of typed unitary covariance, so its conclusion follows by invariance rather than by a perturbative estimate.

Theorem 16.2 (Finite profile-unitary invariance)**.** *Profile-unitarily equivalent finite diagrams have the same analytic shadow. In particular, they have identical ground multiplicities, excitation and Gram spectra, physical channel ranks, magnetic recompression spectra, diagonal lower bounds, exact mixed-corner norms, UV/IR coercive eigenvalues, vacuum-comparison singular values, analytic refinement-defect norms, capacities, and seam reserve vectors.*

Proof. Endpoint spectra and ground ranks are invariant under unitary conjugacy. If $T_D = (I - P_{\mathrm{vac,D}})F_D^W$ and $G_D = T_D^* T_D$, covariance of the marked Wilson map gives $G'_D = S_D G_D S_D^*$; functional calculus therefore conjugates the physicalized channel map and its range projection. The same argument applies to magnetic recompressions and exact block operators. Unitary source–target covariance preserves rectangular singular values and operator norms. The scalar coercive eigenvalues, capacities, and reserve vectors are functions of these invariant quantities. □

16.2 Neutrality of realized relative coboundaries

A change of relative representative has the form

$$(\Delta\zeta_D, \tau_D) \longmapsto (\Delta\zeta_D + d_A\lambda, \tau_D + Q_D\lambda - d_B\mu). \tag{70}$$

Call it comparison-unitarily realized when the displayed coboundary is implemented by a profile-unitary equivalence of the complete finite diagrams, with the H^3 band coordinate fixed.

Theorem 16.3 (Unitary comparison-cell neutrality)**.** *If two representatives of one relative class are related by a comparison-unitarily realized coboundary, then their analytic shadows, capacity–burden ledgers, analytic-defect contributions to* $\mathrm{Curv}_{\mathrm{inv}}$*, relative quotient norms, and total invariant selection costs are identical.*

Proof. The analytic statement is theorem 16.2. The quotient norm is constant on a mapping-fiber cohomology class by proposition 15.3. The band coordinate and its cost are fixed, so substitution in equation (67) proves the remaining assertions. □

The theorem does not assert $r_D = 0$. A locally available unitary comparison may fail to be an admissible global coboundary. Moreover, the one-sided map J_a in equation (69) has not been promoted to a unitary.

Proposition 16.4 (Centrality criterion)**.** *Let $U, V : \mathcal{K}_s \to \mathcal{K}_t$ be unitary intertwiners of two finite-dimensional represented algebra actions. Then V^*U lies in the source commutant. Hence U and V differ by a central phase when the joint represented action is irreducible, and automatically on a one-dimensional vacuum line. In general the comparison need not be central.*

Proof. For every represented observable A, $V^*U\pi_s(A) = V^*\pi_t(A)U = \pi_s(A)V^*U$. Schur's lemma gives a scalar unitary in the irreducible case. □

16.3 Neutrality of a realized coefficient-band change

Let x_0 and x_1 have the same relative class and objectwise physical packages, but realized band classes $b_D^{\mathrm{int},0}$ and $b_D^{\mathrm{int},1}$. A central-analytic band change means that the invariant line-assisted correspondences are fixed; local analytic lifts change by phases $T_a^1 = z_a T_a^0$; pairwise comparison lifts change by

$$U^1_{b,a} = \frac{z_{ba}}{z_b z_a} U^0_{b,a}; \tag{71}$$

and the two normalized reassociators act on triple coefficient-line composites by central unit-modulus cells, without modifying any objectwise analytic operator.

For a pairwise analytic defect

$$\mathfrak{D}^j_{b,a} = U^j_{b,a} T^j_b T^j_a - T^j_{ba}, \qquad j = 0, 1,$$

substitution into equation (71) gives

$$\mathfrak{D}^1_{b,a} = z_{ba}\mathfrak{D}^0_{b,a}. \tag{72}$$

Theorem 16.5 (Central coefficient-band analytic neutrality)**.** *Under the preceding central-unitary covariance conditions,*

$$\mathfrak{A}_D(x_0) = \mathfrak{A}_D(x_1).$$

Thus the two finite diagrams have identical spectra, complete ground projections and ranks, Gram and magnetic spectra, diagonal and mixed block bounds, vacuum-overlap singular values, analytic refinement-defect norms, coercive capacities, and seam reserves. Every finite complete-ground coercive estimate valid for one is valid with the same constant for the other.

Proof. The objectwise data agree by hypothesis. Multiplication by unit-modulus phases preserves vacuum-compression singular values. Equation equation (72) preserves every unitarily invariant defect norm. The reassociator is central and unitary and does not alter an objectwise analytic operator. Every entry of the analytic shadow is therefore unchanged. □

Proposition 16.6 (Band-cost firewall)**.** *For a central-analytic band change,*

$$\mathrm{Curv}_{\mathrm{inv}}(D, x_1) - \mathrm{Curv}_{\mathrm{inv}}(D, x_0) = w_{\mathrm{band}}\big(c_{\mathrm{band}}(b_D^{\mathrm{int},1}) - c_{\mathrm{band}}(b_D^{\mathrm{int},0})\big). \tag{73}$$

Hence analytically indistinguishable band realizations may lie on opposite sides of a selection cutoff.

Proof. The analytic defect terms agree by equation (72); the relative coordinate is fixed. Only the band-cost summand in equation (67) remains. ☐

Corollary 16.7 (Paired fixed-stratum $SU(2)$ band witness)**.** *Consider the strict and nontrivially $\mathbb{Z}_2$-banded fixed-stratum $SU(2)$ depth globalizations constructed with the same objectwise physical packages, Hamiltonians, exact complete-ground projections, magnetic data, mixed blocks, and invariant line-assisted bordisms. Suppose their local analytic lifts differ only by the certified central phases and their comparison compositors obey equation (71). If their relative coordinates agree, then the two branches have one finite analytic shadow and the same coercive floor. Their invariant selection costs can differ only through the band-cost term in equation (73).*

Proof. The stated hypotheses are exactly the central-analytic covariance conditions of theorem 16.5. Apply that theorem and then proposition 16.6. ☐

Warning 16.8 (Scope of neutrality)**.** Neither neutrality theorem says that every formal H^2 or H^3 class has a physical realization. Neutrality may fail when the band change alters an objectwise Hamiltonian or rank stratum, changes the invariant projective correspondence, acts noncentrally or nonunitarily on an analytic channel, changes the relative coordinate, or couples to a higher analytic operator not covered by unitary covariance.

17 Compatible families, cofinal selection, and the persistent finite floor

Pointwise positivity is not persistence. A sequence of positive finite floors may converge to zero, separately selected finite packages may disagree on overlaps, and a restriction may preserve a chosen vacuum vector while losing another ground state. The persistent statement therefore belongs to the full typed inverse system.

Fix a directed collection $\mathcal{J}$ of finite demand diagrams, a semisimple global form, a genuine sector-complete packet, a realized sectorial type, one threshold profile, one invariant cutoff M, and a target floor $\kappa > 0$. Let $X_D^{\mathrm{good}}(M, \kappa)$ be the space of full typed finite certificates over D satisfying all of the following at once:

1. the complete physical predicate and nonzero required sector ranks;
2. the relative H^2 lift, realized H^3 associator, and selected three-sector data of theorem 11.7;
3. the Ward and AQFT seed slots, local plaquette Hamiltonians, and full $\mathsf{HamDesc}_D$ certificate on the compatible gauge–spacetime nerve;
4. the exact complete-ground projection, the three Hamiltonian-descent budgets $\boldsymbol{\varepsilon}_D^H$, and all adjusted UV/IR block estimates at floor κ;
5. provenance, representative-independent selection cost at cutoff M, and every seam reserve needed by a restriction or extension.

Restriction means restriction of this whole package, not merely of endpoint Hamiltonians.

Definition 17.1 (Persistent finite spectral certificate)**.** A compatible family $(x_D)_{D\in\mathcal{J}}$ has persistent finite spectral floor $\kappa > 0$ when its full typed restriction arrows are coherent, preserve the exact complete-ground and Hamiltonian-descent packages, and

$$H_D \geq E_{0,D} I + \kappa(I - P_{\mathrm{vac,D}}) \qquad (D \in \mathcal{J}). \tag{74}$$

Equivalently, the certified local floors satisfy $\inf_{D\in\mathcal{J}} \kappa_D \geq \kappa > 0$.

17.1 Finite compatibility and cofinal recognition

For a finite collection $F \subset \mathcal{J}$, let

$$Y_F(M,\kappa) := \left\{ (x_D)_{D\in F} \in \prod_{D\in F} X_D^{\mathrm{good}}(M,\kappa) : \rho_{D',D}(x_{D'}) = x_D \text{ for every } D \subset D' \text{ in } F \right\}. \tag{75}$$

Theorem 17.2 (Finite-obstruction exhaustion)**.** *Assume every $X_D^{\mathrm{good}}(M,\kappa)$ is compact Hausdorff and every full restriction map is coherent and continuous. Then*

$$\varprojlim_{D\in\mathcal{J}} X_D^{\mathrm{good}}(M,\kappa) \neq \varnothing \quad \Longleftrightarrow \quad Y_F(M,\kappa) \neq \varnothing \text{ for every finite } F \subset \mathcal{J}. \tag{76}$$

If the inverse limit is empty, a finite family of demands already witnesses incompatibility of the fixed full typed data.

