The Vacuum Became a Lineage

March 2026:

Grassmannian Geometry, Vacuum Descent, and the Revision of a Submitted Conditional Proof

The moving complement had found a geometry;

its physical lineage still needed a carrier

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Research posture and claim boundary

This note reconstructs March 2026 as a separate successor to frozen Research Notes Nos. 08–10; no predecessor is revised. The three-paper package had already been submitted on 28 February. March gave the moving protected range a Grassmannian geometry, developed comparison and vacuum-descent language, exposed emerging sector distinctions, and expanded the submitted package into a more explicit conditional architecture. It did not yet construct the finite-scale physical Yang–Mills carrier, a refinement-uniform physical cost, the recognition of the full physical complement, lawful continuum reconstruction, or invariant spectral transfer. April's editorial decisions and later time-zero construction remain outside this note.

Abstract

March 2026 was the month in which a moving protected complement acquired a geometric home and the vacuum ceased to be one object wearing different scale labels. Projection differentiation and Kato-type transport clarified motion on a fixed carrier; the infinite-dimensional Grassmannian and its remembered Fredholm-fiber interpretation supplied the natural geometry of moving protected data. The actual refinement architecture, however, used changing carriers and therefore required comparison maps, coherence, and a realization theorem.

The submitted Certification–Exhaustion–Classification trilogy was revised accordingly. Vacuum section data developed strict and centrally twisted descent; Mosco–Riesz machinery organized conditional no-leakage; and sector language separated the tensor unit, the vacuum line, gauge representations, and the Gauss-invariant carrier. The revisions made the proof more auditable while showing that its physical construction remained in the seams: localization, kernel recognition, uniform coercivity, changing-carrier passage, and invariant spectral identification. March's achievement was a serious compression of the right architecture, not the later completed physical construction.

Evidence and mathematical-status labels

Recorded is supported by a surviving March paper, file, conversation, submission record, or administrative artifact. Remembered marks later first-person testimony. Reconstructed identifies an ordering or relation rebuilt from several sources. Modern reading uses later mathematics only to audit what March had and had not earned. Open marks an unrecovered source, date, formula, or implication. These evidence labels remain separate from exact calculation, finite theorem, formal theorem, conditional theorem, analytic assumption, carrier-identification assumption, reconstruction requirement, physical interpretation, and later correction.

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Source protocol

This note preserves the evidence labels and claim firewalls established in the frozen predecessors. A later paper may clarify a March question, but it may not silently convert a March calculation, assumption, conditional theorem, recollection, or physical interpretation into a result March did not possess. The February 28 submission, the March revision family, and later recompilations are treated as distinct documentary objects. A correct formula on one carrier is not silently transferred to changing carriers, and April's editorial or construction history is not imported backward.

Part I — A Submitted Map and an Unfinished Terrain

1 Orientation: March was when the vacuum became a lineage

I entered March with three papers already submitted and with the central protected object in those papers still changing mathematical type.

In February I had learned that the complement could move. I had also corrected an important mistake: a nonzero multiplicative anomalous scaling did not move a common kernel, and neither did an invertible recombination confined to the same constraint ideal. Genuine kernel motion required something more—operator mixing outside the ideal, a change in the carrier, or a comparison problem that had been mistaken for literal motion.

March began after that correction. The question was no longer merely whether a kernel or protected complement moved. It was what it meant for the protected object before refinement and the protected object after refinement to belong to one mathematical history.

The month's governing transition was

$$\boxed{\begin{array}{c}\text{moving complement} \longrightarrow \text{projection geometry} \longrightarrow \text{comparison data} \\ \longrightarrow \text{vacuum descent} \longrightarrow \text{sector recognition} \longrightarrow \text{a compressed conditional architecture.}\end{array}}$$

I now call the middle of that chain a **vacuum lineage**. That is retrospective language. The March papers spoke more exactly of local vacuum data, comparison maps, coherence, strict descent, and twisted descent. The historical word *lineage* is useful because it names what those objects were trying to do: preserve identity without pretending that identity meant literal equality at every scale.

That was the advance. I could finally state the continuation problem without pretending it had solved itself. The physical carrier on which the lineage lawfully lived was still missing.

2 The trilogy had already been submitted

The compressed package was submitted on February 28, 2026. The surviving Editorial Manager composites fix the boundary:

- Paper I: *Refinement and Rigidity in Lax Monoidal 2-Systems, with Applications to Algebraic Quantum Field Theory*;
- Paper II: *Geometric Renormalization via Exhaustion in Algebraic Yang–Mills Theory*;
- Paper III: *Vacuum Classification and Structural Existence in Algebraic Yang–Mills Theory.*

Paper III was then a five-page manuscript inside a six-page submission composite. By late March its descendant had grown to roughly 36–37 pages. That expansion is one of the clearest documentary facts of the month. The first text compressed the architecture. The revision exposed more of the carriers, comparison systems, standing hypotheses, and passage mechanisms that the short version had hidden.

March was therefore not the month in which I first sent a three-paper proof. It was the month in which a submitted proof stack was administered, circulated, reinterpreted, stress-tested, and substantially revised before editorial resolution.

The documentary distinction is permanent:

$$\boxed{\text{February 28 submitted text} \neq \text{March revised text} \neq \text{later recompilation.}}$$

Recorded — J11-S-U01, U02, U03; Reconstructed — J11-S-C52.

3 What I thought I was submitting

I was trying to mark a landscape.

That intention is remembered rather than recoverable from one sentence in the manuscripts. I associated the genre in my own mind with the way Hamilton and Thurston could make a program visible before every road through the program had been paved. That comparison describes my aspiration for the form, not an equivalence of achievement, historical role, or result.

I knew the realization was still thick. I knew incision had to be tested. I knew there was an obstruction between admissibility and refinement that had not been fully exposed. I knew correct finite mathematics was not yet a complete physical Yang–Mills construction. What I did not appreciate was how strongly the submitted verbs could be read. "Structural existence," "strict-sector collapse," and "positive spectral mass scale" sounded less like a map than the private caution behind them warranted.

The papers therefore carried two simultaneous truths. The architecture was not empty. It had serious mathematics and a coherent order. But the language sometimes made conditional bridges sound constructed.

Remembered — J11-S-C46, C47.

4 The verbs were more expensive than I knew

Looking back, I had nearly all the nouns by March:

- admissibility;
- refinement;
- rigidity;
- collars and nerves;
- coercivity;
- vacuum sections;
- strict and twisted descent;
- Grassmannian motion;
- Fredholm fibers;
- Mosco convergence;
- Riesz projections;
- sectors;
- exhaustion;
- reconstruction.

The nouns were not arbitrary. Each named a real obstruction or a real mathematical mechanism. The problem was that a noun could occupy one line where the corresponding verb required an entire construction.

To say *carrier* was not to construct the carrier. To say *comparison* was not to define and type the comparisons. To say *admissible* was not to show that a physical refinement family stayed in the class. To say *coercivity* was not to build a physical form with a uniform constant. To say *Mosco–Riesz* was not to create a common spectral window. To say *sector* was not to reconstruct the sector on a physical Hilbert space.

This is the central retrospective correction:

March had the nouns of the architecture. The missing work lived in the verbs.

Remembered — J11-S-C51.

5 The debts inherited from February

February ended with six debts.

