The Gap Acquired a Geometry

February 2026: Projection Drift, Refinement Rigidity, and the Search for Coercivity

The gap had not been proved; its failure had become geometrically describable

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Research posture and claim boundary

This note reconstructs February 2026 as a separate successor to frozen Research Notes Nos. 08–09; neither predecessor is revised. February corrected the first projection-drift mechanism, separated admissibility from scale nuclearity, developed exact finite Casimir and graph controls, and made refinement degeneration a first-class coercivity problem. It did not yet possess the later physical Yang–Mills coercive construction, mature admissible-refinement certificates, thick-to-thin reconstruction, or invariant spectral transfer. Later mathematics appears only as an explicitly marked modern reading or correction.

Abstract

February 2026 was the month in which the question "why is there a gap?" acquired a geometry. The month opened by correcting a projection-drift argument: multiplicative anomalous scaling and invertible constraint mixing do not move a common kernel. The correction did not erase the underlying problem. It separated literal kernel motion from comparison between changing physical carriers and made the protected complement itself part of the theorem burden.

The work then moved through finite heat sums, compact-group Casimir isolation, scale nuclearity, admissible refinement, graph Poincaré inequalities, finite-element ideas, tensor-unit isolation, rigidity, and strict or gerbe-type descent. Exact finite mathematics showed how narrow necks, slivers, dumbbells, and growing same-scale diameter could collapse a coercive proxy. What remained missing was the typed bridge from graph, constraint, or categorical carriers to the physical Yang–Mills vacuum complement, together with uniform multiscale control, Mosco–Riesz lineage, and relativistic spectral transfer. February's advance was not a completed mass-gap construction. It was the discovery that creation of cost, preservation of cost, and identification of its carrier were distinct geometric and analytic problems. On February 27, Haag and Wigner became tools for auditing what "existence" could mean, and I provisionally read the strict branch as pseudo-free. The next day's exact submission packet preserved that pressure in a conditional three-paper architecture and a Haag–Wigner transport-groupoid proposal.

Evidence and mathematical-status labels

Recorded is supported by a surviving February paper, file, or conversation fragment. Remembered marks first-person recollection. Reconstructed identifies an ordering or relation rebuilt from several sources. Modern reading uses later mathematics only to audit what February had and had not earned. Open marks an unrecovered source, date, formula, or implication. These evidence labels are kept separate from finite theorem, formal theorem, conditional theorem, analytic assumption, geometric admissibility hypothesis, reconstruction requirement, physical interpretation, and later correction.

Version 1.1 revision record

This revision adds the February 27 Haag–Wigner and pseudo-free interval, replaces the February 28 package's date posture with exact Editorial Manager packet evidence, and records February's early fixed-thickness Wilson-tube terminology. It does not upgrade any February theorem. Version 1.0 remains a frozen predecessor; Research Notes Nos. 08–09 remain unchanged.

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Source protocol

This note preserves the source labels established in Research Notes Nos. 08–09. They concern evidence, not theorem validity. A later paper may clarify a February question, but it may not silently convert a February assumption, theorem attempt, recollection, or conditional architecture into a result February did not possess. A correct formula on one carrier is not silently transferred to another.

The supplied February corpus includes exact-day files, month-fixed papers whose present export metadata does not preserve the original day, detailed author-approved conversation reconstructions, partial fragments, and open source requests. Paper numbers are not treated as chronology because several independent February sequences reused them.

Part I — The inherited problem

1 Orientation: February was when the gap acquired a geometry

January had ended by forcing me to separate two questions I had previously allowed to travel together. One was the creation of a positive cost. The other was the passage of that cost, together with the object it protected, through scale and limit. February did not immediately solve either one. It changed what I understood each question to require.

At the beginning of the month, a finite lower bound could still seem like the central object. By the end, the more difficult questions were visible around it. Which complement carried the lower bound? Was that complement fixed, moving inside one Hilbert space, or represented on changing carriers? Which refinements preserved its meaning? Which geometric degenerations could make a positive proxy collapse? Was a graph constant controlling the same complement as the physical quadratic form? What maps transported the cost, and what theorem allowed the transported objects to survive a limit?

The governing transition was therefore not

$$\text{no gap} \longrightarrow \text{proved gap.}$$

It was closer to

$$\boxed{\begin{gathered}\text{conditional finite positivity} \longrightarrow \text{moving complements and carriers}\\ \longrightarrow \text{typed refinement} \longrightarrow \text{geometric degeneration}\\ \longrightarrow \text{graph or Poincaré control} \longrightarrow \text{the search for uniform coercivity.}\end{gathered}}$$

That change was decisive, but it was still an intermediate architecture. February's last-day packets already used fixed-thickness Wilson-tube terminology, but the mature Wilson-tube construction, the completed three-sector reconstruction, thick-to-thin passage, admissible refinement certificates, and invariant spectral transfer were not present in their later completed forms.

Source status

Status. Reconstructed from the February papers, the dated conversation reconstructions, and the frozen January endpoint. The statement is historical, not a February theorem.

2 The two gaps inherited from January

The inheritance from January can be stated without reopening the frozen predecessor:

gap of creation :	
	why does the physical vacuum complement carry a positive cost?
gap of passage :	
	why do that cost and the protected object survive scale and limit passage?

February subdivided these into a larger bill.

The creation question required a carrier, a projection, a form, and a lower bound. The uniformity question required a declared class of refinements and a reason the constant could not collapse inside that class. The passage question required comparison maps, compactness or no-escape, convergence of forms, and control of kernel projections. Descent required a decision about whether comparisons were strict, inner, central, projective, or higher-coherent. Relativistic interpretation required still more: a Poincaré action, a joint energy–momentum spectrum, and an identification of the constructed generator with the Hamiltonian or invariant mass.

Calling all of these *the gap* made the work sound simpler than it was. February's paper production can be read, in part, as the attempt to give each debt its own place.

Creation gap	Passage gap
Find a positive cost on the correctly identified physical complement.	Carry the complement, form, constant, and spectrum through refinement, limit, descent, and relativistic realization.

Source status

Status. The two-gap distinction is **Recorded** at January's frozen endpoint. The six-way subdivision is **Reconstructed** from the February corpus.

3 A note on sources, memory, and reconstruction

The February archive is unusually dense and unusually uneven. Papers preserve the architectures I was trying to stabilize; they rarely preserve the moment when I realized that an argument was failing and turned in another direction. Some papers have exact February file dates. Some conversations survive through detailed author-approved reconstructions. Four supplied PDFs are fixed by author correction as February papers even though their present export metadata does not preserve the original day. Other files—especially the point-gray calculations, the first SNC/collar-sparsity derivations, the exact OC1 estimate, and the original projection-drift source—remain missing.

The numbering of papers is not reliable chronology. There was a core Papers I–V series, a Papers A–C analytic series, a separate three-avatar realization paper, a proposed Poincaré Paper IV, and a February 28 package again numbered Papers I–III. In the appendices I use stable source identifiers instead of trusting the paper numbers.

The distinction between a document date and an export date matters especially here. Present metadata can show when a file was copied, reopened, or rendered. It cannot by itself establish when the mathematics was composed. Where the exact day is unknown, I leave it unknown.

The same discipline applies to later successful mathematics. A later theorem may explain why a February question mattered. It may not repair a February proof silently. The mature construction appears in this note only in **Modern reading** boxes whose purpose is to mark the distance still to travel.

Open source boundary

Open. The highest-priority missing records are the original `projection_drift_yangmills3.tex`, the complete February 3 conversation export, the original point-gray and finite-element files, the missing Paper C on central recognition, the complete February 13–26 conversation exports, and the original February 27 conversation or working note. The next day's submission packet now supplies documentary corroboration for the Haag–Wigner direction, but not the February 27 wording or sequence.

Part II — Projection drift and the moving carrier

4 February 1–3: projection drift returns

The first days of February returned me to a note on projection drift in Yang–Mills. The file is remembered under the approximate title `projection_drift_yangmills 3.tex`; the surviving conversation is associated with February 3. I do not yet know whether the calculation was first made then or whether an older calculation was reopened then.

The setup was a parameter-dependent family of constraints on a kinematical carrier:

$$\mathcal{H}_{\text{phys}}(\lambda) = \bigcap_a \ker C^a(\lambda). \tag{J10-E-001}$$

The parameter could be a renormalization scale, cutoff, resolution, or refinement level. The physical subspace was defined as a common kernel. The intuitive question was whether the subspace could drift as the constraint presentation changed.

At first I treated anomalous scaling as the mechanism. That was the error the February 3 audit exposed. The importance of the episode lies precisely in keeping the error and the structural question separate.

Source status

Status. Remembered/Reconstructed from J10-S-T03. The original LaTeX source and its creation date remain **Open**.

5 I thought the calculation was wrong

I had thought part of the projection calculation was wrong because I did not yet possess a satisfactory language for what was moving. The February audit showed that one part really was wrong.

Suppose the constraints at two scales are related by an invertible recombination,

$$C^a(\lambda) = U^{ab}(\lambda, \lambda_0) C^b(\lambda_0), \qquad U(\lambda, \lambda_0) \text{ invertible}. \tag{J10-E-002}$$

Then their common kernel does not change:

$$\bigcap_a \ker C^a(\lambda) = \bigcap_a \ker C^a(\lambda_0). \tag{J10-E-003}$$

The scalar case is even more immediate:

$$\ker(\alpha C) = \ker C, \qquad \alpha \neq 0.$$

Multiplicative anomalous scaling therefore does not produce projection drift. Neither does invertible mixing within the same constraint multiplet, once domains are typed consistently.

This was not a cosmetic correction. It removed the mechanism I had used. The right historical sentence is not that I had somehow been correct before I knew the language. I had encountered a structural discomfort before I could type it correctly. Once the bad mechanism was removed, the discomfort became a better question.

Mathematical status and firewall

Status. Finite calculation/Formal theorem on carrier J10-C-01. **Firewall.** Anomalous dimension alone does not imply projection drift.

6 The calculation was wrong; the transport question was real

The corrected evolution allowed two qualitatively different terms:

$$\frac{d}{d\lambda} C^a(\lambda) = M^{ab}(\lambda) C^b(\lambda) + \sum_I K^{aI}(\lambda) D_I(\lambda). \qquad \text{(J10-E-004)}$$

The first term mixes constraints within their existing span or ideal. When that mixing integrates to an invertible transformation, the common kernel is preserved. The second term introduces operator directions not already contained in that family. It is this outside-ideal term that can change the defining zero set.

Even here the statement must remain modest. Mixing outside the original constraint ideal makes genuine kernel motion possible; it does not prove that every such family moves, nor that the dimension of the kernel stays constant, nor that a smooth projection path exists.

The conceptual correction went further. Literal motion of a subspace inside one fixed Hilbert space was not the only possible problem. Different scales might use different physical realizations or carriers. In that case the question becomes whether one has a lawful comparison

$$T_{\lambda_2 \leftarrow \lambda_1} : \mathcal{H}_{\text{phys}}(\lambda_1) \longrightarrow \mathcal{H}_{\text{phys}}(\lambda_2). \qquad \text{(J10-E-005)}$$

The kernel could remain formally unchanged while the preferred identification of its realizations did not. Failure of equality and need for transport were different phenomena.

Source status

Status. **Reconstructed** from the February 3 conversation. **Mathematical status.** Outside-ideal mixing is a **remembered mechanism**; the comparison map is a **formal reconstruction requirement**.

7 What exactly was moving?

The word *vacuum* concealed several non-equivalent objects.

1. A vacuum vector could change phase or representative while the underlying state remained fixed.
2. A vacuum state could change while no preferred vector comparison existed.
3. The orthogonal complement of a vacuum line could move inside one fixed Hilbert space.
4. A spectral projection could vary because an isolated spectral island varied.
5. The whole carrier could change, so that apparent motion was really a comparison problem between different presentations.
6. A constraint kernel could change because the defining operators acquired new directions.

February did not yet possess one theorem that handled all six. The achievement was the beginning of their separation. A calculation about kernels could not be used automatically as a theorem about vacuum states. A comparison map between carriers could not be hidden inside equality notation. A statement about the orthogonal complement of one vector could not stand in for a physical sector reconstruction.

This distinction would become central to the later passage problem. To preserve a lower bound, I would have to know not only that a number stayed positive, but that the complement on which it acted had a lineage.

