NOTE-06-XXIII / WORKING RESEARCH CONTINUATION

Functional-Analytic Spine — Part XXIII

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Research notes — not peer reviewed. Trust, but verify. Always.

Which spaces, domains, estimates, and convergence controls must survive reconstruction?

A separately preserved continuation of the Functional-Analytic Spine. The body retains its original theorem labels, status boundaries, and pagination.

Scope and status

The v0.4.0 source discloses reconstructed sections and unresolved source/literature audits. The separate continuations are not a certified integrated v0.9.0 edition; the incomplete assembly preview is excluded.

This edition adds a reading guide and publication notices. The source’s mathematical status and historical provenance remain attached to its claims.

A way into the note

The central distinction

An estimate belongs to a space, a domain, and a comparison map. Carrying the inequality into a limit requires control of those data as well as the numerical bound.

A concrete example

A sequence of positive finite-dimensional matrices may have positive gaps at every stage while the gaps approach zero. Track a uniform bound and the relation between ground projections before claiming persistence.

What this approach gains

The notebook treats analytic domains, defect controls, compactness, and computational witnesses as part of the mathematical object. This reveals what must be transported for a limiting statement to retain its content.

Reading route

Read this part in sequence with the canonical checkpoint and the other continuation parts. Refer to the section titles and continuation record included in its source archive.

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The Functional-Analytic Spine of the Yang–Mills Reconstruction

Spaces, Inequalities, Admissible Limits, and Computational Witnesses

Part XXIII – Multi-Axis Structural Compactness and No-Leakage Closure

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Standalone Part XXIII Version 0.5.0 — 2026-07-19

Research notes—not peer reviewed. Trust, but verify. Always.

Contents

1 Part XXIII — Multi-Axis Structural Compactness and No-Leakage Closure

Research notes — not peer reviewed. Trust, but verify. Always.

Provenance boundary. The frozen common base is `Functional_Analytic_Spine_v0.4.0_Intrinsic_Thin_Boundary_2026-07-18.zip`, authoritative through Part XXII and §1195. Part XXI in that package is a reconstruction from the authoritative continuation handoff because its original complete prose was not recovered. Nothing in this Part silently upgrades an unrecovered source-specific claim. Such claims remain **SOURCE AUDIT REQUIRED**. Missing scripts, numerical certificates, logs, or precision declarations remain **SOURCE ARTIFACT RECOVERY REQUIRED**.

1.1 1196. Weak convergence is generous

Weak convergence is generous. It will give us a limit long before it gives us the theory we meant to reconstruct. A vector may converge while its energy leaks, its protected subspace rotates, its state leaves the normal folium, its products stop multiplying correctly, and its interface forgets which two sectors it was supposed to join. That is not compactness of the reconstruction. It is merely the survival of some coordinates.

The purpose of this Part is to replace coordinatewise survival by **structural compactness**. The object under study is not one sequence in one Hilbert space. It is a positive-thickness, scale-indexed, three-sector system whose fibers, algebras, forms, states, constraints, actions, and coherence maps may all vary with the admissible index. Compactness must therefore be typed, simultaneous, and compatible with the declared transport architecture.

The governing rule is:

Extract one limit, close every defect channel, and only then call the result a reconstructed structure.

1.2 1197. Scope and non-scope

This Part develops an abstract compactness theory for admissible strict-map-valued and realized correspondence-sensitive scale fields. It gives sufficient conditions under which a single admissible subnet carries compatible limits of all declared structural coordinates. It also proves no-leakage consequences when the complete defect profile vanishes.

The analysis occurs at positive thickness before the intrinsic thin quotient is taken. Part XXII is used only at the end, where the new compactness results supply previously conditional inputs for intrinsic boundary formation.

This Part does **not** construct the intrinsic asymptotic coefficient algebra, the coefficient-valued null module, a self-dual thin correspondence, or a bicategorical universal property. Those belong to Part XXIV. It does not prove fusion-compatible Mosco convergence or bicategorical realization independence; those belong to Part XXV.

Nonclaim. No theorem below asserts compactness over arbitrary scales, charts, regions, Cauchy slices, Cauchy developments, or comparison maps. Every index, localization, transport, extraction, and comparison is admissible.

1.3 1198. Positive thickness precedes structural extraction

Let $\mathfrak{A}$ be the selected directed admissible index set. An index $\alpha \in \mathfrak{A}$ may encode positive thickness, admissible region, chart, refinement order, selected Cauchy datum, approximation order, and sector-compatible auxiliary data. Write $e(\alpha) > 0$ for the thickness coordinate when it is present.

All compactness hypotheses in this Part are imposed on families at $e(\alpha) > 0$. The thin object is not used as an ambient compactification. Instead, one first extracts and identifies a structurally complete positive-thickness limit field along an admissible thin subnet. Only then is the intrinsic boundary functor of Part XXII invoked.

This order matters. Quotienting too early can erase positive energy carried by a Hilbert-null section, collapse interface information, or hide projection rotation inside a null class. The quotient is a destination, not a garbage disposal.

1.4 1199. The admissible three-sector base

For each $\alpha \in \mathfrak{A}$, the bulk sectors are

$$\mathfrak{B}_{s,\alpha} = (\mathcal{M}_{s,\alpha}, \mathcal{H}_{s,\alpha}, Q_{s,\alpha}, P_{s,\alpha}), \qquad s \in \{\mathrm{str}, \mathrm{tw}\},$$

where $\mathcal{M}_{s,\alpha}$ is a represented local von Neumann or controlled operator algebra, $\mathcal{H}_{s,\alpha}$ its selected Hilbert or GNS fiber, $Q_{s,\alpha}$ a nonnegative closed form, and $P_{s,\alpha}$ the protected orthogonal projection.

The obstruction interface is

$$_{\mathcal{M}_{\mathrm{str},\alpha}}\mathcal{K}_{\mathrm{int},\alpha\mathcal{M}_{\mathrm{tw},\alpha}},$$

with seam form $Q_{\mathrm{int},\alpha}$, protected seam projection $P_{\mathrm{int},\alpha}$, and commuting normal left and right actions λ_α and ρ_α on the declared realized class.

The interface is not a third bulk vacuum sector. Its role is relational: it carries the analytic compatibility between the strict and twisted bulks and preserves the relative obstruction

$$[\chi^\Sigma] = [z^{\mathrm{str}}] - [z^{\mathrm{tw}}].$$

1.5 1200. Definition of the complete structural state

A **positive-thickness structural state** at index α is the typed tuple

$$\boxed{\mathfrak{S}_\alpha = \begin{pmatrix} \xi_{\mathrm{str},\alpha}, \xi_{\mathrm{tw},\alpha}, \xi_{\mathrm{int},\alpha}; \\ P_{\mathrm{str},\alpha}, P_{\mathrm{tw},\alpha}, P_{\mathrm{int},\alpha}; \\ \omega_{\mathrm{str},\alpha}, \omega_{\mathrm{tw},\alpha}; \\ \mathbf{A}_\alpha, G_\alpha, W_\alpha, m_\alpha, *_\alpha, 1_\alpha; \\ \sigma_\alpha, c_\alpha, \Gamma_\alpha \end{pmatrix}}.$$

Here:

1. $\xi_{s,\alpha} \in D(Q_{s,\alpha})$ are controlled bulk sections;
2. $\xi_{\mathrm{int},\alpha} \in D(Q_{\mathrm{int},\alpha})$ is a controlled interface section;
3. $P_{s,\alpha}$ and $P_{\mathrm{int},\alpha}$ specify protected directions;
4. $\omega_{s,\alpha}$ are selected normal states or local restrictions of weights;
5. $\mathbf{A}_\alpha$ is a finite or small declared family of controlled local observables and interface action tests;
6. G_α and W_α denote the selected Gauss and Ward graph data;
7. $m_\alpha, *_\alpha, 1_\alpha$ encode multiplication, involution, and units on the controlled algebra class;
8. $\sigma_\alpha \in \{\mathrm{str}, \mathrm{tw}, \mathrm{int}\}$ and its refinements encode sector labels without retyping the interface as bulk;
9. c_α denotes square, triangle, prism, associator, or naturality cells along admissible diagrams;
10. Γ_α denotes the accumulated coherence ledger along admissible chains.

Every entry has a downstream role. Coordinates not used in extraction, identification, or closure are omitted rather than carried as decorative notation.

1.6 1201. Structural test systems

A **structural test system** $\mathscr{T}$ consists of:

- an admissible localization family $\mathscr{O}_{\mathrm{adm}}$;
- controlled bulk test sections $\mathscr{V}_s$;
- controlled interface test sections $\mathscr{V}_{\mathrm{int}}$;
- bounded local algebra tests $\mathscr{A}_s(O)$;
- graph-core tests for Gauss and Ward operators;
- projection tests on controlled sections;
- state tests in the selected local algebras;
- a small family of admissible squares, prisms, and chains;
- sector and relative-obstruction detectors.

The test system is **determining** if equality of every test value forces equality in the declared structural category. It is **small** if its product of coordinate compactifications is a legitimate set-indexed product. Countability is useful for metrization and diagonal sequences but is not required for subnet extraction.

The tests do not identify different fibers. Each test is evaluated only after applying the declared strict map, isometry, correspondence pairing, or admissible realization.

1.7 1202. The structural topology

Fix an admissible comparison realization for the tests in $\mathscr{T}$, not necessarily a global equivalence of fibers. The **structural topology** $\tau_{\mathrm{struc}}(\mathscr{T})$ is the initial topology generated by the following coordinate convergences:

bulk/interface vectors	:	local strong Hilbert convergence and form-weak convergence,
forms	:	Mosco liminf plus controlled recovery values,
projections	:	strong convergence on controlled sections, strengthened when declared,
states	:	$\sigma(\mathcal{M}_*, \mathcal{M})$ convergence plus uniform normality control,
constraints	:	graph convergence on the selected core,
bounded observables	:	local strong-* convergence on uniformly bounded sets,
products/actions	:	convergence of evaluated products and module actions,
sectors	:	convergence in the declared discrete or Hausdorff label space,
coherence	:	vanishing accumulated residual seminorms.

The topology is typed. It is not the norm topology of one giant direct sum, because the interface and the bulk do not have the same analytic role and the fibers are not canonically equal.

1.8 1203. Structural convergence

A subnet $(\mathfrak{S}_{\alpha(\beta)})_\beta$ **converges structurally** to $\mathfrak{S}_0$ if every coordinate in the determining test system converges in its declared topology and every defining compatibility relation belongs to a closed relation in the corresponding product topology.

Thus structural convergence includes more than coordinate convergence. It requires, for example,

$$\lambda_{\alpha(\beta)}(A_{\alpha(\beta)})\xi_{\mathrm{int},\alpha(\beta)} \longrightarrow \lambda_0(A_0)\xi_{\mathrm{int},0},$$

and the right-action analogue; it requires product tests to converge to products rather than to unrelated limits; and it requires the limiting interface square to join the limiting strict and twisted data.

A family of coordinate limits that fails one closed compatibility relation is not a structural limit.

1.9 1204. Structural boundedness

A family $(\mathfrak{S}_\alpha)$ is **structurally bounded** if there exists $C < \infty$, independent of the admissible index on the declared frustum, such that:

$$\sup_\alpha \left(\sum_s \|\xi_{s,\alpha}\|^2 + \sum_s Q_{s,\alpha}[\xi_{s,\alpha}] + \|\xi_{\mathrm{int},\alpha}\|^2 + Q_{\mathrm{int},\alpha}[\xi_{\mathrm{int},\alpha}] \right) \le C,$$

and, on the selected tests,

$$\sup_\alpha \left(\|\mathbf{A}_\alpha\|_{\mathrm{act}} + \|G_\alpha \xi_\alpha\| + \|W_\alpha \xi_\alpha\| + \mathrm{Ent}_\alpha + \mathrm{Tail}_\alpha + \mathrm{Var}(P_\alpha) + \mathrm{CohBudget}_\alpha \right) \le C.$$

The symbols denote the declared action or graph norms, entropy or uniform-integrability controls, regional and nonlocal tails, projection variation, and accumulated coherence budgets. Structural boundedness also includes uniform bounds for multiplication on the selected bounded class and for left and right interface actions.