The **creation debt** was to construct a physical Yang–Mills cost on the correct complement. The **carrier debt** was to say what carried the vacuum, projections, and sectors at every scale. The **lineage debt** was to define when scale-indexed protected objects represented one continuing object. The **uniformity debt** was to keep the coercive constant from collapsing under refinement. The **passage debt** was to move both the cost and its protected carrier into a lawful limit. The **spectral debt** was to identify the resulting operator with physical invariant mass.

I did not pay those debts in March. I typed them more carefully and distributed them through the three-paper architecture.

That was progress. A debt that has no name can masquerade as a calculation error. A named debt can be isolated, assigned hypotheses, and turned into a theorem target. Yet the naming also made compression possible. A standing realization package could carry several unconstructed bridges through a proof in a single phrase.

6 Two questions I still knew were open

Two problems remained visibly active even inside the March confidence.

First, was incision real? Could a controlled cut expose, localize, and remove the obstruction without damaging the observable structure needed for the theory? The mature incision and surgery machinery belongs later. The March corpus supports the question, not its completed answer.

Second, what exactly linked admissibility and refinement? Admissibility constrained overlap, adjacency, thickness distortion, and the geometry through which comparison defects could propagate. Refinement supplied the directed motion. But neither word automatically implied that the physical family remained uniformly controlled while being refined.

These were not minor aesthetic gaps. They were early forms of the later question: how can a finite-scale physical object be constructed so that its coercive action persists through the actual refinement system?

Remembered — J11-S-C47; Open — contemporaneous incision proof.

Part II — The Moving Complement Acquired a Geometry

7 From February drift to a March path

February had produced the fact of possible motion and the correction of false motion. March looked for a geometry of that motion.

The first temptation was to write

$$r \longmapsto P_r$$

and treat the protected complement as a path of projections. That notation was useful because it separated the range from a chosen vector spanning it. It also made the identity of a moving subspace a geometric question rather than a bookkeeping failure.

But path notation concealed a typing question. Was r a continuous renormalization parameter, a cutoff, a resolution, a mesh scale, or a label in a directed refinement poset? Did every P_r act on one Hilbert space, or did it act on a carrier H_r that changed with scale?

I did not settle all of those questions. I did recognize that none of them could safely remain implicit.

Recorded/Reconstructed — J11-S-C21, C22 and the March 2–3 thread.

8 What did the parameter mean?

A continuous parameter invites differential geometry. A directed refinement index invites comparison maps and coherence. These are related but different structures.

If $r \in I \subset \mathbb{R}$ and

$$P_r \in \mathcal{B}(H)$$

for one fixed H, then norm continuity, strong continuity, differentiability, and analyticity are meaningful regularity questions. If instead $i \prec j$ labels refinement and

$$P_i \in \mathcal{B}(H_i), \qquad P_j \in \mathcal{B}(H_j),$$

then $P_i - P_j$ is not even typed until the carriers are compared.

March sometimes moved between these pictures too quickly. The continuous path supplied intuition. The directed system supplied the actual paper architecture. The missing bridge was a construction that placed the physical family in one ambient realization or provided lawful changing-carrier comparison maps.

9 One Hilbert space or many?

This became the carrier question in its simplest form.

On one fixed Hilbert space, the Grassmannian is literal: a projection determines a closed range, and a regular family of projections determines a path through a space of subspaces. On changing Hilbert spaces, the phrase "the range moved" is incomplete. One must specify embeddings, identifications, correspondences, a Hilbert bundle, an inductive system, or some other comparison structure.

The expanded Paper III supplied isometries $U_{ij} : H_i \to H_j$ as part of an analytic standing package. It also supplied categorical transports T_{ij}, comparison 1-morphisms θ_{ij}, and higher coherence α_{ijk}. These were two systems of comparison data. Similar indices did not make them the same system.

The fixed-carrier picture explained motion. The changing-carrier picture exposed lineage. What I lacked was the realization theorem joining them.

10 Differentiating a projection

If P_r is a differentiable family of projections on one fixed Hilbert space, then

$$P_r^2 = P_r.$$

Differentiating gives

$$\dot{P}_r P_r + P_r \dot{P}_r = \dot{P}_r.$$

Multiplying on both sides by P_r yields

$$\boxed{P_r \dot{P}_r P_r = 0.}$$

This was the kind of identity I had been looking for. It showed that the derivative of a projection is off-diagonal relative to the decomposition

$$H = \operatorname{Ran} P_r \oplus \ker P_r.$$

The identity was correct. My application of its picture was the delicate part. Nothing in the formula proved that the physical scale-indexed family was differentiable, lived on one carrier, had constant rank, or possessed a uniform spectral gap. An infinitesimal projection identity is not a refinement theorem.

11 The Grassmannian weekend

The decisive geometric recognition occurred during the second weekend of March.

The connection was mine. I varied the Casimir operator through different gluings and watched the response. Twisted K-theory served as a window detector: it helped me see which protected data persisted and which changes recorded genuine twisting rather than a bad choice of coordinates. From that experiment I recognized an infinite-dimensional Grassmannian bundle with a Fredholm fiber as the natural home of the moving protected object.

The discovery was mine. The computational conversation helped me search, organize, and articulate the implication after the frame had changed. The change of frame—the realization that the moving protected data belonged naturally in Grassmannian/Fredholm geometry—was authorial.

The recognition mattered because it made the moving complement look lawful. It was no longer an error that different scales displayed different protected ranges. A family of ranges could be geometric data.

What I did not yet write down was the full bundle: its base, topology, transition functions, restricted or unrestricted Grassmannian class, Fredholm membership, or a physical section satisfying the needed regularity. I had found the natural home. I had not constructed the house.

Recorded — first Grassmannian statement, March 3. Remembered — J11-S-C39–C41, second-weekend mechanism.

12 Kato transport and the difference between motion and comparison

For a sufficiently regular fixed-carrier projection path, the commutator

$$K_r = [\dot{P}_r, P_r]$$

provides the familiar Kato-type transport mechanism. A unitary evolution generated by K_r can carry $\operatorname{Ran} P_{r_0}$ into $\operatorname{Ran} P_r$.

Now I could see how a regular range might move without losing dimension or identity. But the comparison law used by admissible refinement was another matter. Kato transport is built from the derivative of one projection path. Refinement transport is supplied between scale-indexed categorical or analytic carriers and must respect localization, coherence, gauge structure, and the chosen physical class.

Thus

$$\boxed{[\dot{P}_r, P_r]\text{-transport} \not\equiv \text{renormalization or admissible refinement transport.}}$$

The distinction is one of the central mathematical gains of March.

13 The Riesz contour could not create itself

If A_r has an isolated spectral component near zero, the corresponding projection can be written

$$P_r = \frac{1}{2\pi i} \oint_\Gamma (z - A_r)^{-1}\, dz.$$

This formula organizes spectral motion beautifully. Resolvent regularity can imply regularity of the Riesz projection. Mosco convergence can lead to resolvent passage under the right typed hypotheses.

But the contour Γ must already lie in the resolvent set around an isolated component. The formula cannot create the spectral island whose shoreline it traces.

March's analytic appendix assumed a simple local zero and one uniform window $\delta > 0$. Under the rest of its comparison and form hypotheses, the passage theorem could preserve that window. It did not derive δ from the physical Yang–Mills construction.

