First Glimpses After the Green Field and Fog

The Sock, Measure Mashing, and the Question of What Flows

A historical prelude to the December 2025 vacuum ansatz

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Research posture and claim boundary

This note reconstructs the questions that preceded the December 2025 Yang–Mills ansatz. It is not a priority claim, a proof of a renormalization theorem, or evidence that a dream supplied mathematical warrant. Surviving notes and diagrams are separated from remembered chronology and from modern interpretation. The later Wilson–tube and Geometry of Admissible Transport constructions are not imported backward as facts known in 2024 or 2025.

Abstract

This note reconstructs the formation of a question: what mathematical datum actually flows under renormalization when the vacuum does not appear to admit one ordinary global Fock presentation? The question began with the author's first sustained exposure to renormalization in a 2004 quantum field theory course and was widened by continued study of *Quantum Fields and Strings: A Course for Mathematicians*. From 2016 onward, a provisional non-Fock vacuum picture developed alongside scattering-amplitude geometry. In approximately fall 2024, after a dream of a green field, moving fog, and a familiar mountain, the author woke and wrote a single unexplained word: "Presheaf?" During May–June 2025 an ordinary request to find Sawyer's red baseball sock preceded a striking visual interruption: a red field followed by a shadow tube with strings moving between ceiling and floor. After pausing to check his condition and returning to study, the author recognized the mathematical question as a search for generalized RG equations for geometry. The later amplitude-surface interpretation supplied a candidate source for the geometric payload. In August, Klauder's measure-mashing program supplied a concrete but temporary model for data transported under coarse-graining.

The first-person history deepens that sequence. Present-day recollections are displayed as recollections rather than silently rewritten as contemporaneous quotations. The resulting account follows not only which ideas appeared, but which pressure each idea placed on the next question.

A mathematical audit examines the arrows implicit in the history. It separates parameter flow from transport of a carrier, scattering geometry from local operator-algebraic realization, a local W^*-algebra from a vacuum carrier, presheaf assignment from effective descent, and descent data from an invariant preserved under scale change. These distinctions are presented as modern typing tests, not as structures already constructed in 2004–25.

Section 8 records a further correction: the measure-mashing stage was not only an analogy. Beginning from Klauder's definitions, the author constructed RG equations, worked through their compositions, and varied the mashing terms in order to study how the associated geometric measure data transformed under flow. Present notation reconstructs the type of calculation without claiming to reproduce the still-unrecovered manuscript symbol for symbol.

The purpose here is not to identify those early shadows with the mature theory. It is to preserve their order, their uncertainty, and their separate roles. The note ends before the December Ising calculation, the glued W^*-vacuum toy, Wilson-loop clustering, and the first Clearance conjecture. Those belong to Research Note No. 8. The governing historical conclusion is modest: by late 2025 several independent questions had accumulated enough pressure to demand a toy model, but the carrier, its gluing law, and its scale passage were still unknown.

Evidence labels used throughout

Recorded means supported by a surviving dated source or artifact. Remembered marks first-person historical recollection. Reconstructed identifies an ordering or connection rebuilt from multiple declared sources. Modern reading marks later language not attributed to the historical period. Open identifies a date, identity, quotation, or mathematical connection still requiring documentary support.

Contents

A note on method: logical order is not discovery order

A finished construction is organized so that every object appears before it is used, every map has a declared domain and codomain, and every conclusion rests on hypotheses already earned. Discovery rarely respects that order. A picture appears before its carrier; a word is written before its use is known; a successful calculation is made inside the wrong ambient object; an analogy does indispensable work and then disappears.

This note preserves discovery order. Its subject is not the final architecture of the Yang–Mills construction but the prehistory of the questions that made that architecture thinkable. The distinction is enforced by the rule

Later mathematics may illuminate an early note, but may not be attributed to it without evidence.

The rule has two consequences. First, the word "presheaf" is allowed to appear before it has a mathematical role. Second, the later theory of admissibility, refinement, self-dual Hilbert W^*-modules, associator obstructions, and spectral transfer will appear only in explicitly marked modern-reading sections. The reader is not asked to believe that the road was visible from the field. The road is visible now because it was eventually built.