1.10 1205. Boundedness is not precompactness

Structural boundedness prevents immediate blow-up. It does not prevent translation to infinity, motion along the incision, oscillation across scales, rotation of protected ranges, departure from the normal folium, or accumulation of small coherence errors.

Even in one Hilbert space, the unit ball is only weakly compact. In the present setting, weak compactness of each vector coordinate leaves the projection, state, algebra, interface, and coherence coordinates untouched. A bounded family can therefore have many ambient cluster points and no cluster point in the structural category.

Nonclaim. No result below infers structural precompactness from energy boundedness alone.

1.11 1206. The complete structural defect profile

The Part XXI ledger is retained:

$$\mathbf{D}_\alpha = (\nu_{\mathrm{norm}}, \mu_Q, d_P, d_{\mathrm{ent}}, d_{\mathrm{sector}}, \nu_{\mathrm{int}}, \mu_{\mathrm{int}}, d_\square, \|\gamma_\alpha\|, d_G, d_{\mathrm{Ward}}, d_{\mathrm{prod}}, d_{\mathrm{loc}})\,.$$

For structural closure it is useful to display the dependency refinement

$$\mathbf{D}_\alpha^{\mathrm{full}} = \Big(\mathbf{D}_\alpha, d_{\mathrm{normal}}, d_{\mathrm{act}}^L, d_{\mathrm{act}}^R, d_{\mathrm{scale}}, d_{\mathrm{nonloc}}, d_{\mathrm{kernel}}, d_\chi\Big)\,,$$

where the added coordinates separate normality, left/right actions, scale drift, nonlocal escape, kernel inflation, and relative-obstruction loss when the manuscript requires them to be measured independently. This is a refinement, not a replacement, of Part XXI.

Each coordinate is defined only after its comparison topology and localization family have been fixed.

1.12 1207. The localized defect object

Let $\mathscr{L}$ be the positive cone of the declared admissible localization algebra. A localized defect object is

$$\boldsymbol{\mu}_{\mathrm{def}} = (\nu, \mu_Q, \pi, \eta, \mu_{\mathrm{int}}, \gamma, \mu_G),$$

where each entry is either a positive measure or, before countable additivity is proved, a positive localization functional. The scalar defect coordinates dominate total masses on determining tests:

$$\nu(1) \le \nu_{\mathrm{norm}}, \quad \mu_Q(1) \le \mu_Q^{\mathrm{scalar}}, \quad \pi(1) \le d_P,$$

with analogous inequalities for the interface, coherence, state/sector, and constraint entries.

Calling these objects measures requires the selected additivity theorem. Any manuscript-specific assertion of a genuine Radon, operator-valued, or noncommutative measure representation is **SOURCE AUDIT REQUIRED** unless the source proof is recovered.

1.13 1208. One common subnet is part of the object

Suppose one subnet converges for bulk vectors, another for protected projections, a third for states, and a fourth for interface squares. These are four statements about four cluster configurations. They do not define one reconstructed field.

The common-subnet discipline is encoded by the product map

$$\Phi_{\mathscr{T}} : \mathfrak{A} \longrightarrow \prod_{t \in \mathscr{T}} X_t,$$

where X_t is the compact or relatively compact coordinate target for test t. A structural extraction theorem must find one subnet on which $\Phi_{\mathscr{T}}$ converges in the product topology. Compatibility is then checked by closedness of the structural relations.

This is why nets are natural. Tychonoff compactness extracts a common subnet from a small product of compact coordinate closures without pretending that the test system is countable.

1.14 1209. Block A audit

Last subsection: §1209.

New notation: $\mathfrak{A}$, $\mathfrak{B}_{s,\alpha}$, $\mathfrak{S}_\alpha$, $\mathscr{T}$, $\tau_{\mathrm{struc}}(\mathscr{T})$, $\mathbf{D}_\alpha^{\mathrm{full}}$, $\Phi_{\mathscr{T}}$.

Undischarged hypotheses: existence of determining admissible tests; coordinate compactifications; closed structural relations; uniform bounds; legitimate comparison realizations; manuscript-specific definitions of each defect coordinate.

Status: definitions and product-topology formulation are **RIGOROUS ABSTRACT**. Their Yang–Mills instantiation is **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION**.

1.15 1210. Admissible localization exhaustions

For each admissible region O, let $(\chi_{O,R})_{R \in \mathcal{R}_O}$ be a directed family of admissible localization operators or cutoffs satisfying $0 \le \chi_{O,R} \le 1$ and increasing to the identity in the selected local realization. The parameter R may encode spatial size, distance from a chart boundary, or an admissible geometric exhaustion.

For the interface, use a separate family $(\chi^{\Sigma}_{R,L})$, where R controls ordinary regional extent and L controls escape along the incision or obstruction carrier. The two parameters must not be collapsed: a family can be tight transversely and still run to infinity along Σ.

All cutoffs must be compatible with the admitted transport and regional category. No arbitrary partition of unity or unrestricted chart exhaustion is assumed.

1.16 1211. Regional bulk tightness

A bulk family $(\xi_{s,\alpha})$ is **regionally tight** if for every $\varepsilon > 0$ there exists an admissible localization scale R such that

$$\sup_\alpha \|(1-\chi_{O,R})\xi_{s,\alpha}\|^2 < \varepsilon$$

for every selected region required by the structural test system, with the analogous estimate for the form tail whenever energy localization is needed:

$$\sup_\alpha Q_{s,\alpha}[(1-\chi_{O,R})\xi_{s,\alpha}] < \varepsilon.$$

The estimates may be formulated by positive local functionals rather than literal cutoff operators when the fibers vary. What matters is a transport-compatible tail functional that tends uniformly to zero.

1.17 1212. Interface tightness has two axes

The interface family is **seam tight** if for every $\varepsilon > 0$ there exist admissible R, L such that

$$\sup_\alpha \|(1-\chi^\Sigma_{R,L})\xi_{\mathrm{int},\alpha}\|^2 < \varepsilon, \qquad \sup_\alpha Q_{\mathrm{int},\alpha}[(1-\chi^\Sigma_{R,L})\xi_{\mathrm{int},\alpha}] < \varepsilon.$$

The parameter R controls escape away from the selected regional carrier. The parameter L controls drift along the incision. Interface tightness also includes tail control for the left and right action tests, since a vector can remain norm-tight while its coefficient action becomes increasingly nonlocal.

Nonclaim. Bulk tightness on both sectors does not imply interface tightness.

1.18 1213. Scale tightness and scale drift

Let $r : \mathfrak{A} \to \mathfrak{E}_{\mathrm{adm}}$ be the positive-thickness coordinate. A family is **scale tight** relative to the selected thin direction if, after passage to a cofinal admissible tail, all scale mass and comparison effort remain inside the declared admissible frustum and the scale-drift defect satisfies

$$d_{\mathrm{scale},\alpha} \longrightarrow 0.$$

A useful formulation employs a proper control function $V_{\mathrm{scale}} \geq 0$ on the admissible scale category:

$$\sup_\alpha \mathcal{E}_\alpha[V_{\mathrm{scale}}] < \infty,$$

with compact sublevel sets in the chosen scale topology. This prevents extraction from silently changing the intended thin path or branch.

Scale tightness does not promote comparison maps to equivalences.

1.19 1214. Nonlocal-observable tightness

Let $\mathscr{A}^{\leq R}_{s,\alpha}(O)$ be the selected bounded observables supported or generated within admissible localization radius R. A family A_α is **observable-tight** if for every $\varepsilon > 0$ there exists R and $A^{\leq R}_\alpha \in \mathscr{A}^{\leq R}_{s,\alpha}(O)$ such that

$$\sup_{\alpha} \|A_\alpha - A_\alpha^{\leq R}\|_{\mathrm{act}} < \varepsilon.$$

The action norm may be operator norm, graph norm, form-bounded norm, or a family of seminorms on controlled sections. The choice must be strong enough to pass the products and interface actions used downstream.

This condition excludes escape into increasingly nonlocal observables even when operator norms remain bounded.

1.20 1215. Folium tightness

Let $\mathcal{F}_{s,O}$ be the selected locally normal folium on the limiting local algebra. A state family is **folium-tight** if its local restrictions are uniformly absolutely continuous with respect to a declared faithful normal reference state or weight and have uniformly controlled energy, entropy, or reference-logarithm tails where those quantities are part of admissibility.

For a reference normal state $\varphi_{s,O}$, uniform absolute continuity means:

$$\forall \varepsilon > 0 \ \exists \delta > 0 : \quad \varphi_{s,O}(E) < \delta \Longrightarrow \sup_{\alpha} \omega_{s,\alpha}(E) < \varepsilon$$

for every projection E in the realized local algebra after admissible comparison.

Weak-* boundedness alone is not folium tightness.

1.21 1216. Local compactness gates

A **local compactness gate** is a family of compact maps or compact embeddings that acts after admissible localization. Typical gates are:

$$D(Q_{s,\alpha}) \cap \operatorname{Ran} \chi_{O,R} \hookrightarrow \mathcal{H}_{s,\alpha}^{\mathrm{loc}}$$

with uniform compactness after comparison realization, and an analogous seam embedding for $Q_{\mathrm{int},\alpha}$.

Abstractly, for each fixed localization test t, structural boundedness must place the coordinate values in a relatively compact subset $K_t \subset X_t$. The local compactness gate supplies compactness at fixed R, L; tightness then patches the local compactness to global realized compactness.

The gate may depend on the selected admissible regional system but not on an arbitrary enlargement of it.

1.22 1217. Theorem 1 — common structural extraction

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (common structural subnet extraction). Let $(\mathfrak{S}_\alpha)_{\alpha \in \mathfrak{A}}$ be a structurally bounded admissible family and $\mathscr{T}$ a small determining structural test system. Assume:

1. for every $t \in \mathscr{T}$, the coordinate set $\{\Phi_t(\mathfrak{S}_\alpha)\}$ has compact closure K_t in its declared Hausdorff topology;
2. regional, seam, scale, observable, and folium tightness reduce global tests to the local compactness gates uniformly;
3. every defining compatibility relation is closed in the relevant finite product of coordinate topologies;
4. sector and relative-obstruction labels take values in compact Hausdorff label spaces and the admissible class is closed there.

Then there exists one admissible subnet $(\mathfrak{S}_{\alpha(\beta)})$ and a unique test-determined limit $\mathfrak{S}_0$ such that all declared coordinates converge simultaneously. If all closed compatibility relations are satisfied by the approximants, they are satisfied by $\mathfrak{S}_0$.

1.23 1218. Proof of the common extraction theorem

For each $t \in \mathscr{T}$, let K_t be the compact closure of the coordinate image. By Tychonoff's theorem,

$$K := \prod_{t \in \mathscr{T}} K_t$$

is compact. The structural evaluation map $\Phi_{\mathscr{T}}$ takes the admissible net into K. Compactness gives a convergent subnet

$$\Phi_{\mathscr{T}}(\mathfrak{S}_{\alpha(\beta)}) \longrightarrow x_0 \in K.$$

Because the same product subnet is used for every coordinate, no later diagonal extraction is needed. The determining property of $\mathscr{T}$ identifies at most one typed structural candidate $\mathfrak{S}_0$ with test data x_0.

Each defining compatibility relation is a closed subset of a finite coordinate product. Since every approximant belongs to that relation, the corresponding limiting coordinate tuple also belongs to it. Hence products, actions, sector incidence, and coherence relations that are included among the closed structural relations survive.

Tightness and the local compactness gates are used only to verify the compact-coordinate hypothesis. They do not identify the limit by themselves. This proves the theorem.

1.24 1219. Nonclaim after Theorem 1

The theorem does not assert that arbitrary structurally bounded families satisfy the coordinate compactness hypotheses. It does not assert sequential compactness unless the structural topology is metrizable on the relevant closure. It does not prove that a correspondence-valued limit is intrinsically complete or unique up to bicategorical equivalence. It does not replace manuscript-specific normality, graph, product, action, or coherence estimates.

In particular, the theorem extracts a limit in the declared closed structural class. If the class was declared too large, compactness will faithfully return a limit in the wrong category. Compactness is obedient that way.