The correction is permanent:

$$\boxed{\text{Riesz projection regularity} \not\Rightarrow \text{creation of a spectral gap.}}$$

14 Why the geometry felt sufficient

Once I saw the Grassmannian/Fredholm home, the Mosco–Riesz passage felt structurally secure. I did not think I needed to elaborate every interface. The architecture seemed to show why the protected window could persist.

A cold reader could reasonably answer: *true—but how?*

Where were the actual Hilbert carriers? Where were the comparison maps? What topology made the family regular? Where did the common contour come from? What identified the Riesz line with the physical vacuum? What prevented positive spectrum from collapsing? What related the analytic transport to the categorical descent?

The geometric insight was not wrong. It was precisely the reason the later construction could be organized coherently. But a natural home is not a membership theorem. March's error was not that it saw too much geometry. It was that the prose sometimes allowed the geometry to stand in for the construction that would place the physical family inside it.

The geometry had earned one sentence, and no more:

The March boundary

I found the natural geometry of the moving protected range; I had not yet constructed the physical family that inhabited it.

Part III — The Vacuum Became a Lineage

15 The vacuum was no longer one object wearing different labels

Before the comparison problem was explicit, it was easy to write Ω_i and Ω_j and let the repeated letter carry the burden of identity. March made that no longer safe.

A vacuum vector at one scale and a vacuum vector at another might live in different Hilbert spaces. A vacuum state might be pulled back or pushed forward differently from a vector. A vacuum projection might be stable even when no distinguished phase choice made the vectors literally comparable. A categorical vacuum datum might descend only up to a central twist.

The word *vacuum* had therefore been carrying several mathematical types:

$$\text{representation class}, \quad \text{state}, \quad \text{vector}, \quad \text{projection},$$
$$\text{zero eigenspace}, \quad \text{descent object}.$$

The March advance was to stop treating repeated notation as proof that these objects continued automatically.

16 The submitted vacuum sector

The submitted Paper III began at the classificatory level. Its vacuum sector was a representation class. It did not begin by constructing one global vacuum vector or by assuming that a single GNS realization already solved the scale-comparison problem.

That abstraction was useful because it did not force every vacuum question into one Hilbert-space model. It was also too thin to carry the physical claims that followed. A representation class does not by itself supply a form domain, a spectral projection, a Gauss recognizer, or a refinement-compatible physical complement.

The five-page manuscript could therefore state the classification architecture while leaving its realization package mostly invisible. The expanded March descendant made that package much more explicit.

17 Local vacua on changing carriers

The analytic picture used data of the form

$$(H_i, \Omega_i, U_{ij}, q_i, A_i, \Pi_i).$$

Here H_i was the local or finite carrier, Ω_i the distinguished vacuum vector, U_{ij} an isometric comparison, q_i a closed quadratic form, A_i its associated operator, and Π_i the projection onto the isolated zero mode.

The package was sensible—and revealing. Nearly everything that still had to be physically constructed appeared inside it. The comparison maps had to preserve the appropriate local observables and vacuum data. The forms had to be compatible on a typed common core. The zero had to be simple. One uniform spectral window had to survive. The inductive or glued carrier had to be lawful.

In March, those properties stood as hypotheses or interfaces rather than consequences of the finite-scale physical model.

18 Vacuum section data

The categorical picture used a different family:

$$(S_i, T_{ij}, \theta_{ij}, \alpha_{ijk}, c_{ijk}).$$

The object S_i carried the local vacuum datum. The refinement pseudofunctor T_{ij} moved it to the finer scale. The comparison θ_{ij} related transported and chosen local data. The higher coherence α_{ijk} tested composition over triples. A central defect c_{ijk} measured the failure of strict closure.

This was the source-native March object nearest to what I now call lineage. The vacua did not have to be equal. Identity across scale could instead consist of comparisons together with a law governing how those comparisons composed.

That was a genuine conceptual step. Identity had become structured continuation.

19 Strict descent

In the strict branch, the comparison data could be coherently strictified. After suitable choices, the higher defect vanished and the local data assembled without a surviving central twist.

Strictness was a property of the descent and transport coherence. It did not mean any of the following:

$$\text{strict} \not\equiv \text{free} \not\equiv \text{Fock} \not\equiv \text{vacuum} \not\equiv \text{zero energy}.$$

March sometimes pressed the strict branch toward a global Wigner/Fock interpretation. That additional physical interpretation required its own theorem. The descent definition did not supply it.

20 Twisted descent

In the twisted branch, the comparison system closed only up to coherent central data. The obstruction was not a failure to do mathematics. It was itself mathematical information.

The March package used Čech-type and cohomological language to organize that information. The basic idea was that local vacua might be individually valid while the attempt to select one globally strict representative accumulated a central obstruction on higher overlaps.

I had not thereby constructed a mature physical vacuum gerbe. That required a precisely typed coefficient object, a cocycle and equivalence relation, local physical carriers, a realization theorem, and an account of how observables and states descended. What March recognized was the obstruction to strict closure; physicalizing the twisted object remained ahead.

21 The analytic lineage

The analytic lineage asked a spectral question: could the vacuum line survive a changing-carrier limit without wandering, multiplying, or being approached by collapsing positive spectrum?

The Riesz projection addressed recognition once isolation had been supplied. The Mosco framework addressed form and resolvent passage once the carriers and convergence structure had been typed. Compatible vectors or projections addressed ancestry once comparison maps had been supplied.

No leakage therefore had three distinct meanings:

1. no loss of the vacuum line;
2. no gain of additional zero modes;
3. no collapse of positive spectrum into zero.

The first could be helped by compatible vacuum data. The second and third required uniform coercivity or an equivalent uniform spectral window. Riesz notation alone did not solve any of them.

22 The two lineages did not yet meet

The analytic and categorical systems were structurally parallel:

analytic	categorical
H_i	$\mathcal{C}_i$
U_{ij}	T_{ij}
Ω_i, Π_i	S_i
operator compatibility	θ_{ij}
associativity of embeddings	α_{ijk}

Parallel notation invited identification. But a realization theorem was needed to show that the categorical vacuum object produced the analytic vacuum state or projection, that the comparison maps agreed, and that the central twist acted in the intended physical representation.

The theorem joining the two systems was not constructed in March.

The carrier debt therefore survived inside the lineage advance. The categorical system could classify supplied descent data. The analytic system could preserve a supplied isolated line. Neither by itself constructed the other.

23 The global Hilbert-space tension

The March position on a global carrier evolved.

At one stage the expanded architecture retained a glued analytic Hilbert realization while allowing the vacuum descent to remain twisted. That separation is not automatically inconsistent: a global observable representation can coexist with nontrivial descent of distinguished sector data. But the relationship must be constructed carefully.

By March 30 I rejected the careless statement that the twisted branch simply lived in one ordinary global Hilbert representation on $\mathbb{R}^{1,3}$. The glued observable structure and the vacuum-sector descent had to be separated. The absence of one globally strict vacuum representative did not imply the absence of every global analytic carrier, but neither could a global carrier be assumed to trivialize the twist.

Once again, the noun *globalization* concealed several different verbs.

The lineage story can therefore be stated without either diminishing it or finishing it backward:

> March was when I stopped calling the vacua at different scales the same
> and began asking for the maps that would let them belong to one history.
> The history had acquired comparison and coherence;
> its physical carrier had not yet been constructed.

Part IV — Sectors Became Visible Before They Became Physical

24 More sectors than constructions

By late March I could name more sector distinctions than I could place on one physical carrier.