Proof. Necessity follows by restriction. Conversely, the product of the nonempty compact factors is compact. Each bonding equation defines a closed subset. Every finite family of bonding equations is satisfied by a point of the corresponding Y_F, so the closed bonding sets have the finite-intersection property. Their total intersection is the inverse limit and is nonempty. □

Theorem 17.3 (Compatible-family recognition)**.** *Suppose:*

1. *some bounded full typed good locus contains a physically realized seed with every required detector sector present and floor at least κ;*
2. *every $X_D^{\mathrm{good}}(M,\kappa)$ is compact Hausdorff;*
3. *full restriction is coherent and continuous;*
4. *every inclusion $D \subset D'$ admits a full extension preserving the endpoint packages, relative and band data, selector sector, Ward/AQFT slots, local plaquette Hamiltonians, Beck–Chevalley cells, descent residual budgets, provenance, the single cutoff M, and feasible seam reserve vectors at floor κ; and*
5. *$\mathcal{J}$ tests every demand claimed at the source boundary, or a cofinal comparison preserving the full typed package has been proved.*

Then there exists a compatible family $(x_D)_{D\in\mathcal{J}}$ with persistent finite spectral floor κ. The relative lifts, selector classes, and finite analytic shadows restrict naturally along every certified arrow.

Proof. The extension hypothesis makes every full restriction map surjective and keeps the good loci nonempty. Compactness gives a nonempty inverse limit. Membership in every factor includes the same sectorial type, cutoff, exact complete-ground package, Hamiltonian-descent certificate, adjusted UV/IR floor, and seam inequalities. The selected inverse-limit point therefore satisfies equation (74). Naturality of the cohomological, selector, and Hamiltonian-descent coordinates is part of full typed restriction, not an inference from the endpoint spectra. □

Corollary 17.4 (Capacity–burden recognition)**.** *In theorem 17.3, the analytic part of the extension hypothesis is discharged if every new endpoint carries the fixed profile at floor κ and every required new seam-sector has a feasible reserve vector equation (65). The relative fillings, coefficient lines, reassociators, selector data, Ward/AQFT slots, provenance, Hamiltonian-descent cells and residual budgets, and cutoff remain independent clauses of the extension certificate.*

Proof. Feasibility is exactly the translated positive-semidefinite condition in lemma 15.1. Applying that lemma at each finite new seam preserves the analytic floor. The other listed data are not determined by the endpoint block estimate and must still be extended explicitly. □

17.2 The fixed-data alternative and the exact boundary

Theorem 17.5 (Fixed-data recognition–obstruction alternative)**.** *Under the compactness and continuity assumptions above, exactly one of the following holds for the fixed group, detector packet, physical realization rules, sectorial type, profile, cutoff, floor, selector convention, and comparison calibration, including its Hamiltonian-descent convention and residual budget:*

1. *a compatible full typed family exists; or*
2. *some finite collection of demands has no compatible joint realization.*

The second alternative excludes only the displayed fixed data. It excludes the semisimple group itself only after every genuine sector-complete packet, physical magnetic realization, sectorial type, profile, cutoff, selector, calibration, and permitted finite package has been exhausted.

Proof. The first alternative is nonemptiness of the inverse limit. If it is empty, theorem 17.2 supplies a finite incompatible subdiagram. Mutual exclusivity is immediate. The last statement follows because the entire argument was made after fixing all displayed choices. □

Warning 17.6 (Certificate failure is not group nonexistence)**.** Universal extension and the reserve-vector test are sufficient recognition mechanisms, not logically necessary conditions for every possible branch. Failure to prove compactness makes the exhaustion theorem unavailable; it does not prove absence of a physical family. Likewise, a fixed-data obstruction cannot be promoted to nonexistence for G without exhausting the outer choices named in theorem 17.5.

Warning 17.7 (Reconstruction boundary)**.** The compatible-family theorem produces downstream-ready finite certificates. It does not select a unique GNS realization; decide strict, twisted, or another duality; construct the Grassmannian Fredholm fiber; implement Klauder's square of the Gauss constraints through a window projection; complete a continuum Haag–Kastler net; or turn the persistent finite floor into an invariant relativistic mass gap. These require typed downstream reconstruction theorems. The Grassmannian Fredholm construction belongs to the next paper in the series.

17.3 Semisimple-coercive handoff

Sections 13–17 have discharged the algebraic detector problem for every compact connected semisimple global form and has identified the quadratic Casimir as the universal electric coercive seed. It has retained the literal UV/IR mixed block, given capacity and seam burden their determinant-level meaning, separated analytic burden from cohomological selection cost, and proved the finite compatible-family theorem needed for a persistent floor. The calibrated-interaction construction may therefore introduce calibrated interaction shadows and concrete $SU(2)$, $SU(3)$, and $SU(n)$ witnesses without altering the recognition or coercivity contract established here.

18 Why the mixed square needs a calibration

The two-nerve architecture of section 5 contains a gauge direction and a regional–refinement direction. Their two orders of composition need not agree. That failure, by itself, is not yet an interaction statement. It may come from a declared semidirect action, a twisted structural interchange, different supports, unmatched backgrounds, inequivalent representatives, or incompatible provenance. An interaction-sensitive invariant must first subtract the structural comparison that the same marginals would produce independently.

This is the role of the mixed coherence defect

$$\boxed{\mathbf{\Pi}_\square}.$$

Its construction has four stages:

$$\begin{aligned}\text{matched gauge and regional–refinement marginals} &\longrightarrow \text{independently assembled surface},\\ \text{actual surface} - \text{independent surface} &\longrightarrow \text{closed banded double},\\ \text{closed double} \longrightarrow \text{admissible bordism class} &\longrightarrow U(1)\text{-holonomy}.\end{aligned} \tag{77}$$

The first arrow is the calibration theorem; the last is realized banded 2-transport. Neither is supplied by the existence of a noncommuting square.

Fix a certified finite demand D for a compact connected semisimple global form $G = \widetilde{G}/\Gamma$. Retain the detector, exact-ground, Casimir, UV/IR, cohomological, selector, Ward, and AQFT certificates of Sections 2–17. Let the gauge and regional–refinement routes have common typed endpoints on one selected support:

$$L_x = w^R_{b;ij} w^G_{ab;i}, \qquad R_x = w^G_{ab;j} w^R_{a;ij}. \tag{78}$$

The superscripts G and R are types, not interchangeable labels.

Definition 18.1 (Interaction boundary ledger)**.** The boundary ledger Led_D records the typed source and target, selected support, background, source fingerprint, holonomy and quotient markings, detector grade, sectorial type, coercive profile, complete-ground projection, Gauss–Ward and AQFT slots, provenance, and finite demand label. A comparison is boundary-matched only when these entries are related by the declared typed arrows. Coincident spectra or reused notation do not constitute a boundary comparison.

Let $\mathsf{GauMarg}_D$ and RRMarg_D denote the groupoids of separately certified gauge and regional–refinement marginals. Their matched pairing is the homotopy pullback

$$\mathsf{Marg}_D := \mathsf{GauMarg}_D \times^h_{\mathsf{Led}_D} \mathsf{RRMarg}_D. \tag{79}$$

An object of Marg_D contains both marginals and the comparison arrow between their boundary ledgers.

Warning 18.2 (A random square is not interaction)**.** No class is called an interaction shadow until the actual and independent surfaces have the same typed boundary, support, background, provenance, complement data, and permitted coefficient band. The validity predicate $\mathsf{Val}_{\text{int}} = 1$ below is therefore part of the theorem, not an optional qualification.

19 The independently admissible comparison surface

For $m \in \mathsf{Marg}_D$, write the marginal arrows as

$$g_i : a_i \longrightarrow b_i, \qquad g_j : a_j \longrightarrow b_j, \qquad h_a : a_i \longrightarrow a_j, \qquad h_b : b_i \longrightarrow b_j.$$

The two independently assembled boundary routes are

$$L^0_m = h_b g_i, \qquad R^0_m = g_j h_a. \tag{80}$$

Definition 19.1 (Independently admissible surface)**.** An independently admissible surface over m is a banded comparison cell

$$\mathbb{I}_m : L^0_m \Longrightarrow R^0_m \tag{81}$$

built only from the matched marginal arrows, their previously declared direct, semidirect, or twisted structural action, their independently certified target complements, and the native band unitors, compositors, and associator. It is constructed without access to the actual mixed comparator or its defect.

In a strict direct product the structural surface may be the identity. In a semidirect or twisted product it need not be. Its nontrivial structural interchange is part of the baseline and must be subtracted.

Let $\overline{\mathsf{IndSurf}}_D$ be the groupoid obtained by localizing independent surfaces at certified admissible relative bordisms and their changes-of-bordism. Forgetting the interior gives

$$\overline{p}_D : \overline{\mathsf{IndSurf}}_D \longrightarrow \mathsf{Marg}_D. \tag{82}$$

Definition 19.2 (Independent-calibration certificate)**.** The predicate $\mathsf{Cal}_D = 1$ means:

1. every marginal pair is matched by the full boundary ledger of definition 18.1;
2. the independent surface obeys the nonanticipation rule in definition 19.1;
3. every one-sided inherited image is accompanied by an independently certified target complement;
4. $\overline{p}_D$ is an isofibration and every homotopy fiber $\mathrm{hofib}_m(\overline{p}_D)$ is nonempty and contractible;
5. typed restriction preserves the ledger, structural action, complement, and relative-bordism relations; and
6. the construction is branch-typed and does not assume a common GNS carrier or replace a correspondence or Morita equivalence by a unitary.

19.1 Constructing the contractible calibration fiber

The contractibility clause can be proved from a finite structural presentation rather than postulated.