Modern reading

Modern reading. Under suitable closed-operator regularity and an isolated zero eigenvalue, a projection may be written as a Riesz projection,

$$P(\lambda) = \frac{1}{2\pi i} \oint_{\Gamma} (z - C(\lambda)^* C(\lambda))^{-1} \, dz. \qquad \text{(J10-E-006)}$$

This points toward continuity, differentiability, and connection-like transport. It requires a common or explicitly compared carrier, a stable contour, domain control, and resolvent regularity. February did not establish those hypotheses. The explicit interpretation of a regular path as

$$\lambda \longmapsto P_\lambda \in \mathrm{Gr}(\mathcal{H})$$

belongs to March unless an original February source proves otherwise.

8 A moving complement is not a bookkeeping nuisance

Once the protected complement is allowed to move, the lower-bound problem changes type. A statement such as

$$Q_r(\psi) \geq \kappa_r \|(1 - P_r)\psi\|^2 \tag{J10-E-010}$$

contains more than a positive number. It contains a carrier, a quadratic form and its domain, a projection, a comparison convention, and a range of scales or refinements over which the notation is meant to be coherent.

If (P_r) changes, then the assertion $\inf_r \kappa_r > 0$ has no stable meaning until the family of complements is typed. If the carriers change, the vectors being compared are not even elements of one ambient space without additional structure. If the form domains change, the inequality may not survive passage even when the numerical constants look stable.

This is why projection drift was more than bookkeeping. It made the protected object itself part of the theorem burden. February began with an error in the proposed mechanism and ended with a harder, correctly typed question:

$$\boxed{\text{What comparison law carries the protected complement and its cost together?}}$$

The answer was not yet available. But the question now belonged to mathematics rather than intuition alone.

Creation gap	Passage gap
No positive physical cost was created by the projection calculation.	The need to transport the protected complement became explicit.

Part III — Finite control becomes a proof stack

9 Why I kept returning to finite models

I recognized that correct finite mathematics was not yet physical Yang–Mills. I was asking what kinds of control over the fuller theory might be isolated in finite models, which of those controls could be made uniform, and which could survive a lawful passage.

The finite constructions were diagnostic instruments with different jobs.

- Compact-group heat sums tested representation growth.
- Casimir bounds tested the isolation of nontrivial representation content.
- Gauss penalties tested constraint satisfaction.
- Finite graphs tested how coercivity collapsed under necks and bottlenecks.
- Finite elements suggested local-to-global comparison maps.
- Mosco convergence tested the passage of already-constructed forms.
- Projection calculations tested the stability and comparison of protected complements.

The danger was not that I believed every correct finite calculation was already the physical theory. The danger was that, in the speed of February, I sometimes placed their distinct control roles too close together and wrote implication chains before the carrier bridges were available.

A finite model could tell me exactly how a constant failed. It could show whether a representation sum converged, whether a graph almost disconnected, or whether a constraint penalty vanished. It could not, without additional theorems, identify its form with the physical Hamiltonian or its complement with the physical nonvacuum sector.

Source status

Status. This motive is fixed by the direct author correction J10-S-A03 and supported by the February paper record. It governs the interpretation of every finite result below.

10 Heat, Casimir, and the search for a local cost

One genuine finite ingredient was representation-theoretic. For a fixed normalized compact semisimple group G, the nontrivial irreducible representations are separated from the trivial representation by the quadratic Casimir:

$$c_G := \inf_{\pi \neq \mathbf{1}} C_2(\pi) > 0. \tag{J10-E-011}$$

The associated weighted heat sum also converges for every fixed $\alpha \geq 0$ and $\beta > 0$:

$$Z_\alpha(\beta, G) = \sum_{\pi \in \widehat{G}} (\dim V_\pi)^\alpha e^{-\beta C_2(\pi)} < \infty. \tag{J10-E-012}$$

The proof is robust. Weyl's dimension formula gives polynomial growth in the highest weight, while the Casimir grows quadratically. Gaussian decay dominates every fixed polynomial factor. This part of the February mathematics was not mysterious.

The refinement question was harder. A proposed collar estimate had the form

$$\mathrm{Tr}\Big(P_{\mu \to \lambda} e^{-\beta H_\mu^{\mathrm{col}}/2}\Big) \leq Z_\alpha(\beta/2, G)^{m(\mu)}, \tag{J10-E-013}$$

where $m(\mu)$ counted the effective coarse-visible representation slots. On a fixed finite carrier this was plausible and often straightforward. Uniform refinement control required the separate estimate

$$\sup_{\mu \succeq \lambda} \Big\| P_{\mu \to \lambda} e^{-\beta H_\mu/2} \Big\|_1 < \infty, \tag{J10-E-014}$$

which depended on a correctly defined coarse-visible projection and a uniform slot or collar-sparsity theorem. That theorem was a target, not a recovered February result.

The other temptation was to read Casimir isolation as physical vacuum isolation. It was not. The tensor unit is isolated from nontrivial group representations. The physical Yang–Mills vacuum must also be isolated from gauge-invariant excitations. Those excitations live in the trivial gauge representation and are therefore invisible to the simplest Casimir separation.

Mathematical status

Status. J10-E-011 and J10-E-012 are **finite representation-theoretic theorems**. J10-E-013 is a **conditional finite estimate**. J10-E-014 is an **analytic target**.

11 Admissibility and SNC were different conditions

An important February correction was the separation of admissibility from scale nuclearity or SNC.

Admissibility organized the degrees of freedom across scales. It asked whether refinement maps existed, whether they respected localization and gauge recognition, whether comparison was functorial or coherently non-functorial, and whether the declared protected sector had a lineage.

SNC was analytic. It asked whether the phase-space growth visible through a buffered local inclusion or a compressed heat operator was controlled strongly enough to yield nuclearity or an appropriate compactness property.

The distinction can be stated simply:

> Admissibility organized degrees of freedom across scales; SNC constrained their effective phase-space growth.

Neither condition supplied the other. A beautifully coherent refinement system could have uncontrolled phase-space growth. A nuclear fixed-scale compression could exist in the absence of lawful interscale comparison. Most importantly, neither condition by itself created a positive cost on the physical vacuum complement.

The February Paper A, *Canonical Finite-Scale Coarse-Graining and Scale-Split Inclusions* (J10-S-P01), tried to derive a chain from compression to split inclusions and expectations. Its vector formula was of the form

$$\mathcal{H}_\lambda = \overline{M_\lambda(O)\Omega}, \qquad \Phi_{\mu\to\lambda}(X)\Omega = P_\lambda X\Omega. \tag{J10-E-036}$$

The later audit found several missing steps. Cyclicity could make the declared subspace equal to the ambient GNS space. A vector equation did not automatically define a bounded element of the smaller algebra. Compactness of an induced Hilbert-space map was weaker than the nuclearity needed for the advertised consequences. Split alone did not automatically provide the chosen vacuum-preserving expectation without the required modular hypotheses.

These are not reasons to erase the paper. They identify its historical job. February was attempting to build the analytic foundation needed for transport, and the attempt revealed that the words *compression*, *nuclearity*, *split*, and *expectation* each carried a separate theorem burden.

Source status

Status. The distinction between admissibility and SNC is **Remembered/Reconstructed** from J10-S-T13 and J10-S-T17. The formulas and claims of Paper A are **Recorded**; the deficiencies just listed are **Modern reading**.

12 February 13: every missing bridge became a paper

By February 13, the work had become a stack. I was no longer trying to make one calculation do everything; I was assigning the different burdens to different papers. The architecture was written schematically as

$$-1 \to 0 \to 1 \to A \to B \to C \to D \to E \to F.$$

The labels changed across drafts, but the jobs were recognizable: a constitutional paper fixing dependencies; a finite local-net skeleton; a recognition layer; an analytic gateway; transport and associators; a strict or twisted bifurcation; rigidity or freezing; spectral separation; and Mosco or continuum passage.

That was real progress. It prevented one paper from silently changing the hypotheses of another, separated structural data from analytic input, and kept finite spectral work apart from continuum passage. It also introduced a new danger: a missing implication could acquire a paper title before its theorem had acquired a proof.

The paper trail from this interval therefore has to be read at two levels. It records the decomposition of a difficult problem into honest subproblems. It also records how rapidly an architecture could look complete once every debt had a named place.

The analytic Papers A–C were not the same numbering family as the core Papers I–V. Nor was the later Poincaré Paper IV necessarily the Paper IV of the earlier sequence. By the end of the month there would be another Papers I–III package. Bare paper numbers are therefore almost useless historically.

Paper B, *Admissible Refinement Transport from Conditional Expectations* (J10-S-P02), attempted to turn expectations into coherent comparison. Under sufficiently strong uniqueness hypotheses, one would have

$$E_{\mu\to\lambda} \circ E_{\nu\to\mu} = E_{\nu\to\lambda}. \qquad \text{(J10-E-039)}$$

The same paper also introduced inner implementers measuring composite ambiguity. The later audit found a tension: if the expectations and their standard-form implementations were fully canonical and composed exactly, the source of a nontrivial associator had to be specified with much greater care. The intended reduction from inner ambiguity to central or gerbe-like data was assigned to a missing Paper C.

Source status

Status. Reconstructed from J10-S-T13 and **Recorded** in the surviving February Papers A and B. A paper node is not itself a proved arrow.

13 February 16: the three avatars

On February 16 the large stack suddenly looked smaller. The proposed mechanism compressed to three supports:

1. ultraviolet or refinement control;
2. compact-semisimple representation rigidity;
3. exact sector or kernel recognition.

The compression brought genuine intellectual relief. The analytic pipeline no longer looked like a bag of unrelated techniques. Refinement control was meant to prevent proliferation of independent interface channels. Representation rigidity supplied a nonzero Casimir scale away from the trivial representation. Recognition identified the protected zero sector. Heat trace, nuclearity, coercivity, and Mosco passage were then supposed to follow or transport the result.

The paper *A Refinement–Rigid Operator–Algebraic Construction of Yang–Mills Theory in 3+1 Dimensions* (J10-S-P04) is the clearest surviving realization of this architecture, although its exact February day is still open and its alignment with February 16 is provisional.

Its definitions also expose the central mathematical problem. It declares the gauge-invariant algebra as

$$\mathcal{G}_\lambda(O) = \mathcal{W}_\lambda(O)^G = \bigcap_{v,X} \ker G^X_{\lambda,v}, \tag{J10-E-015}$$

and then defines a quadratic form from the same Gauss generators,

$$Q_\lambda(\psi) = \sum_{v,a} \left\| G^{X_a}_{\lambda,v}\psi \right\|^2. \tag{J10-E-016}$$

For a gauge-invariant observable A, however,

$$Q_\lambda(A\Omega_\lambda) = 0. \tag{J10-E-017}$$

The form is a constraint penalty. It distinguishes gauge-variant directions from the Gauss kernel. It does not distinguish the vacuum from other gauge-invariant excitations inside that kernel. Consequently, the claimed lower bound on every vector orthogonal to the vacuum and the claimed one-dimensional kernel do not follow from these definitions.

This is the most important operator audit in the February corpus. It does not show that the finite work was useless. It shows that three operators had been placed too close together: an edge or configuration-space Casimir, a Gauss-constraint penalty, and the physical Yang–Mills Hamiltonian. Later work would have to separate them.

Source status

Status. The paper and formulas are **Recorded**. The vanishing calculation is a **finite calculation/Later correction**. **Firewall.** Gauge singlet does not mean one-dimensional vacuum.

14 Mosco was passage, not creation

The February 16 compression also clarified the role of Mosco convergence. Mosco was not one of the supports creating the positive cost. It was plumbing: a method for carrying already-constructed quadratic forms through a limit.

The conditional theorem has a clean shape. Suppose forms Q_λ converge to Q_∞ in the Mosco sense, suppose the protected projections converge without leakage, and suppose a uniform lower bound holds on the corresponding complements. Then the limit form inherits the lower bound. Schematically,

$$\begin{gathered}\text{uniform finite coercivity} + \text{Mosco convergence} + \text{stable complements} \\ \Longrightarrow \text{limit coercivity.}\end{gathered} \tag{J10-E-043}$$

This is a conditional reconstruction theorem, not a source of the finite constant. The February corpus did not supply a complete liminf proof, recovery sequences, a no-escape theorem, or strong convergence of the correctly typed physical projections.

One proposed limit formula was

$$Q_\infty(\psi) = \lim_\lambda Q_\lambda(P_\lambda \psi). \tag{J10-E-042}$$

But this notation suppresses the very issues February was learning to see: existence of the limit, comparison of domains, the meaning of P_λ on a common carrier, and the relationship between the protected finite sector and the protected limit sector.

The correction was conceptually valuable. A proof of convergence could not create a lower bound that the approximating forms did not possess. Passage and creation were independent debts.

