
THE GEOMETRY OF ADMISSIBLE TRANSPORT

RESEARCH NOTE II

Source-Recognized Physical Stiffness Geometry

Associator-selected magnetic reduction, physical stiffness matching, and Gromov control of Yang–Mills stiffness sublevels

Folio code	GAT-RN-II
Version	v0.4.0
Branch	Associator-selected magnetic stiffness / ground-reduced Poincare geometry
Status	Active research note; not intended for publication
Claim boundary	Compatibility and discharge architecture. No independent Yang–Mills mass-gap theorem.
Open edge	Whole-nerve identification of the direct cofinal physical completion with the reconstructed thin Yang–Mills system.

CLAIM BOUNDARY

This notebook records a technical mechanism and its remaining certification gates. It does not replace the five-paper Yang–Mills construction, and the magnetic associator-bearing carrier is not asserted to exhaust the complete physical complement.

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Abstract

These notes test whether geometric control of a finite Wilson carrier can be attached to the source-normalized Yang–Mills physical stiffness form without changing the carrier or the form. A GAT 0 recognition packet, a GAT II Klauder-physicalized carrier, and a compatible Čech–module–Grassmannian–Fredholm realization first produce $(\mathcal{H}_n, Q_n, P_n, J_n)$. The magnetic geometry is then restricted to the associator-bearing sector selected by the tensor-selector lineage; it is not required to cover the full physical complement. Its form is defined by restriction of the already identified Q_n . A triangle calculation shows that ordinary edge expansion cannot control the excitation gap above the complete magnetic low sector: the graph can remain fixed while a phase-driven spectral crossing closes the instantaneous gap. This motivates a complete-low-cluster Riesz projection and a ground-reduced, phase-sensitive Poincare geometry. Selector-sector Schur control and cofinal Riesz transport then state precisely when that magnetic floor persists as a physical contribution. Complete off-ground coercivity remains an independent input of the source UV–IR, target-complement, Hamiltonian-residual, and seam ledger. Gromov–Hausdorff control is applied only afterward to compact sublevels of the already identified physical stiffness form. On an exact cofinal comparison subsystem the common null quotient, complete-ground projection, and stiffness form can moreover be completed together, yielding a direct thin physical triple before the later whole-nerve/spacetime identification. These are compatibility and discharge notes, not an independent mass-gap theorem.

1 Introduction and purpose

These notes test whether a Gromov–Hausdorff compactness statement can be attached to the correct Yang–Mills stiffness form without changing the object whose stability is being studied. The order is important. We do not begin with an abstract metric space, a scalar graph energy, or a configuration measure. We first require the finite source-recognition packet, the simultaneous constraint carrier, the Čech realization, the module-valued Grassmannian fibers, and the Fredholm interface to produce the represented physical package

$$(\mathcal{H}_n, Q_n, P_n, J_n). \quad (1.1)$$

Only after this package has been identified do we form stiffness sublevels.

TECHNIQUE NOTE

The note is deliberately source-first. The Gromov argument is not allowed to choose its own carrier or energy. The source-recognition packet, constraint projection, Čech realization, module/Fredholm interface, and complete ground projection must first identify the represented physical package. Only then are metric sublevels formed.

The dependency chain is

$$\begin{aligned} \text{Rec}_n^{GAT0} &\longrightarrow P_{n,*}^K \longrightarrow N_n \longrightarrow \text{Hilbert } W^*\text{-module Grassmannian fibers} \\ &\longrightarrow \text{Fredholm fiber} \longrightarrow (\mathcal{H}_n, Q_n, P_n, J_n) \longrightarrow \mathfrak{B}_n(R). \end{aligned} \quad (1.2)$$

The last object is the metric sublevel space

$$\mathfrak{B}_n(R) = \{\psi \in \mathcal{D}(Q_n) : Q_n[\psi] + \beta\|\psi\|^2 \leq R\},$$

with the physical Hilbert norm as its distance.

The projection hierarchy is part of the point. The Gauss or gauge projection, the maximal Klauder-constrained carrier, and the complete coupled ground projection have different jobs:

$$P_n^G \neq P_{n,*}^K \neq P_n = P_{\Omega,n} \quad \text{in general.} \quad (1.3)$$

The Gromov theorem below is applied only after the Klauder carrier has already been selected and after P_n has been identified as the complete ground projection for the represented stiffness form.

The metric estimate itself is elementary. After a permitted unitary comparison, write

$$a_n[u] = Q_n[J_n^{-1}u]$$

on a common reference form domain. If

$$|a_n[u] - a_{\text{ref}}[u]| \leq \sigma_n(a_{\text{ref}}[u] + \beta\|u\|^2), \quad \sigma_n \rightarrow 0,$$

and if the reference form has compact graph-norm sublevels, then the physical sublevels of Q_n lie close to the reference sublevels in Gromov–Hausdorff distance. The proof uses only radial rescaling in the common Hilbert carrier. It does not manufacture the coercive floor and does not replace the physical form.

CLAIM BOUNDARY

The thin stiffness endpoint and the invariant mass form are not identified in this note. The endpoint used here is the reconstructed physical stiffness form; the invariant-mass statement requires the separate Lorentz-covariant spectral identification.

The Yang–Mills normalization remains separate. In the source mechanism note, the reconstructed stiffness endpoint is

$$Q_{\text{thin}}[\psi] = \|P^0\psi\|^2, \quad K_{\text{thin}} = (P^0)^2, \quad (1.4)$$

within the represented scope of the physical time-generator identification [2, Eq. (10)]. This is not the Hamiltonian expectation form $\langle\psi, P^0\psi\rangle$ and is not, by itself, the invariant-mass form of $M^2 = P_\mu P^\mu$. A surviving stiffness floor first gives an energy threshold; the invariant-mass conclusion requires the separate Lorentz-covariant argument recorded in the source [2, Eqs. (10)–(11)].

The notes are therefore organized as a source-recognized physical stiffness geometry. Gromov control is the final payoff, not the starting assumption.

2 Source-recognized physical stiffness package

This section is intentionally upstream of the metric theorem. It constructs, within the scope of these notes, the intake ladder that must be matched before any stiffness sublevel can be interpreted as a sublevel of the physical Yang–Mills form. The construction is schematic and compatibility-level: the long papers still carry the full verification of each source certificate.

2.1 General intake from GAT 0 and GAT II

The exact-dilation regional test case is the first clean branch on which the metric theorem can be checked, but it is not the most general source interface. The more general intake has two earlier gates.

First, GAT 0 supplies the finite recognized source packet [3]. This packet fixes the compact connected semisimple global form

$$G = \tilde{G}/\Gamma,$$

the sector-complete detector data, the electric/Casimir seed, the native cohomological coordinates, the complete-ground bookkeeping, and the finite coercive ledger. It is still a source-recognition package.

It does not by itself choose a unique GNS realization, a Grassmannian–Fredholm fiber, a Klauder projector, or a continuum relativistic spectrum.

Second, GAT II supplies simultaneous constraint physicalization [4]. A finite constraint packet

$$\{\Phi_{\alpha,n}\}_{\alpha \in A_n}$$

on a common form domain is replaced by one positive quadratic constraint operator

$$K_n \geq 0.$$

The finite Klauder windows are

$$P_{n,\delta}^K = E_{K_n}([0, \delta^2]). \quad (2.1)$$

The spectral width δ is not the geometric thickness ε_n . A window is admitted only after it passes the physical reserve, support-frustum, Ward, complete-ground, and transport tests required by the source package. When the certified seed is nonempty, the feasible windows have a maximal self-admissible element

$$P_{n,*}^K. \quad (2.2)$$

The stiffness-sublevel construction begins only after this constrained carrier has been represented through the same Čech–module–Fredholm realization which produces

$$(\mathcal{H}_n, Q_n, P_n, J_n).$$

Thus the metric theorem is not applied to the unconstrained ambient carrier. It is applied to the source-recognized, Klauder-physicalized, Čech-realized physical stiffness form.

The projection hierarchy must be kept separate:

$$\begin{aligned} P_n^G & \quad (\text{Gauss or gauge reduction}), \\ P_{n,*}^K & \quad (\text{simultaneous constrained carrier}), \\ P_n = P_{\Omega,n} & \quad (\text{complete ground sector}). \end{aligned} \quad (2.3)$$

In particular,

$$P_{n,*}^K \neq P_n$$

in general. The Klauder projection selects the physical carrier on which the remaining theory is represented. The complete-ground projection is then the lowest-sector projection inside that represented carrier.

Assumption 2.1 (Recognized constrained source intake). For the branch under consideration, the finite GAT 0 recognition packet and the GAT II maximal self-admissible Klauder carrier have been identified with the same finite Čech diagram, module-valued carrier, Fredholm-fiber interface, represented Hilbert carrier, complete-ground projection, and source-normalized stiffness form. Equivalently, the chain

$$\text{Rec}_n^{\text{GAT0}} \longrightarrow P_{n,*}^K \longrightarrow \text{Čech--module--Fredholm realization} \longrightarrow (\mathcal{H}_n, Q_n, P_n, J_n)$$

has been certified without changing the form Q_n or the projection P_n .

With this assumption inserted, the exact-dilation ledger below becomes a special case of the general recognized constrained intake. The theorem proved in this note still has the same hypotheses: unitary comparison, common domain, compact reference sublevels, relative-form control, and compatible complete-ground transport. GAT 0 and GAT II enlarge the source class to which those hypotheses may be matched; they do not by themselves prove the Gromov estimate or the electric–magnetic stiffness floor.

2.2 Compatibility with the Čech nerve construction

The sublevel construction in this note is deliberately Hilbert-form geometric. It builds metric spaces from the physical stiffness form Q_n after the source comparison has already realized a represented carrier. The earlier descent construction, however, is organized over a Čech nerve. These two descriptions must be matched before the metric sublevel theorem may be read as a statement about Wilson-tube descent rather than only about an abstract sequence of closed forms.

Let $\mathcal{U}_n = \{U_i^n\}_{i \in I_n}$ be the finite admissible cover of the bounded region O , and let

$$N_n = \text{Nrv}(\mathcal{U}_n)$$

be its Čech nerve. A simplex $I = (i_0, \dots, i_p) \in N_n$ corresponds to a nonempty overlap

$$U_I^n = U_{i_0}^n \cap \dots \cap U_{i_p}^n.$$

The source descent data assign local carrier pieces, local form data, restriction or transport maps, seam terms, and ground projections to this finite diagram [5]. In the source setting the represented carrier is naturally module-valued before it is realized as a Hilbert space: local physical fibers arise as projection ranges in a Grassmannian of self-dual Hilbert W^* -modules. The physical carrier $\mathcal{H}_n$ and physical stiffness form Q_n used above must be the result of this same finite-diagram and module-fiber realization.

We record this as an explicit compatibility gate.

Assumption 2.2 (Čech-nerve compatibility). For the selected region O , branch ξ , and thickness level n , the following data are given on the Čech nerve N_n :

$$\{\mathcal{H}_{n,I}, Q_{n,I}, P_{n,I}\}_{I \in N_n}$$

together with admissible restriction/transport maps

$$r_{JI}^n : \mathcal{H}_{n,I} \longrightarrow \mathcal{H}_{n,J}, \quad J \subset I,$$

and cocycle or seam factors inherited from the vacuum-gerbe and carrier descent data. These data satisfy:

- N1. **Realized carrier.** The Hilbert space $\mathcal{H}_n$ appearing in (2.4) is the represented finite-diagram realization of this nerve data. In schematic notation,

$$\mathcal{H}_n \simeq \text{Real}_{N_n}(\{\mathcal{H}_{n,I}, r_{JI}^n\}),$$

where the symbol Real_{N_n} denotes the source's selected realization/quotient/completion procedure, not an additional construction introduced in this note.

