

Toward Compatibility of Gauge Averaging and Native Yang–Mills Transport

Ground, form, and charged transport under averaging

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Abstract

An averaging map transports both observables and states. To compare it with a native Yang–Mills realization, we must also determine how it carries the ground, the energy form, and the charged coefficient data. We study this demand in a finite $SU(3)$ model whose conditional reversal symmetry is exact while its reference and averaged marginals differ. A unitary density correction preserves multiplication observables, and we compute its ground-projection and full sesquilinear form defects. Both admit explicit small-parameter estimates; a shrinking-angle sequence gives summable paired errors. We then prove a separate reflection-quotient lemma identifying sufficient conditions for averaging to descend isometrically. Together, these results develop a route toward the conjecture that suitably decorated physical averaging realizes an admissible native comparison. Paper IV supplies the thick-to-thin reconstruction once that admission is established. The physical identification must connect the two pairings, the dynamics, and the charged regional coherence on the same selected branch.

1. What must survive averaging?

An averaging map replaces fine configuration data by coarse variables. It therefore gives two related operations: a pushforward of states and a pullback of observables. The comparison with a native Yang–Mills realization begins with these operations. It must then follow the ground and the energy form, which determine the dynamics of the represented states, and the charged coefficient maps and coherence cells, which record how local comparisons compose. The conjecture developed here is that a suitable physical averaging map, with this additional transport retained, enters the native comparison system.

Here *native* refers to the selected positive-thickness observable and represented construction entering Paper IV [4]. Its comparisons carry local states, ground objects, forms, Ward domains, and charged composition data on the same regional and scale diagrams. The conjecture concerns the identification of a Euclidean averaging description with that structure.

We begin by asking whether a symmetry of the conditional law also controls the coarse state. The example shows that it need not. That difference leads us to a density correction and to the calculation of its ground and form costs. We then ask what allows an averaging map to pass to a reflection quotient. These are successive parts of the proposed physical comparison; connecting them on the selected Yang–Mills branch is the central identification problem.

The conditional symmetry reverses relative paths while fixing the averaged variable. The reflection used in reconstruction acts on Euclidean time. We keep their roles distinct, so that the physical spacetime construction can determine how they are related. Paper IV supplies the native thinning theorem once the additional comparison has been admitted.

2. A two-loop model for the state comparison

We want a small system in which a reversal symmetry is exact but the coarse state can still change. Two loop holonomies with a common basepoint allow both quantities to be computed.

Let $G = SU(3)$, with normalized Haar measure dg . Write

$$t(g) = \text{Re Tr}(g), \quad D_\theta = \text{diag}(e^{i\theta}, e^{-i\theta}, 1), \quad |\theta| < \pi.$$

We use the conjugacy orbit of D_θ so that the relative holonomy has a fixed angle and a conjugation-invariant law. Let ν_θ be the law of $R = kD_\theta k^{-1}$, where k is Haar distributed. We choose the equal-weight average $q = Ap$, with $A = R^{1/2} = kD_{\theta/2}k^{-1}$. On the chosen logarithm branch, A is the midpoint factor between I and R . This choice makes exchange of the two loop holonomies compatible with the average.

We interpret p and Rp as loop holonomies based at the same point. A gauge change at that point acts by simultaneous conjugation, under which their traces are invariant. This specifies the gauge symmetry of the model; open paths would require endpoint coefficient data.

To couple the two loops while preserving their exchange symmetry, we choose, for real β ,

$$d\mu_{\beta,\theta}(p, R) = \mathcal{Z}_{\beta,\theta}^{-1} \exp\{\beta[t(p) + t(Rp)]\} dp d\nu_\theta(R).$$

The exponent is real and bounded in absolute value by $6|\beta|$. Thus the partition function is finite and strictly positive. Denote the marginal densities of p and q relative to dg by m_p and m_q . Both integrate to one and are smooth and strictly positive in g . Parameter smoothness follows by writing all orbit integrals over k in the fixed compact group G , including at $\theta = 0$.