1.25 1220. Bulk concentration profiles

For $s \in \{\mathrm{str}, \mathrm{tw}\}$, let $\mu_{Q,s}$ be the form defect on the common subnet. After projection convergence has been established, pass—within the same product extraction—to a subnet on which the scalar nonprotected squared norms converge, and define

$$\nu_{s,\perp} := \lim_{\alpha} \|(1-P_{s,\alpha})u_{s,\alpha}\|^2 - \|(1-P_{s,0})u_{s,0}\|^2 \geq 0.$$

Define $\nu_{s,\|}$ analogously when the protected norm coordinate is required, and write $\nu_s = \nu_{s,\|} + \nu_{s,\perp}$. For a net originally described by a limsup defect, choose the common subnet realizing that limsup before applying the definition. The nonprotected component measures norm lost outside the protected range. The protected component is not controlled by coercivity and remains coupled to projection compactness.

The decomposition must be made after a limiting projection has been identified and on the same subnet as the form defect. Writing $\nu_\perp$ before projection convergence is not an innocent abbreviation; it assumes the very geometry that must be proved stable.

1.26 1221. Theorem 2 — coercive defect domination

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (bulk and seam defect domination). Suppose on one admissible subnet:

$$Q_{s,\alpha}[u] \geq \kappa_s \|(1-P_{s,\alpha})u\|^2 - r_{s,\alpha}\|u\|^2, \qquad \kappa_s > 0,$$

and

$$Q_{\mathrm{int},\alpha}[v] \geq \kappa_{\mathrm{int}} \|(1-P_{\mathrm{int},\alpha})v\|^2 - r_{\mathrm{int},\alpha}\|v\|^2, \qquad \kappa_{\mathrm{int}} > 0,$$

with $r_{*,\alpha} \to 0$. Define the coercive residuals

$$R_{s,\alpha}(u) := Q_{s,\alpha}[u] - \kappa_s \|(1-P_{s,\alpha})u\|^2,$$

and analogously $R_{\mathrm{int},\alpha}$. Assume the projections converge strongly on controlled sections, the vectors converge weakly in the comparison realization, the nonprotected squared norms converge on this same subnet as in §1220, and the residuals are lower semicontinuous along the subnet:

$$R_{s,0}(u_{s,0}) \leq \liminf_{\alpha} R_{s,\alpha}(u_{s,\alpha}), \qquad R_{\mathrm{int},0}(u_{\mathrm{int},0}) \leq \liminf_{\alpha} R_{\mathrm{int},\alpha}(u_{\mathrm{int},\alpha}).$$

Then

$$\boxed{\mu_{Q,s} \geq \kappa_s \nu_{s,\perp}}, \qquad \boxed{\mu_{\mathrm{int}} \geq \kappa_{\mathrm{int}} \nu_{\mathrm{int},\perp}}.$$

Consequently, zero bulk or interface energy defect forces zero nonprotected norm defect.

1.27 1222. Proof of coercive defect domination

Write $u_\alpha = P_\alpha u_\alpha + (1 - P_\alpha)u_\alpha$. Strong convergence of the projections on controlled sections identifies the weak limit of $(1 - P_\alpha)u_\alpha$ with $(1 - P_0)u_0$. Lower semicontinuity of the Hilbert norm gives

$$\liminf_\alpha \|(1 - P_\alpha)u_\alpha\|^2 \geq \|(1 - P_0)u_0\|^2.$$

Decompose the energy exactly as

$$Q_\alpha[u_\alpha] = R_\alpha(u_\alpha) + \kappa\|(1 - P_\alpha)u_\alpha\|^2.$$

Residual lower semicontinuity gives

$$\liminf_\alpha R_\alpha(u_\alpha) \geq R_0(u_0).$$

Using the convergence of the scalar nonprotected norms on the selected common subnet, subtract the corresponding identity at the limit. The excess nonprotected norm is therefore dominated by the excess energy:

$$\mu_Q \geq \kappa \left(\lim_\alpha \|(1 - P_\alpha)u_\alpha\|^2 - \|(1 - P_0)u_0\|^2\right) = \kappa\nu_\perp.$$

The interface proof is identical only at the level of the inequality. Its hypotheses are not identical: the seam realization, seam projection, and left/right action compatibility must already be available. Applying the same argument in the interface comparison space gives the second inequality.

1.28 1223. Nonclaim after Theorem 2

The theorem does not control norm loss inside the protected ranges. It does not prove convergence of the protected projections, equality of limiting kernels, preservation of protected dimension, or survival of sector labels. It does not identify the interface with a bulk sector. It does not prove that the manuscript-specific coercive constants are uniform on the required admissible frustum.

The abstract domination theorem is rigorous. Verification of the exact Yang–Mills inequalities and uniform margins is **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION** and may be **SOURCE AUDIT REQUIRED** where the source proof is not recovered.

1.29 1224. Geometry extracted from the defect inequality

The inequality

$$\mu_Q \geq \kappa\nu_\perp$$

is more than lower semicontinuity with a constant attached. It says that nonprotected concentration has geometric cost. Escape is forced either into the protected geometry or into a positive energy defect.

That is precisely the habit of thought associated with Gromov's influence here: an inequality classifies the shapes of possible failure.

The influence is methodological rather than a claim of direct technical dependence. The lesson is to read compactness failure as structure. If a bounded family escapes, the direction and scale of escape belong in the geometry of the functional object.

1.30 1225. Counterexample: weak convergence with retained energy

Let $\mathcal{H} = \ell^2(\mathbb{N})$, $u_n = e_n$, and $Q[u] = \|u\|^2$. Then $u_n \rightharpoonup 0$ but

$$Q[u_n] = 1 \not\to Q[0] = 0.$$

The norm and energy defects are both one. Weak convergence has extracted a vector limit while retaining positive energy in the defect measure.

This example is not a claim about the Yang–Mills field. It is the minimal warning that weak convergence alone does not recover energy.

1.31 1226. Counterexample: regional tightness without interface tightness

Let both bulk vectors be fixed compactly supported vectors. Model the incision by $\mathbb{R}$ and let the interface vector be $v_n(x) = v(x - n)$ with $v \in H^1(\mathbb{R})$. Bulk regional tails vanish uniformly, and the interface norm and seam energy remain constant, but the interface translates to infinity along the carrier.

Every fixed interface localization sees $v_n \rightharpoonup 0$. The bulks converge perfectly while the joining object disappears. Separate bulk tightness therefore does not control the seam.

1.32 1227. Counterexample: energy control without sector stability

Let two isomorphic Hilbert fibers carry labels str and tw, with identical norm and energy forms. Choose one fixed vector and alternate only the sector label. Norms and energies are constant, but the label net has two incompatible cluster points unless the label topology or an invariant detector forces stability.

In the actual theory the sectors need not be isomorphic, which makes the danger worse, not better. Energy cannot certify which sector survived.

1.33 1228. Block B audit

Last subsection: §1228.

New notation: $\chi_{O,R}$, $\chi^{\Sigma}_{R,L}$, regional/seam/scale/observable/folium tightness, local compactness gates, $\nu_{s,\perp}$, $\nu_{\mathrm{int},\perp}$.

Theorems proved: Theorem 1, common structural subnet extraction; Theorem 2, bulk and seam coercive defect domination.

Undischarged hypotheses: compact coordinate closures; uniform tightness; strong projection control on tested sections; liminf form inequalities; uniform positive coercive margins; closed label spaces.

Status: Theorems 1 and 2 are **RIGOROUS ABSTRACT THEOREMS**. Their full Yang–Mills deployment is **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION**.

1.34 1229. Projection convergence is not one notion

For projections transported into an admissible comparison realization, distinguish:

1. **weak operator convergence:** $\langle P_\alpha x, y\rangle \to \langle Px, y\rangle$;
2. **strong operator convergence:** $P_\alpha x \to Px$ for every controlled realized x;
3. **strong convergence on controlled sections:** $P_\alpha x_\alpha \to Px$ whenever $x_\alpha \to x$ in the selected class;
4. **operator-norm or gap convergence:** $\|P_\alpha - P\| \to 0$;
5. **graph convergence of ranges:** convergence of the closed subspaces through their graphs or orthogonal projections;
6. **restricted-Grassmannian convergence:** ideal-norm control of $P_\alpha - P_{\mathrm{ref}}$ together with component data;
7. **trace/rank convergence:** available only under finite-rank or trace-class hypotheses.

These notions answer different questions. Weak operator convergence gives scalar matrix-element limits. Gap convergence controls the angle between ranges and kernels. Strong convergence on controlled sections is often enough to pass projected vectors, but it may not preserve rank.

1.35 1230. Controlled-section projection convergence

A projection family is **controlled-section compact** on a class $\mathscr{V}$ if every structurally bounded admissible family admits a common subnet and a projection P_0 such that

$$\|P_\alpha x_\alpha - P_0 x_0\| \to 0$$

whenever $x_\alpha \to x_0$ strongly in $\mathscr{V}$. Equivalently, it is enough to have strong operator convergence on a determining dense set plus uniform operator bounds and strong convergence of the vectors.

For form-level statements one also requires

$$\sup_\alpha \|P_\alpha\|_{D(Q_\alpha)\to D(Q_\alpha)} < \infty$$

and a form-intertwining residual tending to zero.

Controlled-section compactness is tailored to the reconstruction. It does not claim global gap convergence on every ambient vector.

1.36 1231. Gap topology and range geometry

For closed subspaces L_α, L with orthogonal projections P_α, P, the symmetric gap is

$$\widehat{\delta}(L_\alpha, L) = \max\left\{ \sup_{x\in L_\alpha, \|x\|=1} \operatorname{dist}(x, L), \sup_{y\in L, \|y\|=1} \operatorname{dist}(y, L_\alpha) \right\}.$$

For orthogonal projections,

$$\widehat{\delta}(L_\alpha, L) = \|P_\alpha - P\|.$$

Thus operator-norm convergence of projections gives simultaneous convergence of ranges and kernels in the gap topology. If $\|P_\alpha - P\| < 1$, the ranges are isomorphic by the canonical angle operator and finite dimensions or codimensions are preserved.

Strong operator convergence is weaker. It identifies the action on each fixed vector but allows protected directions to escape to infinity.

1.37 1232. Projection variation defects

Fix a limiting projection candidate P_0. Define the controlled projection defect by

$$d_P(\alpha; \mathscr{V}) := \sup_{x\in\mathscr{V},\ \|x\|_{\mathscr{V}}\le 1} \|P_\alpha J_\alpha x - J_\alpha P_0 x\|,$$

where J_α denotes the declared admissible realization or comparison of the test section. If a reference polarization is available, one may instead use

$$\|P_\alpha - P_{\mathrm{ref},\alpha}\|_{\mathcal{S}_2}$$

and its tail, together with component or index data.

The defect is not automatically the operator norm of a difference between projections in unrelated fibers. Its definition includes the admissible comparison maps.

1.38 1233. A projection compactness gate

The **projection compactness gate** holds if, on every structurally bounded family, one of the following sufficient packages is available:

- **Gap package:** admissible realizations place the projections in one comparison space and make P_α precompact in operator norm;
- **Restricted-Grassmannian package:** $P_\alpha - P_{\mathrm{ref}}$ is uniformly controlled in $\mathcal{S}_2$, the tails are tight, and the connected component or index is fixed;
- **Controlled-range package:** the union of protected unit balls is relatively compact on the selected local tests, projected sections are strongly precompact, and both range and kernel graph limits are identified;
- **Finite protected package:** ranks are uniformly finite, traces converge, and the ranges are locally precompact.

Each package gives more than weak operator compactness. The manuscript must state which package it actually proves.

1.39 1234. Theorem 3 — projection no-leakage and kernel control

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (projection no-leakage). Let Q_α Mosco-converge to Q_0 in an admissible realization. Assume:

1. the hypotheses of Theorem 2 hold for $(Q_\alpha, P_\alpha, u_\alpha)$, including lower semicontinuity of the coercive residuals and convergence of the nonprotected squared norms on the same subnet;
2. $P_\alpha \to P_0$ strongly on controlled convergent sections;
3. projected bounded sections $P_\alpha u_\alpha$ are strongly precompact;
4. the energy and projection defects vanish on the common subnet.