- N2. **Realized form.** The form Q_n is the closed descended stiffness form obtained from the same finite nerve diagram. Locally it is assembled from the chart forms, transported overlap comparisons, and seam or constraint terms before taking the source closure and normalization. Thus

$$Q_n = \hat{q}_{G,\xi,n;O}$$

only after the Čech-diagram descent and the source normalization have both been applied.

- N3. **Natural comparison.** The common-carrier comparison J_n is induced by a compatible family of chartwise comparison maps

$$J_{n,I} : \mathcal{H}_{n,I} \longrightarrow \mathcal{H}_{\text{ref},I}$$

which commute with the face restrictions up to the same controlled defect that enters the relative-form estimate:

$$J_{n,J}r_{JI}^n - r_{JI}^{\text{ref}}J_{n,I} = E_{n,JI}.$$

The collective contribution of the errors $E_{n,JI}$ is absorbed into $\sigma_n h_{\text{ref}}$ on the realized common domain. In the exact-dilation zero-defect case these errors vanish.

N4. Ground naturality. The complete ground projections are nerve compatible:

$$r_{JI}^n P_{n,I} = P_{n,J} r_{JI}^n$$

exactly, or with an error whose realized effect is included in the compact-sublevel projection defect $\rho_n(R)$. The projection used after realization is the complete coupled ground projection P_n , not a chartwise magnetic witness projection.

N5. Branch markings. Strict and nontrivial branch markings are transported through the same nerve diagram. Passing to the Hilbert-space orthogonal complement $(1 - P_n)\mathcal{H}_n$ is therefore compatible with the source branch marking after quotienting strict vacuum data; it is not a replacement of the Čech branch data by a pointwise support condition.

N6. Sublevel functoriality. The stiffness sublevels

$$\mathfrak{B}_n(R) = \{\psi : Q_n[\psi] + \beta\|\psi\|^2 \leq R\}$$

are formed only after the nerve data have been realized into the physical carrier. Equivalently, the diagram is first descended/realized, then the physical form sublevel is taken. The note does not assert a canonical metric sublevel on each simplex whose realization automatically equals $\mathfrak{B}_n(R)$ without this compatibility.

Under Assumption 2.2, the Gromov–Hausdorff statements above are compatible with the Čech nerve construction in the following precise sense. They apply to the physical carrier and physical form produced by the nerve realization, and the comparison maps used in the proof are the realized maps induced from the same finite diagram. Therefore no descent information is erased by passing to stiffness sublevels.

There is also an important non-claim. The spaces $\mathfrak{B}_n(R)$ and $\mathfrak{B}_n^\perp(R)$ are not themselves the Čech nerve, nor are they the finite-thickness configuration spaces of the Letter-series descent geometry. They are derived metric spaces built from the realized physical form. A future branch-relative Cheeger estimate must still define a nerve-level or configuration-level boundary, measure, and perimeter, and then prove that the resulting boundary form compares to the same realized Q_n . The present compatibility statement only says that the Gromov control proved here is attached to the correct nerve-realized physical stiffness form.

For the source-matching ledger, Assumption 2.2 adds one more gate:

the comparison $(\mathcal{H}_n, Q_n, P_n, J_n)$ must be the realization of the same Čech nerve data.

Only after this gate is closed may the compact-sublevel theorem be cited as a statement about the Wilson-tube Čech descent construction.

2.3 Hilbert-module and Grassmannian-fiber compatibility

The notation above writes $\mathcal{H}_n$ as an ordinary Hilbert space because the Gromov–Hausdorff theorem is a theorem about compact metric sublevels in a Hilbert norm. In the Yang–Mills source construction, however, this Hilbert space is not primitive. It is the represented Hilbert-space realization of a module-valued carrier. Before representation, the admissible physical fibers are selected as ranges of projections in a Grassmannian fiber of self-dual Hilbert W^* -modules.

Let $\mathfrak{M}_{n,I}$ denote the local coefficient W^* -algebra on a Čech simplex $I \in N_n$, and let $\mathcal{E}_{n,I}$ be the corresponding self-dual Hilbert $\mathfrak{M}_{n,I}$ -module. A physical carrier choice is encoded by a projection

$$e_{n,I} = e_{n,I}^* = e_{n,I}^2 \in \mathcal{L}_{\mathfrak{M}_{n,I}}(\mathcal{E}_{n,I}),$$

so that the local module fiber is

$$\mathcal{F}_{n,I} = e_{n,I} \mathcal{E}_{n,I}.$$

The collection $I \mapsto e_{n,I}$ is the Grassmannian-fiber datum. Its admissibility includes compatibility with the face maps, seam factors, central associator, branch markings, and source comparison maps.

A represented Hilbert fiber is then obtained only after choosing the source’s normal representation or GNS realization of the coefficient data. Schematic notation for this step is

$$\mathcal{H}_{n,I} = \text{Rep}_{n,I}(\mathcal{F}_{n,I}),$$

and the global regional carrier used in this note is the realized finite Čech diagram

$$\mathcal{H}_n \simeq \text{Real}_{N_n}(\{\text{Rep}_{n,I}(e_{n,I} \mathcal{E}_{n,I})\}_{I \in N_n}).$$

The physical form Q_n , complete ground projection P_n , and comparison map J_n must descend through the same module-valued diagram before the metric sublevels $\mathfrak{B}_n(R)$ are formed.

Assumption 2.3 (Grassmannian-fiber compatibility). For the selected region, branch, and refinement level, the following compatibilities are part of the source-matching data:

- G1. **Module-valued carrier.** The local carrier over each simplex $I \in N_n$ is a self-dual Hilbert W^* -module $\mathcal{E}_{n,I}$ over a coefficient algebra $\mathfrak{M}_{n,I}$, not merely a Hilbert space assigned ad hoc to the simplex.
- G2. **Grassmannian projection.** The physical local fiber is the range $\mathcal{F}_{n,I} = e_{n,I} \mathcal{E}_{n,I}$ of an admissible projection $e_{n,I}$ in the Grassmannian of module projections. The projection family is compatible with the Čech face maps and with the vacuum-gerbe/carrier seam data.
- G3. **Representation before metric geometry.** The Hilbert spaces $\mathcal{H}_{n,I}$ and $\mathcal{H}_n$ arise by the source representation and finite-diagram realization of these module fibers. The Hilbert norm used in the Gromov–Hausdorff theorem is the represented norm induced from this construction.
- G4. **Form descent.** The stiffness form Q_n is induced from the module-level descent differential, connection, seam, and constraint data after representation and closure. It is not an independently chosen scalar graph form on the Grassmannian.
- G5. **Comparison descent.** The comparison map J_n is induced by module-linear comparison data which intertwine the Grassmannian projections and the Čech restrictions exactly or with the same controlled defect already included in σ_n and $\rho_n(R)$.

G6. Ground-sector descent. The complete ground projection P_n is the represented complete ground projection on the realized physical carrier. It is not replaced by a pointwise module projection, nor by a magnetic witness projection, unless the source construction identifies those objects in the same represented carrier.

Under Assumption 2.3, the metric spaces $\mathfrak{B}_n(R)$ are understood as Hilbert-norm sublevels of a represented module-fiber construction. Thus the Hilbert-space theorem in this note does not erase the Hilbert-module layer; it begins after that layer has supplied the actual carrier, form, projection, and comparison map.

There is an equally important non-claim. This note does not construct the Grassmannian fibration, prove continuity or measurability of the projection field $e_{n,I}$, prove self-dual module descent, or identify the unique physical module fiber. Those are source-construction tasks. The present note only records the compatibility required before its compact-sublevel theorem may be attached to the Yang–Mills Wilson-tube construction.

The corresponding additional gate is

$$\{\mathcal{E}_{n,I}, e_{n,I}, \mathfrak{M}_{n,I}, \text{Rep}_{n,I}\}_{I \in N_n} \Longrightarrow (\mathcal{H}_n, Q_n, P_n, J_n)$$

through the same finite Čech realization. Without this gate, the symbols $\mathcal{H}_n$ and Q_n in the metric theorem would be detached from the module-valued Grassmannian carrier that the source construction uses.

2.4 Fredholm-fiber compatibility

There is one further structural layer that should be recorded before these notes are used in the Yang–Mills direction. The Grassmannian projection selects the local physical module fiber, but the later analytical identification with Yang–Mills also needs a Fredholm-fiber interface. This is where the represented Hilbert carrier becomes tied to the elliptic, constraint, and finite-index analytic data that make the Yang–Mills identification possible.

This note therefore treats the Fredholm fiber as a source-matching gate, not as an object constructed here. On a simplex $I \in N_n$, after the Grassmannian projection has selected

$$\mathcal{F}_{n,I} = e_{n,I} \mathcal{E}_{n,I},$$

we require a Fredholm datum over the same module fiber. Schematically this is an unbounded regular self-adjoint Fredholm-type operator

$$\mathfrak{D}_{n,I} : \mathcal{D}(\mathfrak{D}_{n,I}) \subset \mathcal{F}_{n,I} \longrightarrow \mathcal{F}_{n,I},$$

or its bounded transform

$$\mathfrak{F}_{n,I} = \mathfrak{D}_{n,I}(1 + \mathfrak{D}_{n,I}^2)^{-1/2} \in \mathcal{L}\mathfrak{M}_{n,I}(\mathcal{F}_{n,I}),$$

with Fredholm invertibility understood modulo the appropriate compact or controlled-compact module endomorphisms. The symbol $\mathfrak{D}_{n,I}$ is only a structural placeholder here. In a source realization it must be the actual Fredholm/elliptic/constraint operator supplied by the Yang–Mills construction, not an arbitrary operator placed on the represented Hilbert space afterward.

The Fredholm layer has two jobs in these notes. First, it explains how the module-valued Grassmannian carrier can carry the analytic structure needed for Yang–Mills realization. Second, it can

supply a compactness certificate for the reference form when the represented Fredholm model has compact local resolvent or a compact form-domain embedding. Fredholmness alone is not enough for the Gromov theorem: the compact-reference-sublevel hypothesis used above still requires compact resolvent, compact embedding, or a finite-window certificate.

Assumption 2.4 (Fredholm-fiber compatibility). For the selected region, branch, and refinement level, the source-matching data include a Fredholm-fiber layer satisfying:

- F1. **Same Grassmannian fiber.** The Fredholm datum is defined on the same projected module fiber $\mathcal{F}_{n,I} = e_{n,I}\mathcal{E}_{n,I}$ used in Assumption 2.3. It is not a second, unrelated Hilbert-space model.
- F2. **Module Fredholm structure.** The local operator $\mathfrak{D}_{n,I}$, or its bounded transform $\mathfrak{F}_{n,I}$, is regular and Fredholm in the appropriate self-dual Hilbert W^* -module sense, modulo the source's compact or controlled-compact ideal.
- F3. **Čech compatibility.** The Fredholm data are compatible with face restrictions, seam factors, and central associators. On overlaps, the intertwining errors are zero or controlled by the same comparison defects that enter σ_n and $\rho_n(R)$. No additional unrecorded defect is allowed.
- F4. **Representation compatibility.** After applying the source representation, $\text{Rep}_{n,I}(\mathfrak{D}_{n,I})$ contributes to the same represented carrier $\mathcal{H}_n$, the same physical stiffness form Q_n , and the same complete-ground projection P_n . The Fredholm layer is not permitted to change the form whose sublevels are studied.
- F5. **Compactness certificate.** Whenever the Gromov theorem is cited using a compact reference form, the compactness must be certified either by the represented Fredholm fiber having compact local resolvent, by a compact form-domain embedding, or by an explicitly finite-dimensional verified window. Mere Fredholmness does not imply compactness of the metric sublevels.