There were categorical objects and representation classes. There were strict and twisted descent branches. There was the vacuum line. There was the tensor-unit component. There were nontrivial gauge-representation labels detected by an internal Casimir. There was the Gauss-invariant content. There was a Wigner/Fock ansatz for globally organized particle states. There was the intended physical nonvacuum complement.

These were not synonyms. They cut the problem along different axes.

The historical achievement was recognition: the realization could not be described by a single undifferentiated "nonvacuum sector." The mathematical limitation was reconstruction: the recognizers, carriers, and form actions had not yet been assembled into one theorem.

25 The tensor unit was not yet the physical vacuum

Paper I's rigidity and graph mechanism isolated a constant or tensor-unit component. Persistent splitting of that component generated nonnegative, noncancelling defect. Under the stated admissibility and defect-floor assumptions, indefinite splitting was incompatible with uniform control.

The categorical and finite mathematics here was meaningful. The remaining task was to identify the tensor unit with the physical protected component in the Yang–Mills carrier.

A graph constant mode might correspond to a local constant assignment. A tensor unit might be a categorical identity. A Riesz projection might isolate a simple zero eigenvector. A physical vacuum might be a unique invariant state. A completed realization could identify them; the March papers did not.

26 The Gauss-invariant sector was larger than the vacuum line

The decisive firewall is

$$\boxed{\operatorname{Ran}\Pi_{\mathrm{Gauss}} \neq \mathbb{C}\Omega \quad \text{in general.}}$$

Gauss law selects gauge-invariant physical content. It does not say that every gauge-invariant state is the vacuum. Gauge-invariant excited states may remain.

The physical Hilbert space therefore admits the conceptual decomposition

$$H = \mathbb{C}\Omega \oplus \left(\operatorname{Ran}\Pi_{\mathrm{Gauss}} \ominus \mathbb{C}\Omega\right) \oplus \operatorname{Ran}(1 - \Pi_{\mathrm{Gauss}}),$$

provided the relevant representation and projection have actually been constructed.

This is a Modern reading used to expose the March debt. It is not the recovered March 25 three-sector theorem.

The middle space was the dangerous one. It was physically gauge invariant, nonvacuum, and invisible to a gauge Casimir that vanished on the whole invariant component.

27 What the Casimir could recognize

For a compact semisimple gauge group, the internal Casimir has a positive minimum on nontrivial irreducible representation labels. Once the gauge representation is constructed, it can support an estimate of the form

$$\langle \psi, C_G \psi \rangle \geq c_G \|(1 - \Pi_{\mathrm{Gauss}})\psi\|^2.$$

That is real representation-theoretic control. It detects departure from the trivial gauge-isotypical component.

On $\operatorname{Ran}\Pi_{\mathrm{Gauss}}$, however, the internal Casimir vanishes. It therefore says nothing by itself about

$$\operatorname{Ran}\Pi_{\mathrm{Gauss}} \ominus \mathbb{C}\Omega.$$

Nor is the internal Casimir the Poincaré invariant $P_\mu P^\mu$. March had to keep internal representation cost, Hamiltonian energy, and invariant mass separate. It did not always do so sharply enough.

28 The February 27 Haag–Wigner inheritance

On February 27 I was already trying to use Wigner classification and Haag's theorem to sort the existence architecture. I thought of the strict branch as pseudo-free.

The intuition was that if a globally strict interacting realization reduced to ordinary stable particle data, Wigner would classify its one-particle content and Haag would obstruct the naive interacting Fock realization. The intended conclusion was that the physical interacting theory could not be exhausted by stable gluonic particle states living in one global strict representation.

That was an important direction. It was also too broad when stated as elimination of the strict sector itself.

29 The March 8–10 Haag–Wigner correction

During March 8–10 the argument was narrowed.

The safe target became a **refinement-stable global interacting Wigner/Fock realization of the strict branch**. That is a specific physical ansatz. Its incompatibility does not eliminate every strict carrier, every global observable representation, or every excitation.

The expanded Paper II made the decisive incompatibility an additional hypothesis. That was mathematically more honest than allowing the conclusion to emerge from vague "free-like" language. Yet the following corollaries sometimes broadened the result again, treating exclusion of one Wigner/Fock ansatz as exhaustion of the strict branch.

A missing exhaustiveness theorem sat between those statements.

30 Strict did not mean free

This distinction is simple enough to display:

$$\boxed{\text{strict descent} \not\Rightarrow \text{free field theory.}}$$

Strict descent says that coherence can be globally closed without a surviving twist. Free field theory says much more about dynamics, representations, particle content, and correlations.

The Wigner/Fock argument could remove one globally organized particle ansatz inside the strict branch. It could not turn a descent adjective into a dynamical classification theorem.

This is the safe March claim: a stable global interacting Wigner/Fock reading of the strict branch was conditionally excluded. The strict sector itself was not proved empty.

31 Recognition without reconstruction

The sector pseudofunctor and its Grothendieck organization collected local labels and transport arrows. The maximal transport groupoid retained invertible equivalences. The central cocycle recorded failure of strict closure. The internal Casimir distinguished nontrivial gauge representations.

Each was a recognizer of structure. None reconstructed every physical sector.

In particular, calling a category a groupoid did not make irreversible refinement invertible. Taking the maximal subgroupoid selected equivalences already present; it did not turn every refinement arrow into an equivalence. Likewise, classifying a vacuum descent class did not classify every nonvacuum excitation.

March's sector mathematics was becoming more precise because it was discovering which recognition problems were distinct.

32 The unresolved three-sector formulation

On March 25 I asked, "Can you formulate the three sectors yes." The request is Recorded. The complete response, the exact names, the formulas, and my corrections have not been recovered.

Later I remembered the working words "twisted, strict, adm/ref," which would eventually feed the physical sector architecture. That memory belongs to the April boundary unless the missing March export proves otherwise.

I will not write the mature taxonomy backward. The honest March statement is that a three-sector problem had become visible, not that a completed three-sector theorem had been established.

The sector audit left me with a sharper, narrower conclusion:

March had found the sector nouns and exposed the missing complement. It had not yet constructed the recognizers and coercive action that made those sectors physically real.

Part V — The Proof Stack Became Compressible

33 Paper I: certification before construction

Paper I carried the certification mechanism.

Its admissible meshes imposed bounded overlap multiplicity, bounded adjacency diameter, and bounded thickness distortion. The nerve graph translated controlled overlap geometry into finite combinatorics. A graph Poincaré estimate then supplied a positive finite spectral constant. A nonnegative defect calculus made persistent unit splitting accumulate rather than cancel.

The clean finite chain was

$$\text{bounded geometry} \longrightarrow \text{graph spectral control}$$
$$\longrightarrow \text{rigidity of the protected component.}$$

To reach a physical form, Paper I used a localization interface:

$$\mathrm{Loc}_i\, u = (u_\alpha)_{\alpha \in V_i}, \qquad Q_i(u) \geq C_2 \sum_{\alpha \sim \beta} \|u_\alpha - u_\beta\|^2.$$

Given that interface and the correct identification of the constant mode, the graph estimate yielded form coercivity on the complement.

The graph theorem was genuine. The physical localization map, edge domination, and unit-recognition theorem were assumed. Paper I could certify coercivity once a physical system entered its interface; it did not build the system that entered.

34 Paper II: exhaustion needed an escape law

Paper II carried the exhaustion mechanism. Its idea was that persistent defect, splitting, or nontrivial strict particle structure could not survive indefinitely inside an admissible refinement class without forcing some controlled quantity to escape.