Then every bounded sequence or net with $u_\alpha \rightharpoonup u_0$ and exact energy recovery converges strongly after decomposition into protected and nonprotected parts. Moreover,

$$\ker Q_0 \subseteq \operatorname{Ran} P_0.$$

If additionally $P_0 D(Q_0) \subseteq D(Q_0)$ and $Q_0[P_0 u] = 0$ for every $u \in D(Q_0)$, then

$$\ker Q_0 = (\operatorname{Ran} P_0) \cap D(Q_0).$$

1.40 1235. Proof of Theorem 3

Theorem 2 and zero energy defect give zero nonprotected norm defect. Hence

$$(1 - P_\alpha) u_\alpha \to (1 - P_0) u_0$$

strongly. By projection compactness, every subnet of $P_\alpha u_\alpha$ has a further strongly convergent subnet. Controlled-section convergence of the projections identifies every such cluster point with $P_0 u_0$. Uniqueness of the cluster point in a Hausdorff space implies

$$P_\alpha u_\alpha \to P_0 u_0$$

strongly on the common subnet. Summing the two orthogonal components gives strong convergence of u_α.

For $u \in \ker Q_0$, the limiting coercive inequality gives

$$0 = Q_0[u] \geq \kappa \|(1 - P_0) u\|^2,$$

hence $u \in \operatorname{Ran} P_0$. This proves the inclusion. If the form vanishes on the protected range, the reverse inclusion follows.

1.41 1236. Nonclaim after Theorem 3

The theorem does not prove constancy of protected dimension from strong operator convergence alone. It does not prove gap convergence unless one of the stronger projection packages supplies it. It does not show that every vector in the ambient range of P_0 belongs to the form domain. It does not identify P_0 with a spectral projection without the form-vanishing and representation hypotheses.

Equality of kernels is therefore a theorem only under the additional protected-energy-null statement. Without it, the rigorous conclusion is the inclusion

$$\ker Q_0 \subseteq \operatorname{Ran} P_0.$$

1.42 1237. Counterexample: stable rank with rotating range

In $\ell^2(\mathbb{N})$, let P_n be the rank-one projection onto e_n. Every projection has rank one, but

$$P_n \to 0$$

strongly and weakly. The protected direction has escaped through the basis. Rank stability at each finite index did not produce range compactness.

Thus "the protected space is always one-dimensional" does not identify the limiting protected space.

1.43 1238. Counterexample: kernel inflation under weak operator limits

The same rank-one projections satisfy $P_n \to 0$ in the weak operator topology. Yet

$$\ker P_n = e_n^\perp, \qquad \ker 0 = \ell^2(\mathbb{N}).$$

The limiting kernel is strictly larger than every approximating kernel. Weak operator convergence of projections has inflated the kernel.

The example also shows that weak convergence can preserve the property "is an orthogonal projection" in this case while still losing the geometry of the ranges. In general, a weak operator limit of projections need not even be a projection.

1.44 1239. Selected locally normal state classes

For each admissible local algebra $\mathcal{M}_{s,\alpha}(O)$, let $\omega_{s,\alpha,O}$ be a normal positive functional, normalized when a state is intended. After admissible comparison with a limiting local algebra $\mathcal{M}_{s,0}(O)$, define

$$\mathfrak{F}^{\mathrm{adm}}_{s,O} \subseteq \mathcal{M}_{s,0}(O)^+_*$$

to be the selected locally normal folium, possibly with energy, entropy, or moment constraints.

For weights, convergence is always stated on a common positive core or on bounded spectral cutoffs. No theorem below treats arbitrary semifinite weights as if they were norm-one functionals.

1.45 1240. Uniform absolute continuity

Let φ be a faithful normal state on a local von Neumann algebra $\mathcal{M}$. A family $\Omega \subset \mathcal{M}_*^+$ is uniformly absolutely continuous with respect to φ if

$$\forall \varepsilon > 0\ \exists \delta > 0 : \quad \varphi(E) < \delta \Longrightarrow \sup_{\omega\in\Omega} \omega(E) < \varepsilon$$

for every projection $E \in \mathcal{M}$.

A domination hypothesis

$$\omega \leq C\varphi$$

uniformly in ω is sufficient but stronger than necessary. Uniform entropy or reference-logarithm estimates may imply uniform absolute continuity; the precise implication is manuscript-specific and must be proved rather than cited by atmosphere.

1.46 1241. Weak-* compactness with a normality gate

The state space of a unital C^*-algebra is weak-* compact. The normal state space of a von Neumann algebra need not be weak-* closed inside the full dual state space. Therefore weak-* extraction must be paired with a normality gate.

A useful gate is uniform absolute continuity with respect to one faithful normal state on each selected local algebra. Another is relative weak compactness in the predual $\mathcal{M}_*$. A third may be noncommutative uniform integrability relative to a faithful normal semifinite weight.

The gate must be local and uniform over the admissible scale and regional family used downstream.

1.47 1242. Theorem 4 — state-normality closure

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (local normality under uniform absolute continuity). Let $\mathcal{M}$ be a von Neumann algebra with faithful normal state φ. Let (ω_α) be normal positive functionals such that $\sup_\alpha \omega_\alpha(1) < \infty$ and the family is uniformly absolutely continuous with respect to φ. If

$$\omega_\alpha \xrightarrow{w^*} \omega \quad \text{in } \mathcal{M}^*,$$

then ω is normal. Positivity is preserved, and if $\omega_\alpha(1) = 1$, then $\omega(1) = 1$.

The result applies locally to an admissible net of algebras provided the uniform absolute-continuity constants are compatible with the selected inclusions and the same structural subnet is used.

1.48 1243. Proof of Theorem 4

Positivity and preservation of the value at the unit follow from weak-* convergence. To prove normality, let $E_i \uparrow E$ be an increasing net of projections. Then $F_i := E - E_i \downarrow 0$. Normality of φ gives $\varphi(F_i) \downarrow 0$.

Given $\varepsilon > 0$, uniform absolute continuity provides $\delta > 0$ such that $\varphi(F) < \delta$ implies $\sup_\alpha \omega_\alpha(F) < \varepsilon$. Choose i with $\varphi(F_i) < \delta$. Passing to the weak-* limit gives

$$0 \le \omega(F_i) = \lim_\alpha \omega_\alpha(F_i) \le \varepsilon.$$

Hence $\omega(E_i) \uparrow \omega(E)$. A positive functional on a von Neumann algebra is normal precisely when it is continuous from below on projections. Therefore $\omega \in \mathcal{M}_*^+$.

1.49 1244. Compatibility with local GNS reconstruction

If $\omega_{s,\alpha,O} \to \omega_{s,0,O}$ locally normally and the algebra actions converge on a common controlled core, the limiting cyclic seminorm satisfies

$$\|[A]\|_{\omega_0}^2 = \omega_0(A^*A) = \lim_\alpha \omega_\alpha(A_\alpha^* A_\alpha)$$

whenever product and involution defects vanish. Thus the state limit is compatible with GNS seminorm formation and does not merely define an abstract dual functional unrelated to the limiting action.

A unitary comparison of complete GNS representations requires equality of the resulting state on a determining algebra and the usual universal property. It is not a consequence of weak-* convergence alone.

1.50 1245. Nonclaim after Theorem 4

The theorem does not prove uniform absolute continuity from energy boundedness. It does not show that entropy bounds available at finite scale are uniform in the thin direction. It does not treat a weight that is infinite on the unit without localization or cutoff hypotheses. It does not prove quasi-equivalence of limiting representations unless the selected folium and GNS comparison conditions are separately verified.

Where the Yang–Mills manuscript asserts folium preservation without a recovered uniform-integrability or domination proof, the claim is **SOURCE AUDIT REQUIRED**.

1.51 1246. Counterexample: normal states approaching a singular state

On $\mathcal{B}(\ell^2)$, let $\omega_n(A) = \langle Ae_n, e_n\rangle$. Each ω_n is a normal vector state. Any weak-* cluster point along a free ultrafilter annihilates the compact operators because $\langle Ke_n, e_n\rangle \to 0$ for every compact K, while it takes value one on the identity. Such a cluster point is singular.

Thus normality is not weak-* closed in the full state space of $\mathcal{B}(\ell^2)$. Uniform absolute continuity fails, exactly as it should.

1.52 1247. Gauss and Ward graph systems

Let $G_{a,\alpha}$ be the selected Gauss generators or closed constraint operators on dense domains $D(G_{a,\alpha})$. Let $\delta_{a,\alpha}$ be the Ward derivations on selected invariant algebra cores $\mathscr{A}_{a,\alpha}^{\mathrm{inv}}$.

The structural constraint datum contains:

$$(\xi_\alpha, G_{a,\alpha}\xi_\alpha), \qquad (A_\alpha, \delta_{a,\alpha}(A_\alpha)), \qquad \omega_\alpha(\delta_{a,\alpha}(A_\alpha)).$$

The relevant compactness is graph compactness, not merely convergence of the carrier vectors or observables.

1.53 1248. Graph-liminf closedness

A varying family G_α is **graph-liminf closed** toward G_0 if, whenever

$$x_\alpha \to x, \qquad G_\alpha x_\alpha \to y$$

in the declared admissible realizations, then $x \in D(G_0)$ and $G_0 x = y$.

For Ward derivations, the corresponding condition is: if $A_\alpha \to A$ and $\delta_\alpha(A_\alpha) \to B$ in the selected graph topology, then $A \in D(\delta_0)$ and $\delta_0(A) = B$.

Uniform graph bounds supply precompactness only when paired with a compact graph embedding or resolvent compactness on the selected tests.

1.54 1249. Theorem 5 — Gauss and Ward constraint closure

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (constraint closure on the common subnet). Assume $G_{a,\alpha}$ is graph-liminf closed toward $G_{a,0}$. Let

$$\xi_\alpha \to \xi_0, \qquad \|G_{a,\alpha}\xi_\alpha\| \to 0$$

on the common structural subnet. Then $\xi_0 \in D(G_{a,0})$ and $G_{a,0}\xi_0 = 0$.

Assume likewise that $\delta_{a,\alpha}$ is graph-liminf closed in one declared local realization,

$$A_\alpha \to A_0, \qquad \delta_{a,\alpha}(A_\alpha) \to \delta_{a,0}(A_0)$$

with the second convergence in norm, and $\omega_\alpha \to \omega_0$ weak-* with uniformly bounded functional norms. If

$$\omega_\alpha(\delta_{a,\alpha}(A_\alpha)) \to 0,$$

then

$$\omega_0(\delta_{a,0}(A_0)) = 0.$$

1.55 1250. Proof of Theorem 5

The Gauss statement is immediate from graph-liminf closedness with $y = 0$. The conclusion is not obtained by taking limits of kernels; it is obtained by taking the limit inside the closed graph.

For the Ward statement, graph convergence identifies the limiting derivation value. Put $B_\alpha = \delta_{a,\alpha}(A_\alpha)$ and $B_0 = \delta_{a,0}(A_0)$. Then

$$\begin{aligned}|\omega_\alpha(B_\alpha) - \omega_0(B_0)| &\le \|\omega_\alpha\|\,\|B_\alpha - B_0\| + |\omega_\alpha(B_0) - \omega_0(B_0)| \\ &\longrightarrow 0.\end{aligned}$$

The first term vanishes by norm convergence and uniform boundedness of the functionals; the second vanishes by weak-* convergence. Since the left Ward residual tends to zero, the limiting Ward identity holds.

When the Ward expression contains seam or boundary terms, those terms must be included in the graph datum and interface defect. Omitting them changes the operator whose graph is being closed.

1.56 1251. Nonclaim after Theorem 5

The theorem does not prove graph-liminf closedness from formal commutator identities on finite packets. It does not prove that the domains are common, invariant, or stable under all local products. It does not show that kernels commute with arbitrary scale limits. It does not exclude anomaly terms unless they are included in the Ward defect and shown to vanish.

The exact Gauss-domain and Ward-domain verification in the Yang–Mills manuscripts remains **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION** and may be **SOURCE AUDIT REQUIRED**.

1.57 1252. Block C audit

Last subsection: §1252.

New notation: projection gap $\widehat{\delta}$, controlled projection defect $d_P(\alpha; \mathscr{V})$, selected folia $\mathfrak{F}^{\mathrm{adm}}_{s,O}$, reference state φ, Gauss graphs $G_{a,\alpha}$, Ward derivations $\delta_{a,\alpha}$.