Under Assumption 2.4, the source-matching chain becomes

$$\{\mathcal{E}_{n,I}, e_{n,I}, \mathfrak{D}_{n,I}, \mathfrak{M}_{n,I}, \text{Rep}_{n,I}\}_{I \in N_n} \implies (\mathcal{H}_n, Q_n, P_n, J_n).$$

The metric spaces $\mathfrak{B}_n(R)$ are then understood as sublevels of the represented physical stiffness form that came from a module-valued Grassmannian–Fredholm carrier over the same Čech nerve.

There is a firewall here. This note does not construct the Fredholm fibration, prove Fredholm regularity, prove compact resolvent, or identify the Yang–Mills operator responsible for the Fredholm model. Those are source-construction obligations. The present note records the compatibility needed so that, when those obligations are discharged, the Gromov sublevel theorem is attached to the correct Yang–Mills carrier and not merely to an abstract Hilbert-space model.

2.5 Fredholm–stiffness matching certificate

The Fredholm-fiber layer recorded above is still not enough, by itself, to apply the metric theorem. The theorem controls sublevels of the physical stiffness form

$$Q_n : \mathcal{D}(Q_n) \subset \mathcal{H}_n \rightarrow [0, \infty),$$

so the represented Fredholm datum must be matched to that same form. This is the Fredholm–stiffness matching certificate.

Let

$$\mathfrak{D}_{n,I} : \mathcal{D}(\mathfrak{D}_{n,I}) \subset \mathcal{F}_{n,I} \rightarrow \mathcal{F}_{n,I}$$

be the local Fredholm datum over the Grassmannian module fiber on each simplex $I \in N_n$. After applying the source representation and assembling over the finite Čech diagram, write

$$\mathfrak{D}_n^{\text{rep}}$$

for the represented Fredholm operator or regular operator family on the realized carrier. Let

$$q_n^{\text{Fr}}$$

denote the closed nonnegative form generated by this represented Fredholm package after applying the same source normalization, ground-sector subtraction, and carrier rescaling used to define Q_n .

The certificate is not the assertion that every Fredholm form is a Yang–Mills stiffness form. It is the narrower statement that, on the selected source branch, the represented Fredholm form is the same form as the physical stiffness form, or is relatively close to it by an error already accounted for in the comparison constant.

Assumption 2.5 (Fredholm–stiffness matching). For the selected region, branch, and refinement level, the represented Fredholm package satisfies the following requirements.

FS1. **Same realized carrier.** The assembled represented Fredholm datum acts on the same Hilbert carrier $\mathcal{H}_n$ obtained from the finite Čech–module–Grassmannian realization. No second Hilbert space is introduced.

FS2. **Same source normalization.** The form q_n^{Fr} is formed only after applying the same physical normalization, ground subtraction, and carrier rescaling used in the definition of Q_n . The raw Fredholm form is not compared directly with the normalized stiffness form.

FS3. **Common form core.** There is a descended core

$$\mathfrak{C}_n \subset \mathcal{D}(Q_n) \cap \mathcal{D}(q_n^{\text{Fr}})$$

which is a form core for both Q_n and q_n^{Fr} , or else a specified common core on which a relative-form closure argument is available.

FS4. **Core agreement or recorded relative error.** On $\mathfrak{C}_n$, either

$$Q_n[\psi] = q_n^{\text{Fr}}[\psi]$$

for all $\psi \in \mathfrak{C}_n$, or there is a recorded defect $\epsilon_n^{\text{Fr}} \geq 0$ such that

$$|Q_n[\psi] - q_n^{\text{Fr}}[\psi]| \leq \epsilon_n^{\text{Fr}}(Q_n[\psi] + \beta\|\psi\|^2).$$

In the second case, this defect must be added to the comparison budget that produces σ_n .

FS5. **Ground-sector compatibility.** The lowest or zero sector supplied by the Fredholm realization agrees, after representation and normalization, with the complete physical ground projection P_n . It is not enough for it to agree with a local magnetic kernel, a coefficient-space kernel, or an uncoupled Fredholm zero mode.

FS6. **Comparison-map compatibility.** The same comparison map J_n used in the Gromov theorem transports both Q_n and the represented Fredholm form. If the Fredholm comparison uses a different map, an explicit intertwining estimate must be supplied and included in σ_n or $\rho_n(R)$, according to whether it affects the form or the ground projection.

Proposition 2.6 (Safe use of the Fredholm interpretation). *Assume Assumption 2.5. If the exact core agreement alternative FS4 holds, then $q_n^{\text{Fr}} = Q_n$ as closed forms. If the relative-error alternative in FS4 holds with $\epsilon_n^{\text{Fr}} < 1$, then the Q_n - and q_n^{Fr} -graph norms are equivalent on the common core, and the Fredholm mismatch may be absorbed into the relative-form comparison by replacing σ_n with*

$$\sigma_n^{\text{tot}} = \sigma_n + C_n \epsilon_n^{\text{Fr}},$$

for the corresponding graph-norm comparison constant C_n , on any tail for which $\sigma_n^{\text{tot}} < 1$.

Proof. If the two nonnegative closed forms agree on a common form core, their closures are identical. This proves the exact case. In the relative-error case, the bound in FS4 gives, on the common core,

$$(1 - \epsilon_n^{\text{Fr}})(Q_n[\psi] + \beta\|\psi\|^2) \leq q_n^{\text{Fr}}[\psi] + \beta\|\psi\|^2 + \epsilon_n^{\text{Fr}}\beta\|\psi\|^2,$$

and similarly in the reverse direction after interchanging the two forms and using the same graph-norm comparison. Thus the graph norms are equivalent on the core and the closure domains agree after completion. Any comparison between the transported form and the reference form then acquires the additional Fredholm mismatch. The constant C_n records the chosen comparison of graph norms. The metric theorem applies only after this enlarged defect has been inserted into the total σ -budget. $\square$

The purpose of Proposition 2.6 is defensive. It prevents the following invalid shortcut:

$$\text{Fredholm fiber exists} \implies \text{the sublevels are sublevels of } Q_n.$$

The valid chain is instead

$$\begin{array}{l} \text{Fredholm fiber} \longrightarrow \text{represented Fredholm form} \\ \longrightarrow \text{Fredholm-stiffness matching} \longrightarrow \text{the correct } Q_n. \end{array}$$

Only then may the spaces

$$\mathfrak{B}_n(R) = \{\psi : Q_n[\psi] + \beta\|\psi\|^2 \leq R\}$$

be interpreted as stiffness sublevels of the Čech-module-Fredholm Yang-Mills carrier.

2.6 The actual stiffness form

Fix one bounded controlled regional object O , one permitted represented branch ξ , and one finite allowed Wilson-channel frame for a fixed compact connected semisimple global form G . The intended physical system is 3+1-dimensional. The later abstract metric theorem itself is not spacetime-dimension dependent.

On this selected source carrier, use

$$\mathcal{H}_n = \mathcal{H}_{G,\xi,n;O}, \quad Q_n = \hat{q}_{G,\xi,n;O}, \quad \varepsilon_n = 2^{-n}\varepsilon_0. \quad (2.4)$$

The hats record the source normalization. The form Q_n in (2.4) is the receiving closed stiffness form. A finite magnetic coefficient form, a raw Hamiltonian form, and an auxiliary collar form are not substituted for it. Their physical roles and the distinction between their ground projections are explained in [2, Eqs. (3)–(5), Appendix A].

2.7 Associator-selected magnetic reduction

The magnetic channel is branch-local. Let

$$b_n \in H^3(B\mathcal{B}_n; U(1))$$

be the realized associator and let S_n be the certified tensor selector. Its selected transgression is $s_n^S = \text{trg}_{\ell_n}(b_n)$; the native H^3 and H^2 complexes are not identified. Denote the selected associator-bearing magnetic coefficient sector by $\mathcal{V}_n^{S, \text{mag}}$.

Let

$$J_n^{S, \text{mag}} : \mathcal{V}_n^{S, \text{mag}} \rightarrow \mathcal{H}_n^{\text{thick}}$$

be its thick Wilson realization and define

$$J_n^{\text{mag}} = P_{n,*}^K P_n^G J_n^{S, \text{mag}}.$$

After quotienting certified null directions, put $G_n^{\text{mag}} = (J_n^{\text{mag}})^* J_n^{\text{mag}}$ and

$$\Phi_n^{\text{mag}} = J_n^{\text{mag}} (G_n^{\text{mag}})^{-1/2}. \quad (2.5)$$

The magnetic form is defined from the correct physical form:

$$q_n^{\text{mag}}[v] := Q_n[\Phi_n^{\text{mag}} v]. \quad (2.6)$$

Let L_n^{mag} be its associated operator. For a complete isolated low cluster $\Sigma_{0,n}^{\text{mag}}$, define

$$p_{0,n}^{\text{mag}} = \mathbf{1}_{\Sigma_{0,n}^{\text{mag}}}(L_n^{\text{mag}}) = \frac{1}{2\pi i} \oint_{\Gamma_n} (z - L_n^{\text{mag}})^{-1} dz. \quad (2.7)$$

In general $p_{0,n}^{\text{mag}} \neq P_n$. The ground-reduced carrier and operator are

$$\begin{aligned} \mathcal{C}_n^{\text{mag}, \perp} &= (1 - p_{0,n}^{\text{mag}}) \mathcal{C}_n^{\text{mag}}, \\ L_n^{\text{mag}, \perp} &= (1 - p_{0,n}^{\text{mag}})(L_n^{\text{mag}} - \lambda_{0,n}^{\text{mag}})(1 - p_{0,n}^{\text{mag}}). \end{aligned}$$

Define

$$\delta_n^{\text{mag}} = \inf_{0 \neq z} \frac{\langle z, L_n^{\text{mag}, \perp} z \rangle}{\|z\|^2}. \quad (2.8)$$

Any Poincaré–Cheeger theorem used here must estimate this reduced quantity with its actual measure, phases, boundary convention, admissible subsets, normalization, and complete low-sector projection.

2.8 Triangle obstruction and Grassmannian rank change

For the unit-weight triangle with total holonomy $e^{i\phi}$,

$$\lambda_k(\phi) = 2 - 2 \cos\left(\frac{\phi + 2\pi k}{3}\right).$$

Thus

$$\text{spec } L_0 = \{0, 3, 3\}, \quad \text{spec } L_\pi = \{1, 1, 4\},$$

and for $|\phi| < \pi$ the gap above the one-dimensional lowest sector is

$$\delta(\phi) = 2\sqrt{3} \sin\left(\frac{\pi - |\phi|}{3}\right) \rightarrow 0 \quad (|\phi| \uparrow \pi). \quad (2.9)$$

The graph and its ordinary edge expansion do not change. At $\phi = \pi$ the complete lowest sector becomes two-dimensional and the complementary gap is 3. Hence ordinary graph expansion is an insufficient proxy; the relevant geometry is phase-sensitive and ground-reduced, with the low projection moving between Grassmannian rank strata.

