

Vacuum Readjustment under Gauge-Cell Attachment

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Adding a gauge cell in its isolated interacting vacuum need not preserve the vacuum of the system receiving it. We establish this distinction on a four-edge, three-cell $SU(2)$ graph, without a representation cutoff. Starting from the edge Casimirs, we derive the shared electric interaction and an exact receiving-defect identity: its squared norm is the product of the added vacuum's electric response and the old state's seam-electric energy, with coefficient $4A^2/3$. The native ground state is not a product across the specified cycle-variable partition, and its restriction to a declared gauge-invariant old-system algebra differs strictly from the old pure vacuum. Near zero seam coupling, the old-vacuum excitation probability has a strictly positive quadratic coefficient. We distinguish the fixed cycle-factor vacuum test from the retained old-edge energy and show that vacuum matching cannot preserve the fixed test. An Abelian control separates this attachment mechanism from specifically non-Abelian effects. These are finite-graph results, not a spacetime localization or continuum mass-gap theorem.

The comparison problem.—A new cell arrives with a vacuum of its own. The old graph has one too. There is no reason to assume that putting the two together gives the vacuum of the joined graph: the shared edge still has to be accounted for. Here that question has a direct answer. We derive the electric coupling from the original edge generators and ask which vacuum comparison it actually changes.

Hamiltonian lattice gauge theory provides a setting in which this distinction can be computed directly from edge rotors [1]; related loop and maximal-tree reductions appear in [2]. Gauge constraints also require care in specifying subsystem observables: a tensor product of cycle-variable spaces is not a tensor product of independent physical gauge systems [4]. Here the partition, the observable algebra, and the vacuum comparison are fixed explicitly. The result identifies what fails under isolated-vacuum insertion, and what must change when the insertion is dressed to match the full vacuum.

Edge-derived model.—Let four edges $u_0, u_1, u_2, u_3 \in SU(2)$ join the same two vertices, with endpoint action $u_i \mapsto au_i b^{-1}$. Use normalized Haar measure and cycles

$$x = u_0 u_1^{-1}, \quad y = u_1 u_2^{-1}, \quad z = u_2 u_3^{-1}. \quad (1)$$

The inverse coordinates are $u_0 = xyzg$, $u_1 = yzg$, $u_2 = zg$, and $u_3 = g$; they preserve product Haar measure. Removing the right endpoint action leaves simultaneous conjugation on (x, y, z) . Thus

$$\begin{aligned} \mathcal{K}_2 &= L^2(SU(2)^2), & \mathcal{K}_z &= L^2(SU(2)), \\ \mathcal{H}_3 &= (\mathcal{K}_2 \otimes \mathcal{K}_z)^{SU(2)}. \end{aligned} \quad (2)$$

with old physical space $\mathcal{H}_2 = \mathcal{K}_2^{SU(2)}$. Partial traces below use $\mathcal{K}_2 \otimes \mathcal{K}_z$, not a postulated physical factorization.

Let L^a, R^a be self-adjoint generators of left and right multiplication, with $C = \sum_a (L^a)^2 = \sum_a (R^a)^2$ and $C\chi_j = j(j+1)\chi_j$. Differentiating the four original edges

induces

$$L_x^a, \quad -R_x^a + L_y^a, \quad -R_y^a + L_z^a, \quad -R_z^a. \quad (3)$$

Consequently, the native electric form is

$$\begin{aligned} q_E[F] &= A \sum_a (\|L_x^a F\|^2 + \|(-R_x^a + L_y^a)F\|^2 \\ &\quad + \|(-R_y^a + L_z^a)F\|^2 + \|R_z^a F\|^2). \end{aligned} \quad (4)$$

For $A, B > 0$ and $\chi = \chi_{1/2}$, add the potential $B(6 - \chi(x) - \chi(y) - \chi(z))$. On the smooth core,

$$H_3 = H_2 \otimes I + I \otimes h_z + V, \quad (5)$$

$$\begin{aligned} H_2 &= 2A(C_x + C_y) - 2AR_x \cdot L_y \\ &\quad + B(4 - \chi(x) - \chi(y)), \\ h_z &= 2AC_z + B(2 - \chi(z)), \quad V = -2AR_y \cdot L_z. \end{aligned} \quad (6)$$

The coefficient of V is fixed by the edge squares, not assigned independently. Retaining only the original three edge contributions on the enlarged carrier instead defines

$$H_{2,\text{edge}} = H_2 \otimes I + AC_z + V. \quad (7)$$

These are different comparisons, even though both are defined on the enlarged graph. The supplement proves that the retained old-edge operator has ground projection $\Pi_{2,\text{edge}} = |\Omega_2 \otimes 1\rangle\langle\Omega_2 \otimes 1|$ on the physical Hilbert space. Its test already registers the isolated product state $\Psi_0 = \Omega_2 \otimes \phi$ through $1 - |\langle 1, \phi \rangle|^2 > 0$. This is not the receiving response measured below by $Q_2 \otimes I$, which vanishes on Ψ_0 .