Creation gap	Passage gap
The three-avatar realization did not yet produce the physical nonvacuum cost.	Mosco identified a route for carrying a cost, conditional on form convergence and stable complement lineage.

Part IV — Coherence branches upward

15 Directed RG chains and refinement loops

Almost as soon as the mechanism compressed, the architecture branched upward again. A directed renormalization procedure compares data along a chosen chain. A full refinement system also permits different paths between related resolutions. Comparing those paths creates a loop-level question that a single directed chain cannot see.

The slogan that emerged was:

$$\boxed{\text{RG is chain-level; refinement holonomy is loop-level.}}$$

This distinction organized several new proposals. A refinement path might carry an analytic coarse-graining map together with comparison data. Two composite paths might agree strictly, differ by an inner unitary, or differ by a central phase after passing to a sector. A directed RG subcategory could be trivialized while the full refinement category retained nontrivial holonomy.

The February branch was mathematically ambitious. It proposed a relative bridge between a directed RG subcategory and the full refinement system, and it explored higher coherence through pentagon tests and relative obstruction groups. It also risked becoming detached from the finite coercivity problem. Not every loop-level idea belonged in the proof of a physical gap.

The strongest source-bounded judgment is that February distinguished flow from holonomy. It did not construct the full higher-cohomological theory.

Source status

Status. Remembered/Reconstructed from J10-S-T17. The higher functors and relative classes were **candidate frameworks**.

16 Araki entropy was a witness

During this interval I remembered using the Araki entropy of renormalization curvature to show that admissible refinement paths did not commute. That phrase records the direction of the work, but it needs a mathematical correction.

Relative entropy can witness a difference between two already-defined transported states. Under the required support or faithfulness hypotheses, a Pinsker-type estimate gives

$$S(\varphi\|\psi) \geq \frac{1}{2}\|\varphi - \psi\|^2. \tag{J10-E-034}$$

If a Wilson observable witnesses a central phase,

$$\varphi(W) = \zeta\,\psi(W), \qquad \zeta \neq 1,$$

then

$$S(\varphi\|\psi) \geq \frac{1}{2}|\zeta - 1|^2|\psi(W)|^2. \tag{J10-E-035}$$

This is a useful analytic detector. It separates topology, witness amplitude, and operational admissibility. The phase can be discrete while the witness amplitude degenerates. A nonzero entropy bound then depends on both the phase and a nonvanishing observable response.

What entropy does not do is create the path inequivalence. The two path-transported states must first be defined on a common algebra and shown to differ. The entropy quantifies the already-typed difference.

Source status

Status. The entropy direction is **Remembered** in J10-S-TU1 and **Reconstructed** in J10-S-T17. The inequality is a **conditional analytic theorem**. The exact February definition of “entropy of renormalization curvature” remains **Open**.

17 How high did the obstruction tower go?

February contained two incompatible answers.

Paper B encoded an associator by inner unitaries and placed its obstruction in a degree-two object with nonabelian unitary coefficients. It also argued that no independent higher obstruction survived beyond that level. The February 17 branch, by contrast, proposed a hierarchy:

$$[\alpha] \in H^2(\mathsf{Ref}; Z(G)),$$

$$[\beta_{\text{bridge}}] \in H^3(\mathsf{Ref}, \mathsf{RG}; Z(G)),$$

and a candidate degree-four obstruction measuring failure to fill higher pasting data.

The degree-two Paper-B formula itself required more typing. A class written schematically as

$$[\omega] \in H^2(N(\Lambda), U(M_\lambda(O))) \tag{J10-E-041}$$

uses varying nonabelian coefficient groups. One must specify the coefficient system, its action, and whether the correct home is nonabelian cohomology, a crossed module, or a 2-group. The later reduction from inner ambiguity to a central presheaf was precisely the work assigned to the missing Paper C.

The candidate degree-four story required an equally important correction: ordinary cochains still satisfy $\delta^2 = 0$. A genuine higher obstruction must arise from extension and coherence data, not by declaring the cochain differential to fail.

I therefore leave the conflict visible. February had discovered that the height of the obstruction tower was itself a mathematical question. It had not settled the answer.

18 Poincaré-covariant completion remained separate

The February 20 outline for a Poincaré-covariant completion was an act of separation. It listed geometric pushforward of local scaffolds, exact compatibility with refinement, unitary implementation, strong continuity, the spectrum condition, and strict or centrally twisted covariance.

The outline was detailed enough to show what the finite spectral papers did not yet contain. A positive Dirichlet generator was not automatically the time-translation generator. A Hamiltonian lower bound was not automatically an invariant-mass lower bound. A local net did not become Poincaré covariant merely because its cells could be relabeled geometrically.

The intended relativistic endpoint was

$$P_\mu P^\mu \geq m^2(1 - P_\Omega). \tag{J10-E-049}$$

But this required a strongly continuous Poincaré representation, a joint translation spectrum, the spectrum condition, a typed vacuum projection, and an identification of the constructed quadratic form with invariant mass. None of these was proved by the fixed-scale Casimir or graph estimates.

The outline also exposed a signature problem that a compressed proof could hide. A positive Euclidean sum of squares of translation derivations is not automatically the Minkowski quadratic form $P_0^2 - \mathbf{P}^2$. Relativistic spectral transfer needed its own theorem.

Source status

Status. Recorded/Reconstructed proposed architecture from J10-S-T20. No completed February Poincaré theorem is claimed.

19 February 21: stripping the problem to its invariant core

On February 21 I recognized that one large construction was behaving like several papers. I began stripping away Yang–Mills vocabulary, local-net claims, coercivity avatars, and the refinement selector to ask what invariant remained.

A candidate was the complement Rayleigh quotient of a limit form,

$$\mathfrak{I} = \inf \left\{ \frac{Q_\infty(\psi)}{\|\psi\|^2} : \Pi_\infty \psi = 0,\ \psi \neq 0 \right\}.$$

Positivity of this quantity is equivalent to spectral separation at zero once the form, carrier, projection, and domain are already available. The nontrivial problem is to show that the quantity is presentation-independent, that Π_∞ is canonically selected, and that the invariant survives the refinement limit.

The necessity direction exposed a no-escape problem. A normalized sequence of finite near-zero-energy vectors can converge weakly to zero. A positive limit gap therefore does not, by itself, force uniform finite gaps. Tightness, stable kernel lineage, or a stronger compactness principle is required.

The same day included an aspiration toward a K-theoretic or index-theoretic relation with the gerbe contribution. I treat that honestly as an aspiration. February did not contain the later Fredholm, determinant-line, K/KK, or mapping-spectrum architecture as proved mathematics.

The strategic gain was the decision to ask first: what is invariant under refinement, and what minimal regularity makes spectral separation an invariant? Only afterward should particular Yang–Mills avatars be certified.

Creation gap	Passage gap
The abstract invariant named the desired cost but did not construct its physical realization.	Stable projection, no-escape, and presentation independence became explicit requirements.

Part V — The gap acquires a geometry

20 What counted as an admissible refinement?

The invariant discussion could not remain above the geometry for long. January's ultraviolet tightness had concerned compactness and control of high-frequency behavior. By February, *refinement* was becoming something more specific: a system of changing finite carriers together with maps, localizations, and declared compatibility data.

Several candidate meanings coexisted.

1. A refinement could be a directed relation in a poset.
2. It could be an algebra embedding or pullback of Wilson data.
3. It could include a normal completely positive map or conditional expectation in the reverse direction.
4. It could include comparison maps between local Hilbert presentations.
5. It could be restricted geometrically by overlap, aspect ratio, diameter, neck thickness, or cut connectivity.
6. It could be admitted categorically only when it preserved the tensor unit and the chosen weight or recognition data.

The word *admissible* therefore needed typing. An injective algebra map was not an isometry of quadratic forms. A completely positive map was not a unitary identification of sectors. A functorial pullback of observables did not automatically preserve a vacuum complement. A finite mesh refinement did not automatically preserve a coercive constant.

February's advance was to make geometric restrictions first-class rather than burying them inside a later uniformity claim. The unresolved question was whether those restrictions were additional hypotheses, consequences of stronger locality and bounded geometry, or part of the definition of the refinement category itself.

Source status

Status. Reconstructed from J10-S-T13, J10-S-T22, and J10-S-T26. No single February definition exhausted all uses of *admissible refinement*.

21 Finite diameter, bounded geometry, and excluded pathologies

The proposed geometric packages ranged from elementary graph bounds to finite-element regularity.

One early graph formulation asked for

$$\deg G_k(O) \leq \Delta(O), \qquad \operatorname{diam} G_k(O) \leq D(O). \tag{J10-E-023}$$

Bounded degree or overlap multiplicity is plausible under bounded local complexity. Uniform ordinary graph diameter is far more restrictive. On a fixed interval subdivided into 2^k pieces, the adjacency graph has degree at most two but diameter of order 2^k. Ordinary refinement therefore destroys a uniform same-scale diameter bound.

The more geometric package included quasi-uniform local scale, shape regularity, overlap thickness, bounded multiplicity, and a Lipschitz partition of unity. Such assumptions are familiar in finite-element analysis because they exclude long slivers and arbitrarily anisotropic cells. They also permit continuum Poincaré estimates to be transferred to discrete collar data.

Another proposal targeted cuts directly:

$$|\partial S| \geq \kappa_O |S|, \qquad 0 < |S| \leq \frac{1}{2}|V|. \tag{J10-E-024}$$

This condition forbids a macroscopic part of the graph from being attached through an arbitrarily small interface. It is closer to the actual bottleneck mechanism than bounded degree. Weighted variants are needed when cells represent unequal volumes or interface areas.

These were geometric admissibility hypotheses, not disguised physical gap theorems. They contained no Hamiltonian, Casimir, or mass. Their role was to say which finite degenerations were being excluded before one asked for a uniform coercive constant.

22 The category did not automatically pay the mesh bill

The category supplied a compelling organizing picture. A filtered monoidal system carried a distinguished tensor unit $\mathbf{1}$, and one could assign a nonnegative weight $\lambda(X)$ to objects. Tensor-unit isolation was written as

$$\delta = \inf_{X \not\cong \mathbf{1}} \lambda(X) > 0. \tag{J10-E-031}$$

The temptation was to argue that geometric degeneration creates soft nontrivial objects whose weights approach zero. If so, isolation would exclude bad meshes. Contrapositively,

$$\text{tensor-unit isolation} \Longrightarrow \text{geometric admissibility.} \tag{J10-E-033}$$

The first February proof attempt was not legal as written. It assumed that arbitrary mesh fluctuations could be transported through the refinement system with controlled seminorm. It also assumed that those fluctuations could be lifted to actual Wilson or refinement objects. Neither a bounded transport theorem nor the required surjectivity or density statement had been proved.

There was a deeper correction. I had not intended the categorical weight to be physical energy. The category might instead be defining which finite geometric objects were admitted at all. On that reading, slivers and pathological stacks could lie outside the category by definition. But exclusion by definition is an extra geometric input unless a separate theorem derives the category from more primitive locality data.

Three possibilities remained open at the end of the month:

1. geometric admissibility is an independent hypothesis;
2. bounded-geometry locality proves it;
3. a correctly constructed refinement category admits only geometrically finite objects.

February did not decide among them.

Firewall

Firewall. Categorical tensor-unit isolation does not imply geometric admissibility unless the realization and lifting theorem is proved. Categorical weight is not automatically physical energy.

23 SNC, point-gray, and Wilson–Haar coercivity

Several analytic strands converged on a local mean-zero estimate. The exact point-gray source has not yet been recovered, so I cannot responsibly reproduce its archival definitions or claim its first date. The surviving reconstructions place it near SNC, finite-element, and collar coercivity work.

The Wilson–Haar formulation was conceptually clean. A compact gauge action supplies a Haar conditional expectation

$$E_O(A) = \int_G \alpha_g(A)\, dg,$$

which, under vacuum invariance, extends to the orthogonal projection onto the fixed-point subspace in the local L^2 presentation. The quantity $A - E_O A$ is therefore the analogue of a mean-zero fluctuation.

The target estimate was a local Poincaré or Ward/REG inequality of the form

$$\|A - E_O A\|_{2,\omega}^2 \leq C_O \sum_{\mu=0}^{3} \|\nabla_\mu A\|_{2,\omega}^2.$$

In February this ceased to look like a mysterious Ward identity that produced a gap. It began to look like the operator-algebraic realization of geometric coercivity: Haar averaging selected the mean direction, while admissible overlap geometry controlled local fluctuations.