2.9 Physical coupling and cofinal persistence

Let $P_n^{\text{mag}} = \Phi_n^{\text{mag}}(\Phi_n^{\text{mag}})^*$. Exact form reduction is not assumed. Relative to the selected magnetic range and the rest of the physical carrier, write

$$K_n = \begin{pmatrix} K_n^{\text{mag}} & D_n \\ D_n^* & K_n^{\text{rest}} \end{pmatrix}$$

and define the normalized corner

$$\Theta_n = (K_n^{\text{mag}, \perp})^{-1/2} D_n (K_n^{\text{rest}})^{-1/2}. \quad (2.10)$$

If

$$\delta_n^{\text{mag}} \geq \delta_* > 0, \quad \|\Theta_n\| \leq \vartheta_* < 1,$$

then the Schur complement gives the physical associator-bearing floor

$$\delta_*^{\text{assoc, phys}} = (1 - \vartheta_*^2) \delta_* > 0. \quad (2.11)$$

For a cofinal carry $T_{n'n}$, the selector sector, magnetic form, complete low cluster, and Schur reserve must travel together. In particular require a uniform cluster isolation

$$g_n = \text{dist}(\Sigma_{0,n}^{\text{mag}}, \text{spec } L_n^{\text{mag}} \setminus \Sigma_{0,n}^{\text{mag}}) \geq g_* > 0,$$

a controlled intertwining defect $K_{n'}^{\text{mag}} T_{n'n} - T_{n'n} K_n^{\text{mag}} = R_n^K$, and the Riesz consequence

$$\|T_{n'n} p_{0,n}^{\text{mag}} - p_{0,n'}^{\text{mag}} T_{n'n}\| \lesssim \frac{\|R_n^K\|}{g_*^2} + \epsilon_n^S.$$

The reduced forms are then required to satisfy a relative-form estimate with an absorbable projection-transport defect. This is the cofinal persistence gate for the magnetic floor.

2.10 Complete physical coercivity remains independently typed

LEARNING CHECKPOINT

At this stage three objects must remain distinct:

$$p_{0,n}^{\text{mag}}, \quad P_n, \quad (1 - P_n) \mathcal{H}_n.$$

The first is the complete low magnetic cluster on the selected associator-bearing carrier; the second is the complete physical ground projection; the third is the full physical complement. The magnetic mechanism is branch-local and is not required to cover the third object.

The associator-bearing magnetic carrier is not required to exhaust the source IR block or $(1 - P_n) \mathcal{H}_n$. Complete off-ground coercivity remains the job of the source physical stiffness certificate. Schematically, after all declared target-complement, Hamiltonian-residual, and seam deductions,

$$Q_n[\xi \oplus \eta] \geq u_n \|\xi\|^2 + v_n \|\eta\|^2 - 2m_n \|\xi\| \|\eta\|,$$

with

$$u_n \geq u_* > 0, \quad v_n \geq v_* > 0, \quad m_n^2 \leq \rho u_n v_n, \quad \rho < 1. \quad (2.12)$$

Hence

$$Q_n[\psi] \geq (1 - \sqrt{\rho}) \min\{u_*, v_*\} \|(1 - P_n) \psi\|^2. \quad (2.13)$$

The reduced magnetic floor supplies a geometrically interpretable positive contribution on its own associator-bearing branch; it is not substituted for the independently complete source coercive ledger.

3 Abstract comparison after source matching

All Hilbert spaces below are complex and separable. A closed nonnegative quadratic form is densely defined. We use the same letter for its quadratic value and, with two arguments, its associated Hermitian sesquilinear form.

Assumption 3.1 (Unitary regional form comparison). For each n , let Q_n be a closed nonnegative form on $\mathcal{H}_n$, with associated nonnegative self-adjoint operator K_n . The following data are specified.

(i) There are unitaries

$$J_n : \mathcal{H}_n \longrightarrow \mathcal{H}_{\text{ref}}, \quad I_n = J_n^{-1}, \quad J_n \mathcal{D}(Q_n) = \mathcal{V}. \quad (3.1)$$

The dense common domain $\mathcal{V} \subset \mathcal{H}_{\text{ref}}$ is the domain of a closed nonnegative reference form a_{ref} . Set

$$a_n[u] = Q_n[I_n u], \quad u \in \mathcal{V}. \quad (3.2)$$

(ii) For a fixed $\beta > 0$, the inclusion

$$(\mathcal{V}, \|\cdot\|_{\mathcal{V},\beta}) \hookrightarrow \mathcal{H}_{\text{ref}}, \quad \|u\|_{\mathcal{V},\beta}^2 = a_{\text{ref}}[u] + \beta \|u\|^2, \quad (3.3)$$

is compact. Equivalently, the nonnegative reference operator has compact shifted resolvent.

(iii) There are numbers $0 \leq \sigma_n < 1$, tending to zero, such that

$$|a_n[u] - a_{\text{ref}}[u]| \leq \sigma_n (a_{\text{ref}}[u] + \beta \|u\|^2), \quad u \in \mathcal{V}. \quad (3.4)$$

A family satisfying this condition only on a sufficiently fine tail is restricted to that tail.

In an abstract application, J_n is simply a declared unitary. In the physical application it must be the comparison map that transports the specified physical carrier and form. Unitary equivalence of two unnamed Hilbert spaces is not enough to establish this identification.

Define the auxiliary positive form norms

$$h_{\text{ref}}[u] = a_{\text{ref}}[u] + \beta \|u\|^2, \quad h_n[u] = a_n[u] + \beta \|u\|^2. \quad (3.5)$$

The relative estimate is exactly the two-sided bound

$$(1 - \sigma_n) h_{\text{ref}}[u] \leq h_n[u] \leq (1 + \sigma_n) h_{\text{ref}}[u]. \quad (3.6)$$

Remark 3.2 (The auxiliary shift is not a physical renormalization). The term $\beta \|u\|^2$ makes a sublevel control both norm and stiffness. It is not added to K_n when stating a physical threshold, and it is not substituted for the prescribed ground-energy normalization. Nonnegativity of the actual Q_n is assumed before this auxiliary term is introduced. When stiffness has energy-squared units, β has the same units.

The source comparison is often expressed in another norm on the common domain. The conversion needed here is explicit.

Lemma 3.3 (Conversion of a common-domain defect). *Suppose, on $\mathcal{V}$, that*

$$|a_n[u] - a_{\text{ref}}[u]| \leq t_n \|u\|_*^2, \quad \|u\|_*^2 \leq C_{\mathcal{V}} h_{\text{ref}}[u]. \quad (3.7)$$

Then (3.4) holds with $\sigma_n = C_{\mathcal{V}} t_n$. In particular, $t_n \rightarrow 0$ gives Assumption 3.1(iii) on a tail.

Proof. Substitution in the first inequality gives $|a_n[u] - a_{\text{ref}}[u]| \leq C_{\mathcal{V}} t_n h_{\text{ref}}[u]$. $\square$

This is a comparison of norms for the already normalized forms. Any physical affine rescaling or ground-energy subtraction must be included before the defect in (3.7) is evaluated.

4 Gromov control of the correct stiffness sublevels

Definition 4.1 (Norm-bounded stiffness sublevel). For $R > 0$, define

$$\mathfrak{B}_n(R) = \left\{ \psi \in \mathcal{D}(Q_n) : Q_n[\psi] + \beta \|\psi\|_{\mathcal{H}_n}^2 \leq R \right\}, \quad (4.1)$$

$$\mathfrak{B}_{\text{ref}}(R) = \{u \in \mathcal{V} : h_{\text{ref}}[u] \leq R\}. \quad (4.2)$$

Equip these sets with their ambient Hilbert norm metrics, and write

$$\mathfrak{B}_n^J(R) = J_n \mathfrak{B}_n(R) = \{u \in \mathcal{V} : h_n[u] \leq R\}, \quad r_R = \sqrt{R/\beta}. \quad (4.3)$$

These are sets of vectors, not sets of rays or normalized states. The metric is $\|\psi - \phi\|$, not the square root of a stiffness value. Because the forms are nonnegative, both $\mathfrak{B}_n^J(R)$ and $\mathfrak{B}_{\text{ref}}(R)$ lie in the Hilbert ball of radius r_R .

For nonempty compact subsets A, B of a metric space Z , the Hausdorff distance is

$$d_H^Z(A, B) = \max \left\{ \sup_{a \in A} \inf_{b \in B} d_Z(a, b), \sup_{b \in B} \inf_{a \in A} d_Z(a, b) \right\}. \quad (4.4)$$

The Gromov–Hausdorff distance is the infimum of this quantity over common metric spaces and isometric embeddings of the two compact spaces. The unitary maps in (3.1) therefore give

$$d_{\text{GH}}(\mathfrak{B}_n(R), \mathfrak{B}_{\text{ref}}(R)) \leq d_H^{\mathcal{H}_{\text{ref}}}(\mathfrak{B}_n^J(R), \mathfrak{B}_{\text{ref}}(R)). \quad (4.5)$$

Lemma 4.2 (Compactness of the sublevels). *Under Assumption 3.1, the spaces $\mathfrak{B}_{\text{ref}}(R)$ and $\mathfrak{B}_n(R)$ are compact for every $R > 0$.*

Proof. The set $\mathfrak{B}_{\text{ref}}(R)$ is bounded in the reference form norm, so the compact embedding (3.3) makes it relatively compact in $\mathcal{H}_{\text{ref}}$. Lower semicontinuity of the closed form makes the set closed in that Hilbert space. It is therefore compact.

By (3.6),

$$\mathfrak{B}_n^J(R) \subset \mathfrak{B}_{\text{ref}}\left(\frac{R}{1 - \sigma_n}\right). \quad (4.6)$$

The right-hand side is compact. The left-hand side is closed because a_n is closed under unitary transport. Hence $\mathfrak{B}_n^J(R)$ is compact, and the unitary J_n transfers compactness to $\mathfrak{B}_n(R)$. $\square$

Theorem 4.3 (Quantitative Gromov–Hausdorff stability). *Under Assumption 3.1, for every $R > 0$,*

$$d_{\text{GH}}(\mathfrak{B}_n(R), \mathfrak{B}_{\text{ref}}(R)) \leq g_n(R) := \sqrt{\frac{R}{\beta}}(1 - \sqrt{1 - \sigma_n}). \quad (4.7)$$

The same upper bound holds for the Hausdorff distance of the specified images $\mathfrak{B}_n^J(R)$ and $\mathfrak{B}_{\text{ref}}(R)$ in $\mathcal{H}_{\text{ref}}$.

Proof. We construct a point in the other sublevel in each direction.

Let $u \in \mathfrak{B}_n^J(R)$ and set $s_n = \sqrt{1 - \sigma_n}$. The lower inequality in (3.6) gives

$$h_{\text{ref}}[u] \leq \frac{R}{1 - \sigma_n}, \quad h_{\text{ref}}[s_n u] \leq R.$$

Thus $s_n u \in \mathfrak{B}_{\text{ref}}(R)$. Since $\|u\| \leq r_R$,

$$\|u - s_n u\| \leq r_R(1 - \sqrt{1 - \sigma_n}). \quad (4.8)$$

Conversely, let $v \in \mathfrak{B}_{\text{ref}}(R)$ and set $t_n = (1 + \sigma_n)^{-1/2}$. The upper inequality in (3.6) gives

$$h_n[t_n v] \leq (1 + \sigma_n)t_n^2 h_{\text{ref}}[v] \leq R,$$

so $t_n v \in \mathfrak{B}_n^J(R)$. Therefore

$$\|v - t_n v\| \leq r_R \left(1 - \frac{1}{\sqrt{1 + \sigma_n}}\right). \quad (4.9)$$

For $0 \leq \sigma_n < 1$, $\sqrt{1 - \sigma_n} \leq (1 + \sigma_n)^{-1/2}$, since $(1 - \sigma_n)(1 + \sigma_n) \leq 1$. The bound in (4.9) is consequently no larger than the bound in (4.8). This proves the Hausdorff estimate. Equation (4.5) gives the Gromov–Hausdorff conclusion. $\square$

The proof uses the relative-form estimate to compare the same vectors radially. It does not first infer convergence of eigenvalues, and it does not assume a positive off-ground coercive constant.