Proposition 19.3 (Structural confluence criterion)**.** *Fix $m \in \mathsf{Marg}_D$. Suppose every independent assembly is generated by the marginal arrows, a declared structural interchange*

$$\chi^{\mathrm{str}}_{g,h} : h_b g_i \Longrightarrow g_j h_a,$$

normalized unitors and compositors, the realized associator α^{Ω}, and the fixed independent target complement. Assume:

1. *the interchange satisfies its unit laws and mixed hexagon;*
2. *α^{Ω} satisfies the pentagon;*
3. *these unit, hexagon, and pentagon 3-cells form a complete homotopy basis for the finite structural presentation; and*
4. *the chosen normal-form assembly has no additional central or scalar automorphism.*

Then $\mathrm{hofib}_m(\overline{p}_D)$ is contractible.

Proof. Orient each interchange so that every gauge leg moves to one fixed side of every regional–refinement leg, and orient reassociation by $(xy)z \to x(yz)$. Let $I(w)$ count out-of-order mixed pairs in a word and let $A(w)$ be the sum, over internal binary vertices, of the number of internal vertices in the left subtree. Interchange strictly lowers I; reassociation fixes I and strictly lowers A. The lexicographic measure (I, A) therefore proves termination.

Disjoint interchanges commute by functoriality. An interchange overlapping a reassociation is resolved by the mixed hexagon; two overlapping reassociations are resolved by the pentagon. Thus the rewrite system is locally confluent and has a unique normal form. The complete-homotopy-basis hypothesis identifies any two paths to that normal form, while the final no-isotropy clause removes residual automorphisms. The localized fiber is nonempty, connected, and has trivial isotropy, hence contractible. □

Warning 19.4 (Confluence is not automatically asphericity)**.** Termination, local confluence, the mixed hexagon, and the pentagon determine a normal form and compare reduction paths. Without the complete-homotopy basis and no-isotropy clauses, a residual $U(1)$ automorphism may remain. The calibration is then branch-relative rather than canonical.

Theorem 19.5 (Finite independent-calibration theorem)**.** *If* $\mathsf{Cal}_D = 1$*, then* $\overline{p}_D$ *is an equivalence of groupoids. It therefore admits a quasi-inverse*

$$\mathsf{K}_D : \mathsf{Marg}_D \longrightarrow \overline{\mathsf{IndSurf}}_D, \qquad \overline{p}_D \mathsf{K}_D \simeq 1_{\mathsf{Marg}_D}. \tag{83}$$

Any two such quasi-inverses are joined by a natural admissible relative bordism unique up to modification. Hence an actual mixed square $\mathbb{A}_\square$ *has an independent calibration*

$$\mathbb{I}_\square := \mathsf{K}_D(U\mathbb{A}_\square) \tag{84}$$

canonical up to admissible relative bordism. It depends only on the matched marginals and is natural under typed finite restriction.

Proof. Contractible homotopy fibers give essential surjectivity. The isofibration property supplies lifts of marginal arrows, and fiber connectedness gives fullness. Two lifts of one marginal arrow differ inside a homotopy fiber; contractibility identifies them uniquely after localization, giving faithfulness. Thus $\overline{p}_D$ is an equivalence. Quasi-inverses of an equivalence are naturally isomorphic; here the isomorphism is represented by the unique relative-bordism class in the corresponding fiber. Because K_D is defined on Marg_D, it cannot inspect the actual interior. Restriction naturality follows from the certified commuting square of forgetful maps and fiberwise uniqueness. □

If a calibration fiber is empty, the interaction comparator is undefined. If it is nonempty but not contractible, one must append a calibration-branch label; the resulting defect is relative to that branch and is not canonical without another selection theorem.

Definition 19.6 (Interaction-validity certificate)**.** The predicate $\mathsf{Val}_{\mathrm{int}} = 1$ means that $\mathsf{Cal}_D = 1$ and that $\mathbb{A}_\square$ and $\mathbb{I}_\square$ have the same typed boundary, support, background, source fingerprint, provenance, independently certified target complement, sectorial marking, and permitted coefficient band.

With opposite orientation on the calibration, form the closed double

$$\mathbb{W}_\square := \mathbb{A}_\square \cup_\partial (-\mathbb{I}_\square). \tag{85}$$

Definition 19.7 (Primary mixed bordism defect)**.** The intrinsic primary mixed defect is

$$\mathbf{D}_\square := [\mathbb{W}_\square]^{\mathrm{adm,band}}_{\mathrm{bord}}. \tag{86}$$

Lemma 19.8 (Canonical calibrated surface and double)**.** *Assume* $\mathsf{Val}_{\mathrm{int}} = 1$*. The independent surface is canonical up to admissible relative bordism, and* $\mathbf{D}_\square$ *is invariant under admissible relative 3-cells of the actual surface, changes of calibration quasi-inverse, and admissible homotopies of the common marked boundary.*

Proof. The first claim is theorem 19.5. Let V_A be a relative 3-cell between actual surfaces and V_I the canonical relative bordism between their calibrations. Gluing V_A to the oppositely oriented V_I along the boundary cylinder produces a bordism between the closed doubles. For a boundary homotopy, attach the same cylinder to both open surfaces; its two copies occur with opposite orientations and cancel in the double. □

Together with lemma 11.3, this proves the first two of the three equivalence statements needed by the construction: the selected band loop is canonical up to admissible homotopy, and the calibrated actual/independent surface is canonical up to admissible bordism.

20 Banded 2-transport and the interaction obstruction

The class $b_D \in H^3(B\mathcal{B}_D; U(1))$ supplies an associator, but it is not automatically a surface-holonomy theory. We must realize the coefficient-line part of every certified comparison as a pseudofunctor whose target retains that associator.

Let $\mathbf{Band}_D^{\Omega}$ be the realized central Picard bicategory of graded Hermitian coefficient lines, permitted unitary band maps, and associator

$$\alpha^{\Omega}_{c,b,a} = \Omega_D(|c|,|b|,|a|)I, \qquad \delta\Omega_D = 1. \tag{87}$$

Let $\mathcal{P}^{\mathrm{adm}}_{D,\leq 3}$ be the finite typed computad whose objects are sectorial carriers, 1-cells are certified physical transports or correspondences, 2-cells are certified comparisons, and declared 3-cells are changes of comparison, subdivision prisms, unit and cancellation cells, reassociation cells, and already certified modifications. A physical 1-cell need not be invertible.

Definition 20.1 (Banded 2-transport certificate)**.** The predicate $\mathsf{Band2Tr}_D = 1$ means:

1. every certified 1-cell f has a coefficient-line image $\overline{f}$; this extracts the line-assisted part and does not invert f;
2. every composable pair has a normalized unitary compositor
$$\chi_{g,f} : \overline{g}\widehat{\otimes}\overline{f} \Longrightarrow \overline{gf};$$
3. every composable triple satisfies the pseudofunctor comparison with the native associator equation (87), and fourfold composition satisfies its pentagon;
4. every certified 2-cell $\Sigma : f \Rightarrow f'$ has a unitary line map $t_\Sigma : \overline{f} \Rightarrow \overline{f'}$, covariant under vertical and compositor-corrected horizontal composition;
5. every declared admissible 3-cell is sent to a commuting modification;
6. band-chart change acts by a permitted unitary tensorator;
7. typed restriction preserves the lines, compositors, 2-cell maps, associator, and declared relations; and
8. a geometric cellular presentation is admitted only through a typed cellular-to-band comparison through degree three that preserves boundary, pasting, restriction, carrier, incision, coefficient line, and source marking.

Theorem 20.2 (Finite realization of banded 2-transport)**.** *If $\mathsf{Band2Tr}_D = 1$, the certified assignments descend uniquely up to pseudonatural band equivalence to a normalized pseudofunctor*

$$\mathcal{T}_D^{\mathrm{band}} : \Pi^{\mathrm{adm}}_{\leq 2}(D) \longrightarrow \mathbf{Band}_D^{\Omega}. \tag{88}$$

It does not strictify b_D, invert the underlying physical correspondences, or require a common Hilbert-space or GNS carrier.

Proof. Begin with the free path bicategory on the typed computad. The line images, compositors, and comparison maps define the pseudofunctor on generators and binary pasting. The triple law inserts α^{Ω}; its pentagon removes the only fourfold ambiguity. The declared 3-cell condition makes the assignment constant on the relations defining $\Pi^{\mathrm{adm}}_{\leq 2}(D)$. Chart covariance gives uniqueness up to pseudonatural band equivalence and restriction gives naturality. Only the line image of a physical correspondence is used, so no inverse or common carrier is introduced. □

For parallel boundary-matched surfaces $\Sigma_0, \Sigma_1 : f \Rrightarrow g$, define

$$\mathrm{Hol}_{\mathcal{T}_D}(\Sigma_0 \cup_\partial (-\Sigma_1)) := t_{\Sigma_1}^{-1} t_{\Sigma_0} \in \mathrm{Aut}(\overline{f}) \cong U(1). \tag{89}$$

An open surface is not declared to be a scalar. Only the ratio of two parallel surfaces is canonically $U(1)$-valued.

Definition 20.3 (Calibrated mixed interaction shadow)**.** When $\mathsf{Val}_{\text{int}} = \mathsf{Band2Tr}_D = 1$, define

$$\boxed{\mathbf{\Pi}_\square := t_{\mathbb{I}_\square}^{-1} t_{\mathbb{A}_\square} = \mathrm{Hol}_{\mathcal{T}_D}(\mathbb{W}_\square) \in U(1).} \tag{90}$$

Lemma 20.4 (Transgression and holonomy respect equivalence)**.** *Selected-loop transgression is invariant under admissible homotopy, and* $\mathrm{Hol}_{\mathcal{T}_D}$ *is invariant under admissible banded bordism. Consequently the selector class* s_D^{S} *and the mixed shadow* $\mathbf{\Pi}_\square$ *depend only on* $[\ell_D]_{\text{adm}}$ *and* $\mathbf{D}_\square$*, respectively.*

Proof. The transgression assertion is the prism-homotopy argument of theorem 11.5 and corollary 11.6. For holonomy, check the generators of admissible bordism. Functoriality and normalization preserve identity insertion, subdivision, and inverse-pair cancellation. Reassociation changes the two local evaluations by the two sides of the pentagon, which agree because $\delta\Omega_D = 1$. A certified comparison 3-cell is sent to a commuting modification. Finally, a permitted band-chart change inserts opposite boundary tensorators on the two halves of the closed double, and they cancel. □

This completes the third required equivalence statement: transgression and holonomy respect precisely the admissible homotopy and bordism relations used to define their inputs.