The local bridge was often called OC1. For neighboring collars one wanted

$$\|E_{k,\alpha}(A) - E_{k,\beta}(A)\|_{2,k} \leq C_O h_k \sum_{\mu=0}^{3} \|\nabla_\mu^{(k)} A\|_{2,k} + \delta_{\alpha\beta}^{(k)}(A).$$

The first term was meant to come from a chart-local path and the fundamental theorem of calculus. The defect term was meant to record failure of exact transport or descent. The strict branch hoped for a vanishing defect; the twisted branch allowed a controlled residual.

The exact February OC1 proof and uniform constants have not been recovered. Nor has the original point-gray calculation. Those absences matter. The Wilson–Haar picture was a strong geometric reorganization of the analytic gate, not yet a finished refinement-uniform theorem.

Source status

Status. Remembered/Reconstructed from J10-S-T22 and related imports. The archival point-gray and OC1 formulas are **Open**. The displayed OC1 expression is therefore an unnumbered historical target, not an archival February equation.

24 Slivers, narrow necks, and dumbbells

The title of this note is most literal here. A positive finite gap proxy can collapse because the geometry degenerates.

Take two large clusters connected by a narrow neck. Define a mean-zero function that is almost constant and positive on one cluster, almost constant and negative on the other, and changes only across the neck. Its L^2 norm remains macroscopic, but its edge energy is supported on very few links. As the neck becomes thinner relative to the clusters, the Rayleigh quotient approaches zero.

A long sliver produces a different version of the same effect. Adjacent values can change very slowly along a long chain, so the accumulated variance is large while each local difference is small. Stacked slivers can preserve bounded degree while making the global Poincaré constant deteriorate. A badly weighted mesh can create an equivalent bottleneck even when the unweighted adjacency graph looks harmless.

These examples explained why finite positivity at one mesh did not imply a uniform constant over every refinement. They also clarified that pathological geometry could enter in several ways:

- cut size could collapse;
- aspect ratios could degenerate;
- overlap thickness could vanish;
- adjacency diameter could grow;
- weights could concentrate;
- local comparison constants could depend on scale.

The exclusions had to be stated, derived, or separately typed. They could not be hidden inside the adjective *admissible*.

Mathematical status

Status. Finite examples/Geometric interpretation. The exact first February file containing each example remains **Open**. Modern Cheeger language is used only to clarify the mechanism.

25 Graph and Cheeger control

The small calculation at the center of the February 26 discussion was exact. Let $G = (V, E)$ be a finite connected graph with $n = |V|$, and let x_α be vectors in a Hilbert space. With

$$m = \frac{1}{n} \sum_\alpha x_\alpha,$$

one has the orthogonal decomposition

$$\sum_\alpha \|x_\alpha\|^2 = \frac{1}{n} \left\| \sum_\alpha x_\alpha \right\|^2 + \sum_\alpha \|x_\alpha - m\|^2. \qquad \text{(J10-E-020)}$$

On the mean-zero subspace, the graph Laplacian controls the variance:

$$\sum_\alpha \|x_\alpha - m\|^2 \le \frac{1}{2\lambda_2(G)} \sum_{\alpha\sim\beta} \|x_\alpha - x_\beta\|^2, \qquad \text{(J10-E-021)}$$

with the factor determined by the chosen edge-counting convention.

For a fixed graph this is standard finite mathematics. A bound using maximum degree Δ and diameter D, such as the crude estimate

$$\lambda_2(G) \ge \frac{1}{4}\Delta^{-(2D+1)}, \qquad \text{(J10-E-022)}$$

is positive on an explicitly restricted finite class. Cheeger-type bounds express the same issue through expansion or cut geometry. A narrow neck makes the Cheeger constant small and therefore permits the Laplacian gap to collapse.

The graph theorem answered a precise question: how does one control Hilbert-valued mean-zero data on a finite connected graph? It did not answer the physical question: why is a Yang–Mills excitation represented by such data, why does graph averaging correspond to the vacuum projection, and why does graph edge energy dominate the physical form?

26 What a discrete lower bound could and could not prove

To transfer the graph theorem to a physical or localized Hilbert carrier, one needs a reconstruction map $u \mapsto (u_\alpha)$. The February 28 rigidity paper included an upper localization estimate,

$$\sum_\alpha \|u_\alpha\|^2 \le C_1 \|u\|^2. \qquad \text{(J10-E-026)}$$

This is a Bessel or upper-frame bound. It says localization does not create arbitrarily large total norm. The coercivity proof, however, needs the reverse control

$$\sum_{\alpha} \|u_\alpha\|^2 \geq C_1^{-1} \|u\|^2, \tag{J10-E-027}$$

or an equivalent lower frame or reconstruction theorem. The printed argument used the upper bound in the reverse direction.

Two more bridges were missing. Orthogonality to the global tensor unit or vacuum must imply that the localized graph data have zero mean:

$$u \perp \mathbf{1} \quad \Longrightarrow \quad \sum_{\alpha} u_\alpha = 0. \tag{J10-E-028}$$

And graph edge energy must dominate, or be dominated in the correct direction by, the localized physical form:

$$\sum_{\alpha \sim \beta} \|u_\alpha - u_\beta\|^2 \lesssim Q(u). \tag{J10-E-029}$$

Only after these maps are proved can the graph estimate yield

$$Q(u) \geq \kappa \|u\|^2 \quad \text{on the protected complement.} \tag{J10-E-030}$$

This is the carrier bridge February did not close. A correct graph theorem and a correct finite localization map can still fail to control the same norm, mean, or form.

Firewall

Firewall. Graph or Cheeger positivity does not imply physical Yang–Mills coercivity without a typed carrier bridge.

27 February 26: same-scale diameter failed

The February 26 discussion began with the finite graph inequality and moved almost immediately to uniformity. Bounded overlap and degree looked geometrically plausible. Uniform adjacency diameter did not.

For genuine local refinement, a fixed physical path is represented by more and more same-scale cells. The fine graph diameter therefore grows even when the underlying region remains bounded. The refined interval is the simplest counterexample. The correction can be stated plainly:

uniform ordinary graph diameter is generally false under genuine local refinement.

This failure was useful because it showed that the wrong invariant had been chosen. Literal short paths inside every fine graph were not necessary if the refinement system provided controlled vertical comparison through scale.

The February 28 package nevertheless reused bounded diameter in its admissibility assumptions. I do not harmonize the two documents. The most plausible historical reading is that the pressure to compress the proof reintroduced a hypothesis that the February 26 analysis had already recognized as too strong for ordinary refinement.

Source status

Status. Remembered February correction from J10-S-T26 and **Recorded tension** with J10-S-RR1.

28 Multiscale refinement became the right question

The replacement question was not whether every fine graph had uniformly short horizontal paths. It was whether admissible refinement supplied uniformly controlled paths through scale:

$$\text{fine} \longrightarrow \text{coarse} \longrightarrow \text{coarse comparison} \longrightarrow \text{fine}.$$

The target could be written as a same-scale estimate

$$\inf_k \lambda_2(G_k(O)) > 0, \tag{J10-E-025}$$

but ordinary refined meshes often fail this. The more faithful target was a multiscale Poincaré inequality involving restriction, prolongation, or conditional-expectation maps. One wanted a mean-zero component to contract under controlled passage through the hierarchy, with refinement defects small or summable.

A schematic recursive estimate was

$$\|x - \bar{x}\|^2 \le A\,\mathcal{E}_k(x) + B\,\|T_{k\to k+1}x - \overline{T_{k\to k+1}x}\|^2.$$

If transport contracted the second term or if the accumulated defects were controlled, iteration might produce a refinement-uniform constant. February identified this theorem-shaped possibility but did not prove it.

The governing geometric insight was therefore:

$$\boxed{\text{local diameter may diverge while multiscale admissible control remains uniform.}}$$

This was one of the clearest answers February gave to the historical question. "Why is there a gap?" had become "What degeneration permits soft modes, and what geometry of refinement prevents that degeneration from surviving through scale?"

Creation gap	Passage gap
Graph geometry explained how a finite coercive proxy could collapse.	Multiscale maps were proposed as the carrier of uniformity, but the vertical theorem was still open.

Part VI — Rigidity, descent, and compressed closure

29 Tensor-unit isolation

The abstraction had now become a research problem in its own right. By February 25, a directed family of monoidal categories, functors preserving the unit, and a compatible nonnegative weight made it possible to ask whether the distinguished trivial object was uniformly isolated.

The number

$$\delta(\mathfrak{R}) = \inf_{X \not\cong \mathbf{1}} \lambda(X) \tag{J10-E-031}$$

was the candidate invariant. If $\delta > 0$, no nontrivial object could become arbitrarily soft according to the declared weight. A monoidal equivalence preserving the unit and the weight preserved this number. That was a clean formal invariance statement.

The difficulty was realization. What did $\lambda(X)$ measure? Was it representation weight, refinement defect, geometric complexity, a Rayleigh quotient, or energy? How did a nontrivial categorical object produce a vector in a Hilbert space? Why did the categorical unit correspond exactly to the physical vacuum rather than the larger gauge-singlet sector?

The strongest equivalence proposed during the conversation,

$$\delta > 0 \quad \Longleftrightarrow \quad \text{spectral separation},$$

was corrected before the end of the instance. Categorical isolation could organize the theorem, but analytic realization, stable kernel recognition, no-escape, and limit compatibility were additional hypotheses.

The abstraction was nevertheless significant. The question had changed from "does this model have a gap?" to "under what conditions is separation invariant across an entire refinement system?"

Source status

Status. Remembered/Reconstructed from J10-S-T25. The weight-preserving equivalence is a **formal theorem**; spectral realization is **conditional**.

30 Refinement rigidity, defect, and freezing

Refinement rigidity was the proposed mechanism preventing a nontrivial object or defect from becoming arbitrarily soft along a cofinal chain.

One version assigned a positive defect to persistent nontrivial splitting. If every nontrivial step cost at least $\varepsilon_0 > 0$, then an infinitely persistent splitting would consume an unbounded defect budget unless it stabilized or became inadmissible. Another version defined a detector-based curvature functional and sought two properties:

1. monotonicity or contraction under admissible completely positive transport;
2. discreteness of the values, supplied by finite-center or compact representation data.

The intended conclusion was the elementary rigidity principle

$$\text{eventually monotone} + \text{discrete range} \Longrightarrow \text{eventual constancy.}$$

This principle is correct under an actual discrete/no-accumulation hypothesis. The February papers sometimes moved too quickly from positivity or boundedness to stabilization. Positivity alone does not prevent a sequence from decreasing to zero. A monotone bounded sequence need not stabilize unless its range is suitably discrete. A per-step lower defect requires a separately controlled total defect budget.

The important historical achievement was the recognition of the missing ingredients. Uniformity would not come from one finite lower bound. It would come from a rigidity mechanism that prevented softening along admissible refinement.

Mathematical status

Status. The stabilization principle is a **formal theorem** under explicit discreteness. Its application to the February refinement defects was **conditional**.

31 Strict, inner, central, and gerbe-type comparison

February's descent vocabulary grew rapidly, and several notions were sometimes treated as if they were successive names for one thing. They were not.

- **Strict comparison:** two composites agree exactly.
- **Inner comparison:** they differ by conjugation with a unitary in a larger algebra or implementation space.
- **Central comparison:** the remaining ambiguity lies in a central coefficient system and acts by phases in factorial sectors.

- **Gerbe-type comparison:** local data glue only up to a central 2-cocycle or comparable higher coherence datum.

Paper B attempted to derive inner associators from compositions of conditional expectations. It did not construct the missing reduction from a varying nonabelian family of inner unitaries to a central presheaf. That reduction was promised in Paper C, which has not been recovered.

The local-to-global spectral inference was also incomplete. A central twist may leave a local differential expression or local Casimir form unchanged. It can still alter the global domain, boundary conditions, allowed sections, or multiplicities. Thus

$$\text{local form neutrality} \not\Rightarrow \text{global spectral equality.}$$

One February paper defined a twist energy

$$E_{\mathrm{tw}} := \inf \sigma(L_\infty^{\mathrm{gerbe}}), \tag{J10-E-047}$$

and elsewhere suggested exact equality of strict and twisted spectra. The two statements cannot be combined without a complete domain and descent theorem. February had located the habitat problem; it had not finished the spectral descent.

32 February 27: Haag, Wigner, and the pseudo-free strict branch

On February 27 I used Haag and Wigner to sort through the existence idea. The question was no longer only whether a positive operator or a refinement system could be written down. It was what kind of object would count as an interacting relativistic theory after the carrier, the vacuum, and the comparison maps had all been specified.