Theorems proved: Theorem 3, projection no-leakage and kernel inclusion/equality criterion; Theorem 4, state-normality closure; Theorem 5, Gauss and Ward graph closure.

Undischarged hypotheses: projection compactness package; protected-energy-null property; uniform absolute continuity; graph-liminf closedness; seam terms in constraint graphs; action/product convergence for Ward pairings.

Status: all three are **RIGOROUS ABSTRACT THEOREMS**. Manuscript application is **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION**.

1.58 1253. Product defects must be typed

Let A_α, B_α be controlled observables whose admissible limits are A_0, B_0. If C_α is the chosen representative of the intended product limit, define

$$d_{\mathrm{prod},\alpha}(A,B;\mathscr{V}) := \sup_{x\in\mathscr{V},\|x\|\le 1} \|m_\alpha(A_\alpha,B_\alpha)x - C_\alpha x\|.$$

When a direct limiting multiplication is already available, take C_α to compare with A_0B_0. For interface actions define

$$d^L_{\mathrm{act},\alpha}(A,\xi) = \|\lambda_\alpha(A_\alpha)\xi_\alpha - \text{comparison of } \lambda_0(A_0)\xi_0\|,$$

and similarly on the right.

The topology may be strong, strong-*, graph, or form-bounded. It must be declared before the defect is meaningful.

1.59 1254. Bounded multiplication classes

A controlled algebra class $\mathscr{B}$ is **multiplication bounded** if

$$\sup_{\alpha,A\in\mathscr{B}} \|A_\alpha\| < \infty$$

and the class is closed under the products, adjoints, and units that are actually used. On such a class, the strong-* topology is compatible with multiplication:

$$A_\alpha \to A, \quad B_\alpha \to B \quad \text{strong-*}, \quad \sup_\alpha(\|A_\alpha\| + \|B_\alpha\|) < \infty$$

implies $A_\alpha B_\alpha \to AB$ strong-*.

For unbounded observables, no such statement is made. One must use graph convergence, resolvents, form products, or a common invariant core.

1.60 1255. Theorem 6 — product, involution, and unit closure

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (bounded local algebra closure). In an admissible local realization, let $A_\alpha \to A_0$ and $B_\alpha \to B_0$ strongly-* with uniform operator bounds. Then

$$A_\alpha B_\alpha \to A_0 B_0 \quad \text{strong-*}.$$

If A_α^* and units are included in the structural coordinates, involution and unit relations pass to the limit. More generally, every noncommutative *-polynomial relation with uniformly bounded coefficients passes to the limit in the strong-* topology.

If multiplication is represented by m_α rather than literal operator composition, the same conclusion holds provided $d_{\mathrm{prod},\alpha} \to 0$.

1.61 1256. Proof of Theorem 6

For every vector x,

$$\|(A_\alpha B_\alpha - A_0 B_0)x\| \le \|A_\alpha(B_\alpha - B_0)x\| + \|(A_\alpha - A_0)B_0 x\|.$$

The first term tends to zero by strong convergence of B_α and the uniform bound on A_α; the second tends to zero by strong convergence of A_α. Applying the same estimate to adjoints gives strong-* convergence.

The unit relation passes because $1_\alpha x \to x$ on the determining vectors. Polynomial relations follow by induction on word length. If m_α is not literal composition, insert and subtract the chosen product representative; the product defect controls the extra term.

1.62 1257. Separate convergence is not product convergence

Weak operator multiplication is not jointly continuous. Even strong convergence of one factor and weak convergence of the other must be used with the correct adjoint hypothesis to obtain weak convergence of products. The structural topology therefore either uses strong-* convergence on uniformly bounded classes or carries product evaluations as independent coordinates with vanishing product defect.

A limiting vector space together with separately convergent symbols A_α and B_α is not yet a limiting algebra.

1.63 1258. Unbounded products and graph closure

For unbounded observables X_α, Y_α, product closure requires at least:

- a common or compatibly varying invariant core;
- graph bounds for X_α, Y_α, and the intended product;
- convergence of graph pairs;
- control of domain loss;
- a proof that the limiting core remains a core.

Alternatively, one may pass bounded resolvents or bounded functional calculus elements and reconstruct the unbounded operator afterward. No theorem in this Part treats $X_\alpha Y_\alpha$ by formal symbol manipulation.

1.64 1259. Nonclaim after Theorem 6

The theorem does not make multiplication jointly continuous in the weak operator topology. It does not pass unbounded products, Connes fusion, or operator-valued weights without separate estimates. It does not prove Haag–Kastler locality unless commutation relations are included among the closed *-polynomial tests on the selected admissible regional pairs.

Full fusion-compatible product closure remains Part XXV.

1.65 1260. Counterexample: weakly convergent factors, wrong product

Let S be the unilateral shift on $\ell^2(\mathbb{N})$. Then

$$S^n \xrightarrow{\text{WOT}} 0, \qquad S^{*n} \xrightarrow{\text{WOT}} 0,$$

but

$$S^{*n} S^n = I$$

for every n. The products do not converge to the product of the weak limits.

This is the standard algebraic ambush hidden behind the phrase "pass to the limit in the product."

1.66 1261. Local coherence defects

Let

$$\alpha_0 \to \alpha_1 \to \cdots \to \alpha_N$$

be an admissible chain. For composable transports $T_{j+1,j}$, define a local composition defect

$$R_{k,j,i} = T_{k,j} T_{j,i} - U_{k,j,i} T_{k,i},$$

where $U_{k,j,i}$ is the declared coherence cell. Squares, prisms, associators, and module naturality diagrams have analogous residuals.

A local statement $\|R_{k,j,i}\| \to 0$ at each fixed triple does not control chains whose length grows with the index.

1.67 1262. Accumulated coherence budgets

For an admissible path $p = (\alpha_0, \ldots, \alpha_N)$, define

$$\Gamma_\alpha(p) := \sum_{j=1}^{N-1} M_{\alpha,j}(p) \, \|R_{\alpha,j}(p)\|,$$

where $M_{\alpha,j}(p)$ is the product of operator or action bounds needed to propagate the local residual to the end of the chain.

The family has **summable coherence defects** if

$$\sup_{p \in \mathscr{P}_{\text{adm}}(\alpha)} \Gamma_\alpha(p) \longrightarrow 0.$$

A bounded-chain alternative is valid when admissible chains have uniformly bounded length and local residuals converge uniformly to zero. The summability formulation is stronger and survives unbounded chain lengths.

1.68 1263. Theorem 7 — coherence no-leakage

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (closure of admissible coherence). Suppose all transport and action maps on the selected tests are uniformly bounded, local coherence cells converge in the declared topology, and the accumulated budgets satisfy

$$\sup_{p\in\mathscr{P}_{\mathrm{adm}}(\alpha)} \Gamma_\alpha(p) \to 0.$$

Then every finite limiting admissible diagram commutes with its declared limiting coherence cell. If the approximating data form a lax or pseudofunctor with residuals R_α, the limit is an exact functor or pseudofunctor on the tested admissible category whenever all accumulated residuals vanish. If a nonzero coherence class remains, the limit is only lax with that residual class.

1.69 1264. Proof of Theorem 7

Expand the difference between an iterated composite and the declared direct map by telescoping one local replacement at a time. Each replacement contributes one local residual multiplied on the left and right by bounded transport or action maps. Therefore

$$\|T^{\mathrm{iter}}_{p,\alpha} - U_{p,\alpha}T^{\mathrm{dir}}_{p,\alpha}\| \le \Gamma_\alpha(p).$$

Uniform decay of $\Gamma_\alpha(p)$ implies decay of the complete path residual. Passing to the common structural subnet and using coordinate convergence gives exact limiting commutativity for every tested admissible diagram.

The proof is a quantitative coherence argument. It is not enough that each fixed local defect vanishes; the telescoping sum must vanish uniformly over the chains used in reconstruction.

1.70 1265. Nonclaim after Theorem 7

The theorem does not control untested diagrams or arbitrary refinements. It does not prove a bicategorical universal property. It does not identify a nonzero limiting coherence class as trivial. It does not promote correspondences to equivalences. It proves exactness only in the declared admissible category and only when the accumulated budget vanishes.

1.71 1266. Counterexample: small local errors, order-one global error

Let a chain have length $N = n$, and let every local scalar coherence error equal $1/n$, with all propagation norms one. Each local defect tends to zero, but

$$\Gamma_n = \sum_{j=1}^{n} \frac{1}{n} = 1.$$

The global diagram remains one unit away from commuting. Local smallness without accumulation control is not coherence compactness.

1.72 1267. The interface compactness datum

The interface structural coordinate is

$$\mathfrak{I}_\alpha = \left(\mathcal{K}_{\mathrm{int},\alpha}, Q_{\mathrm{int},\alpha}, P_{\mathrm{int},\alpha}, \lambda_\alpha, \rho_\alpha, \xi_{\mathrm{int},\alpha}, \square_\alpha, [\chi_\alpha^\Sigma]\right).$$

Its compactness requires:

- seam norm and energy tightness;
- a seam local compactness gate;
- protected seam projection compactness;
- strong convergence of controlled interface sections;
- convergence of left and right actions on bounded tests;
- square-defect closure with both bulk limits;
- stability of the relative-obstruction detector.

None of these conditions is implied merely by compactness of the two bulks.

1.73 1268. Seam coercivity and protected seam modes

Assume

$$Q_{\mathrm{int},\alpha}[\eta] \geq \kappa_{\mathrm{int}} \|(1 - P_{\mathrm{int},\alpha})\eta\|^2 - r_{\mathrm{int},\alpha}\|\eta\|^2.$$

As in the bulk, zero seam-energy defect controls only the nonprotected seam component. Protected interface modes require their own projection compactness and energy-null identification. They are not bulk particles merely because they are protected.

The obstruction interface may carry necessary zero-cost relational data without defining a third vacuum representation.

1.74 1269. Left and right action closure

Let $A_{\mathrm{str},\alpha} \to A_{\mathrm{str},0}$ and $B_{\mathrm{tw},\alpha} \to B_{\mathrm{tw},0}$ strongly-* on uniformly bounded classes, and let $\eta_\alpha \to \eta_0$ strongly. If

$$d^L_{\mathrm{act},\alpha} \to 0, \qquad d^R_{\mathrm{act},\alpha} \to 0,$$

then

$$\lambda_\alpha(A_{\mathrm{str},\alpha})\eta_\alpha \to \lambda_0(A_{\mathrm{str},0})\eta_0,$$

$$\rho_\alpha(B_{\mathrm{tw},\alpha})\eta_\alpha \to \rho_0(B_{\mathrm{tw},0})\eta_0.$$

Commutation of the limiting left and right actions passes when the commutator residual is included in the product/coherence defect and tends to zero.

1.75 1270. Square-defect closure

For an admissible scale square, write schematically

$$\mathcal{W}_{f\leftarrow e}(\eta_{\mathrm{str}} \boxtimes \xi_e) - \xi_f \boxtimes \eta_{\mathrm{tw}}.$$

The square defect $d_\square$ is the norm or pairing residual of this expression in the selected realized correspondence test. Interface compatibility survives if:

1. the strict and twisted transport coordinates converge;
2. the interface vectors and actions converge;
3. the square residual tends to zero;
4. accumulated prism defects are summable.

The limit square then joins the limiting bulks. Separate bulk convergence supplies only items 1 and part of 2.

1.76 1271. Preservation of the relative obstruction

Let $\mathfrak{X}_\chi$ be a Hausdorff detector space for the admissible relative class. Assume

$$\chi_\alpha := [z_\alpha^{\mathrm{str}}] - [z_\alpha^{\mathrm{tw}}] \longrightarrow \chi_0$$

and that the detector map is closed under the selected admissible comparisons. If every $\chi_\alpha = [\chi^\Sigma]$, then $\chi_0 = [\chi^\Sigma]$.

This is a stability theorem for a declared invariant. It is not a proof that the class is trivial, nor a proof that every realization detects it faithfully. The relevant source-specific cocycle representatives and detector maps are **SOURCE AUDIT REQUIRED** unless recovered.