Definition 20.5 (Finite admissible decoupling)**.** Finite admissible decoupling of the validated square means a marked factorization of the actual mixed comparison through the separately certified gauge and regional–refinement marginals, together with an admissible relative 3-cell from $\mathbb{A}_\square$ to its independently calibrated surface $\mathbb{I}_\square$.

Theorem 20.6 (Calibrated mixed-interaction obstruction)**.** *Assume* $\mathsf{Val}_{\text{int}} = 1$ *and* $\mathsf{Band2Tr}_D = 1$*. Then*

$$\mathbf{D}_\square = 0 \quad \Longrightarrow \quad \mathbf{\Pi}_\square = 1.$$

Moreover,

$$\boxed{\mathbf{\Pi}_\square \neq 1 \quad \Longrightarrow \quad \textit{finite admissible decoupling fails.}} \tag{91}$$

Thus a nontrivial calibrated holonomy is a finite interaction shadow. The converse is not asserted.

Proof. If $\mathbf{D}_\square = 0$, the closed double bounds an admissible banded 3-cell. By lemma 20.4, its holonomy equals that of the empty spherical diagram, namely 1. If finite admissible decoupling exists, its relative 3-cell becomes a null-bordism of the double after gluing to its orientation reverse. Hence decoupling implies $\mathbf{D}_\square = 0$ and then $\mathbf{\Pi}_\square = 1$. Taking the contrapositive proves equation (91). □

Warning 20.7 (Vanishing is not complete decoupling)**.** $\mathbf{\Pi}_\square = 1$ need not imply finite decoupling. Holonomy has not been assumed injective on admissible bordism classes, and stable, cofinal, higher-descent, or represented interaction shadows may survive. GAT 0 exports a calibrated first obstruction, not a complete interaction avatar.

21 The explicit $SU(n)$ witness family

We now evaluate the detector, Casimir, associator, selector, and interaction coordinates in one finite $SU(n)$ family. The construction is a witness of the abstract theorem, not a claim that every $SU(n)$ Wilson–tube realization has the same filling or mixed comparator.

Set

$$\zeta_n = e^{2\pi i/n}, \qquad Z(SU(n)) = \langle \zeta_n I \rangle \cong \mathbb{Z}_n.$$

For $1 \le r \le n-1$, let $V_r = \bigwedge^r \mathbb{C}^n$, and let $V_0 = \mathfrak{su}(n)_{\mathbb{C}}$ be the adjoint representation.

Proposition 21.1 ($SU(n)$ detector and Casimir witness). *The packet*

$$\Pi_n^{\mathrm{sec}} = \{V_0, V_1, \dots, V_{n-1}\}$$

contains a nontrivial genuine representation in every center grade. With $\mathrm{tr}(T_a T_b) = \frac{1}{2}\delta_{ab}$,

$$C_2(V_r) = \frac{r(n-r)(n+1)}{2n}, \qquad C_2(V_0) = n, \qquad c_{SU(n)} = \frac{n^2-1}{2n}. \tag{92}$$

For the symmetric finite UV/IR block $u = v = c_{SU(n)}$ and $0 \le m < c_{SU(n)}$, the certified floor is

$$\kappa = c_{SU(n)} - m > 0. \tag{93}$$

Proof. V_r has highest weight ω_r and center character r modulo n; the adjoint supplies a nontrivial neutral object. The type-A_{n-1} highest-weight Casimir formula gives equation (92), minimized at $r = 1$ or $n-1$. The comparison matrix with equal diagonal entries has eigenvalues $c_{SU(n)} \pm m$, giving equation (93). □

21.1 Pure center band: nontrivial associator, silent selector

For $k \in \mathbb{Z}_n$ and representatives $a, b, c \in \{0, \dots, n-1\}$, define

$$\Omega_k(a,b,c) := \exp\left(\frac{2\pi i k}{n} a \left\lfloor \frac{b+c}{n} \right\rfloor\right). \tag{94}$$

Proposition 21.2 (Cyclic center associator). *Ω_k is a normalized $U(1)$-valued 3-cocycle representing the class k in*

$$H^3(B\mathbb{Z}_n; U(1)) \cong \mathbb{Z}_n.$$

It is trivial exactly when $k = 0$ and has order $n/\gcd(n,k)$.

Proof. The cocycle equation reduces to the carry identity

$$\left\lfloor \frac{b+c}{n} \right\rfloor + \left\lfloor \frac{[b+c]_n + d}{n} \right\rfloor = \left\lfloor \frac{c+d}{n} \right\rfloor + \left\lfloor \frac{b + [c+d]_n}{n} \right\rfloor.$$

The normalized periodic resolution of $\mathbb{Z}_n$ identifies the resulting classes with the nth roots of unity, giving the stated label and order. □

On the inertia component labelled by $g \in \mathbb{Z}_n$, loop transgression is represented by

$$\theta_{k,g}(x,y) = \exp\left(\frac{2\pi i k g}{n} \left\lfloor \frac{x+y}{n} \right\rfloor\right). \tag{95}$$

Theorem 21.3 (Pure-center selector evaluation). *Let*

$$\beta_{k,g}(x) = \exp\left(\frac{2\pi i k g x}{n^2}\right).$$

Then $\theta_{k,g} = \delta\beta_{k,g}$. Thus the loop-transgression class vanishes on every pure cyclic center component. When the native relative scale class is zero, the witness has

k	b_D	s_D^{S}	*classification*
0	0	0	I,
$k \neq 0$	$[\Omega_k] \neq 0$	0	$\mathrm{III}_{\mathrm{silent}}$.

Proof. The identity $x + y = [x+y]_n + n\lfloor (x+y)/n \rfloor$ gives

$$\frac{\beta_{k,g}(x)\beta_{k,g}(y)}{\beta_{k,g}([x+y]_n)} = \theta_{k,g}(x,y).$$

Hence the displayed 2-cocycle is a coboundary on every inertia component. The sector statement follows from whether $[\Omega_k]$ itself vanishes. □

The cochain can still take a visible nontrivial value: for $g = 1$, $x = n-1$, and $y = 1$, one has $\theta_{k,1}(n-1, 1) = \zeta_n^k$. This is why the selector is classified by its cohomology class, not by one displayed phase.

21.2 Gauge–regional–refinement enhancement

Let one certified finite subdiagram have a typed band-isotropy quotient

$$A_{n,D} = \langle e_G, e_R, e_S \rangle \cong \mathbb{Z}_n^G \times \mathbb{Z}_n^R \times \mathbb{Z}_n^S, \tag{96}$$

where the superscripts retain gauge, regional/spacetime, and scale/refinement provenance. Define

$$\Omega_k^{\mathrm{mix}}(a,b,c) := \zeta_n^{k a_G b_R c_S}. \tag{97}$$

Theorem 21.4 (Detected Sector-III witness)**.** *Ω_k^{mix} is a normalized 3-cocycle. For $k \neq 0$, its class is nonzero. On the loop component $g = e_G$, its transgression is*

$$\theta_{k,e_G}(x,y) = \zeta_n^{k x_R y_S}, \tag{98}$$

which represents a nonzero degree-two class. If the selected filling space contains a certified regional–refinement 2-cycle T_{RS} mapped to (e_R, e_S), then

$$\langle s_D^{\mathsf{S}}, [T_{RS}] \rangle = \zeta_n^k \neq 1.$$

Thus the enriched package realizes $\mathrm{III}_{\mathrm{det}}$*.*

Proof. Trilinearity modulo n proves normalization and the cocycle identity. The standard loop-transgression formula gives equation (98). Its alternating commutator is

$$\zeta_n^{k(x_R y_S - y_R x_S)}.$$

At $(x,y) = (e_R, e_S)$ this equals $\zeta_n^k \neq 1$. A 2-coboundary on an abelian group has trivial alternating commutator, so the transgressed class is nonzero. A zero 3-class would transgress to zero, so the mixed 3-class is also nonzero. Pullback to T_{RS} gives the stated evaluation. □

The pointed Picard bicategory of $A_{n,D}$-graded Hermitian lines with associator Ω_k^{mix} realizes the banded 2-transport certificate: route grades add, comparison cells are unitary line maps, and the cocycle identity is exactly the pentagon. This is a genuine coefficient-line model; it does not identify the full carrier maps with lines.

21.3 Clock–shift evaluation of the mixed shadow

On a certified local n-dimensional carrier, define

$$C = \mathrm{diag}(1, \zeta_n, \ldots, \zeta_n^{n-1}), \qquad S e_j = e_{j+1},$$

with indices modulo n. Both have determinant $(-1)^{n-1}$. Put

$$\eta_n = e^{-\pi i (n-1)/n}, \qquad \widehat{C} = \eta_n C, \qquad \widehat{S} = \eta_n S.$$

Then $\widehat{C}, \widehat{S} \in SU(n)$ and $\widehat{C}\widehat{S} = \zeta_n \widehat{S}\widehat{C}$.