To measure the energy cost of the state correction, we equip G with the bi-invariant metric for which the positive Laplacian has fundamental Casimir eigenvalue $4/3$. On $\mathcal{H}_i = L^2(G, m_i dg)$, we use the weighted Dirichlet forms

$$\mathcal{E}_i(f, g) = \int_G m_i \nabla \bar{f} \cdot \nabla g dg, \quad D(\mathcal{E}_i) = H^1(G).$$

Smooth positive weights on the compact group give equivalent weighted and unweighted Sobolev norms. Thus $C^\infty(G)$ is a common form core. Since G is connected, each form has the one-dimensional kernel of constant functions, and 1 is normalized. These choices fix the ground and energy form used in the following calculation. Their identification with a physical Hamiltonian is a separate step in Section 8.

3. Reversal symmetry and unequal coarse marginals

The chosen density lets us test whether exact reversal of the conditional law also fixes the coarse marginal.

Proposition 3.1. The map

$$\mathcal{R}(p, R) = (Rp, R^{-1})$$

is a measure-preserving involution for $\mu_{\beta, \theta}$ and fixes q . Its conditional action on q -fibers therefore preserves the conditional law. Nevertheless, for $\theta = \pi/3$ and all sufficiently small nonzero β , $m_p \neq m_q$.

Proof. The map is an involution because its second application sends (Rp, R^{-1}) back to (p, R) . The conjugacy orbit is invariant under inversion, and left translation $p \mapsto Rp$ preserves Haar measure for each R . Hence the reference law is preserved. The two terms $t(p)$ and $t(Rp)$ are exchanged, so the interacting law is preserved as well. The averaged variable satisfies

$$q(\mathcal{R}(p, R)) = R^{-1/2}Rp = R^{1/2}p = q(p, R).$$

Measure preservation and invariance of q give conditional-law invariance for almost every q . A version defined for every $q = y$ has the density

$$d\nu_{\beta, \theta; y}(R) = \frac{\exp\{\beta[t(A^{-1}y) + t(Ay)]\}}{F_q(\beta, \theta, y)} d\nu_{\theta}(R),$$

whose numerator is unchanged by $R \mapsto R^{-1}$. Here $F_q(\beta, \theta, y)$ is the integral of that numerator against ν_{θ} .

We now compute the two marginals. Before normalization, their densities are

$$\begin{aligned} F_p(\beta, \theta, g) &= \int_G e^{\beta[t(g) + t(kD_{\theta}k^{-1}g)]} dk, \\ F_q(\beta, \theta, g) &= \int_G e^{\beta[t(kD_{-\theta/2}k^{-1}g) + t(kD_{\theta/2}k^{-1}g)]} dk. \end{aligned}$$

The change of variables $p = A^{-1}q$ shows that both densities have the same partition function $\mathcal{Z}$. The conjugation average of D_{θ} commutes with the irreducible fundamental representation, so it is scalar; its trace determines that scalar:

$$\int_G kD_{\theta}k^{-1} dk = a(\theta)I, \quad a(\theta) = \frac{1 + 2\cos\theta}{3}.$$

Since $\int_G t dg = 0$, the partition function has no term linear in β . Taylor expansion therefore gives, in $C^r(G)$ for each fixed finite r ,

$$\begin{aligned} m_p &= 1 + \beta[1 + a(\theta)]t + O(\beta^2), \\ m_q &= 1 + 2\beta a(\theta/2)t + O(\beta^2), \\ m_p - m_q &= \beta\delta(\theta)t + O(\beta^2), \quad \delta(\theta) = 1 + a(\theta) - 2a(\theta/2). \end{aligned}$$

At $\theta = \pi/3$, $\delta = (3 - 2\sqrt{3})/3 \neq 0$. Evaluating at $g = I$ gives a nonzero coefficient of β . It follows that the two marginal densities differ for every sufficiently small nonzero β . $\square$

4. Correcting the state and measuring its cost

Before constructing the correction, it is useful to see why exact ground preservation is restrictive. Suppose a unitary intertwines the full scalar multiplication algebra in the two scalar multiplicity-one spaces and sends 1 to 1. It then sends each bounded f to itself. Equality of its norms on indicator functions forces $m_p dg = m_q dg$. Thus, when the marginals differ, this observable correspondence requires a ground cost.