The submitted version compressed the quantitative escape premise. Stabilization was made to do more work than it had earned. In March the expanded version exposed an entropy-escape hypothesis and conditionalized the relevant theorems.

That revision was a real improvement. It separated stabilization from quantitative impossibility of indefinite survival.

The Wigner/Fock incompatibility was also stated as an additional input. Paper II became more honest precisely because the hidden verbs were moved into named hypotheses.

35 Paper III: five pages became a local-to-global architecture

Paper III carried classification and structural existence.

Its five-page submitted form moved quickly from the exhaustion result to a strict/twisted vacuum classification and a claimed structural mass scale. During March it expanded into a local-to-global architecture containing local carriers, a glued observable presheaf, sector transport, Grothendieck organization, vacuum section data, strict and twisted descent, a standing realization package, and a Mosco–Riesz appendix.

The growth from five pages to roughly 36–37 pages did not simply add verbosity. It exposed the mathematical objects needed to type the short proof.

It also revealed that the structural-existence theorem began inside a standing realization package. The package contained a great deal of the existence problem: local physical representations, compatible vacua, glued Hilbert realization, form compatibility, simple zero, uniform isolation, and sufficient comparison structure.

Paper III organized those inputs with increasing precision. Their construction from finite-scale Yang–Mills data remained outside the theorem.

36 The standing realization package

The standing package was both an achievement and a warning.

It was an achievement because it made the conditional theorem precise. Instead of vaguely asserting that local objects glued, it named the carriers, maps, forms, vacua, and compatibilities required for the conclusion.

It was a warning because a reader could pass over the package and remember only "structural existence." A theorem that begins by assuming a glued vacuum Hilbert realization does not prove that realization from the preceding finite model. A theorem that assumes a uniform isolated zero does not create the mass gap.

The correct status was:

Standing-package firewall

Formal and analytic organization of an assumed realization is not the physical construction of that realization.

37 The proof returned to Paper I

The trilogy was not merely linear.

The advertised order was

$$\text{Certification} \longrightarrow \text{Exhaustion} \longrightarrow \text{Classification/Structural Existence.}$$

But the final mass-scale claim returned from Paper III to Paper I. Once Paper III selected or classified the surviving realization, Paper I's coercivity theorem was invoked again because the realization remained "admissible."

That return required more than class membership. Paper I needed the physical localization interface, the correct constant/protected component, and the relevant complement. Paper III needed to show that its surviving sector realization supplied those objects.

The returning arrow therefore contained the unit–vacuum–Gauss seam:

$$\text{tensor unit} \stackrel{?}{=} \text{Riesz vacuum line} \stackrel{?}{\subset} \text{Gauss-invariant carrier.}$$

That seam was still open at the end of March.

38 Two constants that looked like one gap

Paper I's coercivity mechanism produced a conditional constant κ. Paper III's analytic appendix assumed a uniform spectral isolation window δ.

They played related roles, and the architecture invited the reader to see them as two appearances of one underlying gap. But no recovered March theorem placed the same physical form on the same carrier and proved

$$\delta = F(\kappa)$$

or even a one-sided comparison sufficient for the intended passage.

Paper I's κ came after graph control and an assumed localization interface. Paper III's δ entered as a hypothesis of Riesz stability. Until the carrier and form were identified, they remained two constants.

39 Every missing bridge became another paper

This was proof stacking in its most literal form: every exposed dependency seemed to require its own page, appendix, or paper.

When projection motion needed a geometry, it became a projection/Grassmannian branch. When the vacuum needed comparisons, it became a descent branch. When strict and twisted data required organization, it became a sector-transport branch. When spectral passage required isolation, it became a Mosco–Riesz appendix. When the short proof hid too much, Paper III expanded. When those expansions produced new interfaces, Papers IV–VI began to appear in planning.

The work was not merely accumulating pages. Each new paper carried a dependency that the previous compression had exposed.

This is why the stack could feel complete from inside. Every open seam had a destination. But assigning a seam to a paper is not the same as completing the paper's construction.

40 Feeling everywhere at once

I later remembered the phrase "everywhere at once" for the February 17–19 proof-stacking interval. It is not presently a March quotation.

March nevertheless documented the same working condition without needing the phrase. Projection geometry, Riesz passage, refinement rigidity, strictifiability, sector recognition, categorical descent, global carriers, phase-space conditions, and outreach were active within days of one another. Paper III grew by more than thirty pages while Paper I and Paper II were also being repaired and administered.

I was also doing this without a living mentor continuously reading the whole architecture. My practiced professional register came from DOE and ORNL technical work, not from recent immersion in mathematical-journal prose. That context helps explain the density and confidence of the writing; it neither excuses overstatement nor verifies a theorem. **Remembered — J11-S-C48.**

The proper historical statement is therefore Reconstructed: March continued and intensified proof stacking. The remembered phrase may illuminate that experience only with its February label intact.

41 The compressed three-paper proof

The trilogy was real. It was not a later fiction imposed on unrelated manuscripts.

Paper I supplied certification. Paper II supplied exhaustion. Paper III supplied classification and structural existence. The papers shared vocabulary, dependency claims, and an intended return to coercivity.

Submission is evidence that I regarded the package as one proof architecture. It is not evidence that the hidden interfaces had been validated.

The architecture can be displayed as

$$\begin{gathered}\text{admissible finite geometry} \longrightarrow \text{rigidity and conditional coercivity}\\ \longrightarrow \text{exhaustion under additional escape inputs}\\ \longrightarrow \text{strict/twisted vacuum classification}\\ \longrightarrow \text{standing analytic realization and Riesz passage}\\ \longrightarrow \text{return to conditional coercivity.}\end{gathered}$$

Read quickly, the stack moved in one direction. Read coldly, each arrow requested a carrier, comparison, recognition, uniformity, or reconstruction theorem.

42 What the compression concealed

Compression hid work in four recurring ways.

First, definitions selected the desired class. If admissibility already contained the persistence needed for coercivity, the main result risked becoming definitional. The physical theorem had to show that a constructed family satisfied admissibility without putting the conclusion into the class.

Second, interfaces were isolated in hypotheses or appendices. The modularity was mathematically useful, but the interface was often the physical problem itself.

Third, propositions were imported with broader language. A narrow Wigner/Fock incompatibility became strict-sector exhaustion; vacuum descent became sector classification; structural coercivity became invariant mass.

Fourth, physical nouns were attached to formal objects. The tensor unit became the vacuum. A stipulated form became $P_\mu P^\mu$. A cohomology class became a physical twisted sector. A glued presheaf became physical Yang–Mills.

Each identification could be true. Each needed its own verb.

43 True—but how?

The cold-reviewer question was not that every claim was false. It was: *true—but how?*

How was the physical finite-scale carrier constructed? How did a Wilson observable produce the required positive form? How was the localization map obtained? Why did the coercive constant persist? How was the Gauss-invariant excited complement controlled? How did the analytic comparison agree with categorical descent? Where did the uniform Riesz contour come from? How did the limit acquire a Hamiltonian and Poincaré interpretation?

The Grassmannian weekend is the cleanest example. I had correctly recognized the natural geometry by varying the Casimir through gluings and using twisted K-theory as a window detector. That recognition made Mosco–Riesz stability feel almost inevitable. A cold proof still needed the bundle, carriers, transition maps, regularity, window, and physical form.