Proposition 21.5 (Typed clock–shift interaction witness)**.** *Let the gauge and regional–refinement marginal arrows act on the certified carrier by $U_G = \widehat{C}^a$ and $U_R = \widehat{S}^b$. Orient the carrier routes by*

$$U_{\mathrm{LR}} = U_R U_G, \qquad U_{\mathrm{RL}} = U_G U_R.$$

Their carrier commutator is central:

$$U_{\mathrm{RL}} = \zeta_n^{ab} U_{\mathrm{LR}}, \qquad U_{\mathrm{RL}}^* U_{\mathrm{LR}} = \zeta_n^{-ab} I. \tag{99}$$

Under the certified centrality comparison, this scalar descends to the actual coefficient-line 2-cell $\Theta_{\mathrm{act}} = \zeta_n^{-ab}$. If the independently calibrated structural cell is $\Theta_{\mathrm{ind}} = \zeta_n^{q_{\mathrm{str}}}$, then

$$\boxed{\mathbf{\Pi}_\square = \Theta_{\mathrm{act}} \Theta_{\mathrm{ind}}^{-1} = \zeta_n^{-ab-q_{\mathrm{str}}}.} \tag{100}$$

For a validated strict direct-product calibration $q_{\mathrm{str}} = 0$, the shadow is nontrivial exactly when $ab \not\equiv 0$ (mod n)*. If a declared semidirect or twisted calibration has $q_{\mathrm{str}} \equiv -ab$* (mod n)*, the structural phase is correctly subtracted and $\mathbf{\Pi}_\square = 1$.*

Proof. The determinant adjustment is scalar and leaves the clock–shift relation unchanged. Iteration gives $\widehat{C}^a\widehat{S}^b = \zeta_n^{ab}\widehat{S}^b\widehat{C}^a$, proving equation (99). Because the commutator is scalar, proposition 16.4 permits its coefficient-line descent on the declared irreducible carrier. The calibrated shadow is the ratio of the parallel actual and independent line cells. The final obstruction statement follows from theorem 20.6. □

Warning 21.6 (Carrier operator versus band cell)**.** $\widehat{C}^a$ and $\widehat{S}^b$ are carrier operators, not coefficient-line images. Only their certified central commutator descends to the band cell used by $\mathbf{\Pi}_\square$. Without the centrality comparison, the scalar formula is not a typed interaction certificate.

22 The $SU(2)$ and $SU(3)$ specializations

The two smallest cases make the independent coordinates visible without changing the general construction.

22.1 The $SU(2)$ witness

For $n = 2$, $\zeta_2 = -1$. The packet is

$$\Pi_2^{\mathrm{sec}} = \{\mathbf{3}, \mathbf{2}\},$$

where the adjoint $\mathbf{3}$ is a nontrivial neutral object and the fundamental $\mathbf{2}$ has odd center grade. In the chosen normalization,

$$c_{SU(2)} = \frac{3}{4}, \qquad \kappa_{SU(2)} = \frac{3}{4} - m \quad (0 \le m < \tfrac{3}{4}).$$

Corollary 22.1 ($SU(2)$ interaction ledger)**.** *The nonzero pure center class $k = 1$ gives $b_D \neq 0$ but $s_D^{\mathsf{S}} = 0$, hence $\mathrm{III}_{\mathrm{silent}}$. The enriched cocycle on $(\mathbb{Z}_2)^3$ evaluates to -1 on T_{RS} and gives $\mathrm{III}_{\mathrm{det}}$. For the strict clock–shift calibration with $a = b = 1$,*

$$\mathbf{\Pi}_\square = -1,$$

so finite admissible decoupling fails.

Proof. Specialize theorems 21.3 and 21.4 and equation (100) to $n = 2$, $k = a = b = 1$, and $q_{\mathrm{str}} = 0$. □

This witness does not assert that every $SU(2)$ realization is twisted or interacting. It shows that strict, silent-banded, detected-banded, and nontrivial calibrated interaction coordinates are all available as separately typed finite possibilities.

22.2 The $SU(3)$ witness

For $n = 3$, let $\zeta_3 = e^{2\pi i/3}$. The packet is

$$\Pi_3^{\mathrm{sec}} = \{\mathbf{8}, \mathbf{3}, \overline{\mathbf{3}}\},$$

with triality grades $0, 1, 2$. Its Casimir seed is

$$c_{SU(3)} = \frac{4}{3}, \qquad \kappa_{SU(3)} = \frac{4}{3} - m \quad (0 \le m < \tfrac{4}{3}).$$

Corollary 22.2 ($SU(3)$ interaction ledger)**.** *For $k = 1$ or 2, the pure cyclic center band has $b_D = [\Omega_k] \neq 0$ but $s_D^{\mathsf{S}} = 0$, giving $\mathrm{III}_{\mathrm{silent}}$. The enriched cocycle evaluates on T_{RS} as ζ_3^k and gives $\mathrm{III}_{\mathrm{det}}$. With strict calibration and $a = b = 1$,*

$$\mathbf{\Pi}_\square = \zeta_3^{-1} = e^{-2\pi i/3} \neq 1, \tag{101}$$

which obstructs finite admissible decoupling.

Proof. Apply the general formulas with $n = 3$. The exterior powers are $\bigwedge^1 \mathbb{C}^3 = \mathbf{3}$ and $\bigwedge^2 \mathbb{C}^3 = \overline{\mathbf{3}}$, while the adjoint is $\mathbf{8}$. The remainder follows from theorems 21.3 and 21.4 and equation (100). $\square$

Warning 22.3 (What the current triangular $SU(3)$ data force)**.** The cohomological-selector construction proved that the currently certified marked fusion-frame comparison has $r_D = 0$. Triality and ordinary change of fusion basis do not force the enriched mixed 3-cocycle or the clock–shift comparator. The present $SU(3)$ construction is an explicit admissible witness branch: it becomes a certificate only when the additional regional–refinement cycle, banded 2-transport, physical comparison cells, and validity ledger are realized.

22.3 Independence of the three coordinates

Corollary 22.4 (The internal firewall witnessed in $SU(n)$)**.** *For strict calibration, the associator label k and clock–shift label ab can vary independently:*

1. *$k = 0$ and $ab \neq 0$ gives $b_D = 0$, $s_D^{\mathsf{S}} = 0$, and a potentially nontrivial $\mathbf{\Pi}_\square$;*
2. *$k \neq 0$ and $ab = 0$ gives $b_D \neq 0$, pure-center $s_D^{\mathsf{S}} = 0$, and $\mathbf{\Pi}_\square = 1$; and*
3. *$k \neq 0$ and $ab \neq 0$ gives a nontrivial associator and a nontrivial mixed interaction shadow; the selector is silent or detected according to whether the pure-center or enriched three-axis band is realized.*

No equation between k and ab is canonical.

Proof. The associator and selector follow from theorems 21.3 and 21.4; the mixed shadow follows independently from equation (100). The formulas contain no identification between their parameters. $\square$

23 The downstream-ready mixed certificate

The interaction construction exports the finite record

$$\begin{aligned}\mathsf{Cert}_D^{\mathrm{mix}} = \big(&r_D, b_D, \widehat{\kappa}_D^{\mathrm{cs}}, [\ell_D]_{\mathrm{adm}}, s_D^{\mathsf{S}},\\ &\mathsf{Cal}_D, \mathsf{Val}_{\mathrm{int}}, \mathsf{Band2Tr}_D, \mathcal{T}_D^{\mathrm{band}}, \mathbf{D}_\square, \mathbf{\Pi}_\square\big),\end{aligned} \tag{102}$$

together with the semisimple detector packet, exact-ground projection, Casimir and UV/IR profile, seam reserves, provenance, support projections, Ward/AQFT slots, and typed restriction arrows established earlier.

Theorem 23.1 (Finite mixed-certificate recognition)**.** *Suppose one finite GAT 0 package satisfies the full physical predicate, cohomological and selector certificates, semisimple detector and coercive certificates, $\mathsf{Cal}_D = \mathsf{Val}_{\mathrm{int}} = \mathsf{Band2Tr}_D = 1$, and the declared restriction and provenance checks. Then equation (102) is a well-defined finite calibrated interaction certificate. Its nontrivial value $\mathbf{\Pi}_\square \neq 1$ obstructs finite admissible decoupling of that validated package.*

Proof. Sections 9–12 supply the native H^2 and H^3 data, canonical selected loop, and selector class. The independent-calibration theorem supplies the canonical surface up to admissible bordism. The banded 2-transport theorem supplies holonomy, and lemma 20.4 proves invariance under the declared equivalences. The obstruction is theorem 20.6. $\square$

Under theorem 17.3, the certificate may be included in every bounded full typed good locus. Persistence then requires restriction naturality of the marginal ledger, calibration, banded 2-transport, and actual surface. Objectwise nontrivial phases chosen independently do not constitute a persistent interaction family.

Warning 23.2 (Interaction reconstruction firewall)**.** The mixed certificate is a finite calibrated interaction shadow. It does not construct the complete interaction avatar, select a unique GNS realization, choose strict rather than twisted duality, prove Haag–Kastler reconstruction, construct the Grassmannian Fredholm fiber, implement Klauder's squared Gauss constraint window, or infer an invariant relativistic mass gap. Those are downstream reconstruction tasks. The Grassmannian Fredholm fiber is reserved for the next paper in the series.