Corollary 4.4 (Thickness rate and the zero-defect case). *Suppose the normalized common-domain defect has the form $t_n = C_{G,\xi,O} \varepsilon_n^\alpha$ in Lemma 3.3, where $\alpha > 0$ and $\varepsilon_n \rightarrow 0$. Then, on the tail where $\sigma_n = C_V C_{G,\xi,O} \varepsilon_n^\alpha < 1$,*

$$d_{\text{GH}}(\mathfrak{B}_n(R), \mathfrak{B}_{\text{ref}}(R)) \leq \sqrt{\frac{R}{\beta}} C_V C_{G,\xi,O} \varepsilon_n^\alpha. \quad (4.10)$$

If the prescribed physical normalization makes $a_n = a_{\text{ref}}$ exactly, then J_n restricts to an isometry $\mathfrak{B}_n(R) \rightarrow \mathfrak{B}_{\text{ref}}(R)$.

Proof. Use $1 - \sqrt{1 - s} = s/(1 + \sqrt{1 - s}) \leq s$ for $0 \leq s < 1$. In the zero-defect case the transported sublevels coincide. $\square$

The constants in (4.10) may depend on the region, represented branch, and selected finite carrier frame. No uniformity in those labels follows from this estimate. Exact dilation up to an unspecified positive affine rescaling does not by itself imply the zero-defect conclusion: the forms must agree after the normalization used in the theorem.

5 Complete-ground-compatible sublevels

The metric theorem controls the whole stiffness sublevel. To identify the nonvacuum part, it must be supplied with the complete ground projections of the same forms.

Let K_{ref} be the operator associated with a_{ref} , and define

$$P_n = \mathbf{1}_{\{0\}}(K_n), \quad P_{\text{ref}} = \mathbf{1}_{\{0\}}(K_{\text{ref}}), \quad \hat{P}_n = J_n P_n J_n^{-1}. \quad (5.1)$$

These projections retain the full zero sector, including its degeneracy. For a nonnegative closed form, a vector in its kernel belongs to its form domain and has zero sesquilinear pairing with every form-domain vector. Consequently

$$a_n[(1 - \hat{P}_n)u] = a_n[u], \quad a_{\text{ref}}[(1 - P_{\text{ref}})u] = a_{\text{ref}}[u], \quad u \in \mathcal{V}. \quad (5.2)$$

The projections preserve $\mathcal{V}$, and removing a ground component cannot increase the corresponding h_n or h_{ref} . In a physical application, identifying a supplied vacuum projection with the projections in (5.1) is part of the source input.

Assumption 5.1 (Complete-ground transport). In the specified common Hilbert carrier,

$$\hat{P}_n \longrightarrow P_{\text{ref}} \quad \text{strongly on } \mathcal{H}_{\text{ref}}. \quad (5.3)$$

Define the nonvacuum sublevels

$$\mathfrak{B}_n^\perp(R) = \mathfrak{B}_n(R) \cap \ker P_n, \quad \mathfrak{B}_{\text{ref}}^\perp(R) = \mathfrak{B}_{\text{ref}}(R) \cap \ker P_{\text{ref}}, \quad (5.4)$$

and the compact-sublevel projection defect

$$\rho_n(R) = \sup_{u \in \mathfrak{B}_{\text{ref}}(R)} \|(\hat{P}_n - P_{\text{ref}})u\|. \quad (5.5)$$

The term “nonvacuum” here refers to orthogonality in the physical Hilbert space. It is not a pointwise support condition on a proposed vacuum subset of a configuration space.

Lemma 5.2 (Uniform ground transport on a compact sublevel). *Under Assumptions 3.1 and 5.1, $\rho_n(R) \rightarrow 0$ for every fixed $R > 0$.*

Proof. The set $\mathfrak{B}_{\text{ref}}(R)$ is compact by Lemma 4.2. Given $\delta > 0$, choose a finite δ -net $\{u_1, \dots, u_m\}$ in this set. For every u in the set, choose u_j with $\|u - u_j\| < \delta$. Because $\|\hat{P}_n - P_{\text{ref}}\| \leq 2$,

$$\|(\hat{P}_n - P_{\text{ref}})u\| \leq \max_{1 \leq j \leq m} \|(\hat{P}_n - P_{\text{ref}})u_j\| + 2\delta.$$

The finite maximum tends to zero by strong convergence. Take a supremum in u , then let $\delta \downarrow 0$. $\square$

Theorem 5.3 (Ground-compatible Gromov–Hausdorff stability). *Under Assumptions 3.1 and 5.1, the sets in (5.4) are compact and*

$$d_{\text{GH}}(\mathfrak{B}_n^\perp(R), \mathfrak{B}_{\text{ref}}^\perp(R)) \leq \sqrt{\frac{R}{\beta}}(1 - \sqrt{1 - \sigma_n}) + \rho_n(R). \quad (5.6)$$

Again, this bounds the Hausdorff distance of the specified common-carrier images as well.

Proof. Compactness follows by intersecting the compact full sublevels with closed projection kernels. We prove both Hausdorff inclusions.

Take $u \in J_n \mathfrak{B}_n^\perp(R)$ and set $v = s_n u$, where $s_n = \sqrt{1 - \sigma_n}$. Theorem 4.3 places v in $\mathfrak{B}_{\text{ref}}(R)$. Put $v_\perp = (1 - P_{\text{ref}})v$. By (5.2) and orthogonality, $v_\perp \in \mathfrak{B}_{\text{ref}}^\perp(R)$. Since $\hat{P}_n v = 0$,

$$\|v - v_\perp\| = \|P_{\text{ref}} v\| = \|(P_{\text{ref}} - \hat{P}_n)v\| \leq \rho_n(R).$$

Together with (4.8), this gives $\|u - v_\perp\| \leq g_n(R) + \rho_n(R)$.

Conversely, take $v \in \mathfrak{B}_{\text{ref}}^\perp(R)$ and put $w = t_n v$, where $t_n = (1 + \sigma_n)^{-1/2}$. Then $w \in \mathfrak{B}_n^J(R)$. Set $u = (1 - \hat{P}_n)w$. Equation (5.2) and orthogonality give $u \in J_n \mathfrak{B}_n^\perp(R)$. Since $P_{\text{ref}} v = 0$,

$$\|w - u\| = t_n \|\hat{P}_n v\| = t_n \|(\hat{P}_n - P_{\text{ref}})v\| \leq \rho_n(R).$$

Combining this with (4.9) yields the same bound in the reverse direction. The Hausdorff estimate and then the Gromov–Hausdorff estimate follow. $\square$

The two terms in (5.6) have different origins. The first measures stiffness-form mismatch. The second measures mismatch of the complete ground sectors under the actual comparison. A power-law estimate for σ_n alone gives no asserted power-law rate for $\rho_n(R)$. Strong convergence gives the latter's vanishing, not a rate. Nor does the compact-sublevel lemma assert operator-norm convergence of the projections on the whole Hilbert space.

Example 5.4 (A small form change can lift a ground vector). On $\mathbb{C}^2$, take $J_n = I$ and

$$K_{\text{ref}} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad K_n = \begin{pmatrix} n^{-1} & 0 \\ 0 & 1 \end{pmatrix}.$$

For $\beta = 1$ and $n \geq 2$, the relative-form bound holds with $\sigma_n = n^{-1}$. The full sublevels converge by Theorem 4.3. Nevertheless, $P_n = 0$ while $P_{\text{ref}} = \text{diag}(1, 0)$, so Assumption 5.1 fails. In fact, $\mathfrak{B}_n^\perp(R) = \mathfrak{B}_n(R)$ converges to the full reference sublevel, which is not the reference nonvacuum sublevel. A Riesz projection around a small cluster could retain the nearly zero eigenvalue; it would not thereby be the exact kernel projection. This elementary example distinguishes the two hypotheses without invoking a field-theoretic model.

6 Form convergence and separately supplied floor

The same relative-form estimate gives the usual variational convergence. We include the short proof to specify what is carried by the form estimate and what is carried by compactness. Extend each form by $+\infty$ outside $\mathcal{V}$.

Proposition 6.1 (Mosco convergence on the straightened carrier). *The common-domain and relative-defect hypotheses in Assumption 3.1 imply that a_n Mosco converges to a_{ref} on $\mathcal{H}_{\text{ref}}$. Compactness of the reference embedding is not needed for this conclusion.*

Proof. Let $u_n \rightharpoonup u$ in $\mathcal{H}_{\text{ref}}$. If $\liminf_n a_n[u_n] = +\infty$, the lower-limit inequality is immediate. Otherwise choose a subsequence attaining a finite lower limit. Weak convergence makes the norms bounded. The lower relative-form estimate gives

$$a_{\text{ref}}[u_n] \leq \frac{a_n[u_n] + \sigma_n \beta \|u_n\|^2}{1 - \sigma_n}. \quad (6.1)$$

Thus $a_{\text{ref}}[u_n]$ is bounded along this subsequence, and (3.4) gives $a_n[u_n] - a_{\text{ref}}[u_n] \rightarrow 0$. A closed nonnegative form is convex and weakly lower semicontinuous when extended by $+\infty$. Hence

$$a_{\text{ref}}[u] \leq \liminf_n a_{\text{ref}}[u_n] = \liminf_n a_n[u_n]$$

on the selected subsequence, which proves the lower-limit condition.

For $u \in \mathcal{V}$, the constant sequence $u_n = u$ converges strongly and (3.4) gives $a_n[u] \rightarrow a_{\text{ref}}[u]$. This is a recovery sequence. These two properties are Mosco convergence. $\square$

Compactness adds a strong-subsequence statement: any family whose h_n values are uniformly bounded has, after transport, a strongly convergent subsequence. This follows from (4.6) on a tail where σ_n is bounded away from one. The metric theorem gives more than this subsequence existence: it controls the whole sublevel at fixed R .

Corollary 6.2 (Transfer of an imported off-ground floor). *Suppose Assumptions 3.1 and 5.1 hold and, independently, a constant $\kappa_* > 0$ satisfies*

$$Q_n[\psi] \geq \kappa_* \|(1 - P_n)\psi\|_{\mathcal{H}_n}^2 \quad \text{for every } \psi \in \mathcal{D}(Q_n) \quad (6.2)$$

on a tail. Then

$$a_{\text{ref}}[u] \geq \kappa_* \|(1 - P_{\text{ref}})u\|^2, \quad u \in \mathcal{V}. \quad (6.3)$$

Proof. Apply (6.2) to $I_n u$ and use unitarity: $a_n[u] \geq \kappa_* \|(1 - \hat{P}_n)u\|^2$. The recovery equality from Proposition 6.1 and strong ground convergence give

$$a_{\text{ref}}[u] = \lim_n a_n[u] \geq \kappa_* \lim_n \|(1 - \hat{P}_n)u\|^2 = \kappa_* \|(1 - P_{\text{ref}})u\|^2.$$

$\square$

The proof uses recovery equality, not the Mosco lower-limit inequality in the reverse direction. The value of κ_* is not supplied by the Gromov–Hausdorff estimate. On one fixed compact-resolvent carrier an exact kernel is spectrally isolated whenever there is a nonzero complementary sector. That local spectral fact does not supply a region-independent physical coercive constant. In the Wilson-tube application, the required uniform physical floor belongs to the coupled electric–magnetic estimates and their comparison data [2, Eqs. (5)–(8)].