The two states give different L^2 norms on the same scalar observables. We seek a positive multiplication map $Uf = hf$ that preserves those norms. The condition $m_q h^2 = m_p$ determines

$$h = \sqrt{m_p/m_q}, \quad Uf = hf.$$

Theorem 4.1. The map U is a unitary $\mathcal{H}_p \rightarrow \mathcal{H}_q$, preserves $H^1(G)$ domains in both directions, and intertwines every bounded scalar multiplication observable. With $b = \nabla \log h$, its full sesquilinear form defect is

$$\mathcal{E}_q(Uf, Ug) - \mathcal{E}_p(f, g) = \int_G m_p [g \nabla \bar{f} \cdot b + \bar{f} b \cdot \nabla g + \bar{f} g |b|^2] dg.$$

Writing $s = \|b\|_\infty$ and $\|f\|_{\mathcal{E}_p}^2 = \|f\|_p^2 + \mathcal{E}_p(f, f)$,

$$|\mathcal{E}_q(Uf, Ug) - \mathcal{E}_p(f, g)| \leq (2s + s^2) \|f\|_{\mathcal{E}_p} \|g\|_{\mathcal{E}_p}.$$

To compare the constant grounds, let $P_i = |1\rangle\langle 1|_{m_i}$, let $Z = P_q - UP_p U^*$ be their transported projection difference, and set $\eta = \|U1 - 1\|_q$. Then

$$\eta^2 = \int_G (\sqrt{m_p} - \sqrt{m_q})^2 dg, \quad \|Z\|^2 = \eta^2 - \eta^4/4.$$

For fixed θ , as $\beta \rightarrow 0$,

$$\eta^2 = \frac{\beta^2 \delta(\theta)^2}{8} + O(|\beta|^3), \quad \mathcal{E}_q(U1, U1) = \frac{\beta^2 \delta(\theta)^2}{6} + O(|\beta|^3).$$

Proof. The identity $m_q h^2 = m_p$ proves unitarity, and multiplication by h commutes with multiplication by any scalar observable. Smoothness and strict positivity on the compact group give bounded h , h^{-1} , and their derivatives, proving the H^1 domain assertions. Substituting $\nabla(hf) = h(\nabla f + fb)$ proves the sesquilinear identity on smooth functions and then on H^1 by continuity. Cauchy-Schwarz gives the bound.

The vectors 1 and h are normalized in $\mathcal{H}_q$. Their positive overlap is

$$c = \langle 1, h \rangle_q = \int_G \sqrt{m_p m_q} dg, \quad \eta^2 = 2(1 - c).$$

For two normalized vectors, restricting the difference of their rank-one projections to their two-dimensional span gives eigenvalues $\pm\sqrt{1 - c^2}$, with the coincident-vector case obtained by continuity. Hence $\|Z\|^2 = 1 - c^2 = \eta^2 - \eta^4/4$.

The first-order density expansion gives $h = 1 + \beta\delta t/2 + O(\beta^2)$ in C^1 . Character orthogonality gives $\int t^2 dg = 1/2$: the fundamental and conjugate fundamental are distinct irreducibles, $\int |\text{Tr } g|^2 dg = 1$, and $\int (\text{Tr } g)^2 dg = 0$. The metric normalization gives $-\Delta t = (4/3)t$, so integration by parts gives $\int |\nabla t|^2 dg = 2/3$. Substituting these identities in the vector norm and in $\mathcal{E}_q(h, h)$ yields the coefficients. $\square$

The correction preserves the scalar observable action while moving the constant ground and changing its form energy by explicit amounts. The projection calculation uses a one-dimensional ground space. Extending it to a physical complete-ground object, or strengthening the form estimate to a generator or Ward graph norm, requires the corresponding operators and domains.