23.1 Calibrated-interaction handoff

The paper now has a concrete first interaction theorem. Its "why" is the need to distinguish actual mixed continuation from the structural behavior of the same separately certified marginals. The construction uses independent calibration, oriented doubling, admissible bordism, and banded 2-holonomy. The resulting implication is deliberately one-sided:

$$\mathsf{Val}_{\mathrm{int}} = 1, \qquad \mathbf{\Pi}_{\square} \neq 1 \quad \Longrightarrow \quad \text{finite admissible decoupling fails.}$$

The explicit $SU(2)$, $SU(3)$, and $SU(n)$ witnesses show that this shadow is nonvacuous and independent of both the native relative H^2 coordinate and the internal H^3 associator coordinate. The remaining task for GAT 0 is the global theorem ledger, introduction, dependency audit, and final unification of notation across all sections.

24 The unified finite GAT 0 certificate

The preceding sections deliberately kept the source, physical, cohomological, analytic, selector, and interaction coordinates separate. The global theorem now assembles them without identifying them. For one finite demand D, the native recognition coordinates are

$$o_D^{\mathrm{fill}}, \qquad r_D \in H^2(\mathrm{Fib}(Q_D)), \qquad b_D \in H^3(B\mathcal{B}_D; U(1)), \qquad (C_D^{\mathrm{el}}, P_{\mathrm{vac,D}}, \kappa_D). \tag{103}$$

The derived selector class is obtained only through the certified map

$$b_D \xmapsto{\mathrm{trg}_{\ell_D}} s_D^{\mathsf{S}},$$

and the mixed interaction shadow is obtained only through the calibrated actual/independent double. Neither is a notational identification of the native coordinates.

Definition 24.1 (Unified finite GAT 0 certificate)**.** A unified finite certificate is the typed tuple

$$\mathfrak{C}_D^{\mathrm{GAT0}} = (\mathcal{C}_D^{\mathrm{src}}, \mathcal{C}_D^{\mathrm{loc}}, \mathcal{C}_D^{\mathrm{coh}}, \mathcal{C}_D^{\mathrm{an}}, \mathcal{C}_D^{\mathrm{mix}}), \tag{104}$$

with the following fields.

1. $\mathcal{C}_D^{\mathrm{src}}$ contains the demand, semisimple global form, genuine detector packet, formal proposal, source fingerprint, background, support convention, quotient marking, and provenance.
2. $\mathcal{C}_D^{\mathrm{loc}}$ contains the physically admissible realization, finite Yang–Mills package, detector readout, regional algebras, two typed Čech directions, local plaquette Hamiltonians, the Hamiltonian descent certificate $\mathsf{HamDesc}_D$, mixed seam data, Ward seed, and AQFT seed.
3. $\mathcal{C}_D^{\mathrm{coh}}$ contains the filling obstruction, realized relative lift r_D, coefficient lines, associator class b_D, selected band loop, selector class, and realized three-sector label.
4. $\mathcal{C}_D^{\mathrm{an}}$ contains the exact complete-ground projection, weighted Casimir seed, literal UV/IR block, independent complement bounds, the Čech-adjusted coercive triple $(u_D^0 - \varepsilon_D^{\mathrm{UV}}, v_D^0 - \varepsilon_D^{\mathrm{IR}}, m_D^0 + \varepsilon_D^{\mathrm{mix}})$, capacity and seam-reserve ledger, selection cost, and finite floor κ_D.
5. $\mathcal{C}_D^{\mathrm{mix}}$ contains the matched marginal ledger, independent calibration, actual and calibrated surfaces, banded 2-transport, primary bordism defect, interaction-validity status, and $\mathbf{\Pi}_{\square}$.

Every entry carries its type, domain, support, representative-equivalence relation, status, and permitted restriction arrows.

Definition 24.2 (Unified certification clauses)**.** The tuple in equation (104) is certified when the following seven clauses hold.

(UC1) **Genuine semisimple detection.** The packet consists of genuine representations of $G = \widetilde{G}/\Gamma$; its common lifted kernel is exactly Γ; its degrees generate $\widehat{Z(\widetilde{G})/\Gamma}$; and every declared residual degree has nonzero realized physical detector rank.

(UC2) **Physical lift and finite local interface.** The formal proposal has a full admissible physical realization satisfying the physical predicate of Sections 2–4. Its localization, readout, two-nerve, Gauss–Ward, seam, local plaquette Hamiltonian, Hamiltonian descent, and AQFT probe data are present with their honest finite statuses.

(UC3) **Native cohomology and selected tensor sector.** The filling obstruction vanishes for the realized point; the actual relative class r_D and coefficient-band class b_D exist in their declared native complexes; the selector-comparison certificate passes; and exactly one of the realized Sectors I, II, III of theorem 11.7 is recorded, with silent or detected subtype in Sector III.

(UC4) **Exact-ground Casimir coercivity.** $P_{\mathrm{vac,D}}$ is the exact complete-ground projection. Every required sector is nonzero, the Hamiltonian two-nerve residual has been charged by equation (58), every resulting literal UV/IR block satisfies the strict determinant test, and every independent target complement and seam reserve has been certified. The resulting estimate is

$$H_D \geq E_{0,D} I + \kappa_D (I - P_{\mathrm{vac,D}}), \qquad \kappa_D > 0. \tag{105}$$

(UC5) **Calibrated mixed coherence.** $\mathsf{Cal}_D = \mathsf{Band2Tr}_D = 1$; actual and independent surfaces are compared only after the full boundary ledger matches; and $\mathbf{\Pi}_\square$ is interpreted as an interaction shadow only when $\mathsf{Val}_{\mathrm{int}} = 1$.

(UC6) **Restriction-ready comparison data.** Every proposed restriction has separately typed components for carriers, regional algebras, projections, forms, coefficient bands, cohomology, selector data, mixed surfaces, Ward/AQFT slots, and defect ledgers. A unitary, isometry, completely positive map, correspondence, partial isometry, or band cell is used only with its declared type and validity theorem.

(UC7) **Finite demand and provenance completeness.** Every finite demand claimed by the certificate is tested, one-sided new complements are independently certified, all source fingerprints and backgrounds remain visible, and no Čech subsystem is called cofinal without a full-package cofinal comparison theorem.

Theorem 24.3 (Unified finite GAT 0 recognition)**.** *For fixed finite data, clauses* (UC1)–(UC7) *are sufficient to construct a unified finite certificate* $\mathfrak{C}_D^{\mathrm{GAT0}}$. *It has the following invariant conclusions.*

1. *It recognizes the marked compact connected semisimple global form at the stated genuine packet and physical realization.*
2. *It retains the relative* H^2 *lift and internal* H^3 *associator as independent native coordinates, together with the certified selector image of the latter and the realized three-sector label.*
3. *It carries a finite Gauss–Ward, plaquette-Hamiltonian descent, and AQFT probe interface on the two typed Čech directions without promoting any open status.*
4. *It carries an exact complete-ground projection and the positive finite coercive estimate equation* (105).
5. *If* $\mathsf{Val}_{\mathrm{int}} = 1$ *and* $\mathbf{\Pi}_\square \neq 1$, *finite admissible decoupling of the matched mixed square fails.*

Proof. Clause (UC1) combines theorem 13.3 with the nonzero physical-rank ledger. Clause (UC2) invokes the realization and local interface theorems of Sections 2–8 together with proposition 14.4. Clause (UC3) applies the filling theorem, the internal associator construction, and theorems 11.5 and 11.7. These results retain the two native complexes while constructing the selector through an explicit transgression.

Clause (UC4) combines proposition 14.7, theorem 14.8, corollary 14.9, and lemma 15.1. Taking the minimum of the certified sector blocks and independent complement bounds gives equation (105). Exactness of $P_{\mathrm{vac,D}}$ identifies the estimated carrier with the complete excitation complement.

Clause (UC5) invokes the independent-calibration and banded 2-transport theorems. Invariance under the declared equivalences follows from lemmas 19.8 and 20.4, and the interaction conclusion is theorem 20.6. Clauses (UC6)–(UC7) retain the typed arrows and provenance required by downstream consumers. Collecting the five fields in definition 24.1 proves the theorem. □

Corollary 24.4 (General semisimple recognition route)**.** *For every compact connected semisimple global form, the algebraic detector packet and universal electric Casimir seed required by definition 24.2 exist. Hence all remaining obligations in the unified theorem are explicitly physical, cohomological-comparison, magnetic/IR, seam, calibration, or extension obligations. No separate case-by-case theorem is required for the classical or exceptional Lie families at the algebraic and electric levels.*

Proof. Apply theorem 13.3 and proposition 14.7. The factor list in equation (41) includes every compact simply connected simple classical and exceptional factor; the central quotient is retained by the common-kernel condition. □

25 The theorem dependency ledger

The proof is not a linear chain in which every finite construction already contains its continuum use. It is a directed acyclic dependency architecture. The central spine is

$$\begin{gathered}\text{typed proposal} \longrightarrow \text{physical realization} \longrightarrow \begin{matrix}\text{local Ward/AQFT interface,}\\ \text{native } H^2 \text{ and } H^3 \text{ coordinates}\end{matrix}\\ \longrightarrow \begin{matrix}\text{selector and calibrated mixed shadow,}\\ \text{Hamiltonian two-nerve descent and exact-ground coercivity}\end{matrix} \longrightarrow \text{unified finite certificate}\\ \longrightarrow \text{compatible family} \longrightarrow \text{typed reconstruction input.}\end{gathered} \tag{106}$$

The selector branch and coercive branch meet only in the unified certificate; neither is used to prove the native existence of the other.

Definition 25.1 (Theorem ledger entry)**.** A theorem ledger entry is a quadruple

$$(\mathsf{Input}, \mathsf{Result}, \mathsf{Output}, \mathsf{Firewall})$$

recording the hypotheses consumed, theorem applied, data produced, and stronger conclusion explicitly not produced.

The principal ledger is as follows.