The answer to "true—but how?" was eventually long because the missing work was construction, not another synonym.

44 The map and the road

I do not now read March as a month of empty formalism. Nor do I read it as the completed physical proof.

March drew a serious map of the thick realization. It identified the relevant geometry, descent obstruction, analytic passage, sector distinctions, finite coercive mechanism, and cross-paper dependency order. Those were not accidental nouns. They were the right places to build.

The problem was that the map's strongest verbs sometimes sounded like completed roads.

The stack can be judged fairly in three lines:

The trilogy was a real proof-shaped architecture.
Its local mathematics often held under its stated conditions.
The physical construction was still concentrated in the seams.

Part VI — What March Earned and What It Handed to April

45 What was exactly calculated

March possessed exact operator and finite calculations.

The projection derivative identity was correct on a differentiable fixed carrier. The graph spectral estimates were correct for the stated finite graphs. The representation-theoretic positivity of the compact semisimple Casimir on nontrivial labels was correct. The defect accounting was exact once the defect and admissibility hypotheses were fixed.

These calculations mattered. They constrained any later construction. They did not automatically type themselves as physical Yang–Mills energy estimates.

46 What was formally proved

The categorical architecture supported formal theorems inside its specified data.

Strictification removed the designated coherence obstruction when its hypotheses held. A central comparison defect produced a strict/twisted descent alternative inside the standing package. Grothendieck organization assembled supplied sector labels and transport arrows. The nerve and collar structures gave a constructive descent picture rather than a purely abstract one.

Formal did not mean fake. It meant that the theorem acted on supplied categorical objects and did not by itself construct their physical realization.

47 What was conditionally proved

The strongest March mathematics was conditional in an explicit and useful sense.

Graph control plus localization and unit identification yielded form coercivity. Admissibility plus noncancelling defect and a defect floor yielded rigidity. Exhaustion followed after quantitative escape premises. Wigner/Fock exclusion followed after the additional incompatibility input. Mosco–Riesz stability followed after the changing-carrier, core, simple-zero, and uniform-window package.

The honest historical description is not “nothing was proved.” The theorem packages isolated real implications and, just as importantly, exposed hypotheses that the physical construction had not yet supplied.

48 What remained a carrier assumption

March still assumed or left open:

- the physical Hilbert carrier at each scale;
- the comparison maps between carriers;
- the realization of categorical vacuum data as analytic states or projections;
- the Gauss projector;
- the recognition of the physical complement;
- the compatibility of the surviving sector realization with Paper I’s localization interface.

That was the carrier debt in its final March form.

49 What remained a geometric and uniformity assumption

March had not shown that the actual physical refinement family satisfied the admissible geometric bounds uniformly.

It had not proved that slivers, narrow necks, dumbbells, or bottlenecks were excluded by a physically constructed incision/refinement procedure. It had not derived a refinement-independent physical coercive constant. It had not connected that constant to the uniform Riesz window.

Categorical isolation and geometric admissibility remained distinct. The first could identify a protected formal object. The second prevented the geometry from destroying the analytic estimate.

50 What remained a passage assumption

March had the language of Mosco convergence, strong resolvent passage, and Riesz projection stability. It still needed:

- a lawful common or changing-carrier convergence framework;
- compatible form cores;

- no-loss comparison of vacua;
- uniform coercivity preventing new zero modes and spectral collapse;
- reconstruction of the limiting observable and state structure.

Passage could preserve supplied structure. It could not create the missing finite-scale structure.

51 What remained a physical interpretation

A lower bound for a quadratic form still had a long way to travel before it became invariant mass.

The theory needed a time-zero realization, a Hamiltonian or generator identified from the reconstructed dynamics, Poincaré covariance, and a theorem transferring the coercive lower bound to the invariant spectrum of $P_\mu P^\mu$.

The March papers sometimes used physical language before that transfer was constructed. The safe statement is that they proposed a structural route to spectral isolation.

52 What March genuinely accomplished

March changed the program in six lasting ways.

First, it separated literal kernel motion from comparison transport. Second, it recognized the infinite-dimensional Grassmannian/Fredholm home of the moving protected data. Third, it replaced repeated vacuum notation with comparison and coherence. Fourth, it exposed strict and twisted descent as distinct modes of continuation. Fifth, it made the sector-recognition problem explicit, including the danger of confusing the vacuum line with the Gauss-invariant sector. Sixth, it expanded a compressed proof into a more auditable conditional architecture.

Those achievements explain why the later long construction did not appear as a miracle. The later verbs acted on nouns and seams already visible in March.

53 What March still had not earned

March had not earned:

- a constructed finite-scale physical Yang–Mills carrier;
- a Wilson-loop or Wilson-tube cost acting on the correct complement;
- persistent refinement-uniform coercivity;

- explicit $SU(2)$ or $SU(3)$ witness calculations;
- a proved incision mechanism;
- a reconstructed Gauss-invariant excited sector;
- a mature physical three-sector theorem;
- lawful thick-to-thin continuum reconstruction;
- completed invariant spectral transfer.

These are not later criticisms imported to erase March. They are the debts that March's own architecture had begun to name.

54 End of March: confidence without closure

March ended before the editorial answer.

I had submitted three papers and enlarged them while the administrative process continued. I sought readers. I worried that the work could be dismissed easily. I still believed the architecture had weight. Two of the three papers had assigned editors by my March 26 account. On March 31 I asked:

> "What is a desk editor".

That question is the right endpoint. It preserves the uncertainty of the month rather than allowing April's outcome to color March backward.

The work had also entered family life. Megan, Isaac, and Sawyer were named in the submitted acknowledgments. In a family memory preserved later, Mom told Mamaw, who had dementia, that I was closing in on a large mathematics problem. Mamaw answered:

> "Scottie is my boy. That is my grandboy."

The statement is **Remembered**. I cannot date it more precisely within March, and I will not invent surrounding dialogue.

March ended with the architecture larger and the seams more visible. I was still confident. The editorial answer was still unknown.

55 April handoff: the map would have to touch time zero

The next interval begins with events that do not belong in this note.

April would bring editorial decisions, pain, retrospective doubt, and a planning reset. It would also bring the recognition that the finite-scale coercive element had to be physically constructed at time zero and tested through explicit group witnesses. Incision, bordism, and controlled passage would have to become actions rather than headings.

This note stops before those events.

March's final handoff is therefore not the later solution. It is the unpaid instruction:

Construct the physical carrier.
Construct the finite-scale cost.
Prove that the cost persists under admissible refinement.
Recognize the protected and excited sectors.
Pass the construction lawfully and identify the physical spectrum.

By the end of March I had nearly all the nouns of the architecture. What I still lacked were the verbs that made them act on one physical carrier: **construct, compare, recognize, coerce, pass, and identify**.