5. Quadratic improvement as the relative angle shrinks

The single-step correction is useful for refinement only if its cost improves. In this model the relative angle supplies a parameter with which to test that improvement.

Theorem 5.1. Fix $0 \leq B < \infty$ and $0 < \theta_0 < \pi$. There is a constant C depending on B , θ_0 , and the fixed metric such that for $|\beta| \leq B$, $|\theta| \leq \theta_0$,

$$\|\log h\|_\infty + \|\nabla \log h\|_\infty \leq C|\beta|\theta^2.$$

Consequently $\eta, \|Z\| \leq C'|\beta|\theta^2$ and $\mathcal{E}_q(h, h) \leq C'\beta^2\theta^4$.

Proof. Write $L = (\log F_p - \log F_q)/2$. The common normalization cancels. Both marginal integrals are even in θ , and they agree at $\theta = 0$; they also agree at $\beta = 0$. Thus L and its first θ -derivative vanish at $\theta = 0$, while $L(0, \theta, g) = 0$. For F_p , evenness follows by changing the Haar variable using a fixed element that conjugates D_θ to $D_{-\theta}$; for F_q , it follows by exchanging the two terms. Smoothness and positivity hold on the fixed compact parameter region.

The integral Taylor formula gives

$$L(\beta, \theta, g) = \beta\theta^2 \int_0^1 \int_0^1 (1-u) \partial_\beta \partial_\theta^2 L(v\beta, u\theta, g) dv du.$$

The mixed derivative and its spatial gradient are bounded on the compact region, proving the C^1 estimate. Bounded L implies $|e^L - 1| \leq e^{\|L\|_\infty} |L|$; integration against the probability measure $m_q dg$ gives the vector bound. The projection estimate follows from Theorem 4.1, and $\mathcal{E}_q(h, h) = \int m_p |\nabla L|^2 dg$ gives the form bound. $\square$

Near $(\beta, \theta) = (0, 0)$, the same smooth factorization, together with the first β -derivative of L , yields

$$L = \frac{1}{2}\beta\delta(\theta)t + O_{C^1}(\beta^2\theta^2).$$

Since $\delta(\theta) = -\theta^2/6 + 7\theta^4/288 + O(\theta^6)$,

$$\begin{aligned} \eta^2 &= \frac{\beta^2\theta^4}{288} + O(|\beta|^3\theta^4 + \beta^2\theta^6), \\ \mathcal{E}_q(h, h) &= \frac{\beta^2\theta^4}{216} + O(|\beta|^3\theta^4 + \beta^2\theta^6). \end{aligned}$$

For $\theta_n = \theta_0 L_0^{-n}$, $L_0 > 1$, and bounded β_n , the vector and projection errors and the coefficients $2s_n + s_n^2$ are summable. This controls the disagreement within each pair. Convergence across scales also depends on how the endpoint states are related, as the following example shows.

Paired agreement and variation across scales. Let $\beta_n = 0$ for even n and $\beta_n = b > 0$ for odd n , while $\theta_n \rightarrow 0$. The discrepancy within each pair tends to zero. Along the even subsequence both marginals tend to Haar measure; along the odd subsequence they tend to the density

$$\frac{e^{2bt(g)}}{\int_G e^{2bt} dg}.$$

Because t is nonconstant, these two limiting states differ. The comparison estimates control paired disagreement, while the construction of a limiting family requires compatible states and maps across scales. In the proposed application, that latter structure is supplied on the native side by Paper IV.

6. Passing the averaging map through reconstruction

The preceding estimates compare weighted configuration-space norms. Euclidean reconstruction introduces a different pairing, whose null vectors must also be respected by the averaging map. We now isolate a sufficient condition for that passage. This lemma concerns independently specified reflection-positive data; applying it to a physical averaging family requires identifying those data.