1.77 1272. Theorem 8 — interface structural compactness

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (interface compactness and seam closure). Let $(\mathfrak{I}_\alpha)$ be structurally bounded and seam-tight. Assume a seam local compactness gate; the seam hypotheses of Theorems 2 and 3, including coercive-residual lower semicontinuity and strong precompactness of protected projected sections; Mosco convergence of the seam forms; vanishing seam norm, energy, and projection defects;

strong-* convergence of bounded bulk action tests; vanishing left/right action defects; vanishing square defects; summable prism defects; and stability of $[\chi^{\Sigma}]$. Then one common admissible subnet converges to a realized interface datum $\mathfrak{I}_0$ compatible with both limiting bulks. The limit retains commuting left and right actions on the tested class, the seam coercive inequality, the protected seam projection, the limiting square relations, and the relative obstruction.

1.78 1273. Proof of Theorem 8

Seam tightness plus the local compactness gate yields strong precompactness of localized interface sections. Theorem 1 extracts the interface vectors, actions, projections, squares, and labels on the same subnet as the bulks. Theorem 2 suppresses nonprotected seam leakage; Theorem 3 closes the protected seam component. Theorem 6 and the vanishing action defects pass left and right actions. Theorem 7 passes the accumulated square and prism coherence. Closedness of the relative-obstruction detector preserves $[\chi^{\Sigma}]$.

Every conclusion is formulated in the declared realization. The intrinsic coefficient-valued completion is not used. Hence the theorem supplies a structurally compatible realized interface, which is the correct input for Part XXIV but not yet the intrinsic thin correspondence itself.

1.79 1274. Nonclaim after Theorem 8

The theorem does not prove that the limiting correspondence is self-dual, full, unique up to unitary module equivalence, or independent of the coefficient realization. It does not prove Connes-fusion continuity. It does not turn the interface into a third bulk sector. It does not trivialize $[\chi^{\Sigma}]$.

Those stronger statements remain **PROGRAMMATIC** for Parts XXIV–XXV.

1.80 1275. Counterexample: convergent bulks, failing interface square

Take constant strict and twisted bulk spaces and constant identity transports. Let the interface space be one-dimensional, but let the square comparison be multiplication by $(-1)^n$. Both bulks converge trivially. The interface vectors may also be constant, yet the square has two incompatible subnet limits and no full-net limit.

Alternatively, fixing the intended limiting square as the identity gives a square defect equal to two on odd indices. Separate bulk compactness has not joined the sectors.

1.81 1276. Block D audit

Last subsection: §1276.

New notation: product defect d_{prod}, action defects $d_{\mathrm{act}}^{L,R}$, coherence budget $\Gamma_\alpha(p)$, interface datum $\mathfrak{I}_\alpha$, relative detector $\mathfrak{X}_\chi$.

Theorems proved: Theorem 6, bounded product/*/unit closure; Theorem 7, coherence no-leakage; Theorem 8, realized interface structural compactness.

Undischarged hypotheses: uniform strong-* boundedness; graph control for unbounded observables; summable coherence budgets; seam local compactness; action no-leakage; square/prism closure; faithful relative-obstruction detector.

Status: Theorems 6 and 7 are **RIGOROUS ABSTRACT THEOREMS**. Theorem 8 is a **RIGOROUS ABSTRACT THEOREM** whose type is correspondence-sensitive in a declared realization; its Yang–Mills application is **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION**.

1.82 1277. The closed structural class

Let $\mathfrak{C}^{\mathrm{struc}}_{\mathrm{adm}}$ be the class of typed positive-thickness structural states satisfying:

- the strict/twisted/interface incidence relations;
- the declared local algebra and action relations;
- the selected sector and relative-obstruction conditions;
- form-domain and projection-domain compatibility;
- Gauss and Ward graph conditions;
- admissible square, prism, and chain coherence;
- local normality in the selected folia.

The class is **structurally closed** if it is closed in $\tau_{\mathrm{struc}}(\mathscr{T})$. The closure theorem is not automatic: each relation must be shown closed in the exact topology assigned to it. Theorems 3–8 provide sufficient coordinate closure mechanisms.

1.83 1278. The structural compactness hypothesis package

Write **SC** for the following package:

$$\mathbf{SC} = (\mathbf{B}, \mathbf{T}, \mathbf{L}, \mathbf{P}, \mathbf{N}, \mathbf{G}, \mathbf{M}, \mathbf{C}, \mathbf{I}, \mathbf{S}),$$

where:

- **B**: structural boundedness;
- **T**: regional, seam, scale, observable, and folium tightness;
- **L**: local bulk and seam compactness gates;
- **P**: bulk and seam projection compactness;
- **N**: state-normality compactness;
- **G**: relative compactness of the selected graph coordinates together with graph-liminf closure of Gauss and Ward data;
- **M**: relative compactness in the selected strong-* or graph topology together with product/action continuity on the selected class;
- **C**: relative compactness of the tested local coherence cells together with summable accumulated defects;
- **I**: relative compactness and closure of the interface square, action, and relative-class coordinates;
- **S**: relatively compact Hausdorff sector-label and twisted-coefficient coordinates with closed admissibility conditions.

Every letter has one job. Removing a letter reopens a specific defect channel.

1.84 1279. The complete structural topology as a product topology

For each test $t \in \mathscr{T}$, let X_t be the Hausdorff coordinate target. Define

$$\mathbb{X}_{\mathscr{T}} = \prod_{t\in\mathscr{T}} X_t$$

and let $\mathfrak{R} \subseteq \mathbb{X}_{\mathscr{T}}$ be the intersection of all closed structural relations. Then

$$\mathfrak{C}^{\mathrm{struc}}_{\mathrm{adm}} \cong \Phi^{-1}_{\mathscr{T}}(\mathfrak{R})$$

on the determining class.

This answers the first hostile-referee question: compactness is in the typed initial topology induced by the coordinate targets, with the admissible structural class realized as a closed relation space. It is not compactness in a single weak Hilbert topology.

1.85 1280. Theorem 9 — common-subnet structural compactness

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (multi-axis common-subnet structural compactness). Let $(\mathfrak{S}_\alpha) \subset \mathfrak{C}^{\mathrm{struc}}_{\mathrm{adm}}$ be an admissible family satisfying **SC**. Assume the structural test system is small and determining, the coordinate realizations are faithful on the tested relations, and every coordinate compactness statement is uniform on the same admissible frustum. Then there exists one admissible subnet $(\mathfrak{S}_{\alpha(\beta)})$ and one structural limit

$$\mathfrak{S}_0 = (\mathfrak{B}_{\mathrm{str},0}, \mathfrak{B}_{\mathrm{tw},0}, \mathfrak{I}_0, \omega_0, G_0, W_0, m_0, c_0)$$

such that:

1. both bulk vectors and forms converge in the declared local/form topology;
2. all protected projections converge on controlled sections;
3. the states remain in the selected locally normal folia;
4. Gauss and Ward constraints close;
5. bounded products, involutions, units, and tested local relations close;
6. the interface converges compatibly with both bulks, including left/right actions and square relations;
7. sector labels and $[\chi^\Sigma]$ persist;
8. accumulated coherence defects vanish in the limit.

Thus the limit is one typed admissible three-sector field, not a collection of separately convergent components.

1.86 1281. Proof of Theorem 9

Structural boundedness together with the tightness and local-compactness package makes the localized bulk and seam vector/form coordinates relatively compact. The remaining clauses of **SC** explicitly supply relative compactness in their declared coordinate topologies: projection coordinates through **P**, locally normal state restrictions through **N**, graph coordinates through **G**, product and action coordinates through **M**, local cells and accumulated residual coordinates through **C**, interface coordinates through **I**, and label/coefficient coordinates through **S**. The closure portions of those same clauses identify the structural relations after extraction.

Theorem 1 then yields one product subnet carrying every coordinate. Theorems 3–8 show that all defining relations are closed in the selected coordinate topologies. Hence the product limit belongs to $\mathfrak{R}$ and defines $\mathfrak{S}_0 \in \mathfrak{C}^{\mathrm{struc}}_{\mathrm{adm}}$.

The same subnet is used throughout because all coordinates were extracted by the single product map $\Phi_{\mathscr{T}}$. No later theorem is permitted to replace it without explicitly passing all previous conclusions to a further common subnet.

1.87 1282. Hypothesis-to-conclusion ledger for Theorem 9

Hypothesis	Defect channel closed	Structural conclusion
B	blow-up	bounded coordinate sets; compactness still supplied by the relevant gate
T	regional, seam, scale, nonlocal, folium escape	global tightness
L	local concentration/dichotomy	local relative compactness
P	projection rotation, protected leakage	stable protected decomposition
N	state-normality loss	locally normal limit state
G	Gauss/Ward loss	constraint kernels close
M	product/action loss	limiting algebra and actions
C	coherence accumulation	exact tested limiting diagrams
I	interface concentration/square failure	compatible seam limit
S	sector/relative-class drift	preserved three-sector type

This table is a proof dependency object. It is not a substitute for verification of the hypotheses.

1.88 1283. Nonclaim after Theorem 9

The theorem does not assert that **SC** follows from a single energy estimate. It does not prove compactness in an untested larger category. It does not supply an intrinsic coefficient algebra, fusion product, self-dual module completion, or bicategorical universal property. It does not prove that the scale correspondences are equivalences. It does not reconstruct over arbitrary Cauchy data.

The stronger universal theorem "every bounded admissible coercive correspondence field is structurally compact" is explicitly **not claimed**.

1.89 1284. Scalar-to-measure determination

Assume each localized defect functional is positive and is determined by a family $\mathscr{L}_0 \subset \mathscr{L}$ containing an increasing admissible approximate unit. Suppose its total variation on every determining localization is bounded by the corresponding scalar defect coordinate:

$$0 \le \mu_j(\ell) \le C_\ell d_j, \qquad \ell \in \mathscr{L}_0.$$

If $d_j \to 0$, then $\mu_j(\ell) = 0$ for every determining ℓ. Positivity and determination imply $\mu_j = 0$.

This is the bridge from the scalar ledger $\mathbf{D}_\alpha$ to the localized defect object $\boldsymbol{\mu}_{\mathrm{def}}$.

1.90 1285. Theorem 10 — complete no-leakage closure

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (complete structural no-leakage). Assume Theorem 9 and, on its common admissible subnet,

$$\mathbf{D}^{\mathrm{full}}_{\alpha(\beta)} \longrightarrow 0.$$

Assume also that the scalar-to-measure determination condition of §1284 holds. Then

$$\boxed{\boldsymbol{\mu}_{\mathrm{def}} = 0.}$$

More precisely:

1. bulk vectors converge strongly off the protected ranges;
2. interface vectors converge strongly off the protected seam range;
3. bulk and interface energies are recovered exactly;
4. projected components converge strongly and no protected rotation remains on controlled sections;
5. no sector or relative-obstruction drift remains;
6. no state mass leaves the selected locally normal folia;
7. Gauss and Ward residuals vanish in the limit;
8. bounded products, involutions, units, local relations, and left/right actions pass to the limit;
9. accumulated coherence residuals vanish;
10. $\ker Q_{s,0} \subseteq \operatorname{Ran} P_{s,0}$ and $\ker Q_{\mathrm{int},0} \subseteq \operatorname{Ran} P_{\mathrm{int},0}$.

Kernel equality holds only under the additional protected-energy-null hypotheses of Theorem 3.

1.91 1286. Proof of Theorem 10

The scalar-to-measure determination lemma annihilates every localized defect component. Theorem 2 converts zero energy defect into zero nonprotected norm defect. The projection compactness gate and zero d_P identify the protected components and exclude rotation on controlled sections. Weak-plus-exact-norm convergence then gives strong convergence of the full controlled vectors whenever both protected and nonprotected pieces are covered.

Mosco liminf together with zero μ_Q and μ_{int} gives exact energy recovery. Theorem 4 and zero state defect retain normality. Theorem 5 and zero graph residuals retain Gauss and Ward constraints. Theorem 6 and zero product/action defects retain the local algebraic operations. Theorem 7 and zero accumulated coherence defect retain the tested diagrams. Theorem 8 and zero seam/square/relative-class defects retain the obstruction interface.

Finally, limiting coercivity gives the kernel inclusions. Equality requires vanishing form on the protected ranges; this is not inferred from defect vanishing alone.