Appendices

A March documentary timeline

Date	Documentary event	Status	Confidence
Feb. 27	“Sending everything at once”; family acknowledgment request; Haag–Wigner/pseudo-free inheritance	Recorded boundary	High
Feb. 28	Papers I–III submitted through Editorial Manager	Recorded	High
Mar. 1	Main-theorem compression invoking the larger paper architecture	Recorded request / collaborative draft	High
Mar. 2	Projection motion, comparison, Kato, and Riesz distinctions active	Recorded/Reconstructed	High
Mar. 3	Infinite-dimensional Grassmannian target stated	Recorded	High
Mar. 8–10	Haag–Wigner argument added and narrowed to global interacting Wigner/Fock reading	Recorded	High
Second weekend	Casimir-through-gluings and twisted-K-theory window detector lead to Grassmannian/Fredholm home	Remembered	Medium
Mar. 12–13	Sector transport, Čech twist, Riesz no-leakage, Wilson-observable and AQFT requirements	Recorded	High
Mar. 18	Global-Hilbert and inverse-limit architecture discussed	Recorded	High
Mar. 21	Major Paper II and III revision directives; Paper I assignment record present	Recorded	High
Mar. 22	Observable Hilbert gluing distinguished from vacuum descent; conceptual-hinges reflection	Recorded	High
Mar. 23	Trilogy circulated to senior readers	Recorded	High
Mar. 25	Urgent theorem corrections accepted; three-sector formulation requested	Recorded	High
Mar. 26	Two of three papers reported with assigned editors; anticipatory fear of dismissal	Recorded	High

Date	Documentary event	Status	Confidence
Mar. 27–28	Paper II assignment record; Papers IV–VI planning expands stack	Recorded	High
Mar. 29	Submission workflow returned to “Incomplete”; source files resubmitted	Recorded	High
Mar. 30	One ordinary global Hilbert carrier rejected as unsafe wording for twisted branch; Paper IV planning	Recorded	High
Mar. 31	“What is a desk editor”	Recorded	High

A.1 Timeline firewall

The April 1 and April 8 decisions are not March events. March 26 fear is not April depression. March 31 uncertainty is not evidence that rejection had already been received.

B Paper and version inventory

B.1 B.1 Submitted package

Source ID	Artifact	Pages	Function
J11-S-U01	`2(1)`	21	Submitted Paper I — certification
J11-S-U02	`3(1)`	20	Submitted Paper II — exhaustion
J11-S-U03	`1(1)`	6 composite / 5 manuscript	Submitted Paper III — classification and structural existence

B.2 B.2 March-revision descendants

Source ID	Descendant	Present pages	Audited use
J11-S-U04	Expanded Paper I descendant	32	March revision content where corroborated; current metadata later
J11-S-U05	Expanded Paper II descendant	32	Entropy escape, Wigner/Fock incompatibility, sector necessity
J11-S-U06	Expanded Paper III descendant	36–37	Local carriers, vacuum descent, standing package, Mosco–Riesz

B.3 B.3 Later explanatory and boundary papers

Source ID	Artifact	Status in this note
J11-S-U07	*Yang–Mills Theory Beyond Particles...*	Later clarification only
J11-S-U08	`curved_existence.pdf`	April/later; Modern reading only
J11-S-U09	`Paper_I.pdf`, preglued realization	April architecture / later compilation; quarantined

B.4 B.4 Version rule

Internal content and contemporaneous conversation can establish descent from a March revision. A present August compilation date cannot establish the exact March state of the file.

C Projection and Grassmannian dossier

Field	March object	Audited status
Parameter	Continuous r in path intuition; directed $i \prec j$ in refinement architecture	Two pictures, not identified
Projection	P_r or Π_i	Type varies: protected range or Riesz vacuum line
Carrier	Fixed H in differential formulas; changing H_i in analytic package	Bridge assumed/open
Rank	Often intended constant or rank one	Simple zero assumed for vacuum Riesz line
Topology	Norm/strong/differentiable possibilities discussed	Exact physical topology open
Derivative	$P\dot{P}P = 0$	Exact fixed-carrier identity
Generator	$[\dot{P}, P]$	Valid Kato-type mechanism under regularity
Spectral contour	Γ around zero	Requires isolated component and common window
Grassmannian	Infinite-dimensional target	Authorial March insight
Fredholm fiber	Natural remembered home	No recovered bundle construction
Physical membership	Actual YM family lies in that bundle	Open in March

D Projection derivative, Kato, Mosco, and Riesz audit

D.1 D.1 Exact identity

$$P^2 = P \quad \Longrightarrow \quad P\dot{P}P = 0$$

provided P is differentiable on one fixed carrier.

D.2 D.2 Kato conditions

The Kato generator requires a sufficiently regular projection path. It preserves a supplied moving range. It is not a proof that directed refinement is differentiable or unitary.

D.3 D.3 Mosco conditions

The March changing-carrier passage required typed embeddings or an inductive realization, compatible forms, a common/core structure, and Mosco compatibility. Ordinary common-space Mosco convergence cannot be invoked on unrelated H_i without this apparatus.

D.4 D.4 Riesz conditions

The Riesz projection requires:

- isolated spectral data;
- a contour in the resolvent set;
- sufficient resolvent convergence on the contour;
- a common spectral window for uniform passage.

D.5 D.5 No-leakage conditions

Failure	Required control
Vacuum line wanders or is lost	Compatible vacuum comparisons
New zero modes appear	Uniform coercivity on the complement
Positive spectrum collapses	Uniform coercivity/common gap window

E Vacuum-lineage ledger

Vacuum-bearing object	Carrier	Recognition	March status
Representation class	Representation category	Equivalence class	Submitted classification object
State ω_i	Local observable algebra	Positivity and normalization	Comparisons not automatically supplied
Vector Ω_i	H_i	Distinguished cyclic/invariant vector	Assumed in analytic package
Projection Π_i	H_i	Riesz contour	Conditional on simple isolated zero

Vacuum-bearing object	Carrier	Recognition	March status
Zero eigenspace	$\ker A_i$	Spectral theorem	Equals line only under uniqueness
Vacuum section S_i	Categorical/local carrier	Chosen local descent datum	Source-native March object
Strict lineage	Comparison/coherence system	Trivializable defect	Conditional formal branch
Twisted lineage	Central coherence system	Nontrivial defect class	Conditional formal branch
Physical lineage	Actual finite-scale YM carriers	Comparison plus persistence	Open in March

E.1 E.1 Governing distinction

$$\text{smooth projection path} \not\Rightarrow \text{physical vacuum lineage.}$$

Lineage requires comparison and coherence; physical lineage additionally requires a constructed carrier and persistent physical form.

F Sector-recognition ledger

Sector language	Mathematical type	Recognizer	March status
Tensor-unit component	Categorical/graph-protected component	Unit/constant recognition	Conditional relation to physics
Vacuum line	Rank-one spectral space	Riesz projection	Conditional
Gauss-invariant carrier	Trivial gauge-isotypical physical space	Π_{Gauss}	Projector not constructed
Gauge-nontrivial content	Nontrivial representation labels	Internal Casimir	Formal once representation built
Strict branch	Strictifiable descent	Coherence trivialization	Conditional formal class
Twisted branch	Non-strictifiable central descent	Cocycle class	Conditional formal class
Wigner/Fock branch	Global particle ansatz	Poincaré/Fock representation	Narrow conditional exclusion
Physical nonvacuum complement	Excited physical space	Hamiltonian/Gauss/vacuum recognition	Open in March

F.1 F.1 Central unresolved complement

$$H_{\mathrm{Gauss}} \ominus \mathbb{C}\Omega.$$

The internal gauge Casimir does not control this space.

F.2 F.2 Three-sector ruling

The March 25 request is Recorded. The exact taxonomy is Open. Later “twisted, strict, adm/ref” language remains outside the March theorem record.