Wigner supplied one side of the audit: particle carriers are organized by irreducible unitary representations of the Poincaré group. Haag supplied the other: under the theorem's hypotheses, an interacting theory cannot simply be identified with a free Fock representation through the usual interaction picture. Neither result constructed Yang–Mills. Together they made it harder to hide an existence claim inside a convenient Hilbert-space presentation.

At the time I called the strict branch *pseudo-free*. I was trying to name a worry, not state a theorem. If strict descent returned one globally coherent vacuum and particle-like carrier, perhaps the branch had recovered something too close to the free organization to carry the interaction I was seeking. The non-strict branch, by contrast, seemed able to preserve interaction through a failure of global strictification.

That interpretation was premature. Strictifiability removes a coherence obstruction; it does not prove that a theory is free, pseudo-free, interacting, or physically existent. Non-strict descent does not prove interaction either. Haag's theorem is not a theorem that interacting local theories do not exist, and Wigner's classification does not provide a global colored one-particle carrier for Yang–Mills. The February value of the episode was diagnostic: it forced the existence problem to separate representation, interaction, strictification, and observable content.

The February 28 Paper III packet contains a section titled "The Yang–Mills Haag–Wigner Transport Groupoid" and describes the theory through a gerbe-refined transport groupoid rather than a single Hilbert space or particle spectrum. It is therefore documentary evidence that the Haag–Wigner pressure had entered the written architecture by the next day. The claim that the February 27 audit caused that exact formulation is a reconstruction, not a recorded causal fact.

Source status

Remembered. The date, the use of Haag and Wigner, and the phrase "pseudo-free" are fixed by direct author testimony J10-S-A04. The next-day Haag–Wigner transport-groupoid formulation is **Recorded** in J10-S-SUB3. Their causal relation is **Reconstructed**. The limits on what strictification, Haag, and Wigner prove are **Modern reading** firewalls.

33 February 28: the compressed three-paper package

The exact February 28 Editorial Manager packets gave documentary form to the sense that a highly compressed proof was near. Their recorded creation times fall between 11:22 and 11:30 that morning. Three papers now stood where many more had stood only days before: refinement rigidity, geometric exhaustion, and vacuum classification.

Paper I was *Refinement and Rigidity in Lax Monoidal 2-Systems, with Applications to Algebraic Quantum Field Theory*. Its cover letter called it the first paper in a three-part sequence and described the sequence as intentionally structural and conditional. Paper II was *Geometric Renormalization via Exhaustion in Algebraic Yang–Mills Theory*; it explicitly disclaimed universality and the existence of particular realizations. Paper III was *Vacuum Classification and Structural Existence in Algebraic Yang–Mills Theory*. It made its dependence on Papers I and II explicit and stated that it did not resolve the Jaffe–Witten problem.

The first used finite graph control to argue from tensor-unit isolation toward a uniform form bound. Its graph theorem was valid on the stated finite class. Its bridge to the global form was not: an upper localization estimate was used as though it were a lower frame bound, and unit orthogonality was not proved to become graph mean zero.

The second sought an exhaustion principle. Roughly, noncoercive behavior was meant to force either geometric degeneration or an entropy/defect signal. The proposed dichotomy was too strong without compactness and classification hypotheses. A bounded positive sequence can fail to stabilize without diverging. A defect can disperse or escape the chosen detectors.

The third classified strict and twisted vacuum structures under the preceding architecture. Its conclusion depended on the unproved central reduction, the exhaustion theorem, and the spectral-descent bridge.

Papers I and II also used the expression "Wilson-tube" for observables or nets at fixed thickness. This corrects an overly clean retrospective division: the vocabulary and an early fixed-thickness ansatz were already present in February. What was not present was the mature May–July construction: there was no completed thickness-removal theorem, physical coercive carrier, thick-to-thin reconstruction, or invariant spectral transfer. The early term is an ancestor, not the later construction under another date.

The package was not empty. It unified the month's central ideas:

$$\text{isolation} \to \text{admissible geometry} \to \text{coercivity} \to \text{rigidity} \to \text{vacuum classification.}$$

Its weakness was that nearly every arrow was carrying more than the preceding paper had proved. The February 26 diameter correction was also lost in the compression, as bounded same-scale diameter reappeared among the admissibility conditions.

Source status

Status. The submission, titles, sequence, disclaimers, and exact packet times are **Recorded** in J10-S-SUB1, J10-S-SUB2, and J10-S-SUB3. The manuscript contents remain cross-audited through J10-S-RR1, J10-S-GR1, and J10-S-VC1. The author remembers administrative work and a substantial technical revision to Paper III before desk-editor review (J10-S-A05); the exact revision diff is **Open**. The theorem-status audit is **Modern reading**.

34 The terminal spectral paper returned to the gluon question

The month-only February paper *Spectral Isolation and the Absence of Massless Gauge Particles* (J10-S-P03) returned the architecture to the physical question from which much of the program had grown: where are the gluons in the Poincaré irreducible-representation picture?

Its terminal claim was that invariant-mass coercivity would exclude massless Wigner representations carrying the relevant gauge degrees of freedom:

$$P_\mu P^\mu \geq m^2(1 - P_\Omega) \quad \Longrightarrow \quad \text{no massless gauge-particle carrier.} \qquad \text{(J10-E-050)}$$

As a conditional theorem shape, this expresses a genuine endpoint. As a February result, it depended on nearly every unpaid bridge in the stack: certification of the physical form, exhaustion of soft alternatives, stable vacuum classification, Poincaré covariance, joint spectral control, invariant-mass identification, and a typed map from gauge degrees of freedom to Wigner sectors.

The paper therefore belongs in the history as an endpoint aspiration, not as evidence that those layers had already been constructed. Its importance is strategic. It kept the full physical question visible while the intermediate mathematics multiplied.

Creation gap	Passage gap
Rigidity and categorical isolation proposed reasons a cost might resist softening, but the physical form was still not constructed.	Central descent, continuum passage, and invariant-mass transfer remained conditional.

Part VII — Human interval and historical judgment

35 Proof stacking: work, family, fatigue, and fidelity to the question

February was not a secluded mathematical retreat. The work took place around ordinary employment, Megan, Isaac, Sawyer, fatigue, and the practical obligations of family life. Looking back, the month feels compressed almost past recognition. I produced papers rapidly, rejected branches rapidly, and often discovered that an implication I had treated as one step was two or three different theorems. The papers show where an idea eventually landed; they do not always show the succession of "this is not working" moments that sent it there.

> *I was not everywhere in one proof. I was moving rapidly among several proofs that shared objects and vocabulary but had different theorem burdens.*

That distinction matters. Projection drift, SNC, graph coercivity, tensor-unit isolation, gerbe descent, Mosco convergence, Poincaré covariance, and invariant mass could all use words such as *sector*, *transport*, *rigidity*, and *gap*. Their shared vocabulary made the entire structure feel closer to closure than it was.

The repeated hostile-referee or "Clay attack" passes were part of how I tried to remain faithful to the question. I associated that discipline with the workmanship ethic I had learned from my grandfathers: whatever I did, I wanted to know that I had done it to the best of my ability. I also thought about the example the work might give my children—not an example of fame or prize-seeking, but of remaining loyal to difficult questions for the integrity and joy of pursuing them.

My faith belonged to that private register of fidelity. I sometimes experienced the work as tending something entrusted. That experience explains persistence; it is not a premise of the mathematics, and I do not ask the reader to share it.

The paper production itself is documentary evidence of compression. By the middle of the month there were constitutional, recognition, analytic, transport, rigidity, spectral, continuum, Poincaré, categorical, and physical-endpoint papers or outlines. By February 28 the compressed architecture had become a three-paper package.

The highly compressed proof felt near because nearly every necessary idea was on the page. What I could not yet see clearly was that several of those ideas belonged to different theorems.

Nearly every needed noun had acquired a paper. What remained unpaid were the verbs connecting them.

Source status

Status. Remembered through J10-S-T13, T16, T26, and TU1. No private family screenshot or invented domestic scene is used. Human context explains tempo; it is not evidence of correctness.

36 What February genuinely accomplished

February accomplished more than an inventory of gaps. It changed the kind of questions I was able to ask.

First, it corrected the projection-drift mechanism. Multiplicative scaling was removed, outside-ideal mixing was distinguished from invertible recombination, and literal kernel motion was separated from comparison between changing physical carriers.

Second, it made the protected complement part of the theorem. A lower bound could no longer be discussed responsibly without naming the projection, form domain, carrier, and comparison maps.

Third, it separated admissibility from SNC. Coherence across scales and control of local phase-space growth became independent analytic requirements.

Fourth, it isolated several exact finite mechanisms: compact-group Casimir separation, weighted heat summability, the Hilbert-valued graph Poincaré theorem, and explicit bottleneck degenerations.

Fifth, it recognized geometry as the source of uniformity failure. Slivers, necks, dumbbells, cut degeneration, and same-scale diameter growth explained how a positive finite proxy could soften.

Sixth, it separated categorical isolation from geometric admissibility and from physical energy. The failed attempt to derive one from the other was itself clarifying.

Seventh, it demoted Mosco to passage and separated Poincaré completion and invariant mass as later debts. A finite Hamiltonian-like generator no longer automatically carried the full relativistic conclusion.

Finally, February made creation and passage independently auditable. By the end of the month it was possible to ask of each arrow: what carrier does this act on, what theorem supports it, and is it creating a cost or merely transporting one?

37 What remained conditional

The physical creation gap remained open. I had not constructed a uniformly positive quadratic form on the correctly identified nonvacuum physical Yang–Mills complement. The Gauss penalty vanished on the gauge-invariant core. The Casimir separated nontrivial gauge representations, not gauge-invariant excitations from the vacuum. The graph theorem controlled mean-zero graph data, not automatically the physical complement.

Uniformity was a separate open theorem. No complete proof showed that admissible refinement excluded every soft geometric mode. The coarse-visible slot bound required for SNC was not recovered. OC1 and its scale-uniform constants remained unproved. The lower frame and unit-to-mean compatibility needed for the graph bridge were missing.

Passage remained open as well. Mosco convergence was proposed, but its liminf, recovery sequence, domain compatibility, no-escape, and projection-lineage hypotheses were incomplete. Strong convergence of the physical protected projections was not established.

The descent problem had not closed. Inner associators had not been reduced to a central coefficient presheaf. Paper B's degree-two truncation conflicted with the higher-obstruction branch. Local twist neutrality did not prove global spectral equality.

Nor had the relativistic interpretation closed. The Poincaré action, joint spectrum, spectrum condition, Hamiltonian recognition, invariant-mass transfer, and the map to Wigner particle carriers were separate proposed theorems.

Mosco was passage, not creation. Entropy was a witness, not the obstruction. The category organized isolation, but it did not automatically supply geometry or energy. Finite mathematics illuminated the physical problem, but did not become physical Yang–Mills by analogy.

38 End of February: a mechanism was becoming visible, but its carrier and range were not closed

At the end of February I could see a mechanism in pieces. A protected complement might move or require transport. A finite cost could live on representation, graph, or localized carriers. Degenerating geometry could produce soft modes. Admissible refinement would have to exclude or control those modes. Rigidity would have to prevent a positive defect from dissolving along scale. Mosco and Riesz methods would have to carry the form and projection. Poincaré reconstruction would have to turn the resulting generator into invariant mass.

That was not the mature construction. The carrier bridge from discrete geometry to the physical Yang–Mills complement was not closed. The admissible refinement range was not closed. The continuum and invariant-spectral passages were not closed.

The February accomplishment was not that the gap had been proved. It was that failure of the gap had become geometrically describable.

February made the creation gap geometrically intelligible,
but did not yet construct a uniformly typed physical cost;
it reorganized the passage gap,
but did not yet complete continuum or invariant-spectral transfer.

The calculation was wrong; the question was real. By the opening of March, the next problem was no longer merely whether a projection moved. It was how moving projections, vacuum lineages, and emerging sectors could be given a geometry without silently identifying carriers that the proof had not yet connected.

Appendices

A February documentary timeline

The chronology below distinguishes exact-day records, author-dated conversation reconstructions, month-only placements, and open intervals. Confidence is documentary confidence, not theorem validity.