7 Regional source matching and first test case

7.1 The declared common-carrier comparison

The v1.1.2 submission edition of the long reconstruction paper specifies a bounded-region exact-dilation Friedrichs comparison class [6, Sec. XI.C]. Its local operator has Laplace-type principal part and a pointwise Hermitian lower-order term:

$$A_{G,\xi,n} = \eta \nabla_{G,\xi,n}^* \nabla_{G,\xi,n} + M_{V_{G,\xi,n}}. \quad (7.1)$$

The source states compact local resolvent on its bounded regional carriers, and records the scaling $\hat{A}_{G,\xi,n} = \varepsilon_n^2 A_{G,\xi,n}^{\text{phys}}$. The source describes the zero-defect orbit as stationary up to positive affine rescaling; the larger declared class uses the relative coefficient estimate in its Eq. (48).

The general reconstruction comparison does not invert J_n . In the exact-dilation specialization its comparison and reconstruction maps are mutual unitaries [6, Sec. IX.A]. This note applies only to that unitary common-carrier setting, or to another setting in which the same unitary and common-domain conditions have actually been verified.

In the notation of Section 3, the needed source comparison is

$$|Q_n[I_n u] - a_{\text{ref}}[u]| \leq C_{G,\xi,O} \varepsilon_n^\alpha \|u\|_*^2, \quad \|u\|_*^2 \leq C_V (a_{\text{ref}}[u] + \beta \|u\|^2). \quad (7.2)$$

The first inequality has the form of Paper IV, Eq. (48). The second is the norm comparison needed to use that estimate for the normalized stiffness; it is isolated in Appendix B of the physical-mechanism Letter [2, Eq. (B3)]. Nonnegativity, ground normalization, and any remaining rescaling are checked before (7.2) is invoked.

Once these identifications hold, Theorem 4.3 gives

$$d_{\text{GH}}(\mathfrak{B}_{G,\xi,n;O}(R), \mathfrak{B}_{G,\xi,\text{ref};O}(R)) \leq \sqrt{\frac{R}{\beta}} \left(1 - \sqrt{1 - C_V C_{G,\xi,O} \varepsilon_n^\alpha}\right). \quad (7.3)$$

Compatible complete-ground transport then gives Theorem 5.3. The source's complete-ground form structure and moving-family transport are separate from the coefficient estimate [6, Theorem IX.2 and Sec. IX.C]. They are not replaced by smallness of the lower-order coefficients.

7.2 What is imported, and what is proved here

The long source supplies the declared regional comparison class and the form estimate. Its Theorem XI.5 explicitly distinguishes analytical comparison non-vacuity from identification with every independently produced physical refinement [6]. We retain that distinction. Applying (7.3) to a physical branch requires matching its actual forms, normalization, domains, comparison maps, and ground projections with the objects in Assumptions 3.1–5.1. Merely sharing the label $G = SU(3)$, or carrying a finite $SU(2)$ witness, does not establish this match.

The new result in this note is the explicit compact-sublevel metric comparison, together with its ground-compatible version. No claim is made that the long source already states these Gromov–Hausdorff inequalities. Nor does this note reprove the long source's physical realization or its uniform electric–magnetic lower bound.

The result is objectwise on the selected represented carrier. Constants may depend on O , ξ , and the finite frame. A compatible countable exhaustion does not by itself control states whose support escapes through that exhaustion. Likewise, a separate nonunitary coarse-comparison problem still needs the source's retained-carrier, reconstruction, and domain controls. The one-sided collar estimate in [2, Appendix A] is not promoted here to a unitary identification on every fine state.

7.3 Source-matching ledger

The preceding abstract theorem applies to a Yang–Mills source branch only when its objects are matched to the theorem's objects without changing the physical form. The following ledger records the required identifications.

L1. GAT 0 source-recognition packet. The source branch begins with a recognized finite packet

$$\text{Rec}_n^{\text{GAT0}},$$

including the chosen compact connected semisimple global form, the genuine sector-complete detector data, the electric/Casimir seed, the native cohomological coordinates, and the finite coercive ledger. This is the finite cargo to be consumed downstream. It is not yet a represented physical Hilbert space and does not by itself provide the compact stiffness sublevels used in this note.

- L2. **GAT II maximal Klauder carrier.** The finite constraint packet must be physicalized by the positive quadratic constraint operator K_n , and the represented carrier used here must come from the maximal self-admissible finite Klauder window

$$P_{n,*}^K = \max\{E_{K_n}([0, \delta^2]) : \text{the source certificate admits the window}\}.$$

The width δ is constraint-spectral width, not geometric thickness. The projection $P_{n,*}^K$ selects the constrained physical carrier; it is not the complete-ground projection P_n .

- L3. **Hilbert-module/Grassmannian/Fredholm origin.** The receiving carrier is not taken as a primitive abstract Hilbert space. It must be the represented Hilbert fiber obtained from the source's Hilbert-module-valued descent data after restriction to the admitted Klauder carrier: local self-dual Hilbert W^* -modules, physical projection ranges in the appropriate Grassmannian fiber, the Fredholm-fiber interface used by the Yang–Mills realization, and the chosen representation or GNS completion. This note begins after those constructions have supplied the Hilbert carrier, but it records that origin as part of the source-matching gate.

- L4. **Receiving carrier.**

$$\mathcal{H}_n = \mathcal{H}_{G,\xi,n;O}.$$

This is imported from the selected represented region, branch, finite Wilson-channel frame, and admitted Klauder carrier after the module-valued Grassmannian–Fredholm fiber has been represented as a Hilbert space. It is not replaced by a coefficient graph carrier unless the source comparison identifies that graph carrier with the same constrained physical Hilbert carrier.

- L5. **Correct stiffness form.**

$$Q_n = \hat{q}_{G,\xi,n;O}.$$

This is the closed, ground-normalized, positive-thickness stiffness form of the selected source carrier. It is not the magnetic graph form $\langle c, L_\theta c \rangle$, not the raw Hamiltonian expectation, and not the auxiliary collar form. Any graph or collar estimate must be transported into this form before the metric theorem is invoked.

- L6. **Uniform electric–magnetic and UV–IR floor.** The same source-normalized form Q_n must carry the finite coercive floor after electric–magnetic coupling, UV–IR mixing, ground subtraction, channel normalization, and seam reserves have all been accounted for:

$$Q_n[\psi] \geq \gamma_* \|(1 - P_n)\psi\|^2, \quad \gamma_* > 0.$$

Equivalently, the branch ledger must include the two-channel constants $c_n^{\text{el}}, c_n^{\text{mag}}, \eta_n, \chi_n$ or the UV–IR constants u_n, v_n, m_n , together with a positive cofinal infimum after all residual budgets are deducted. This note records the algebraic two-block calculation; it does not prove the source floor independently of the long construction.

- L7. **Fredholm–stiffness matching.** The Fredholm-fiber form associated with the represented module data must either agree with the source-normalized physical stiffness form on a common form core or differ from it by a defect already recorded in the relative-form constant. Without this certificate, the Fredholm fiber is only a related analytic carrier; it has not been identified with the particular form Q_n whose metric sublevels are being studied.

L8. Comparison map.

$$J_n : \mathcal{H}_n \rightarrow \mathcal{H}_{\text{ref}}, \quad I_n = J_n^{-1}.$$

In this note J_n is a unitary straightening of the selected regional comparison. This is a specialization. A nonunitary coarse comparison does not satisfy the hypotheses unless an additional argument provides an isometric common-carrier replacement.

L9. Transported form.

$$a_n[u] = Q_n[I_n u], \quad u \in \mathcal{V}.$$

This is new notation for the same physical stiffness form in reference coordinates. It is not a new dynamics.

L10. Reference form and common domain.

$$a_{\text{ref}} : \mathcal{V} \rightarrow [0, \infty), \quad J_n \mathcal{D}(Q_n) = \mathcal{V}.$$

The compact-reference carrier is imported from the bounded-region Friedrichs comparison branch or from an independently verified common-core realization. Compactness is regional and does not assert global infinite-volume compact resolvent.

L11. Auxiliary shift.

$$h_n = a_n + \beta \|\cdot\|^2, \quad h_{\text{ref}} = a_{\text{ref}} + \beta \|\cdot\|^2, \quad \beta > 0.$$

The shift chooses compact norm-and-stiffness sublevels. It is a bookkeeping device for the geometry, not a physical mass, ground-energy renormalization, or perturbation of the source operator.

L12. Relative defect.

$$|a_n[u] - a_{\text{ref}}[u]| \leq \sigma_n h_{\text{ref}}[u], \quad \sigma_n \rightarrow 0.$$

In the source application this is obtained only after converting the coefficient estimate to the normalized physical form. If the source gives $|a_n - a_{\text{ref}}| \leq t_n \|\cdot\|_*^2$ and $\|u\|_*^2 \leq C_{\mathcal{V}} h_{\text{ref}}[u]$, then $\sigma_n = C_{\mathcal{V}} t_n$.

L13. Complete ground projection.

$$P_n = P_{\Omega, n; O}, \quad \hat{P}_n = J_n P_n J_n^{-1}.$$

This is the complete coupled ground projection on the physical stiffness carrier. It is not the lowest-eigenvalue projection of an isolated magnetic coefficient problem unless the source comparison has identified the two within the coupled physical carrier.

L14. Ground transport error.

$$\rho_n(R) = \sup_{u \in \mathfrak{B}_{\text{ref}}(R)} \|(\hat{P}_n - P_{\text{ref}})u\|.$$

This is a new compact-sublevel error term. It follows from strong ground projection convergence together with compactness of the reference sublevel; it is not an operator-norm convergence claim on the whole Hilbert space.

L15. New metric objects.

$$\mathfrak{B}_n(R) = \{\psi : Q_n[\psi] + \beta \|\psi\|^2 \leq R\}, \quad \mathfrak{B}_n^{\perp}(R) = \mathfrak{B}_n(R) \cap \ker P_n.$$

These are constructed in this note from the correct physical stiffness form and physical Hilbert metric. They are not configuration spaces equipped with a measure or perimeter.

L16. **New consequences proved here.** The compactness of the sublevels, the quantitative Gromov–Hausdorff estimate, the ground-compatible estimate, and the Mosco consequence are proved in this note from the ledger assumptions. The uniform off-ground stiffness floor is not proved by this note; it must be supplied by the coupled electric–magnetic coercivity package.

L17. **Thin physical endpoint.** Where the source reconstruction and time-generator identification apply,

$$Q_{\text{thin}}[\psi] = \|P^0\psi\|^2, \quad K_{\text{thin}} = (P^0)^2.$$

This is an energy-squared stiffness identity. It is not the assertion $Q_{\text{thin}} = q_{M^2}$. The invariant-mass conclusion requires the separate forward-cone and Lorentz-covariance argument.

The gate for applying the theorem to a new branch is therefore simple: every entry in this ledger must be identified on that branch before the Gromov–Hausdorff estimate is cited as a statement about the source physical stiffness.