1. **Physical realization.** Input: a well-typed finite proposal and physical predicate. Result: finite realization and complete defect ledger. Output: a physical package and source-native comparison slots. Firewall: no comparison between arbitrary admissible fibers.
2. **Two-nerve local interface.** Input: a physical package, gauge cover, spacetime-region cover, and seam data. Result: typed mixed faces, finite Gauss–Ward seed, local plaquette Hamiltonians with $\mathsf{HamDesc}_D$, and AQFT probe seed. Output: status-bearing local and Hamiltonian-descent interface. Firewall: no completed Ward hierarchy or Haag–Kastler net.
3. **Relative lift.** Input: strict defect coefficients and raw refinement mismatch. Result: filling obstruction and, after vanishing, a selected mapping-fiber class r_D. Output: the native relative H^2 coordinate. Firewall: no internal associator follows from this class.
4. **Coefficient band.** Input: realized coefficient lines and comparison compositors. Result: the normalized associator class b_D. Output: the native internal H^3 coordinate. Firewall: no physical realization of an arbitrary formal class.
5. **Tensor selector.** Input: b_D, a selector-comparison certificate, and a canonical selected loop. Result: s_D^{S} and the three realized sectors. Output: the H^3-indexed selector sector. Firewall: no quotient or identification of the native H^2 and H^3 complexes.

6. **Semisimple coercivity.** Input: genuine detector packet, nonzero physical sector ranks, exact ground, weighted Casimir, magnetic/IR bound, literal mixed corner, and the three Hamiltonian-descent residual budgets. Result: the Čech-adjusted UV/IR lower-eigenvalue theorem and seam reserve criterion. Output: a finite positive floor. Firewall: no continuum or relativistic mass gap.
7. **Calibrated interaction shadow.** Input: matched marginals, nonanticipatory contractible calibration, actual surface, and banded 2-transport. Result: $\mathbf{D}_{\square}$ and $\mathbf{\Pi}_{\square}$. Output: an obstruction to finite admissible decoupling when the phase is nontrivial. Firewall: trivial phase is not complete decoupling.
8. **Compatible-family recognition.** Input: compact full typed good loci, coherent continuous restrictions, one cutoff, full extension preserving $\mathsf{HamDesc}$, and feasible common-floor seams. Result: a nonempty inverse limit. Output: a compatible family with persistent finite floor. Firewall: no continuum reconstruction without its own comparison theorem.

Theorem 25.2 (Dependency closure and absence of circularity)**.** *Every conclusion of theorem 24.3 is supported by an earlier ledger entry, and no ledger entry uses as a hypothesis the global conclusion to which it contributes. In particular:*

1. *the H^3 associator is constructed before its selector transgression;*
2. *the independent interaction calibration is constructed before the actual/calibrated holonomy ratio;*
3. *the exact complete-ground projection is assumed and verified before the excitation lower bound is interpreted as a spectral floor; and*
4. *Hamiltonian two-nerve residuals are charged to the UV, IR, and mixed budgets before the coercive determinant is tested; and*
5. *compatible-family persistence is proved from full extension and compactness, not inferred from pointwise positivity.*

Proof. Order the ledger entries by the arrows in equation (106). The physical realization precedes every realized cohomological or analytic coordinate. The two native cohomologies are defined independently; selector transgression consumes the already realized band class and selected loop. Calibration consumes only matched marginals and the previously declared structural action, while holonomy consumes the resulting closed double. The coercive theorem consumes the exact-ground, Hamiltonian-descent, and adjusted block ledger. Finally, the inverse-limit theorem consumes the complete finite certificates. Every edge moves forward in this order, so the graph is acyclic and each global conclusion has an earlier source. □

25.1 Universal results and witness-only results

Proposition 25.3 (Quantifier ledger)**.** *The following scope distinctions are exact.*

1. *The finite genuine detector-packet theorem and universal electric Casimir floor hold for every compact connected semisimple global form.*
2. *Physical survival, magnetic/IR bounds, selected fillings, banded 2-transport, Hamiltonian-descent coherence and residual budgets, interaction calibration, and full extension are certificate hypotheses unless a named witness theorem supplies them.*
3. *The $SU(2)$, $SU(3)$, and $SU(n)$ calculations of Sections 9–12 and 18–23 are finite witnesses at their stated carriers and do not universalize their sectorial types or interaction phases to all realizations of those groups.*
4. *A fixed-data finite obstruction excludes only the fixed packet, physical realization rules, type, profile, cutoff, floor, selector, and calibration; it excludes the group only after all allowed outer choices have been exhausted.*

Proof. The first assertion is theorem 13.3 and proposition 14.7. The second is the hypothesis list in definition 24.2. The witness firewalls are stated in Sections 9–12 and 18–23. The last assertion is theorem 17.5. □

26 Typed export to downstream reconstruction

There are two notions of readiness. *Interface readiness* means that every field and comparison slot has a type, provenance, and honest status. *Theorem readiness* means that the complete hypotheses of one named downstream theorem have passed. GAT 0 constructs the first; it asserts the second only relative to an explicit requirement profile.

Definition 26.1 (Finite reconstruction input)**.** The finite reconstruction input exported by $\mathfrak{C}_D^{\mathrm{GAT0}}$ is

$$\begin{aligned}\mathsf{RecIn}_D = (&\mathsf{Key}_D, \mathsf{Carr}_D, \mathsf{Alg}_D, \mathsf{Proj}_D, \mathsf{Form}_D, \mathsf{Coh}_D,\\ &\mathsf{Sel}_D, \mathsf{Int}_D, \mathsf{Ward}_D, \mathsf{Net}_D, \mathsf{CmpSlot}_D).\end{aligned} \tag{107}$$

Its fields have the following meanings.

1. Key_D: demand, group, quotient, packet, background, support, source fingerprint, provenance, and sector label.
2. Carr_D: localization and detector carriers and their readout relation. Carriers may be Hilbert spaces, self-dual Hilbert W^*-modules, correspondences, or other declared objects.
3. Alg_D: finite region poset, supported algebras, internal inclusions, separation relation, and cover convention.
4. Proj_D: separately typed Gauss, vacuum, charge, Riesz, and selector projections.
5. Form_D: local plaquette Hamiltonians or closed forms, their domains, exact grounds, $\mathsf{HamDesc}_D$, Hamiltonian Beck–Chevalley cells, the residual budget ε_D^H, the adjusted block/complement/seam ledger, and the finite floor.
6. Coh_D: o_D^{fill}, r_D, b_D, native complexes, representatives, and permitted equivalences.
7. Sel_D: selected loop, transgression, selector class, and three-sector label.
8. Int_D: matched marginals, actual and independent surfaces, validity status, banded 2-transport, bordism defect, and mixed phase.
9. Ward_D and Net_D: the finite Ward and AQFT seeds with every status retained.
10. $\mathsf{CmpSlot}_D$: the types of comparison arrows and Hamiltonian form inequalities a later regional, refinement, cofinal, represented, spectral, interaction, or Fredholm reconstruction may request.

Definition 26.2 (Typed reconstruction comparison)**.** For an inclusion $i : D \hookrightarrow D'$, a typed comparison

$$\mathsf{R}_i : \mathsf{RecIn}_{D'} \longrightarrow \mathsf{RecIn}_D$$

is a tuple of carrier, algebra, projection, Hamiltonian-descent/form, band, cohomology/selector, mixed-surface, Ward, AQFT, provenance, and defect-ledger components. It must:

1. preserve source, target, variance, support, background, and provenance or provide their displayed comparison cells;
2. preserve internal algebra inclusions without identifying them with the external comparison;
3. state every exact projection intertwining and form inequality used downstream, placing every failure in the defect ledger;
4. preserve the gauge and spacetime Hamiltonian comparisons, their higher coherence cells, and the three residual budgets, or record the resulting change in the adjusted coercive triple;
5. preserve the normalized associator and banded 2-transport up to the declared coherent cells;
6. preserve both native cohomology complexes and commute with certified selector transgression;
7. preserve no Ward or AQFT status by default; each preservation is proved or recorded as a defect; and

8. carry invertible compositors for successive restrictions, satisfying unit and pentagon identities componentwise.

The carrier component is not required to be unitary. Its admissible type is whatever the relevant comparison theorem actually proves.

Proposition 26.3 (Bicategory of finite reconstruction inputs)**.** *Finite reconstruction inputs, typed comparisons, and componentwise invertible modifications form a bicategory* $\mathbf{RecIn}^{\mathrm{fin}}$*. If every restriction compositor is an identity, they form an ordinary category.*

Proof. Compose like-typed components and paste their validity cells. Defect ledgers compose by the declared propagation rules rather than by deletion. Identity comparisons have identity components and zero defects. The componentwise unit and pentagon laws supply the bicategory axioms. □

Theorem 26.4 (No-loss finite export)**.** *Every unified finite certificate determines a finite reconstruction input* RecIn_D*. The export preserves, as separately recoverable data:*

1. *the native* H^2 *and* H^3 *coordinates and the certified selector map;*
2. *the actual, independently calibrated, and closed mixed surfaces;*
3. *the exact complete-ground projection, local plaquette Hamiltonians,* $\mathsf{HamDesc}_D$*, Casimir seed, descent-residual budgets, adjusted UV/IR ledger, seam reserves, and finite floor;*
4. *every Ward and AQFT status without promotion; and*
5. *every absent comparison as an empty typed slot rather than an implicit unitary or common GNS realization.*

Proof. Each field in equation (107) is a named subtuple of $\mathfrak{C}_D^{\mathrm{GAT0}}$ with its type and provenance. The export takes no quotient identifying the mapping-fiber, band, selector, or mixed-surface coordinates. It copies the projection, form, defect, and status ledgers entrywise and manufactures no comparison. □

Theorem 26.5 (Compatible-family export)**.** *Let* $(\mathfrak{C}_D^{\mathrm{GAT0}})_{D\in\mathcal{J}}$ *be a compatible family from theorem 17.3. Its no-loss exports and full typed restrictions define a contravariant pseudofunctor*

$$\mathsf{RecIn}_{\mathcal{J}} : \mathcal{J}^{\mathrm{op}} \longrightarrow \mathbf{RecIn}^{\mathrm{fin}}. \tag{108}$$

The persistent finite floor is a restriction-compatible lower-bound field. The Hamiltonian-descent certificate, selector class, and calibrated mixed shadow are pseudonatural fields when their corresponding restriction cells pass.