1.92 1287. Coordinatewise failure map

The complete no-leakage theorem answers which coordinate controls which failure:

Failure	Primary defect coordinate
norm concentration or vanishing	ν_{norm}
energy retained or lost	μ_Q
protected rotation	d_P, π
state-normality loss	$d_{\text{ent}}, d_{\text{normal}}$
sector drift	$d_{\text{sector}}, d_\chi$
interface concentration	$\nu_{\text{int}}, \mu_{\text{int}}$
interface square failure	$d_\square$
long-chain coherence escape	$\|\gamma_\alpha\|$, Γ_α
Gauss loss	d_G, μ_G
Ward loss	d_{Ward}
product or action loss	$d_{\text{prod}}, d_{\text{act}}^{L,R}$
regional/nonlocal escape	$d_{\text{loc}}, d_{\text{nonloc}}$
scale drift	d_{scale}
kernel inflation	d_{kernel} plus coercivity/projection data

No one coordinate closes the entire table.

1.93 1288. Kernel inflation after complete defect closure

Under Theorem 10, no **nonprotected** kernel inflation occurs:

$$\ker Q_0 \subseteq \operatorname{Ran} P_0.$$

To exclude protected kernel inflation or loss, one needs one of:

- gap convergence of protected projections plus form vanishing on their ranges;
- finite-rank trace convergence and exact protected energies;
- restricted-Grassmannian component stability plus an exact kernel theorem;
- a direct spectral-projection identification.

Thus complete defect closure relative to the declared protected subspace prevents hidden zero modes outside that space. It does not prove that the protected taxonomy itself is complete unless the projection and energy-null identification are complete.

1.94 1289. Counterexample: incompatible subnet structures

Let the even indices carry sector label str, projection P, and state ω; let the odd indices carry label tw, projection Q, and state η, with all vector norms and energies equal. The even subnet converges to one structural object and the odd subnet to another.

Choosing the vector limit from the even subnet and the state limit from the odd subnet manufactures a tuple that was never approximated by any common subnet. It may even violate sector incidence. This is why separate extraction cannot be repaired by editorial assembly.

1.95 1290. Complete failure diagnostics

The ten required model mechanisms are now located as follows:

1. weak limit with retained energy: §1225;
2. stable-rank projection rotation: §1237;
3. kernel inflation under weak operator limits: §1238;
4. normal states with singular cluster state: §1246;
5. separately weakly convergent factors with wrong product: §1260;
6. accumulating local coherence defects: §1266;
7. separately convergent bulks with failing interface square: §1275;
8. incompatible limiting structures from different subnets: §1289;
9. regional bulk tightness without interface tightness: §1226;
10. energy control without sector stability: §1227.

These are abstract examples. None is asserted to occur in the Yang–Mills manuscripts. Their role is to show that the corresponding hypotheses cannot simply be deleted.

1.96 1291. Nonclaim after Theorem 10

The theorem does not say that $\mathbf{D}_\alpha^{\mathrm{full}} \to 0$ has been verified for the manuscript. It does not claim that the defect taxonomy is metaphysically exhaustive. It does not prove spectral-gap transfer, forbidden intervals, or physical units. Those are downstream results requiring the Part XXII boundary form, kernel identification, and later spectral arguments.

It does prove the abstract implication from verified structural compactness and verified zero defects to a structurally complete admissible limit.

1.97 1292. Block E audit

Last subsection: §1292.

New notation: closed structural class $\mathfrak{C}_{\mathrm{adm}}^{\mathrm{struc}}$, hypothesis package $\mathbf{SC}$, product test space $\mathbb{X}_{\mathscr{T}}$, relation space $\mathfrak{R}$.

Theorems proved: Theorem 9, common-subnet structural compactness; Theorem 10, complete no-leakage closure.

Undischarged hypotheses: every coordinate of **SC**; scalar-to-measure determination; full defect vanishing; protected-energy-null condition for kernel equality; faithful test realizations.

Status: Theorems 9 and 10 are **RIGOROUS ABSTRACT THEOREMS**. Their Yang–Mills application is **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION**.

1.98 1293. The Admissible Structural Rellich Property

An admissible three-sector scale field has the **Admissible Structural Rellich Property**, abbreviated ASRP, on a structural class $\mathfrak{C}$ if every structurally bounded family in $\mathfrak{C}$ that is regionally, seam, scale, observable, and folium tight has a structurally convergent admissible subnet, and every zero-defect limit belongs to the same closed structural class.

Equivalently, the inclusion

$$(\mathfrak{C}, \|\cdot\|_{\text{structural bound}}) \hookrightarrow (\mathfrak{C}, \tau_{\text{struc}})$$

is compact on tight subsets, with compatibility relations closed and defect-zero classes preserved.

The term "Rellich" refers to the local-compactness-plus-tightness mechanism, not to a claim that one universal Sobolev embedding controls the entire reconstruction.

1.99 1294. Exact ingredients of ASRP

The ASRP is supplied by the conjunction:

$$\boxed{\begin{aligned}&\text{local compactness} + \text{tightness} + \text{coercivity}\\ &+ \text{projection compactness} + \text{state/graph/product closure}\\ &+ \text{coherence control.}\end{aligned}}$$

Local compactness controls bounded families on fixed admissible regions. Tightness prevents escape beyond those regions or along the seam. Coercivity converts energy control into nonprotected norm control. Projection compactness handles the directions coercivity intentionally leaves free. State, graph, and product closure retain the physical and algebraic category. Coherence control prevents local approximations from accumulating into global failure.

Leaving any term out produces a weaker property and reopens the associated defect channel.

1.100 1295. Theorem 11 — Structural Rellich theorem

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (admissible structural Rellich compactness). Let $\mathfrak{C} \subset \mathfrak{C}^{\text{struc}}_{\text{adm}}$ be structurally closed. Assume:

1. localized bulk and interface form-bounded sets are relatively compact in the local strong topology;
2. the five tightness conditions of §1278 hold uniformly;
3. uniform bulk and seam coercive inequalities hold;

4. the projection compactness, state-normality, graph-closure, product/action, sector, and coherence gates hold;
5. the structural test system is small, determining, and faithful on $\mathfrak{C}$.

Then $\mathfrak{C}$ has ASRP. In particular, every structurally bounded tight family is relatively compact in τ_{struc}. If its complete defect profile vanishes, every structural cluster point is a complete no-leakage limit.

1.101 1296. Proof of Theorem 11

For every fixed localization test, the local compactness hypothesis makes the localized bulk and seam coordinates relatively compact. Uniform tightness approximates each global coordinate, uniformly in the family, by a localized coordinate. A standard compactness-by-tightness argument therefore gives global relative compactness of the vector and form tests.

By definition, the remaining gates include relative compactness of the projection, state, graph, product, action, label, interface, and local-coherence coordinates in their declared topologies, in addition to the closure properties needed for identification. Theorem 1 yields one common subnet. Structural closedness and Theorems 3–8 identify the limit inside $\mathfrak{C}$. Hence bounded tight subsets are relatively compact in τ_{struc}.

If the complete defect profile vanishes, Theorem 10 upgrades the cluster point to a no-leakage structural limit. This proves ASRP.

1.102 1297. Nonclaim after Theorem 11

The theorem does not assert a universal compact embedding for all admissible scale fields. It does not provide quantitative entropy numbers, approximation numbers, or rates of convergence. It does not prove compactness without tightness. It does not prove compactness of arbitrary unbounded algebra families or arbitrary fusion products.

The stronger statement “the structural unit ball is compact” is explicitly not claimed.

1.103 1298. Inputs required by Part XXII

Part XXII’s strict intrinsic boundary theorem used the following conditional inputs:

1. controlled Cauchy consistency of asymptotic pairings;
2. an exact-energy controlled core;
3. form-null compatibility;
4. intrinsic boundary closability;
5. controlled diagonal recovery for Mosco limsup;
6. protected projection descent and projection–form compatibility;
7. faithful realization comparison.

Part XXIII does not redo the quotient or universal property. It determines which of these inputs can be supplied by structural compactness and which remain independent assumptions.

1.104 1299. Pairing-control coordinates

Enlarge the structural test system by the asymptotic Hilbert and form pairings used in Part XXII:

$$h_{f,e}(\xi_f, \eta_e), \qquad a_{f,e}(\xi_f, \eta_e).$$

Define their Cauchy residuals

$$d_h(\alpha, \beta) = |h_{\alpha,\beta}(\xi_\alpha, \eta_\beta) - h_0(\xi_0, \eta_0)|,$$

$$d_a(\alpha, \beta) = |a_{\alpha,\beta}(\xi_\alpha, \eta_\beta) - a_0(\xi_0, \eta_0)|.$$

If these residuals are included in the coherence/defect budget and vanish uniformly on the exact-energy core, structural compactness supplies controlled Cauchy consistency on the extracted family.

1.105 1300. Form-null compatibility from no-leakage

Let n be an intrinsic Hilbert-null controlled section. To infer form-nullness, assume:

- exact energy recovery on the common subnet;
- zero energy defect for n;
- coercivity on the nonprotected complement;
- projection compactness;
- the protected-energy-null property.

Hilbert nullity gives strong boundary norm convergence to zero after no-leakage closure. The non-protected energy vanishes by coercivity and exact recovery; the protected energy vanishes by the protected-energy-null hypothesis. Therefore

$$a_0(n, n) = 0.$$

Thus form-null compatibility is supplied for the zero-defect exact-energy class. Without the protected-energy-null hypothesis it is not supplied.

1.106 1301. Intrinsic boundary closability from structural Rellich

Suppose $q(\xi^{(n)}) \to 0$ in intrinsic boundary norm and $(\xi^{(n)})$ is Cauchy in the asymptotic form seminorm. Assume the sequence is structurally bounded and tight and that ASRP applies to its differences.

Every subnet has a structurally convergent further subnet. Boundary norm convergence forces the only Hilbert cluster point to be zero. Exact energy recovery and form-null compatibility force the asymptotic energy of each difference cluster to vanish. Since the form sequence is Cauchy, its energy norm has a unique limit, which must be zero. Hence the intrinsic pre-form is closable.

This is a genuine supply mechanism, but only on the class for which structural boundedness, tightness, and zero-defect recovery have been proved.

1.107 1302. Controlled diagonal recovery

Let $\{u^k\}$ be a countable or small determining exact-energy core. Suppose each u^k admits an admissible recovery field u^k_α with uniform structural bounds and defects tending to zero, and suppose the recovery choices satisfy a common coherence budget.

The product compactness theorem extracts one subnet on which all recovery fields needed for a finite stage converge simultaneously. A directed diagonal over the finite stages then yields a common admissible recovery subnet. This supplies the controlled diagonal recovery used in intrinsic Mosco limsup arguments.

For an uncountable core, a smallness or compact-generation hypothesis is required. The word "diagonal" does not make a proper class countable.

1.108 1303. Projection–form compatibility

Assume $P_\alpha D(Q_\alpha) \subseteq D(Q_\alpha)$, uniform form-norm bounds for P_α, projection compactness, and vanishing form-intertwining residual

$$|Q_\alpha(P_\alpha u_\alpha, v_\alpha) - Q_\alpha(u_\alpha, P_\alpha v_\alpha)| \to 0.$$

Then the limiting projection preserves the limiting form domain and is symmetric for the descended form on the exact-energy core. Closure extends the compatibility to the intrinsic form domain.

Thus Part XXIII can supply projection–form compatibility when the residual is an explicit structural coordinate. It cannot infer the residual from Hilbert-space projection convergence alone.

1.109 1304. Faithful realization comparison is partly independent

Structural compactness can show that two faithful admissible realizations implementing the same asymptotic pairings receive the same zero-defect controlled sections and forms. It cannot prove that a proposed realization is faithful merely because limits exist inside it.

Therefore Part XXIII supplies the compactness and compatibility side of faithful realization comparison. The hypotheses

- faithfulness on the determining algebra;
- fidelity of the asymptotic Hilbert and form pairings;
- preservation of the protected and sector detectors

remain separate realization assumptions. Host independence in Part XXII then applies once those assumptions are verified.

1.110 1305. Theorem 12 — supply theorem for Part XXII

Status: RIGOROUS ABSTRACT THEOREM.