7.4 First populated branch ledger: the exact-dilation regional test case

We now record the first clean test case for the ledger. This is not a new physical branch and not an enlargement of scope. It is the bounded-region, unitary-straightened, exact-dilation comparison case already isolated as the simplest setting for the abstract theorem.

Fix

$$G = SU(3), \quad O \Subset \Sigma, \quad \xi = \xi_{\text{ed}}, \quad (7.4)$$

where O is one bounded controlled region in the time-zero carrier and ξ_{ed} denotes the represented exact-dilation branch after the source comparison has been straightened by its declared unitary maps. The symbol ξ_{ed} is only a local label for this note; it is not a new sector in the source construction.

The ledger is then populated as follows.

E1. **Recognized source packet.** Use the recognized GAT 0 source packet for the selected regional branch,

$$\text{Rec}_{SU(3), \xi_{\text{ed}}, n; O}^{GAT0}.$$

This entry is *source-imported*. The group label alone is not the certificate; the finite sector data, electric seed, complete-ground ledger, and coercive bookkeeping must be the ones consumed by the represented branch.

E2. **Maximal Klauder carrier.** Use the GAT II constrained carrier

$$P_{SU(3), \xi_{\text{ed}}, n; O, *}^K$$

obtained from the admitted finite spectral windows of the quadratic constraint operator. This entry is *source-imported after constraint physicalization*. It is not the complete-ground projection; it is the carrier on which the complete-ground projection is later marked.

E3. **Carrier.** Use the source carrier

$$\mathcal{H}_n = \mathcal{H}_{SU(3), \xi_{\text{ed}}, n; O}. \quad (7.5)$$

This entry is *source-imported*. The regional and branch labels are kept because the constants below are not claimed uniformly over all regions or all branches.

E4. **Physical stiffness form.** Use

$$Q_n = \hat{q}_{SU(3), \xi_{\text{ed}}, n; O}. \quad (7.6)$$

This entry is *source-imported after normalization*. The hat is part of the data: it includes the source's ground-energy normalization and positive scaling. The form in (7.6) is the only form whose sublevels are used below.

E5. Unitary straightening. Use the exact-dilation comparison maps

$$J_n : \mathcal{H}_{SU(3),\xi_{\text{ed}},n;O} \rightarrow \mathcal{H}_{\text{ref}}, \quad I_n = J_n^{-1}, \quad (7.7)$$

with common form domain

$$J_n \mathcal{D}(Q_n) = \mathcal{V}. \quad (7.8)$$

This entry is a *specialization*. If one leaves the exact-dilation unitary branch and works with a coarse nonunitary comparison, the present Gromov–Hausdorff theorem does not automatically apply.

E6. Transported form. Define

$$a_n[u] = Q_n[I_n u], \quad u \in \mathcal{V}. \quad (7.9)$$

This is *new notation only*. It is the same physical stiffness form in the common reference coordinates.

E7. Reference form. Let a_{ref} be the exact-dilation reference Friedrichs form on $\mathcal{V}$, with associated compact-resolvent operator K_{ref} . In the local model one writes

$$K_{\text{ref}} = \eta \nabla_{\text{ref}}^* \nabla_{\text{ref}} + M_{V_{\text{ref}}} \quad (7.10)$$

with the regional boundary condition inherited from the source branch. The compact-resolvent property is *source-imported regionally*; it is not a global infinite-volume assertion.

E8. Relative defect. In the zero-defect exact-dilation orbit one has, after the source normalization,

$$a_n[u] = a_{\text{ref}}[u] \quad (u \in \mathcal{V}), \quad (7.11)$$

and therefore

$$\sigma_n = 0. \quad (7.12)$$

In the larger vanishing-defect branch, replace (7.11) by

$$|a_n[u] - a_{\text{ref}}[u]| \leq C_{\mathcal{V}} C_{SU(3),\xi_{\text{ed}},O} \varepsilon_n^\alpha (a_{\text{ref}}[u] + \beta \|u\|^2), \quad (7.13)$$

so that

$$\sigma_n = C_{\mathcal{V}} C_{SU(3),\xi_{\text{ed}},O} \varepsilon_n^\alpha. \quad (7.14)$$

The zero-defect statement is a *specialized source match*; the vanishing-defect estimate is a *source-import plus norm-conversion match*.

E9. Sublevel spaces. The metric objects for this branch are

$$\mathfrak{B}_{SU(3),\xi_{\text{ed}},n;O}(R) = \{\psi \in \mathcal{D}(Q_n) : Q_n[\psi] + \beta \|\psi\|^2 \leq R\}, \quad (7.15)$$

with the physical Hilbert metric. The reference object is

$$\mathfrak{B}_{SU(3),\xi_{\text{ed}},\text{ref};O}(R) = \{u \in \mathcal{V} : a_{\text{ref}}[u] + \beta \|u\|^2 \leq R\}. \quad (7.16)$$

These are *new objects constructed in this note* from the matched source form.

E10. **First metric conclusion.** In the exact zero-defect case,

$$J_n : \mathfrak{B}_{SU(3),\xi_{\text{ed}},n;O}(R) \longrightarrow \mathfrak{B}_{SU(3),\xi_{\text{ed}},\text{ref};O}(R) \quad (7.17)$$

is an isometry. In the vanishing-defect case,

$$d_{\text{GH}}(\mathfrak{B}_{SU(3),\xi_{\text{ed}},n;O}(R), \mathfrak{B}_{SU(3),\xi_{\text{ed}},\text{ref};O}(R)) \leq \sqrt{\frac{R}{\beta}} \left(1 - \sqrt{1 - C_{\mathcal{V}} C_{SU(3),\xi_{\text{ed}},O} \varepsilon_n^\alpha}\right) \quad (7.18)$$

on the tail where the expression under the square root is positive. This is *proved in this note* once E1–E6 are verified.

E11. **Complete ground projection.** Use the source complete coupled ground projection

$$P_n = P_{\Omega, SU(3), \xi_{\text{ed}}, n; O}, \quad \hat{P}_n = J_n P_n J_n^{-1}. \quad (7.19)$$

The word “complete” is essential. This is not replaced by the lowest magnetic coefficient projection of a finite witness graph.

E12. **Ground matching.** If the exact-dilation branch identifies the complete ground sector exactly, then

$$\hat{P}_n = P_{\text{ref}}, \quad \rho_n(R) = 0. \quad (7.20)$$

If only moving-family or comparison-strong convergence is supplied, the note uses instead

$$\rho_n(R) = \sup_{u \in \mathfrak{B}_{\text{ref}}(R)} \|(\hat{P}_n - P_{\text{ref}})u\| \rightarrow 0. \quad (7.21)$$

This entry is *source-imported for convergence* and *proved here only as a compact-sublevel uniformization*.

E13. **Nonvacuum metric conclusion.** Define

$$\mathfrak{B}_{SU(3),\xi_{\text{ed}},n;O}^\perp(R) = \mathfrak{B}_{SU(3),\xi_{\text{ed}},n;O}(R) \cap \ker P_n. \quad (7.22)$$

Then

$$d_{\text{GH}}(\mathfrak{B}_{SU(3),\xi_{\text{ed}},n;O}^\perp(R), \mathfrak{B}_{SU(3),\xi_{\text{ed}},\text{ref};O}^\perp(R)) \leq \sqrt{\frac{R}{\beta}} \left(1 - \sqrt{1 - C_{\mathcal{V}} C_{SU(3),\xi_{\text{ed}},O} \varepsilon_n^\alpha}\right) + \rho_n(R). \quad (7.23)$$

This is the first fully populated ground-compatible Gromov statement for the regional test branch.

The populated ledger has an important negative conclusion. It does not produce a new value for the physical coercive constant. It gives a metric stability theorem for bounded sublevels of the already specified physical stiffness form. The off-ground floor, if used later, is still imported from the coupled electric–magnetic coercivity estimate on the same branch.

7.5 Failure modes and certification gates for the test case

The populated exact-dilation ledger gives a clean first test case, but it also identifies the ways in which the argument can fail when transported to a less rigid branch. The point of this subsection is not to add hypotheses silently, but to mark the certification gates that must be closed before the metric statement may be used as a statement about the physical stiffness.

- F1. Wrong source packet, wrong constraint carrier, wrong Fredholm fiber, or wrong form.** The theorem applies to sublevels of the physical stiffness form $Q_n = \widehat{q}_{SU(3),\xi,n;O}$ on the represented Hilbert carrier obtained from the recognized GAT 0 source packet, the admitted GAT II Klauder carrier, and the module-valued Grassmannian–Fredholm fiber, after the source normalization has been made. It does not apply to an unrelated source packet, an unconstrained ambient carrier, a different Klauder window, an auxiliary graph Laplacian, a magnetic witness operator, or a collar comparison form unless that carrier and form have first been matched to the source physical stiffness by the comparison maps. This is the most important gate:

$$\mathfrak{B}_n(R) = \{\psi : Q_n[\psi] + \beta\|\psi\|^2 \leq R\} \quad \text{must be built from the actual } Q_n. \quad (7.24)$$

- F2. Zero-defect versus vanishing-defect.** In the exact zero-defect orbit the transported forms agree:

$$a_n = a_{\text{ref}}, \quad \sigma_n = 0, \quad (7.25)$$

so the sublevels are isometric after applying J_n . In the larger vanishing-defect case, the theorem only gives convergence with modulus

$$\sqrt{R/\beta}(1 - \sqrt{1 - \sigma_n}). \quad (7.26)$$

Thus a zero-defect branch and a vanishing-defect branch should not be reported as the same statement. The first is exact isometry; the second is metric convergence controlled by the relative-form error.

- F3. Loss of unitarity.** The proof uses unitary straightening to place $\mathcal{H}_n$ and $\mathcal{H}_{\text{ref}}$ in a common Hilbert carrier without distorting the physical norm. If a comparison map is only coarse, partially defined, noninjective, or one-sided on bounded-energy families, the radial Hausdorff proof no longer applies unchanged. A nonunitary branch requires a separate distortion estimate of the form

$$|\|\psi - \phi\|_{\mathcal{H}_n} - \|J_n\psi - J_n\phi\|_{\mathcal{H}_{\text{ref}}}| \leq \delta_n(R) \quad (7.27)$$

on the relevant sublevel. Without such a metric distortion certificate, the relative-form estimate alone does not imply Gromov–Hausdorff convergence.

- F4. Failure of compact reference sublevels.** The Gromov–Hausdorff estimate is a compact metric-space statement. The regional theorem obtains compactness from compact resolvent of the reference operator, equivalently from compactness of the embedding of the reference form domain into $\mathcal{H}_{\text{ref}}$. If $\mathfrak{B}_{\text{ref}}(R)$ is not compact, the radial comparison still gives inclusions between sublevels, but it does not give a compact Gromov–Hausdorff target. A global infinite-volume carrier therefore requires a separate exhaustion or tightness argument; it is not covered by this regional certificate.

- F5. Ground projection mismatch.** The off-ground spaces use the complete coupled ground projection P_n , not a magnetic witness projection and not a pointwise vacuum-region indicator. The projection error

$$\rho_n(R) = \sup_{u \in \mathfrak{B}_{\text{ref}}(R)} \|(\widehat{P}_n - P_{\text{ref}})u\| \quad (7.28)$$

must tend to zero on the compact reference sublevel. If ground transport is known only weakly, or only after passing to graph-bounded moving families, then the off-ground Gromov estimate must be restricted to that same admitted class.

F6. **No new coercive constant.** The compact-sublevel theorem does not create the off-ground floor

$$Q_n[\psi] \geq \gamma_n \|(1 - P_n)\psi\|^2. \quad (7.29)$$

That floor must be supplied by the coupled electric–magnetic comparison on the same branch. The present note proves stability and geometric control of bounded sublevels; it does not prove the initial stiffness lower bound.