Date or interval	Documentary statement	Confidence	Boundary
February 1–3	Projection-drift material returned; the February 3 discussion corrected the multiplicative-scaling mechanism and separated kernel motion from transport	High for the conversation reconstruction	Original file creation date open
February 4–12	Heat, Casimir, SNC, point-gray, finite-element, and refinement questions continued	Medium–low	Exact order and original files open
February 13	The large $-1 \to F$ proof stack and admissible-refinement architecture were articulated	Medium–high	Paper nodes do not prove their arrows
February 16	The proposed mechanism compressed to refinement control, representation rigidity, and recognition; Mosco became passage	Medium–high	P04 alignment is provisional
February 17	RG/coherence, entropy, pentagon, higher-obstruction, and instrument branches opened	Medium	Much of the branch was candidate theory
February 17–22	Proof stacking, feeling "everywhere at once," and compressed-proof nearness	Remembered	Exact day open
February 20	Poincaré-covariant completion was separated as its own paper/debt	High for outline	Not a completed theorem
February 21	The monolithic construction was separated into layers; invariant core and K/KK aspiration identified	High for reconstruction	No February index theorem
February 22	Category-to-mesh turn, GAT alternatives, OC1, pathologies, and legality audit	High for reconstruction	Exact OC1 source open
February 23	Finite-scale positivity and one-step transport papers	Exact file date	Positivity conditional; transport inequality defined, not proved

Date or interval	Documentary statement	Confidence	Boundary
February 24	Several architectures likely coexisted	Open	No event asserted
February 25	General refinement–tensor isolation program	Author-assigned date	Import contains an internal "early February" tension
February 26	Exact finite graph calculation; same-scale diameter correction; multiscale target	High	Vertical refinement theorem not proved
February 27	Haag–Wigner existence audit; strict branch provisionally called pseudo-free	Remembered exact day	Original conversation or working note open
February 28	Editorial Manager packets for the three-paper refinement-rigidity/ exhaustion/classification submission	Exact packet timestamps	Several proof seams and the later Paper III revision diff remain open
Month-only	P01–P04 and terminal spectral paper belong to February	Author-fixed month	Exact days open

A.0.1 Chronological tensions retained

1. **February 25 dating.** The imported reconstruction is assigned to February 25 but calls the conceptual stage "early February." The assigned date is retained; the tension is not resolved without an original record.
2. **Entropy.** The entropy-of-renormalization-curvature remembrance fits February 17–22. The exact day is open.
3. **P04.** Its three-avatar architecture aligns closely with February 16, but no original day has been recovered.
4. **Diameter.** February 26 rejects uniform ordinary diameter as generally false. The February 28 package reuses it. Both records are preserved.
5. **Obstruction degree.** Paper B truncates at degree two; the February 17 branch proposes relative degree three and a candidate degree four.
6. **Wilson-tube terminology.** The February 28 packets use fixed-thickness Wilson-tube language. This is documentary evidence for an early ancestor, not for the mature May–July construction.

B February paper and conversation inventory

B.1 Stable source register

Source ID	Source	Date posture	Principal use
J10-S-H0	Research Note No. 10 handoff	Retrospective control	Scope and firewalls only
J10-S-M09	Frozen Research Note No. 09, v1.0	Frozen January endpoint	Two inherited gaps
J10-S-FS1	*Paper I — Finite-Scale Yang–Mills*	February 23	Fixed-scale proxy and conditional positivity
J10-S-AT1	*Paper II — Admissible Transport Across Scales*	February 23	One-step transport and deficit notation
J10-S-AT1d	Apparent duplicate of AT1	February 23	Duplicate control only
J10-S-RR1	February 28 package Paper I	February 28	Refinement rigidity and graph theorem
J10-S-GR1	February 28 package Paper II	February 28	Exhaustion/entropy proposal
J10-S-VC1	February 28 package Paper III	February 28	Vacuum classification proposal
J10-S-IMG1	Unclassified screenshot	February 28	No current mathematical use
J10-S-C17	Partial conversation fragment	February 17	Coherence and overclaim boundary
J10-S-C20	Partial conversation fragment	February 20	Poincaré separation and paper layering
J10-S-C22	Partial conversation fragment	February 22	Tensor-unit isolation and geometry
J10-S-C25	Partial conversation fragment	February 25	Certification and graph bridge
J10-S-T03	Author-imported reconstruction	February 3	Projection correction and transport distinction
J10-S-T13	Author-imported reconstruction	February 13	Paper stack, admissibility, SNC
J10-S-T16	Author-imported reconstruction	February 16	Three avatars and Mosco role
J10-S-T17	Author-imported reconstruction	February 17	RG, entropy, pentagon, higher obstruction
J10-S-T20	Author-imported reconstruction	February 20	Poincaré completion outline
J10-S-T21	Author-imported reconstruction	February 21	Invariant core and no-escape
J10-S-T22	Author-imported reconstruction	February 22	Mesh geometry and legality audit
J10-S-T25	Author-imported reconstruction	February 25	General refinement–tensor theory
J10-S-T26	Author-imported reconstruction	February 26	Graph calculation and multiscale correction
J10-S-TU1	Author remembrance	February 17–22 provisional	Entropy, proof stacking, compressed-proof nearness
J10-S-PD-R	Later Mountain reconstruction	Retrospective	Projection history; no Grassmannian backdating
J10-S-TR1	Later PDE-to-graph reconstruction	Retrospective	Modern clarification of diameter failure
J10-S-A01	Author correction	Present	First three supplied PDFs are February, not August
J10-S-A02	Author placement	Present	Fourth construction PDF belongs to February corpus
J10-S-A03	Author correction	Present	Finite models were probes of control, not physical identification

Source ID	Source	Date posture	Principal use
J10-S-A04	Author placement	Present	February 27 Haag–Wigner audit and pseudo-free strict-branch reading
J10-S-A05	Author clarification	Present	February 28 submission administration and later Paper III technical revision before desk review
J10-S-SUB1	Editorial Manager packet, Paper I	February 28, 11:30:42 +06	Three-part sequence; conditional refinement-rigidity paper
J10-S-SUB2	Editorial Manager packet, Paper II	February 28, 11:22:37 +06	Conditional exhaustion/geometric-renormalization paper
J10-S-SUB3	Editorial Manager packet, Paper III	February 28, 11:27:35 +06	Vacuum classification and Haag–Wigner transport groupoid
J10-S-P01	*Canonical Finite-Scale Coarse-Graining and Scale-Split Inclusions*	February, day open	Paper A analytic route
J10-S-P02	*Admissible Refinement Transport from Conditional Expectations*	February, day open	Paper B expectation/associator route
J10-S-P03	*Spectral Isolation and the Absence of Massless Gauge Particles*	February, day open	Terminal conditional synthesis
J10-S-P04	*A Refinement–Rigid Operator–Algebraic Construction of Yang–Mills...*	February, day open	Three-avatar realization and operator audit

B.2 Paper genealogy

B.2.1 Core February 23 family

The recovered Papers I and II belong to a projected Papers I–V sequence on finite scale, transport, refinement, continuum, and spectral consequences. Their paper numbers must always be qualified.

B.2.2 Analytic Papers A–C

P01 is Paper A and P02 is Paper B. Paper C, intended to reduce recognized inner ambiguity to central data, is missing. These are not Papers I and II of the February 23 family.

B.2.3 Three-avatar realization

P04 is a separate self-contained construction manuscript. It is provisionally aligned with February 16 but has no recovered exact day.

B.2.4 Poincaré Paper IV

The February 20 completion paper is a proposed Poincaré sequel. Its “Paper IV” label need not coincide with the core-series Paper IV.

B.2.5 February 28 package

SUB1–SUB3 are the exact Editorial Manager packets for the February 28 Papers I–III submission. RR1, GR1, and VC1 are the corresponding manuscript-level sources used for the mathematical audit. The two source layers must not be collapsed: the packets establish submission chronology and declared scope; the manuscripts expose the proofs and gaps.

B.2.6 Terminal spectral synthesis

P03 is downstream of certification, exhaustion, classification, and Poincaré assumptions. Its exact February day is open.

B.3 Missing source requests

Highest priority:

1. `projection_drift_yangmills 3.tex` and version history;
2. original February 3 export;
3. point-gray definitions and calculations;
4. SNC/collar-sparsity files;
5. first finite-element coercive statement;
6. complete February 13, 16, 17, 20, 21, 22, 25, and 26 exports;
7. original February 27 conversation, notes, or drafts recording the Haag–Wigner audit and pseudo-free terminology;
8. missing Paper C;
9. exact graph-to-Wilson or graph-to-physical localization map;
10. original OC1 proof and uniform constants;
11. central-holonomy, relative H^3, candidate H^4, and depth-four pentagon files;

12. certification, exhaustion, spectrum-nonconcentration, and Poincaré sources assumed by P03;
13. repository histories, original filesystem dates, and version diffs, especially the Paper III revision made before desk-editor review.

C Projection-drift calculation dossier

C.1 Carrier distinctions

Carrier	Protected object	Permitted claim
Fixed Hilbert space with constraint family	Common kernel	Invertible recombination preserves kernel
Fixed Hilbert space with isolated spectral island	Spectral projection	Riesz methods apply under regularity
Varying Hilbert presentations	Physical fibers	Explicit comparison map required
State space	Vacuum state	Vector phase and carrier language are insufficient
Fixed vacuum line	Orthogonal complement	Projection motion can be discussed if line is regular

C.2 Exact correction

For a common domain and invertible matrix U,

$$C^a(\lambda) = U^{ab} C^b(\lambda_0) \quad \Longrightarrow \quad \bigcap_a \ker C^a(\lambda) = \bigcap_a \ker C^a(\lambda_0).$$

The proof is two inclusions. If every old constraint annihilates ψ, every new one does. Conversely, invertibility expresses each old constraint as a linear combination of the new constraints.

C.3 Genuine motion mechanism

An evolution containing directions outside the original constraint span can alter the common zero set:

$$\dot{C}^a = M^{ab} C^b + K^{aI} D_I.$$

This is a possible mechanism, not an automatic drift theorem. Rank changes, loss of isolation, domain changes, and bifurcation of the kernel remain possible.

C.4 Motion versus comparison

Even when the abstract common kernel is stable, scale-dependent physical realizations may require

$$T_{21} : \mathcal{H}_{\mathrm{phys}}(\lambda_1) \to \mathcal{H}_{\mathrm{phys}}(\lambda_2).$$

One must state whether T_{21} is injective, isometric, unitary, completely positive after passing to observables, or merely comparison data. February did not supply one universal transport class.

C.5 Modern operator-theoretic direction

If zero is uniformly isolated in $C(\lambda)^* C(\lambda)$, then a Riesz contour can define $P(\lambda)$. Differentiability of the resolvent yields differentiability of the projection and, under additional hypotheses, a Kato-style transport generator involving $[\dot{P}, P]$. This is later mathematical clarification. It is not evidence that February had already formulated a Grassmannian connection.

D Refinement and geometric-admissibility ledger

Condition or object	February role	Status	Nonclaim
Directed refinement poset	Organize scale relations	Structural datum	Directedness alone gives no uniformity
Algebra pullback/embedding	Move observables	Formal map	Does not transport forms automatically
CP map or expectation	Coarse-grain states/algebras	Conditional analytic datum	Not unitary sector transport
Bounded overlap/degree	Local complexity control	Plausible bounded-geometry consequence	Does not exclude long chains
Uniform same-scale diameter	Early graph shortcut	Geometric hypothesis	Generally false under ordinary refinement
Quasi-uniform scale	Finite-element regularity	Geometric hypothesis	Does not alone exclude all bottlenecks
Shape regularity	Exclude slivers	Geometric hypothesis	Requires weights and interfaces to be typed
Overlap thickness	Prevent vanishing interfaces	Geometric hypothesis	Not proved from category
Cut connectivity	Exclude dumbbells/necks	Geometric target	Not proved from locality in February
Partition of unity	Transfer continuum Poincaré to mesh	Analytic/geometric device	Original full proof not recovered
OC1	Compare neighboring collar expectations	Conditional analytic target	Uniform constants missing

Condition or object	February role	Status	Nonclaim
Multiscale vertical path	Replace short same-scale paths	Open theorem target	No contraction/defect iteration theorem
Category membership	Possibly exclude pathological meshes	Conceptual proposal	Exclusion by definition is extra input
Mature GAT certificate	Later architecture	Modern reading only	Must not be backdated

D.0.1 D.1 Degeneration types

Degeneration	Soft mode mechanism	Candidate control
Long sliver or whisker	Slow variation along a long chain	Shape regularity; multiscale contraction
Dumbbell	Opposite constants on large lobes, change across small cut	Cheeger/cut lower bound
Narrow neck	Macroscopic variance supported by few interface edges	Overlap thickness; weighted cut control
Stacked slivers	Bounded degree with growing effective distance	Vertical refinement theorem
Anisotropic cells	Large physical change hidden in small graph energy	Aspect-ratio and weight control
Unbalanced weights	Apparent unweighted expansion but vanishing physical interface	Weighted Poincaré/Cheeger control

D.0.2 D.2 Three February statuses of geometric admissibility

1. **Independent hypothesis:** declare an admissible bounded-geometry class.
2. **Locality consequence:** derive mesh regularity from stronger spacetime locality and finite-element geometry.
3. **Categorical membership:** construct the refinement category so pathological objects are absent.