Let X_i carry probability measures μ_i and measurable involutions ϑ_i . Let $\mathcal{A}_{i,+}$ be specified complex vector spaces of bounded positive-time observables containing 1. Assume the fine form

$$(F, G)_f^{\text{OS}} = \int \overline{F(\vartheta_f x)} G(x) d\mu_f(x)$$

is positive semidefinite on $\mathcal{A}_{f,+}$. Suppose $q : X_f \rightarrow X_c$ is measurable, $\mu_c = q_* \mu_f$, $q\vartheta_f = \vartheta_c q$ almost everywhere, and $q^* \mathcal{A}_{c,+} \subset \mathcal{A}_{f,+}$.

Lemma 6.1. The coarse reflection form is positive semidefinite. Pullback induces an isometry of completed reflection-quotient spaces and sends $[1]_c$ to $[1]_f$.

Proof. Substitution and the pushforward identity give

$$(q^* F, q^* G)_f^{\text{OS}} = \int \overline{F(\vartheta_c q(x))} G(q(x)) d\mu_f(x) = (F, G)_c^{\text{OS}}.$$

The coarse form is therefore positive. Its null vectors map to fine null vectors, so $[F]_c \mapsto [F \circ q]_f$ is a well-defined isometry on the quotient and extends to the completion. Constants pull back to constants. $\square$

The lemma constructs an isometric map of the reflected state spaces. To identify the physical dynamics, we must also compare their time translations. Neither surjectivity nor complete-ground coverage follows from the isometry alone.

For an approximate linear comparison B , the first question is whether a coarse null vector remains null after transport. A sufficient bound in the reflected seminorm is

$$(BF, BF)_f^{\text{OS}} \leq C(F, F)_c^{\text{OS}}$$

for every coarse test function, with B mapping into the fine positive-time space. If the coarse reflected norm is zero, the bound forces the fine reflected norm to vanish as well. An additive error measured only in ordinary L^2 does not have this implication. Thus the form estimates of Section 4 must be connected to the reflected pairing before they can be used here.

The next comparison concerns evolution. Once the two reconstructed Hamiltonians and their time units have been fixed, the relevant discrepancy is

$$e^{-tH_f}T - Te^{-s(t)H_c}.$$

Here $s(t)$ identifies coarse and fine physical time. To obtain a form comparison from this discrepancy, one needs its behavior near $t = 0$ and control of the relevant domains. Establishing those estimates is the dynamical part of the proposed identification.

7. Charged pairings and the averaging paths

Charged observables carry coefficient vectors as well as scalar dependence on the configuration. Their comparison therefore includes a map $D(x)$. In a trivial finite-dimensional Hermitian coefficient model, write $BF(x) = D(x)F(q(x))$. The reflected inner product then contains $D(\vartheta_fx)^*D(x)$, so, subject to integrability and positive-time support, the condition

$$D(\vartheta_fx)^*D(x) = I$$

recovers the pairing identity of Lemma 6.1. This tests the relation between x and its reflection, whereas ordinary unitarity tests only one point. For nontrivial coefficient objects, the physical reflection pairings K_i lead to the local condition

$$D(\vartheta_fx)^*K_f(x)D(x) = K_c(q(x)),$$

with the source and target fibers identified in the coefficient construction. The pairings K_i , their positivity, and the selected pointed objects are part of the physical data. Verifying this identity must therefore use those pairings and must be accompanied by the comparison of the pointed objects.

We can now ask how the averaging paths behave under the reflection required above. Bałaban's published formula [1] has the structure

$$q(U)_c = e^{Y_c(U)}U(c), \quad Y_c(U) = \sum_x w_x \log(U(\Gamma_{c,x})U(c)^{-1}).$$

Bałaban defines the pushed-forward density and proves gauge covariance and block locality for this average [1, equations (10), (11), (22), (42), and discussion after (43)]. Reflection compatibility additionally depends on how the chosen paths and reference bonds transform.

If reflection preserves the whole reference-path prescription and weights, substitution gives reflection equivariance.