Write $H(t) = H_0 + tV$, $H_0 = H_2 \otimes I + I \otimes h_z$, with $t = 1$ the native graph. The kinetic forms satisfy

$$q_{E,t}[F] \geq A(2 - \sqrt{2}) \sum_{w=x,y,z} \langle F, C_w F \rangle, \quad 0 \leq t \leq 1. \quad (8)$$

The common closed form domain is $H^1(SU(2)^3)$; the associated operators have compact resolvent and a simple,

smooth, strictly positive ground state. Gauge invariance follows from uniqueness. The self-contained reduction, ellipticity estimate, and domain details are supplied in the Supplemental Material [5].

Exact receiving defect.—Let Ω_2 and ϕ be the normalized positive ground states of H_2 and h_z , with energies E_2, e_z . Define

$$J_\phi f = f \otimes \phi, \quad P_\phi = J_\phi J_\phi^*, \quad Q_\phi = I - P_\phi. \quad (9)$$

Since ϕ is a class function, J_ϕ is gauge equivariant and sends $\mathcal{H}_2$ into $\mathcal{H}_3$.

Theorem 1 (Shared-edge receiving identity). *For smooth $f \in \mathcal{K}_2$, the native off-diagonal block is*

$$K_\phi f := Q_\phi H_3 J_\phi f = -2A \sum_a (R_y^a f) \otimes (L_z^a \phi), \quad (10)$$

with exact norm

$$\|K_\phi f\|^2 = \frac{4A^2}{3} \langle \phi, C_z \phi \rangle \langle f, C_y f \rangle. \quad (11)$$

The resulting first-order operator closes on $D(C_y^{1/2})$. For $B > 0$, its kernel consists precisely of functions independent of y .

Proof. The decoupled Hamiltonian preserves $\text{Ran } J_\phi$. Real positivity and Haar integration give $\langle \phi, L_z^a \phi \rangle = 0$, so the mixed term is orthogonal to that range. Adjoint invariance of the class function makes its electric covariance isotropic:

$$\langle L_z^a \phi, L_z^b \phi \rangle = \frac{\delta_{ab}}{3} \langle \phi, C_z \phi \rangle. \quad (12)$$

This proves the norm identity. Its first factor cannot vanish: otherwise ϕ would be constant, whereas $h_z 1 = B(2 - \chi(z))$ is nonconstant. The domain and kernel follow from the closed y -Casimir form. $\square$

In particular, for $\Psi_0 = \Omega_2 \otimes \phi$,

$$b := V\Psi_0 \neq 0. \quad (13)$$

Indeed, Ω_2 cannot be independent of y : its eigenvalue equation would then contain an uncanceled term $-B\chi(y)\Omega_2$.

Vacuum readjustment at native coupling.—Declare the old observable algebra to be

$$\mathfrak{A}_{\text{old}} = \{O \in \mathcal{B}(\mathcal{K}_2) : [O, U_2(g)] = 0 \ \forall g \in \text{SU}(2)\}, \quad (14)$$

represented on $\mathcal{H}_3$ by $(O \otimes I)|_{\mathcal{H}_3}$. Here U_2 is simultaneous conjugation. In particular,

$$P_2 = |\Omega_2\rangle\langle\Omega_2|, \quad Q_2 = I - P_2 \quad (15)$$

belong to $\mathfrak{A}_{\text{old}}$. This fixes a cycle-factor comparison. It admits a gauge-invariant lift supported on the original edge

set, but is not asserted to preserve the enlarged native old-edge Hamiltonian or to define a spacetime-exterior local algebra. The witness already belongs to the old Hamiltonian's spectral algebra, since $P_2 = s\text{-}\lim_{\tau \rightarrow \infty} e^{-\tau(H_2 - E_2)}$.

Theorem 2 (Strict native vacuum readjustment). *Let Ω_3 be the normalized positive ground state of H_3 . It is not a product across $\mathcal{K}_2 \otimes \mathcal{K}_z$. For $\rho_2 = \text{Tr}_z |\Omega_3\rangle\langle\Omega_3|$,*

$$p_{\text{exc}} := \langle \Omega_3, (Q_2 \otimes I) \Omega_3 \rangle > 0, \quad (16)$$

$$D(\rho_2, P_2) := \frac{1}{2} \|\rho_2 - P_2\|_1 \geq p_{\text{exc}} > 0, \quad (17)$$

$$E_3 < E_2 + e_z. \quad (18)$$

Proof. If $\Omega_3 = \psi \otimes \eta$, its strict positivity permits smooth normalized positive factors. Every individual electric generator has zero expectation in a real factor. Projecting the eigenvalue equation onto η and onto ψ therefore makes each factor a positive eigenfunction of its isolated Hamiltonian. Uniqueness forces $\psi = \Omega_2$ and $\eta = \phi$. But $V\Psi_0 = b \neq 0$ is orthogonal to Ψ_0 , so this product is not an eigenvector. This proves nonfactorization. If $p_{\text{exc}} = 0$, the full vector would belong to $\Omega_2 \otimes \mathcal{K}_z$, again a product. The effect Q_2 gives (17). Finally $\langle \Psi_0, H_3 \Psi_0 \rangle = E_2 + e_z$; equality with the ground energy would make Ψ_0 a ground vector, which it is not. $\square$