Proof. Objectwise export is theorem 26.4. The compatible-family theorem and clause (UC6) supply the typed components of every restriction. Their full coherence supplies the compositors and pentagon, so proposition 26.3 gives equation (108). The common exact-ground inequality gives the persistent lower-bound field. Hamiltonian-descent, selector, and mixed-shadow naturality hold precisely when their named comparison components are part of the compatible family. □

27 Target-relative readiness and the exact endpoint

Definition 27.1 (Downstream requirement profile)**.** A downstream requirement profile Req is a finite list of fields, identities, inequalities, convergence properties, and acceptable statuses in the signature of definition 26.1. Write

$$\mathsf{Pass}_{\mathsf{Req}}(\mathsf{RecIn}) = 1$$

only when every required entry is proved, or every dependency of a conditional entry is included and discharged. Open, minimal, or unposed entries do not pass a requirement asking for a theorem.

The principal exposed profiles are:

1. $\mathsf{Req}_{\mathrm{Ward}}$: Gauss domain and kernel, observable preservation, post-reduction locality, boundary matching, finite variation, covariance, ground-state identity, and restriction/readout clauses;
2. $\mathsf{Req}_{\mathrm{AQFT}}$: isotony, locality, generation, dynamics/covariance, vacuum, readout, Gauss–Ward, and transport clauses;
3. $\mathsf{Req}_{G\text{-phys}}$: nonzero realized detector ranks, magnetic physicalization, exact Gauss reduction, seam matching, exact complete ground, and compatible extension for the declared compact semisimple global form;
4. $\mathsf{Req}_{\mathrm{cof}}$: a full compatible family, a proved full-package cofinal comparison, one cutoff, compatible $\mathsf{HamDesc}$ cells, uniform descent-residual budgets, and uniform seam reserves;
5. $\mathsf{Req}_{\mathrm{spec}}$: closed semibounded forms, exact grounds, persistent Čech-adjusted floor, Mosco comparison, convergence of the exact ground projections, and the Riesz/nonleakage data demanded by the chosen spectral theorem;
6. $\mathsf{Req}_{\mathrm{Ham/id}}$: a compatible strongly continuous time-translation family, its Stone generators, convergence of the reconstructed forms on the physical Gauss-reduced domains, and proof that the limiting form operator equals the physical Stone Hamiltonian;
7. $\mathsf{Req}_{\mathrm{GNS/dual}}$: the chosen representation family and the strict, twisted, sectional, or other duality hypotheses of the target theorem, together with domain-preserving equivariance of the local Hamiltonian family under the certified H^3 reassociator;
8. $\mathsf{Req}_{\mathrm{Kl}}$: the domain, squared Gauss constraints, window, projection estimates, convergence to the declared Gauss kernel, and form compatibility needed for Klauder's implementation;
9. $\mathsf{Req}_{\mathrm{Fr}}$: module, polarization, restricted Grassmannian, Fredholm, determinant-line, and comparison data reserved for the next paper;
10. $\mathsf{Req}_{\mathrm{Int},\infty}$: restriction-natural actual and independent calibrations, compatible doubled bordisms, banded holonomy, full-package cofinal comparison, and any represented or spectral realization claimed for the persistent interaction shadow; and
11. $\mathsf{Req}_{\mathrm{cont}}$: completion over the declared region class, compatible continuum algebras and dynamics, Poincaré covariance, spectrum condition, vacuum data, and every comparison demanded jointly by $\mathsf{Req}_{\mathrm{AQFT}}$, $\mathsf{Req}_{\mathrm{cof}}$, $\mathsf{Req}_{\mathrm{spec}}$, and $\mathsf{Req}_{\mathrm{Ham/id}}$.

No converse interaction theorem is hidden in $\mathsf{Req}_{\mathrm{Int},\infty}$. A claim that $\mathbf{\Pi}_{\square} = 1$ implies decoupling requires an additional completeness profile. Likewise, $\mathsf{Req}_{\mathrm{Ham/id}}$ separates convergence to a closed coercive operator from identification of that operator with physical time translation.

Two composite interfaces make the later splice points explicit:

$$\begin{aligned} \mathsf{Req}_{\mathrm{YM,cont}} &:= \mathsf{Req}_{G\text{-phys}} \cup \mathsf{Req}_{\mathrm{cof}} \cup \mathsf{Req}_{\mathrm{spec}} \cup \mathsf{Req}_{\mathrm{Ham/id}} \cup \mathsf{Req}_{\mathrm{Kl}} \cup \mathsf{Req}_{\mathrm{AQFT}}, \\ \mathsf{Req}_{\mathrm{Int,rec}} &:= \mathsf{Req}_{\mathrm{Int},\infty} \cup \mathsf{Req}_{\mathrm{GNS/dual}} \cup \mathsf{Req}_{\mathrm{Fr}}. \end{aligned} \tag{109}$$

The first is the complete exposed interface for a continuum physical Hamiltonian theorem; the second is the interface for represented persistence of the mixed interaction shadow. These unions collect hypotheses and do not assert that their downstream conclusions have already been proved.

Theorem 27.2 (Target-relative reconstruction readiness)**.** *Let $\mathfrak{R}$ be a downstream theorem whose domain is specified by $\mathsf{Req}_{\mathfrak{R}}$. A finite certificate or compatible family is admissible input to $\mathfrak{R}$ exactly when*

$$\mathsf{Pass}_{\mathsf{Req}_{\mathfrak{R}}}(\mathsf{RecIn}) = 1. \tag{110}$$

In particular, a finite AQFT seed is not a completed net; a pointwise finite floor is not a persistent or continuum gap; a compatible family is not a cofinal completion without a proved cofinal comparison; an empty GNS/duality slot chooses no representation; and a typed Fredholm slot is not a Grassmannian Fredholm realization. Passing $\mathsf{Req}_{\mathrm{spec}}$ *without* $\mathsf{Req}_{\mathrm{Ham/id}}$ *does not identify the limiting operator as physical time translation, and a finite* $\mathbf{\Pi}_{\square}$ *without* $\mathsf{Req}_{\mathrm{Int},\infty}$ *is not a persistent represented interaction shadow.*

Proof. Necessity is the definition of the target theorem's domain. Sufficiency means only that the already-stated theorem may be applied because its full hypotheses have passed. Each final assertion is the corresponding failed profile with one or more required fields absent. □

Corollary 27.3 (Exact GAT 0 handoff)**.** *The natural endpoint of GAT 0 is*

$$\boxed{(\mathsf{RecIn}_{\mathcal{J}}, \{\mathsf{Pass}_{\mathsf{Req}}\}_{\mathsf{Req}})\,.} \tag{111}$$

The first coordinate is the complete typed finite cargo; the second is its target-relative theorem ledger. Every downstream paper can therefore state exactly what it imports, what additional theorem it proves, and which slots remain open.

Warning 27.4 (Final source boundary)**.** GAT 0 proves finite recognition, compatible-family selection under explicit hypotheses, persistent finite coercivity, and a calibrated first interaction shadow. It does not prove a unique GNS realization, strict or twisted duality, a continuum AQFT net, Grassmannian Fredholm reconstruction, Klauder's constraint projector, Mosco–Riesz continuum convergence, an invariant relativistic spectrum, identification with a physical Stone Hamiltonian, cofinal represented persistence of $\mathbf{\Pi}_{\square}$, or a Yang–Mills mass-gap theorem. Those conclusions require the downstream profiles above.

28 Acknowledgements and responsibility statement

The author developed the mathematical direction, conjectures, witness architecture, and theorem program of this work. Computational language-model tools were used as editorial and organizational aids in consolidating notation, checking internal dependencies, and preparing parts of the exposition. All mathematical claims and responsibility for the manuscript remain with the author.

29 Conclusion

GAT 0 identifies the finite source object required by the later Geometry of Admissible Transport architecture. The general semisimple theorem supplies a genuine finite detector packet and universal Casimir seed for every compact connected semisimple global form. The physical predicate then separates algebraic availability from realized nonzero rank. The two Čech directions retain gauge/admissibility and spacetime-localization provenance. The relative H^2 class records the realized refinement–admissibility lift; the internal H^3 class records the associator; and certified transgression lets that associator select among three tensor sectors without collapsing the native complexes.

Analytically, the quadratic Casimir is the universal electric coercive agent, but the theorem keeps the IR diagonal, exact mixed corner, target complement, and seam determinant visible. Compact full-package selection and extension then promote one finite certificate to a compatible family with a persistent finite floor. Interaction enters through a separate calibrated comparison: the actual mixed surface is doubled against a nonanticipatorily assembled independent surface, and nontrivial banded holonomy obstructs finite admissible decoupling.

The final object is consequently neither a bare gap number nor a single GNS representation. It is the typed pseudofunctor of reconstruction inputs and its requirement ledger in equation (111). That is the precise finite cargo handed to the remainder of the series.

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