Theorem (supply of intrinsic-boundary hypotheses). Let a strict admissible scale field satisfy ASRP on an exact-energy controlled class. Assume:

1. the complete defect profile, including pairing and form-intertwining residuals, vanishes on one common admissible subnet for the controlled fields and for every structurally bounded tight form-Cauchy difference family used in the closability test;
2. the protected-energy-null property holds on that class;
3. every element of the declared compactly generated recovery core admits a uniformly structurally bounded zero-defect admissible recovery field, and the recovery fields share one controlled coherence budget;
4. the asymptotic pairing realizations are faithful wherever comparison with a host is desired.

Then:

1. controlled Cauchy consistency holds on the extracted class;
2. the intrinsic Hilbert null space is form-null on the exact-energy core;
3. the intrinsic boundary pre-form is closable on structurally bounded tight form-Cauchy families;
4. controlled diagonal recovery is available on the declared compactly generated core;
5. the protected projection descends and is compatible with the closed intrinsic form;
6. intrinsic Mosco recovery applies on that class;
7. every faithful realization implementing the same asymptotic pairings yields the Part XXII host-independent boundary.

1.111 1306. Proof of Theorem 12

Item 1 follows from inclusion of the asymptotic pairings in the structural coordinate system and vanishing pairing/coherence residuals. Item 2 is §1300. Item 3 is §1301. Item 4 is §1302. Item 5 is §1303 together with Part XXII's projection descent theorem. Item 6 combines structural liminf compactness, exact energy recovery, and the common recovery subnet. Item 7 invokes the universal realization theorem of Part XXII after the separate faithfulness hypotheses have been verified.

No coefficient-valued pairing is constructed. The theorem operates in the strict map-valued model and in faithful realized tests of correspondence-sensitive data.

1.112 1307. Nonclaim after Theorem 12

The theorem does not supply an exact-energy core if none has been constructed. It does not prove protected-energy-nullness from projection compactness. It does not prove faithfulness of a host realization. It does not construct the asymptotic coefficient algebra or null submodule. It does not establish fusion-compatible Mosco convergence.

Accordingly, the supply theorem converts several Part XXII hypotheses into conclusions **on the verified ASRP zero-defect class**, not on every admissible controlled section.

1.113 1308. Block F audit

Last subsection: §1308.

New notation: ASRP, pairing residuals d_h, d_a, exact-energy recovery class.

Theorems proved: Theorem 11, Admissible Structural Rellich theorem; Theorem 12, supply theorem for Part XXII hypotheses.

Part XXII hypotheses supplied: controlled Cauchy consistency; form-null compatibility; closability on the ASRP class; controlled diagonal recovery on a compactly generated core; projection–form compatibility; compactness side of faithful realization comparison.

Still independent: construction of the exact-energy core; protected-energy-nullness; faithfulness of realizations; coefficient-valued pairing data; correspondence completion and fusion continuity.

Status: Theorem 11 is a **RIGOROUS ABSTRACT THEOREM**. Theorem 12 is a **RIGOROUS ABSTRACT THEOREM**; its Yang–Mills application remains **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION**.

1.114 1309. Three theorem-status levels

Every result in this Part belongs to exactly one class:

$$\boxed{\text{RIGOROUS ABSTRACT THEOREM}}$$

means the conclusion follows from the stated abstract hypotheses by the proof given here.

$$\boxed{\text{CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION}}$$

means the abstract theorem is available but the Yang–Mills source must still verify its hypotheses on the declared admissible frustum.

$$\boxed{\text{PROGRAMMATIC}}$$

means the mathematical target is identified but a required construction—typically coefficient, fusion, self-duality, or bicategorical universality—has not yet been supplied.

No result is described as "essentially proved" or "morally clear." Those phrases are excellent at lowering blood pressure and terrible at closing theorems.

1.115 1310. Rigorous abstract results in Part XXIII

The following are rigorous abstract theorems under their explicit hypotheses:

1. common product-subnet extraction;
2. bulk and seam coercive defect domination;
3. projection no-leakage and kernel inclusion;
4. normality closure under uniform absolute continuity;
5. Gauss and Ward graph closure;
6. bounded strong-* product, involution, and unit closure;
7. coherence closure under vanishing accumulated budgets;
8. realized interface compactness under seam/action/square hypotheses;
9. common-subnet structural compactness;
10. complete no-leakage closure;
11. the Admissible Structural Rellich theorem;

12. the Part XXII supply theorem on the verified ASRP class.

Each theorem is strict-map-valued unless explicitly marked correspondence-sensitive in a declared realization.

1.116 1311. Manuscript-specific verification obligations

The Yang–Mills manuscripts must still verify, on one declared admissible frustum:

- uniform local compactness estimates for strict, twisted, and seam form balls;
- regional and incision-direction tightness;
- projection compactness in a stated topology;
- uniform normality or domination of the state family;
- graph-liminf convergence of Gauss and Ward data including incision terms;
- strong-* or graph product closure on the chosen observable class;
- convergence of left and right interface actions;
- summable square, prism, and associator budgets;
- stability of twisted coefficient and relative-obstruction detectors;
- a common exact-energy recovery core;
- protected-energy-nullness;
- faithful realization comparison.

Until those are verified, the corresponding application is **CONDITIONAL ON MANUSCRIPT-SPECIFIC VERIFICATION**.

1.117 1312. Source-audit markers

The following claims require source-specific recovery or direct rederivation before journal use:

- the exact Yang–Mills localization and seam compactness estimates;
- the uniform ranges of all coercive constants;
- the precise topology and component data for the protected projections;
- the reference states or weights and the proof of uniform absolute continuity;
- the exact Gauss and Ward graph domains after surgery;
- the product and action continuity estimates on the selected local algebra cores;
- the cocycle representatives and detector proving persistence of $[\chi^{\Sigma}]$;
- any statement that a correspondence realization is faithful, full, or canonical.

Each unrecovered claim is **SOURCE AUDIT REQUIRED**.

1.118 1313. Artifact-recovery markers

A computational compactness certificate should include:

- the exact admissible index and subnet data;
- localization-tail tables;
- form and norm defect evaluations;

- projection-angle, gap, or Schatten-tail diagnostics;
- state-normality or entropy-tail diagnostics;
- Gauss and Ward residual tables;
- product and action residuals;
- square, prism, and accumulated coherence budgets;
- sector and relative-class detectors;
- tolerances, precision, software versions, and reproducible scripts.

No such manuscript-specific certificate is manufactured in this abstract Part. Missing artifacts are **SOURCE ARTIFACT RECOVERY REQUIRED**.

1.119 1314. Hostile-referee answers I: topology and boundedness

1. What is the compactness topology? The typed initial topology $\tau_{\mathrm{struc}}(\mathscr{T})$ induced by local strong/form, projection, normal-state, graph, strong-*, action, label, and coherence coordinates, with the structural class a closed relation space.

2. Why is bounded energy insufficient? It controls neither tightness, protected rotation, folium escape, products, constraints, sector labels, nor coherence accumulation.

3. How is projection rotation excluded? By a declared projection compactness package plus vanishing d_P, not by rank constancy or WOT convergence.

4. How is kernel inflation excluded? Coercivity gives $\ker Q_0 \subseteq \operatorname{Ran} P_0$; equality requires protected-energy-nullness and adequate projection geometry.

1.120 1315. Hostile-referee answers II: states, constraints, and products

5. Why does the limit state remain normal? Uniform absolute continuity or an equivalent predual compactness gate makes the weak-* limit normal.

6. Why do Gauss and Ward constraints survive? The graphs are liminf closed and the residuals vanish on the common subnet.

7. Why does multiplication survive? Strong-* convergence on uniformly bounded classes is multiplicatively stable; otherwise products are carried as coordinates and $d_{\mathrm{prod}} \to 0$.

8. Why is the interface compatible with both bulks? Seam compactness, action closure, square-defect closure, and summable prism coherence are imposed together.

1.121 1316. Hostile-referee answers III: coherence and common subnets

9. Why do local coherence defects not accumulate? The accumulated path budget Γ_α tends uniformly to zero.

10. Why do all components belong to one subnet? They are coordinates of one map into a compact product; Tychonoff extraction produces one product subnet.

11. Which defect controls each failure? The map is recorded in §1287.

12. Which Part XXII hypotheses become conclusions? On the verified ASRP zero-defect class: Cauchy consistency, form-null compatibility, closability, controlled recovery, and projection–form compatibility. Faithfulness remains separately assumed.

1.122 1317. Hostile-referee answers IV: remaining boundaries

13. Which claims remain manuscript-specific? Every estimate and topology listed in §1311.

14. Is the final limit strict, lax, or unrelated? In the strict map-valued coordinates it is strict where all residuals vanish. In correspondence-sensitive realized coordinates it is exact on the tested admissible diagrams. If a nonzero coherence class remains, the honest conclusion is lax. Without square/action closure it is only a family of unrelated limits.

15. What stronger theorem is not claimed? No universal compactness of all bounded admissible correspondence fields; no arbitrary-chart or arbitrary-Cauchy reconstruction; no intrinsic correspondence-valued boundary; no fusion-compatible realization-independence theorem.

1.123 1318. Strict, lax, and correspondence-sensitive outcomes

The status of a limit is determined by its residuals:

$$\begin{array}{ll} \mathbf{D}^{\mathrm{full}} \to 0 & \Rightarrow \text{strict tested structural limit,} \\ \mathbf{D}^{\mathrm{full}} \not\to 0 \text{ but residuals converge} & \Rightarrow \text{lax limit with recorded defect,} \\ \text{incompatible subnets or unclosed relations} & \Rightarrow \text{unrelated coordinate limits.} \end{array}$$

Correspondence-sensitive data remain realized until Part XXIV constructs the intrinsic coefficient-valued boundary. Calling the realized interface “the intrinsic correspondence” at this stage would erase precisely the distinction Part XXII taught us to respect.

1.124 1319. Methodological note on Gromov’s influence

The compactness theory here follows a geometric habit of mind strongly associated with Mikhail Gromov: inequalities are read as geometry, scales are treated as active variables, compactness failure is classified rather than hidden, and rigidity is extracted from the cost of deformation.

No direct technical dependence on a particular Gromov theorem is asserted. The influence is methodological. The defect ledger asks what shape an escaping family must take. Coercivity says which directions are expensive. Projection geometry says which directions may rotate. Tightness says where mass may travel. Coherence budgets say how local errors accumulate. The resulting compactness theorem is therefore a geometry of controlled failure.

1.125 1320. Boundaries reserved for Part XXIV and Part XXV

Part XXIV is reserved for:

- the intrinsic asymptotic coefficient algebra;
- coefficient-valued Hilbert or semi-inner-product pairings;
- coefficient-valued null submodules;

- self-dual or otherwise specified module completions;
- limiting normal left and right actions;
- preservation of the strict–twisted–interface architecture;
- the universal property appropriate to correspondence-valued boundaries.

Part XXV is reserved for fusion-compatible Mosco convergence, compatibility with Connes fusion or its selected replacement, and bicategorical realization independence.

No subsection of either Part is begun here.

1.126 1321. Closing perspective

Ordinary compactness asks whether something survives. Structural compactness asks whether the relations that made the thing meaningful survive with it.

The answer developed here is deliberately conditional and deliberately exact. Local compactness and tightness extract one admissible subnet. Coercivity controls nonprotected concentration. Projection compactness controls the directions coercivity leaves free. Normality gates keep states inside the physical folium. Graph closure preserves Gauss and Ward constraints. Strong-* and defect-controlled multiplication preserve the local algebra. Summable coherence keeps a thousand small approximations from becoming one large lie. Seam compactness preserves the obstruction interface and the relative class it carries.

Under those hypotheses, the limit is structurally complete. The intrinsic strict boundary machinery of Part XXII can then be applied without asking a convenient host to perform acts of creation.

The next problem is coefficient-valued. The compactness and no-leakage machinery has supplied a coherent limiting field. Part XXIV must now construct the intrinsic correspondence-valued boundary that this field deserves.

Extract one limit, close every defect channel, and only then call the result a reconstructed structure.

Next: Part XXIV — Intrinsic Correspondence-Valued Thin Boundaries and Asymptotic Coefficient Algebras.

1.127 1322. Block G audit and continuation boundary

Final subsection of Part XXIII: §1322.

Exact next subsection: §1323.

Final theorem count: twelve major abstract theorems.

Programmatic boundary: intrinsic coefficient algebra, coefficient-valued null module, self-dual thin correspondence, bicategorical universal property, and fusion-compatible realization independence.

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