Both ρ_2 and P_2 commute with U_2 . Thus the optimal trace-distance effect can itself be chosen in $\mathfrak{A}_{\text{old}}$; the distinction in (17) does not require gauge-noninvariant measurements.

The response is also quantitative near decoupling. Let $E_* = E_2 + e_z$ and let R_* be the inverse of $H_0 - E_*$ on $\Psi_0^\perp$, extended by zero on Ψ_0 . Analytic form perturbation theory [6] gives

$$\Omega(t) = \Psi_0 - tR_*b + O(t^2), \quad (19)$$

$$\langle \Omega(t), (Q_2 \otimes I) \Omega(t) \rangle = t^2 c_{\text{exc}} + O(t^3),$$

$$c_{\text{exc}} = \|R_*b\|^2 > 0. \quad (20)$$

Here b and R_*b lie in $\Omega_2^\perp \otimes \phi^\perp$. This coefficient has the standard entanglement-susceptibility spectral form [3]; our result is its exact edge-derived, untruncated realization. Theorem 2 is proved directly at $t = 1$; it is not an extrapolation of this expansion.

What dressing can preserve.—The nonzero receiving block does not mean that the exact vacuum leaks from its own eigenspace. It also does not forbid a dressed vacuum map.

Corollary (vacuum matching versus the old test). There exists a gauge-equivariant isometry $J_{\text{dr}} : \mathcal{K}_2 \rightarrow \mathcal{K}_2 \otimes \mathcal{K}_z$ with $J_{\text{dr}} \Omega_2 = \Omega_3$. No isometry $\tilde{J}$ with this vacuum-matching property can also satisfy

$$\tilde{J}^*(Q_2 \otimes I) \tilde{J} = Q_2. \quad (21)$$

The incompatibility already holds after restriction to the old physical space.

Proof. A unitary rotation W in $\text{span}\{\Psi_0, \Omega_3\}$, extended by the identity on its orthogonal complement, can send Ψ_0 to Ω_3 . Both vectors are gauge invariant, so W commutes with the gauge action. Then $J_{\text{dr}} = WJ_\phi$ is the required isometry; its explicit rank-two formula is given in [5]. Taking the Ω_2 expectation of (21) would equate $p_{\text{exc}} > 0$ with zero. $\square$

The constructed dressed map preserves a transported copy of the old algebra:

$$J_{\text{dr}}^* W(O \otimes I) W^* J_{\text{dr}} = O. \quad (22)$$

This construction uses the full ground state. It is neither a local attachment rule nor a dynamically intertwining refinement map. On the matched vacuum alone,

$$(I - J_{\text{dr}} J_{\text{dr}}^*) H_3 J_{\text{dr}} \Omega_2 = 0. \quad (23)$$

Abelian control and scope.—For $U(1)$ rotors, $N = -i\partial_\theta$ and $V_{\text{ab}} = -2AN_y N_z$ yield

$$\|K_\phi^{\text{ab}} f\|^2 = 4A^2 \langle \phi, N_z^2 \phi \rangle \langle f, N_y^2 f \rangle. \quad (24)$$

With a nonconstant cosine potential, the isolated vacua have nonzero electric response. Thus attachment-induced readjustment is not uniquely non-Abelian. No susceptibility of a specifically non-Abelian commutator observable is asserted.

What changes is not the existence of a vacuum, but the compatibility of two ways of identifying it. The isolated product is not the native vacuum, and the shared-edge calculation says precisely where its mismatch enters. Neither a dressed vacuum map nor a change in the chosen old observable test can erase that distinction without also

changing the comparison. A spatial localization claim requires a specified exterior measurement algebra and a vacuum-relative comparison, not just nonfactorization; related distinctions occur in photon-localization analyses [7]. This letter constructs no exterior net, continuum refinement limit, physical associator obstruction, or Yang–Mills mass gap. The native shared electric edge forces vacuum readjustment under the specified cycle-factor comparison; exact vacuum matching cannot preserve that fixed vacuum-test observable.

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 - [5] See the accompanying Supplemental Material, v0.3.3, for the full gauge reduction, domains, receiving calculation, perturbative response, observable algebra, dressed isometry, and Abelian control.
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