For orientation reversal, suppose reversed bonds use reversed paths with matched weights. Put $V = U(c)$, $W_x = U(\Gamma_{c,x})$, and $R_x = W_xV^{-1}$. The reversed relative holonomy is $V^{-1}R_x^{-1}V$. On a branch where $\log(R^{-1}) = -\log R$,

$$Y_{-c} = -V^{-1}Y_cV, \quad q(U)_{-c} = q(U)_c^{-1}.$$

The inversion identity holds on a logarithm branch for which $\log(R^{-1}) = -\log R$; the source interval $(-\pi, \pi]$ requires separate treatment at eigenvalue -1 . For a general $SU(3)$ extension, one must also check that the logarithmic average remains in the required Lie algebra, since principal logarithms need not be traceless away from the identity chart. The orbit used in Section 2 avoids these difficulties. Their treatment for the selected physical paths belongs to the source-level comparison.

Positive-time support requires the endpoint blocks and selected paths to lie in the allowed half-space. If the path calculation is restricted to a regularity region, extending it to the full measure also requires control of the reflected pairing. Conditioning on that region need not preserve reflection positivity, even when its complement has small probability. The finite orbit model chooses its support inside a regularity region; a physical application must establish the needed measure control.

7.1. Central lifts and the reflected charged pairing

The density correction controls a positive scalar weight. A charged comparison also carries phases. To see why these require their own calculation, fix the reflection pairings above and work in a local trivialization of a charge- r coefficient space, where $r \in \{0, 1, 2\}$. Write $\omega = e^{2\pi i/3}$, with central action $\rho_r(\omega I) = \omega^r I$. Suppose two candidate lifts differ by

$$D'(x) = \omega^{ra(x)} D(x), \quad a(x) \in \mathbb{Z}_3.$$

This comparison keeps the physical pairings fixed. A simultaneous change of coefficient frames would also transform those pairings and must be treated separately. We assume the displayed maps have the required integrability and positive-time support; identifying such lifts in the physical averaging construction is a further task.

Proposition 7.1 (central-lift residual). Suppose $D(\vartheta_f x)^* K_f(x) D(x) = K_c(q(x))$. Then the residual for the second lift is

$$R_r(x) := D'(\vartheta_f x)^* K_f(x) D'(x) - K_c(q(x)) = \left(\omega^{r[a(x) - a(\vartheta_f x)]} - 1 \right) K_c(q(x)).$$

It vanishes on the neutral coefficient space. For $r = 1, 2$, wherever $K_c(q(x)) \neq 0$, it vanishes exactly when $a(x) = a(\vartheta_f x)$ in $\mathbb{Z}_3$. Otherwise,

$$\|R_r(x)\| = \sqrt{3} \|K_c(q(x))\|.$$

Proof. Substitution of the two central factors gives the phase $\omega^{-ra(\vartheta_f x)} \omega^{ra(x)}$. The assumed identity for D then gives the residual. Both nontrivial cube roots of unity have distance $\sqrt{3}$ from 1, proving the norm formula. $\square$

Now let h be a strictly positive scalar correction. Its effect on the second lift is

$$(hD')(\vartheta_f x)^* K_f(x) (hD')(x) = h(\vartheta_f x) h(x) \omega^{r[a(x) - a(\vartheta_f x)]} K_c(q(x)).$$

A positive real factor cannot turn a nontrivial cube root of unity into one. Thus a positive density correction cannot remove this pointwise central mismatch. The charged transport and its

independently specified reflection pairing must account for the phase. This is a statement about the local pairing condition; equality after integration on a restricted test space could be weaker.

The integrated error can also be bounded explicitly. Put

$$E_r = \{x : r[a(x) - a(\vartheta_f x)] \neq 0 \text{ in } \mathbb{Z}_3\},$$

and assume $\|K_c(q(x))\| \leq M$. For coefficient-valued test functions define

$$\Delta_r(F, G) = \int F(q(\vartheta_f x))^* R_r(x) G(q(x)) d\mu_f(x).$$

The pointwise norm estimate gives

$$|\Delta_r(F, G)| \leq \sqrt{3}M \int_{E_r} \|F(q(\vartheta_f x))\| \|G(q(x))\| d\mu_f(x).$$

For bounded test functions, this implies

$$|\Delta_r(F, G)| \leq \sqrt{3}M \mu_f(E_r) \|F\|_\infty \|G\|_\infty,$$

where the norms bound the displayed pullbacks. A small exceptional-set probability therefore controls bounded-column pairings. To obtain quotient descent, the error must instead be controlled in the reflected seminorm, or reflection-null preservation must be proved separately. The probability estimate alone does not supply that conclusion.