The third was conceptually deepest and mathematically least complete. The first was most explicit but carried the risk of assuming away the difficult uniformity problem.

E SNC, point-gray, graph, Cheeger, and finite-element dossier

E.1 Fixed-group heat summability

For a fixed compact semisimple G, Weyl's dimension formula and Casimir growth give constants $A, p, c, b > 0$ such that

$$\dim V_\pi \le A(1+|\lambda_\pi|)^p, \qquad C_2(\pi) \ge c|\lambda_\pi|^2 - b.$$

Therefore, for every fixed $\alpha \ge 0$ and $\beta > 0$,

$$(\dim V_\pi)^\alpha e^{-\beta C_2(\pi)} \lesssim (1+|\lambda_\pi|)^{\alpha p} e^{-\beta c|\lambda_\pi|^2},$$

and the sum over dominant integral weights converges. The exponent α must include the multiplicities actually present in the chosen Peter–Weyl decomposition; the full $L^2(G)$ heat trace contains dimension factors beyond an unweighted representation sum.

This proves a fixed-carrier heat estimate. Under refinement, the full graph heat trace can still grow without bound as the number of edges increases. The missing theorem was a uniform bound on the *coarse-visible* or collar-effective slots.

E.2 Heat trace and nuclearity

If the reduced heat operator on a fixed Hilbert carrier is trace class,

$$(1-\Pi)e^{-\beta L/2} = \sum_n s_n |u_n\rangle\langle v_n|, \qquad \sum_n s_n < \infty,$$

then the algebra-to-Hilbert map

$$X \longmapsto (1-\Pi)e^{-\beta L/2} X\Omega$$

has the nuclear decomposition

$$X \longmapsto \sum_n s_n \langle v_n, X\Omega\rangle u_n.$$

This is a correct fixed-carrier lemma. It does not identify the appropriate buffered local inclusion, prove uniformity over refinement, or imply a lowest positive eigenvalue uniformly bounded away from zero. A trace norm can remain bounded while the lowest positive eigenvalue approaches zero if multiplicities and the rest of the spectrum compensate appropriately; any explicit uniform-gap inference requires extra low-energy counting or coercivity input.

E.3 Exact graph calculation

For Hilbert-valued vertex data, the mean decomposition J10-E-020 is an exact orthogonal identity. On the mean-zero subspace, J10-E-021 follows from the Rayleigh quotient of the combinatorial Laplacian. The finite theorem extends without change from scalar to Hilbert-valued data by expanding in an orthonormal basis or using the tensor product of the scalar Laplacian with the identity.

The uniform question depends on the family G_k. Bounded degree alone is insufficient: path graphs have degree two and gaps of order $|V|^{-2}$. Degree plus fixed diameter gives a positive finite-class bound, but ordinary local refinement need not preserve diameter. Cheeger or cut lower bounds target the bottleneck directly.

E.4 Continuum-to-discrete finite-element route

Suppose a bounded Lipschitz region has a shape-regular, quasi-uniform cover $\{U_\alpha\}$ with bounded multiplicity and a subordinate partition of unity $\{\varphi_\alpha\}$. Given Hilbert-valued collar data ξ_α, define

$$F(x) = \sum_\alpha \varphi_\alpha(x)\xi_\alpha.$$

The continuum Poincaré inequality controls $F - \bar{F}$. Since $\sum_\alpha \nabla\varphi_\alpha = 0$, the gradient can be written in terms of local differences $\xi_\alpha - \xi_\beta$. Bounded overlap and shape regularity then compare continuum variance with a weighted discrete variance.

This is a plausible finite-element mechanism for mesh coercivity under explicit bounded-geometry hypotheses. The original February proof, constants, boundary conventions, and relationship to the physical Wilson carrier have not yet been recovered.

E.5 Point-gray status

The point-gray or SNC calculation is repeatedly remembered as an intermediate coercive component. No original February definition has yet been recovered. It must therefore remain **Open**. The note does not infer its formula from later Wilson-tube or finite-element constructions.

E.6 Cheeger terminology

When a February source says neck, cut, dumbbell, or bottleneck, modern Cheeger language offers a concise interpretation:

$$h(G) = \min_{0<|S|\le|V|/2} \frac{|\partial S|}{|S|}.$$

Cheeger inequalities relate $h(G)$ to $\lambda_2(G)$ under stated conventions. Unless an original February Cheeger formula is recovered, this terminology is **Modern reading**, not an archival claim.

F Coercivity and uniformity hypothesis audit

F.1 The three operators that must remain distinct

Operator/form	Carrier	What it controls	What it does not establish
Edge or configuration Casimir	Compact-group representation variables	Nontrivial representation content and heat decay	Vacuum isolation inside the gauge-singlet sector
Gauss penalty $\sum \|G_v^X\psi\|^2$	Kinematical gauge carrier	Violation of the Gauss constraint	Physical energy of gauge-invariant excitations
Physical Yang–Mills Hamiltonian or invariant mass	Reconstructed physical/Poincaré carrier	Dynamics and physical spectral separation	Not constructed by naming either finite operator

F.2 Fixed-scale positivity paper

J10-S-FS1 defines

$$c_k = \inf \sigma(H_k|_{\Omega_k^\perp})$$

and proves $c_k > 0$ under a confinement-type assumption that every nonvacuum gauge-invariant excitation already carries a uniformly positive fixed-scale energy contribution. The theorem is exact conditional mathematics. It states the creation hypothesis; it does not derive it.

F.3 One-step transport paper

J10-S-AT1 introduces defects $a_k^* \in [0,1)$, $b_k^* \ge 0$, and the target

$$c_{k+1} \ge (1-a_k^*)c_k - b_k^*.$$

To iterate this inequality one needs its proof, control of the products $\prod(1-a_j^*)$, control or summability of the additive defects, and a typed identification of the complements being compared.

F.4 Gauss-form audit

If the local observable algebra is defined as the common Gauss kernel and the form is the sum of squared Gauss generators, then the form vanishes on the natural observable core. The conclusion $\ker L = \mathbb{C}\Omega$ requires an independent energy form or an independent theorem that the only gauge-invariant zero-energy vector is the vacuum. Gauge invariance alone supplies neither.

F.5 Graph-to-form audit

A lawful graph-to-form theorem needs at least:

1. a localization map into vertex or cell data;
2. an upper bound preventing localization blow-up;
3. a lower frame or reconstruction bound preventing norm loss;
4. compatibility of the global protected projection with graph averaging;
5. domination of graph edge energy by the physical form in the correct direction;
6. constants uniform over the declared refinement class.

The February 28 rigidity proof supplies only part of this list and reverses the upper localization inequality at the load-bearing step.

F.6 Uniformity and no escape

Uniform finite coercivity is stronger than fixed-scale positivity. Passing it to a limit also requires that normalized low-energy vectors cannot disappear weakly or escape into unobserved degrees of freedom. Suitable hypotheses may include compact embeddings, uniform tightness, collective compactness of resolvents, or a correctly formulated SNC/no-escape theorem. February identified the problem but did not select and prove one complete package.

F.7 Mosco and projection passage

Mosco convergence of closed forms yields strong-resolvent convergence of the associated operators under the standard framework. Stable spectral projection passage at zero additionally requires a uniform isolated spectral island or a compatible kernel theorem. A continuous cutoff or Riesz contour can then give strong projection convergence. Without a uniform island, new near-zero spectrum can enter and the protected complement can leak.

F.8 Descent and invariant spectrum

Central transition phases may act trivially on local observables while changing the space of global sections. Spectral equality therefore requires an explicit unitary equivalence of global operators or a theorem comparing their closed forms and domains. The later step from a positive generator to invariant mass additionally requires Poincaré covariance and joint spectral analysis.

F.9 Final coercivity verdict

February contained correct finite cost proxies and degeneration estimates, but not a proved, refinement-uniform, physically typed Yang–Mills coercive form.

G Equation-provenance audit

G.1 Carrier ledger

ID	Carrier	Protected object	Principal firewall
J10-C-01	Fixed kinematical Hilbert space with constraints	Common kernel	Changed formula is not changed kernel
J10-C-02	Scale-dependent physical Hilbert fibers	Physical fiber/lineage	Comparison cannot be hidden in equality
J10-C-03	Fixed-scale Hilbert proposal	Vacuum line and complement	Positivity may be assumed rather than created
J10-C-04	Nested local von Neumann algebras/standard form	Smaller algebra and vacuum	Compression/expectation chain needs modular hypotheses
J10-C-05	Gauge-invariant Wilson GNS carrier	Declared vacuum line	Gauss invariance is larger than the vacuum
J10-C-06	Compact-group representation sectors	Trivial irrep/tensor unit	Casimir isolation is not physical vacuum isolation
J10-C-07	Finite graph or overlap nerve	Constants versus mean-zero data	Graph complement is not physical complement
J10-C-08	Localized Hilbert family	Unit-orthogonal localized data	Lower frame and mean compatibility required
J10-C-09	Filtered monoidal/tensor system	Tensor unit	Weight is not automatically energy
J10-C-10	Refinement-limit or glued Hilbert carrier	Limit kernel projection	No-escape and domain passage required
J10-C-11	Strict or centrally twisted realizations	Glued vacuum/sector data	Local formula invariance is not spectral equality
J10-C-12	Poincaré-covariant representation	Joint spectrum and invariant mass	No finite paper constructs this carrier

G.2 Equation register J10-E-001–064

ID	Short description	Carrier	Source/status
J10-E-001	Physical space as common constraint kernel	C-01	T03; reconstructed definition
J10-E-002	Invertible constraint recombination	C-01	T03; finite/formal calculation
J10-E-003	Common-kernel invariance	C-01	T03; formal theorem
J10-E-004	Evolution with outside-ideal directions	C-01	T03; remembered mechanism
J10-E-005	Transport between physical fibers	C-02	T03; reconstruction requirement
J10-E-006	Riesz projection formula	C-01/02	T03; later-understood direction
J10-E-007	Fixed-scale gap proxy c_k	C-03	FS1; exact definition
J10-E-008	$c_k > 0$	C-03	FS1; conditional finite theorem
J10-E-009	One-step deficit inequality	C-03 plus comparison	AT1; defined/formal, unproved
J10-E-010	Typed finite coercive target	Declared finite carrier	T13/H0; safe historical target
J10-E-011	Compact-group Casimir isolation	C-06	T16/P04; finite theorem
J10-E-012	Weighted heat-sum convergence	C-06	T13/T16; finite theorem
J10-E-013	Coarse-visible collar trace estimate	Finite collar sector	T17; conditional estimate
J10-E-014	Refinement-uniform trace norm	Scale-indexed presentations	T13/T17; analytic target
J10-E-015	Gauge-invariant algebra as Gauss kernel	C-05	P04; printed definition
J10-E-016	Sum-of-squared-Gauss form	C-05	P04; finite form attempt
J10-E-017	Form vanishes on gauge-invariant core	C-05	Modern audit of P04; finite correction
J10-E-018	Claimed complement lower bound	C-05	P04; not implied
J10-E-019	Claimed one-dimensional kernel	C-05	P04; independent theorem missing
J10-E-020	Hilbert-valued mean decomposition	C-07	T26; exact finite calculation
J10-E-021	Graph Poincaré inequality	C-07	T26; finite theorem
J10-E-022	Degree–diameter graph lower bound	C-07	RR1; finite theorem on restricted class
J10-E-023	Uniform degree/diameter hypothesis	C-07	T26/RR1; geometric hypothesis, partly false for ordinary refinement
J10-E-024	Uniform cut-connectivity target	C-07	T22; geometric target
J10-E-025	Uniform graph gap or multiscale analogue	C-07 plus refinement	T26; open target
J10-E-026	Upper localization bound	C-08	RR1; analytic hypothesis
J10-E-027	Needed lower frame bound	C-08	Audit of RR1; missing requirement
J10-E-028	Unit orthogonality to graph mean zero	C-08/09	RR1; unproved compatibility