F7. **No Cheeger structure yet.** The metric spaces $\mathfrak{B}_n(R)$ and $\mathfrak{B}_n^\perp(R)$ carry the Hilbert metric and a stiffness constraint. They are not equipped here with a canonical branch measure, perimeter, or boundary operator. Therefore the present Gromov theorem is not a Cheeger theorem. A branch-relative Cheeger constant requires additional geometric data and a separate comparison from that boundary form into the physical Q_n .

F8. **Exhaustion drift.** The populated ledger is regional: $O \in \Sigma$ is fixed. On an increasing exhaustion O_j , the constants $C_{V,j}$, $C_{SU(3),\xi,O_j}$, β_j , and the compactness moduli may depend on j . A diagonal sequence can control every fixed region, but it does not automatically control families whose support escapes through the exhaustion. Global use therefore needs an additional no-escape certificate attached to the physical comparison maps.

F9. **Physical endpoint.** Even after sublevel convergence and even after a transferred stiffness floor, the source endpoint is the energy-squared identity

$$Q_{\text{thin}}[\psi] = \|P^0\psi\|^2, \quad K_{\text{thin}} = (P^0)^2. \quad (7.30)$$

The invariant-mass statement is a later consequence of the time-generator identity, forward-cone spectrum, and Lorentz covariance. It is not obtained from the Gromov–Hausdorff theorem alone.

These gates define the safe use of the note. The exact-dilation regional case closes the gates by design: the form is the normalized source stiffness, the comparison is unitary, the reference carrier has compact resolvent, and the complete-ground projection is transported in the same carrier. Any extension beyond that case must say which of F1–F9 is being replaced by a new certificate.

7.6 The spectral interpretation remains separate

The metric spaces constructed here carry the Hilbert metric and the stiffness sublevel constraint. They are not supplied with a canonical configuration measure or a perimeter. No Cheeger constant is defined for them in this note. A magnetic carrier estimate must still be related to the coupled physical stiffness before it contributes a spectral bound.

Where the source reconstruction identifies the reference limit with the physical thin stiffness, and where the physical time-generator theorem is available, (1.4) gives

$$\mathcal{D}(Q_{\text{thin}}) = \mathcal{D}(P^0), \quad Q_{\text{thin}}[\psi] = \|P^0\psi\|^2. \quad (7.31)$$

A transferred stiffness floor consequently yields an energy threshold. The subsequent invariant-mass statement uses the forward-cone spectrum and full Lorentz covariance in the represented category allowed by the source [2, Eqs. (10)–(11), Appendix A]. Neither covariance nor $Q_{\text{thin}} = q_{M^2}$ follows from convergence of stiffness sublevels; the latter is not the source normalization.

8 Direct cofinal physical completion

The preceding refinement analysis suggests a stronger endpoint than separate post hoc convergence of carrier, ground, and form. On a cofinal comparison subsystem for which the relevant maps are exact on the neutral physical data, the thin physical triple can be defined as one decorated completion. This section records that construction and its precise boundary.

Definition 8.1 (Compatible null-ground-form packet). Let $\mathcal{C}_*$ be a cofinal subsystem of bounded Wilson packets. At each object λ let $\mathcal{E}_\lambda$ be the pre-Hilbert or module carrier, $\mathcal{N}_\lambda$ its null subspace, $P_{\Omega,\lambda}$ the complete-ground projection, and Q_λ the source-recognized physical stiffness form. A comparison $T_f : \mathcal{E}_\lambda \rightarrow \mathcal{E}_\mu$ is *null-ground-form compatible* when

$$T_f^{-1}\mathcal{N}_\mu = \mathcal{N}_\lambda, \quad (8.1)$$

$$T_f P_{\Omega,\lambda} = P_{\Omega,\mu} T_f, \quad (8.2)$$

$$Q_\mu[T_f \xi, T_f \eta] = Q_\lambda[\xi, \eta] \quad (8.3)$$

for vectors in the common form domain. Charged representatives may compose projectively by a certified central unitary, provided that unitary preserves all three structures above.

Theorem 8.2 (Direct cofinal physical completion). *Assume $\mathcal{C}_*$ is cofinal and every admitted comparison is null-ground-form compatible. Assume also that one constant $\gamma_* > 0$ satisfies*

$$Q_\lambda[\xi] \geq \gamma_* \|(1 - P_{\Omega,\lambda})\xi\|^2 \quad (8.4)$$

for every object of $\mathcal{C}_*$. Then the algebraic direct limit of the null quotients

$$\mathcal{E}_{\text{alg}} = \varinjlim_{\lambda \in \mathcal{C}_*} (\mathcal{E}_\lambda / \mathcal{N}_\lambda) \quad (8.5)$$

contains a well-defined projection P_Ω and a well-defined positive form Q_{alg} determined by

$$P_\Omega[\xi_\lambda] = [P_{\Omega,\lambda}\xi_\lambda], \quad Q_{\text{alg}}[[\xi_\lambda], [\eta_\lambda]] = Q_\lambda[\xi_\lambda, \eta_\lambda]. \quad (8.6)$$

After Hilbert completion and form closure one obtains a canonical physical triple

$$(\mathcal{H}_{\text{thin}}, P_\Omega, Q_{\text{thin}}) \quad (8.7)$$

with

$$Q_{\text{thin}}[\psi] \geq \gamma_* \|(1 - P_\Omega)\psi\|^2. \quad (8.8)$$

Proof. Equation (8.1) makes the comparison maps descend to the null quotients and prevents the norm from depending on a representative. Equation (8.2) makes the complete-ground projection well-defined on equivalence classes. Equation (8.3) does the same for the sesquilinear stiffness form. Thus all three structures are present on the same algebraic limit before completion. Inequality (8.4) descends there verbatim. Completing the norm and closing the positive form preserves the inequality, giving (8.8). $\square$

Remark 8.3 (Projective carrier, strict physical shadow). If charged transports satisfy

$$T_g T_f = Z_{g,f} T_{g \circ f}$$

with $Z_{g,f}$ central and unitary, the charged/module carrier may retain a nontrivial associator. The direct physical completion above remains strict whenever

$$Z\mathcal{N} = \mathcal{N}, \quad ZP_\Omega = P_\Omega Z, \quad Q[Z\xi, Z\eta] = Q[\xi, \eta].$$

Thus observable or form-level single-valuedness does not imply cohomological triviality of the charged refinement system.

LEARNING CHECKPOINT

The null ideal, vacuum projection, and stiffness form are not three objects that are independently guessed after the limit. On an exact cofinal comparison subsystem they are part of the equivalence data defining the completion itself. The later spectral theorem must identify the descended P_Ω with the zero spectral projection of the thin time generator; it does not create the vacuum lineage retroactively.

CLAIM BOUNDARY

Boundedness of the Wilson-packet comparisons alone does not imply the direct completion theorem. Exact null compatibility and ground compatibility are required, and exact form compatibility must either hold on the selected cofinal subsystem or be replaced by a separate asymptotic/Mosco theorem. In the merely relative-form case, Q_{thin} cannot be defined by (8.6) without further work.

WHAT BECAME VISIBLE

The thin stiffness triple can be an invariant of the certified transport system rather than a structure reconstructed after the transport has been forgotten. This reframes the downstream problem: whole-nerve Mosco and spacetime reconstruction become identification and verification theorems for the same completed physical object, not independent attempts to manufacture its null space, vacuum, and stiffness form.

9 Conclusion and discharge map

The note now separates three jobs that should not be conflated. First, source recognition, constraint physicalization, and the Čech-module–Grassmannian–Fredholm realization identify the correct physical carrier and stiffness form Q_n . Second, the associator-selected Wilson sector supplies a branch-local magnetic mechanism:

$$H^3 \longrightarrow \text{tensor selector} \longrightarrow \Phi_n^{\text{mag}} \longrightarrow p_{0,n}^{\text{mag}} \longrightarrow L_n^{\text{mag},\perp} \longrightarrow \delta_n^{\text{mag}}.$$

The triangle calculation shows why the complete low projection and phase data are essential. Selector-sector Schur control and cofinal Riesz transport then determine whether this magnetic floor persists as a physical contribution. Third, the source UV–IR, target-complement, Hamiltonian, and seam ledger remains independently responsible for complete off-ground coercivity.

Only after those objects have been matched is Gromov control applied. A relative-form estimate places transported stiffness sublevels between radial rescalings of one compact reference sublevel and yields

$$d_{\text{GH}}(\mathfrak{B}_n(R), \mathfrak{B}_{\text{ref}}(R)) \leq \sqrt{R/\beta}(1 - \sqrt{1 - \sigma_n}).$$

Thus Gromov control stabilizes the geometry on which stiffness is measured; it does not manufacture the stiffness floor.

The remaining discharge items are therefore finite and explicit: selector survival through P_n^G and $P_{n,*}^K$; magnetic Gram nondegeneracy; complete low-cluster isolation; a carrier-specific ground-reduced Poincaré–Cheeger estimate; a uniform selector-sector Schur reserve; cofinal Riesz/form persistence; and whole-nerve identification of the direct cofinal physical completion $(\mathcal{H}_{\text{thin}}, P_\Omega, Q_{\text{thin}})$ with the reconstructed thin Yang–Mills system satisfying $Q_{\text{thin}}[\psi] = \|P^0\psi\|^2$. The invariant-mass conclusion remains a later Lorentz-covariant gate.

OPEN EDGE

The direct cofinal physical completion is now the natural endpoint of this notebook on an exact null-ground-form compatible subsystem. The remaining downstream task is to identify that same completed triple

$$(\mathcal{H}_{\text{thin}}, P_\Omega, Q_{\text{thin}})$$

with the whole-nerve/spacetime thin Yang–Mills reconstruction, while retaining the persistent associator on the charged/module side and without identifying the associator-selected magnetic subform with the whole physical stiffness form. The merely relative-form case still requires the separate comparison–Mosco passage.

A A spectral compactness certificate

For reference, the compactness input has a direct finite-mode expression. Suppose K_{ref} has compact resolvent on an infinite-dimensional carrier, and let

$$K_{\text{ref}}e_j = \lambda_j e_j, \quad 0 \leq \lambda_1 \leq \lambda_2 \leq \cdots, \quad \lambda_j \longrightarrow \infty,$$

with every multiplicity retained. For $u = \sum_j u_j e_j \in \mathfrak{B}_{\text{ref}}(R)$,

$$\sum_j (\lambda_j + \beta) |u_j|^2 \leq R, \quad \sum_{j>N} |u_j|^2 \leq \frac{R}{\lambda_{N+1} + \beta}. \quad (\text{A.1})$$

The finite-dimensional projections are bounded and the tails vanish uniformly. This proves total boundedness; closedness of the form completes the compactness argument. In finite dimension the compactness is immediate. No numerical off-ground floor enters this certificate.

Compactness is stronger than a gap. For example, on an infinite-dimensional Hilbert space with a unit vector e_0 , let P project onto $\mathbb{C}e_0$ and let $K = 1 - P$. Then $Q[u] = \|(1 - P)u\|^2$ has an exact off-ground gap of one, but the sublevel $Q[u] + \beta\|u\|^2 \leq R$ contains an infinite orthogonal family of vectors of one fixed positive norm. It is not compact. Thus a global spectral gap alone cannot justify the compact-resolvent hypothesis used for the regional Gromov theorem.

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