The calculation identifies a discrete residual that the shrinking-angle estimates do not themselves control. Its physical evaluation requires the central factor of the admitted path lift and its behavior under reflection. Preservation of the vacuum-gerbe associator additionally requires comparison of the actual composition cells.

8. The compatibility conjecture and its constructive proof program

Compatibility conjecture. Fix a Euclidean Yang–Mills family and the canonical native positive-thickness branch as independently specified physical data for the proposed comparison. There exist identifications of the selected states and represented observable actions, together with charged coefficient maps and composition data decorating the Euclidean averaging pullbacks, such that the pullbacks descend through reconstruction and induce admissible comparisons in the native Paper-IV system.

This is an existence conjecture for the selected physical pair. It does not quantify over arbitrary averaging prescriptions. Its source-level formulation must specify that pair, the averaging maps, and the coefficient objects; the finite model here supplies comparison mechanisms rather than selecting those physical data.

The physical matching must identify the Euclidean state and its reconstructed observable action with the selected native branch. The calculations above show how a state discrepancy can be corrected while its ground and form cost remains visible. The conjecture asks whether this analytic comparison can be carried out together with the charged transport and coherence already retained by the native construction.

Regional comparison asks whether the correction assigned to a larger region restricts to the correction assigned on an overlap. The global density ratio alone does not answer this question. Likewise, charged path transport must be compared on the composition diagrams where the tensorators and associator act. These are the reasons the same packet and its regional coherence must remain in the calculation.

Paper IV [4, Theorem IV.1 and Scope IV.2] identifies the canonical branch’s reconstruction inputs through the Paper-III certificates. Its Theorem Q.1 supplies the thick-to-thin reconstruction, exact thin ground kernel, and coercive form bound within the declared framework. The present comparison seeks to place the additional averaging maps in that framework; it does not require a new native thinning construction.

The first task is to identify the physical pairing on a finite charged Wilson packet. The Euclidean state, its reflected pairing, and the native represented columns must give the same comparison problem. This fixes what the map acts on and how its discrepancy is measured.

With that pairing fixed, the density or state correction can be evaluated in the receiving norms. The comparison must respect reflection-null vectors where the reflection route is used, and it must control the ground and the full form. Sections 4–6 supply formulas and a sufficient descent condition for this stage.

The same map must then agree with regional restriction, preserve the prepared Ward domains, and carry the charged central action and mixed cells. These comparisons place the analytic estimate in the native system to which Paper IV applies.

9. Relation to earlier work

Balaban [1] provides the averaging operation considered here. Ziebell [2] develops a rigorous functional-RG setting with reflection-positive regularization and convergence conditions. Gross [3] treats the passage from a Schrödinger operator with a suitable positive ground to a weighted Dirichlet-form operator. These works locate three operations relevant to our comparison: averaging, reconstruction, and the relation between a state and its energy form. Here the weighted forms are specified directly; identifying them with physical reconstructed dynamics is part of the conjectured comparison.

The density-unitary identity, rank-one projection formula, and reflection-pushforward argument serve as elementary tools. The $SU(3)$ calculation brings their distinct roles together: it exhibits an exact conditional symmetry, evaluates the remaining ground and form costs, and identifies the additional pairing needed for reconstruction.

10. What the comparison now makes possible

The averaging map determines a coarse state, but comparison with a selected realization also asks how that state carries the ground and the energy form. In the finite example, an exact conditional symmetry leaves a marginal discrepancy whose correction has an explicit cost. The reflected-pairing calculation identifies a further condition for carrying the map into a reconstructed state space. These results give the compatibility conjecture a concrete direction: evaluate those comparisons on the physical Wilson packet and retain its charged regional composition throughout. Paper IV supplies the thinning theorem once that transport has been admitted.

References

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