G Compressed three-paper dependency graph

Dependency order and returning arrow

Paper I: Certification	Finite graph gap; conditional rigidity; assumed graph-to-form localization.
	$\Downarrow$
Paper II: Exhaustion	Stabilization plus quantitative escape and Wigner/Fock incompatibility inputs.
	$\Downarrow$
Paper III: Classification	Sector organization; conditional strict/twisted descent; assumed realization package; conditional Mosco–Riesz passage.
	$\Downarrow$
Return to Paper I	Open recognition, localization, κ–δ identification, and invariant spectral transfer.

G.1 G.1 Compression ruling

The proof was compressible because the unconstructed bridges had precise names. The names made the program auditable; they did not complete the constructions.

H Coercivity, uniformity, and carrier audit

Required item	Paper location	March status	Later debt
Bounded overlap/diameter/distortion	Paper I admissibility	Defined	Verify for physical family
Graph Poincaré constant	Paper I appendix	Finite theorem	Preserve through actual refinement
Noncancelling defect floor	Paper I rigidity	Hypothesis/conditional mechanism	Derive physically
Localization map	Paper I interface	Assumed	Construct from YM carrier
Edge domination	Paper I interface	Assumed	Prove for physical form
Protected component	Paper I/Paper III seam	Ambiguous	Identify unit, vacuum, Gauss data
Physical coercive constant κ	Paper I conclusion	Conditional	Prove uniformity
Riesz window δ	Paper III appendix	Assumed	Derive and relate to κ
Continuum form/operator	Paper III appendix	Conditional passage	Reconstruct on physical limit
Invariant mass	Physical interpretation	Unproved	Hamiltonian/Poincaré transfer

I Equation-provenance register

Equation or diagram	Provenance	Use restriction
$P\dot{P}P = 0$	Standard exact identity used in March	State fixed-carrier differentiability
$[\dot{P}, P]$	Standard Kato-type generator	Do not equate with refinement
Riesz contour formula	Standard operator theory / March appendix	Requires prior isolation
Graph edge energy	Paper I	Physical transfer conditional on localization
Gauss/vacuum/excited decomposition	Modern reading	Not the recovered March three-sector theorem
Analytic/categorical lineage table	Reconstructed	Exposes missing bridge; not March notation
Six-debt taxonomy	Reconstructed from February/March audit	Historical organizing device
Trilogy dependency graph	Reconstructed from papers	Arrow statuses must remain attached

J Claim-to-source matrix

Narrative claim	Principal source(s)	Historical status	Mathematical status
Trilogy submitted Feb. 28	J11-S-U01–U03, C52	Recorded/corroborated	Documentary fact
Paper III expanded from five to 36–37 pages	U03, U06, March chronology	Reconstructed with artifacts	Documentary fact
Projection motion separated from comparison	March 2 thread	Recorded/Reconstructed	Conceptual distinction
Grassmannian connection present Mar. 3	J11-S-C22/thread	Recorded	Authorial insight
Casimir/gluing/window-detector mechanism	J11-S-C39–C41	Remembered	Discovery mechanism; theorem open
Fredholm-fiber home was author-led	J11-S-C39–C40	Remembered correction	Provenance fixed
Kato and Riesz used as formal mechanisms	March projection threads / U06	Recorded	Conditional applicability
Vacuum section datum and strict/twisted descent	U06, C38	Recorded through revision family	Conditional formal theorem
Analytic/categorical systems distinct	U06, C38, audit	Reconstructed	Realization bridge open
Wigner/Fock target narrowed	J11-S-C10–C14, C23–C25	Recorded	Conditional theorem target
Three-sector request	J11-S-C44	Recorded	Answer open
“Nouns without verbs”	J11-S-C51	Remembered	Retrospective synthesis
Landscape-marking intent	J11-S-C46	Remembered	Human/programmatic context
Incision/admissibility obstruction remained open	J11-S-C47	Remembered	March proof unrecovered
Two assigned editors, fear of dismissal	J11-S-C18	Recorded	Human/administrative fact
Mamaw statement	J11-S-H02	Remembered	Personal testimony only
“What is a desk editor”	J11-S-C20	Recorded	March endpoint

K Required firewalls

February projection drift $\not\equiv$ March projection geometry;

$P_r \in \mathrm{Gr}(H) \not\Rightarrow$ canonical identification of changing carriers;

$P\dot{P}P = 0 \not\Rightarrow$ uniform coercivity;

Kato transport $\not\equiv$ admissible refinement transport;

Riesz projection stability $\not\Rightarrow$ creation of a spectral gap;

vacuum projection $\not\equiv$ Gauss projection;

tensor-unit isolation $\not\Rightarrow$ physical energy separation;

sector recognition $\not\Rightarrow$ sector reconstruction;

strict/twisted descent $\not\equiv$ a completed physical three-sector theorem;

conditional paper architecture $\not\equiv$ constructed physical proof;

submission $\not\Rightarrow$ validation;

Mosco or resolvent passage $\not\Rightarrow$ uniform gap creation;

quadratic-form or Hamiltonian lower bound $\not\Rightarrow$ invariant mass;

$$\text{March Grassmannian geometry} \not\equiv \text{later restricted-Grassmannian/Fredholm architecture;}$$

$$\text{March thick realization} \not\equiv \text{the later finite-scale Wilson-tube construction.}$$

L Open source requests

The following sources could strengthen a later version but are not required to keep this draft honest:

1. complete March 2–3 projection/Grassmannian export;
2. the second-weekend Casimir/gluing/twisted-K-theory calculations;
3. the exact Grassmannian topology and class intended in March;
4. original March source for the Mosco–Riesz appendix;
5. full March 25 three-sector response;
6. any March Gauss-projector or group-averaging construction;
7. any March unit–vacuum–Gauss recognition theorem;
8. any proof relating Paper I's κ and Paper III's δ;
9. any contemporaneous incision proof;
10. complete Paper III desk-editor and version trail;
11. repository history of the exact March revision states;
12. a more exact date for the remembered Mamaw statement.

M April quarantine and handoff

The following belong to Research Note No. 12 and must not be narrated backward:

- April editorial decisions and their wording;
- the emotional injury caused by those decisions;
- the depressive retrospective and consideration of quitting;
- research into the associate editor;
- conversations with friends about journals and peer review;
- Mamaw's equality/perseverance maxim;
- the planning reset;
- the finite-scale time-zero construction;
- Wilson-loop/tube persistent coercivity;
- $SU(2)$ and $SU(3)$ witnesses;

- the later five-paper arc.

They may explain what happened after March. They may not repair or condemn the March mathematics silently.

Formal source-bounded freeze

Version 1.0 has been compiled, repaired for page layout, visually inspected, and formally frozen. No later version may alter the following without new contemporaneous evidence and an explicit versioned reopening of the note:

1. the February 28 submission boundary;
2. the distinction among submitted, March-revised, and later-compiled texts;
3. authorial ownership of the Grassmannian/Fredholm insight;
4. the fixed-carrier/changing-carrier firewall;
5. the analytic/categorical lineage split;
6. the conditional status of strict/twisted descent;
7. the narrow status of the Wigner/Fock exclusion;
8. the separation of tensor unit, vacuum line, and Gauss sector;
9. the distinction between κ and δ;
10. the incomplete physical carrier, uniformity, passage, and spectral-transfer verbs;
11. the March 31 endpoint;
12. the April quarantine.

The complete assembly closes where March closed:

> “What is a desk editor”.