ID	Short description	Carrier	Source/status
J10-E-029	Edge/form domination	C-08	RR1/T22; analytic assumption
J10-E-030	Desired physical coercivity	C-08/12	RR1; conditional attempt
J10-E-031	Tensor-unit isolation constant	C-09	T22/T25; categorical condition
J10-E-032	Positive defect per nontrivial step	C-09	RR1; conditional lemma
J10-E-033	Isolation implies geometry	C-09 to C-07	T22; failed/incomplete attempt
J10-E-034	Pinsker-type entropy bound	Common state carrier	T17; conditional analytic theorem
J10-E-035	Wilson phase entropy witness	Common local algebra	T17; conditional estimate
J10-E-036	Compression-vector construction	C-04	P01; printed attempt
J10-E-037	Claimed SNC/compactness equivalence	C-04	P01; incomplete claim
J10-E-038	Compactness to split/expectation chain	C-04	P01; missing nuclearity/modularity
J10-E-039	Exact expectation composition	C-04	P02; conditional formal theorem
J10-E-040	Inner composite implementer	Standard-form data	P02; canonicity tension
J10-E-041	Nonabelian degree-two obstruction	Varying unitary groups	P02; coefficient system missing
J10-E-042	Proposed limit form	C-10	P04; unproved limit definition
J10-E-043	Uniform coercivity plus Mosco implies limit coercivity	C-10	T16/P04; conditional theorem
J10-E-044	Strong projection passage	C-10	T16/T21; reconstruction requirement
J10-E-045	Spectral containment above threshold	C-10	Formal conditional consequence
J10-E-046	Claimed full interval equality	C-10	P04; not implied by coercivity
J10-E-047	Twist-energy definition	C-11	P04; printed definition
J10-E-048	Claimed equality of twisted/strict spectra	C-11	P04; descent theorem missing
J10-E-049	Invariant-mass lower bound	C-12	P03/T20; relativistic target
J10-E-050	No massless gauge Wigner carrier	C-12	P03; conditional physical claim
J10-E-051	Refinement-uniform categorical isolation	C-09	T21/T25; reconstructed general form
J10-E-052	Asymptotically soft categorical lineage	C-09	T21/T25; formal failure mode
J10-E-053	Normalized analytic soft sequence	Form carriers	T21; no-escape warning
J10-E-054	Detector defect $F_{\min}$	Recognized central witness	T13; remembered schematic formula
J10-E-055	Uniform accumulated-defect budget	Refinement defect system	Audit of RR1; missing hypothesis
J10-E-056	Sector-visible central holonomy	Central coefficient plus character	T17; conditional formal proposal
J10-E-057	Strict descent iff sector phase trivial	Same	T17; conditional criterion

ID	Short description	Carrier	Source/status
J10-E-058	Entropy of renormalization curvature	Scale-compared states	T22/TU1; candidate definition
J10-E-059	Center-valued H^2	Full refinement category	T17; candidate coherence class
J10-E-060	Relative H^3 bridge	Pair $(\mathsf{Ref}, \mathsf{RG})$	T17; candidate extension obstruction
J10-E-061	Candidate relative H^4	Higher refinement pasting	T17; candidate only
J10-E-062	Sector pentagon holonomy	Finite move calculus	T17; conditional target
J10-E-063	Pentagon fillability criterion	Finite move calculus	T17; calculus and independence missing
J10-E-064	Weight preservation under monoidal equivalence	C-09	T25; formal invariance condition

G.3 Unnumbered formulas

Several displayed formulas in the narrative are explanatory restatements or theorem targets and do not receive a J10-E identifier. In particular, the OC1 estimate remains unnumbered because its original February formula and constants have not been recovered. No new stable equation identifier is created in this assembly.

H Claim-to-source matrix and required firewalls

H.1 Main claim matrix

Claim in the note	Allowed strength	Principal sources
Multiplicative scaling does not move a common kernel	Strong finite/formal correction	J10-S-T03
Outside-ideal mixing can permit kernel motion	Mechanism, not universal theorem	J10-S-T03
Changing carriers create a transport problem even without literal kernel motion	Strong conceptual distinction	J10-S-T03
February distinguished admissibility from SNC	Strong historical claim	J10-S-T13, T17
Fixed-group Casimir isolation and weighted heat summability are correct	Strong on typed finite carrier	J10-S-T13, T16, P04
P04's Gauss form vanishes on its gauge-invariant core	Strong finite audit	J10-S-P04
Paper A's compression/split chain is incomplete	Strong modern audit	J10-S-P01
Paper B leaves canonicity, coefficients, and central reduction unresolved	Strong modern audit	J10-S-P02
Degenerating geometry can collapse a graph coercive constant	Strong finite/graph statement	J10-S-T22, T26, RR1

Claim in the note	Allowed strength	Principal sources
The finite graph Poincaré theorem is exact	Strong finite theorem	J10-S-T26
The graph-to-physical bridge needs lower frame, mean compatibility, and form domination	Strong audit judgment	J10-S-RR1
Same-scale diameter generally grows under ordinary refinement	Strong correction	J10-S-T26
Multiscale admissible control was the replacement target	Strong historical reconstruction	J10-S-T26
Tensor-unit isolation organized the problem but did not prove geometry or energy	Strong audit judgment	J10-S-T22, T25
Entropy can witness already-distinct path states	Conditional analytic statement	J10-S-T17, TU1
February had conflicting obstruction-degree architectures	Strong documentary judgment	J10-S-P02 versus T17
Poincaré completion and invariant mass were separate debts	Strong historical claim	J10-S-T20, T21
February 27 used Haag and Wigner to audit existence and provisionally read the strict branch as pseudo-free	Direct retrospective historical claim; not a theorem	J10-S-A04
February 28 contained an exact three-paper submission architecture	Strong recorded claim	J10-S-SUB1, SUB2, SUB3; RR1, GR1, VC1
Fixed-thickness Wilson-tube terminology appeared in the February 28 packet	Strong lexical documentary claim; not a construction theorem	J10-S-SUB1, SUB2
Paper III underwent a technical revision before desk-editor review	Remembered administrative claim; exact diff open	J10-S-A05
Physical uniform Yang–Mills coercivity was constructed	Forbidden	No February source supports it
Continuum reconstruction and invariant spectral transfer were completed	Forbidden	No February source supports them
Massless gluon Wigner carriers were excluded	Terminal conditional aspiration only	J10-S-P03

H.2 Governing-arrow classification

Arrow	February status	Axis
Constraint evolution to kernel question	Corrected finite calculation	Carrier/selection
Kernel question to fiber transport	Formal reconstruction requirement	Carrier/passage
Transport deficit to admissible geometry	Additional geometric hypothesis	Uniformity
Admissible finite geometry to graph control	Finite theorem under explicit hypotheses	Creation proxy
Graph control to localized physical coercivity	Missing reconstruction theorem	Creation
Casimir isolation to physical Yang–Mills cost	Invalid without energy/carrier realization	Creation
Heat summability to SNC	Fixed-carrier route; uniform slot theorem missing	Analytic uniformity
SNC to Paper-A expectation package	Incomplete claimed chain	Passage foundation

Arrow	February status	Axis
Expectations to inner associator	Claimed; canonicity tension	Descent
Inner associator to central gerbe class	Missing Paper C	Descent
Tensor-unit isolation to geometry	Failed legality audit	Uniformity
Positive defect to freezing	Conditional on contraction, discreteness, budget	Uniformity
Finite coercivity to limit coercivity	Conditional Mosco/no-leakage theorem	Passage
Local central neutrality to global spectral equality	Not established	Descent/passage
Positive generator to invariant mass	Missing Poincaré recognition	Relativistic interpretation
Invariant mass to no massless gauge carrier	Conditional typed physical theorem	Physical interpretation

H.3 Required firewalls

$$\text{projection drift} \not\equiv \text{March's Grassmannian interpretation;}$$

$$\text{graph or Cheeger positivity} \not\Rightarrow \text{physical Yang–Mills coercivity}$$
$$\text{without a typed carrier bridge;}$$

$$\text{finite-scale coercivity} \not\Rightarrow \text{uniform coercivity over all refinements;}$$

$$\text{categorical tensor-unit isolation} \not\Rightarrow \text{geometric admissibility unless proved;}$$

$$\text{categorical weight} \not\equiv \text{physical energy;}$$

$$\text{inner associator} \not\Rightarrow \text{central gerbe cocycle;}$$

$$\text{relative entropy} \not\Rightarrow \text{path noncommutativity without prior state/path inequivalence;}$$

$$\text{geometric admissibility} \not\equiv \text{the mature GAT certificate architecture;}$$

$$\text{refinement rigidity} \not\Rightarrow \text{continuum reconstruction without compactness and lineage;}$$

$$\text{local twist neutrality} \not\Rightarrow \text{global spectral equality;}$$

$$\text{Hamiltonian lower bound} \not\Rightarrow \text{invariant mass without relativistic spectral transfer;}$$

$$\text{strictifiability} \not\Rightarrow \text{freeness, pseudo-freeness, or interaction;}$$

$$\text{Haag obstruction} \not\Rightarrow \text{nonexistence of an interacting local theory;}$$

$$\text{Wigner particle classification} \not\Rightarrow \text{a global colored one-particle Yang–Mills carrier;}$$

$$\begin{gathered}\text{February fixed-thickness Wilson-tube language} \\ \not\equiv \text{the mature May–July Wilson-tube construction.}\end{gathered}$$

H.4 Modern-reading quarantine

The following may clarify the February debts only when explicitly marked **Modern reading**:

- the March Grassmannian interpretation of a regular projection path;
- Hilbert-bundle or Hilbert–W^*-module language for changing carriers;
- the mature Wilson-tube construction beyond February's fixed-thickness terminology;
- thick-to-thin reconstruction and invariant spectral transfer;
- mature GAT certificates and coefficient-band descent;
- the three-operator distinction among edge Casimir, Gauss penalty, and physical Hamiltonian;
- the $U(1)$ sanity check;
- later K/KK, Fredholm, determinant-line, or mapping-spectrum architecture.

No later theorem may upgrade the status of a February claim without a contemporaneous source.

I March handoff: projection geometry, lineage, and the emerging sector problem

February ended with three related problems ready to pass into March.

I.1 Projection geometry

The February correction had separated literal kernel motion, spectral projection motion, and carrier comparison. March could now ask for regularity of a family of projections and for the geometry of their ranges. The explicit infinite-dimensional Grassmannian interpretation belongs to that next interval, not to February.

I.2 Vacuum lineage

A scale-indexed vacuum could no longer be treated as one object merely wearing different labels. March needed a definition of lineage: a family of vectors, states, projections, or sectors together with comparison maps and coherence laws. The lineage had to be strong enough to formulate complement stability and weak enough not to assume a global Hilbert trivialization.

I.3 Emerging sector problem

February's compact-group work had separated the trivial representation from nontrivial irreducible representations. The Gauss audit showed that the gauge-singlet sector was larger than the vacuum line. March therefore inherited an emerging sector problem: which sector decomposition was physically protected, how were sectors recognized across refinement, and what cost acted on each complement?

I.4 Debts carried forward

At March's opening, the following were still unavailable:

1. a constructed physical Yang–Mills quadratic form with positive cost on the nonvacuum gauge-invariant complement;
2. a lawful graph/category-to-physical carrier bridge;
3. a uniform multiscale refinement-coercivity theorem;
4. a complete SNC/collar-sparsity result;
5. OC1 with uniform constants;
6. Mosco liminf, recovery, no-escape, and protected-projection passage;
7. central recognition and gerbe spectral descent;

8. Poincaré covariance, Hamiltonian recognition, and invariant-mass transfer;
9. a completed sector reconstruction;
10. the mature Wilson-tube and thick-to-thin mechanisms.

March would not begin from nothing. It would begin from a better-typed mountain: the carrier could move, geometry could degenerate, the category could isolate without yet realizing energy, and passage had to preserve an identified lineage. February had made those facts visible.

Formal freeze declaration

Version 1.1 — formally frozen

Research Note No. 10, *The Gap Acquired a Geometry*, is formally frozen at Version 1.1 on 23 August 2026. This revision adds the February 27 Haag–Wigner interval and the exact February 28 submission packets without upgrading the mathematics. Version 1.0 remains a frozen historical predecessor. The present freeze includes the title, narrative, chronology, source register J10-S-, carrier ledger J10-C-, equation-provenance register J10-E-, claim-to-source matrix, and mathematical firewalls. Later source recovery may require a separately numbered revision or addendum; it may not silently repair or upgrade a February claim. Research Notes Nos. 08–09 remain independently frozen and unchanged.