Prologue: before the first calculation

The December 2025 notes begin abruptly. An Ising toy is placed beside a renormalization question; local vacuum data become gluing data; Wilson loops enter; a W^*-algebraic vacuum picture is guessed; and a holonomy estimate is given the hopeful name *Clearance*. Read in isolation, the sequence can look like a single night's improvisation.

It was a single night's improvisation, but it was not a beginning.

For more than twenty years the renormalization group had carried an unresolved question. For nearly a decade the vacuum had seemed unlikely to fit inside a single ordinary Fock representation. Scattering geometry had suggested that some datum survived on a surface and descended away from it. Klauder's work had supplied a measure-level example in which the choice of measure was not a passive background. And, before the conscious geometry was assembled, one unexplained word had been written after a dream.

This is the story of those questions before they became a calculation.

1 First exposure: QFT II, 2004

1.1 Renormalization as working field theory

REMEMBERED. The author's first sustained encounter with the renormalization group occurred in QFT II in 2004. The setting was working quantum field theory: divergent expressions, regularization, running parameters, scale dependence, and the construction of effective descriptions. The renormalization group was not introduced as a decorative symmetry. It was the machinery that explained why the same physical theory could be described differently at different resolutions.

At that stage the operational picture was enough. One begins with a scale μ, a collection of couplings $g_i(\mu)$, and beta functions

$$\mu \frac{dg_i}{d\mu} = \beta_i(g_1, \ldots, g_n). \tag{1}$$

The flow reorganizes the effective description. Fixed points, relevant and irrelevant directions, and universality explain how distinct microscopic systems may share large-scale behavior. This was already a profound change of viewpoint: scale was not an external label placed on a completed theory. It participated in what could be observed and how the theory was presented.

The question that survived the course was less computational:

$$\boxed{\text{What, underneath the changing parameters, is actually flowing?}} \tag{2}$$

The question was not yet geometric. It did not yet ask for a category, a bundle, a presheaf, or a transport law. It was the discomfort produced when a successful formalism seems to describe motion but leaves the identity of the traveler unclear.

Retrospective recollection (2026)

I always wanted to know what it looked like with what flowed. The beta functions told me how parameters moved, but I kept wondering what the passage itself would look like if one could see the object being carried through it.

This wording is contemporary, not a quotation from a 2004 notebook. It nevertheless identifies the durable residue of the course with unusual precision. The unresolved issue was not whether the renormalization group worked. It was whether its successful parameter-level description was the whole ontology of scale change or only a visible coordinate trace.

1.2 Three meanings already hiding in the word "flow"

Even the conventional picture contains several nonequivalent statements:

(i) the couplings change with the subtraction scale;

(ii) an effective action changes when degrees of freedom are integrated out;

(iii) observables at separated resolutions are compared by an induced rule.

The first is a differential equation on parameter space. The second is a transformation of functionals or measures. The third asks for a comparison between physical descriptions. In 2004 these distinctions were not yet the author's research program. They are listed here because the later question cannot be reconstructed honestly if every use of "RG" is treated as though it meant the same thing.

The distinction would eventually matter in three separate ways. A differential equation can control a trajectory without identifying the carrier of that trajectory. A coarse-graining map can remove degrees of freedom without specifying which physical distinctions are licensed to survive. And a family of effective descriptions can agree observationally without yet being assembled into one geometric object. These are modern separations of the original pressure, not claims about what was formalized in the course.

Audit 1: a vector field on coupling space is not yet transport of a carrier

Writing

$$\beta = \sum_i \beta_i(g) \frac{\partial}{\partial g_i}$$

declares a vector field on a parameter space. It does not by itself declare an object $\mathfrak{C}_\mu$ at each scale, a comparison $\Phi_{\mu_1 \to \mu_2} : \mathfrak{C}_{\mu_1} \to \mathfrak{C}_{\mu_2}$, or a law for composing such comparisons. The first formalism answers how coordinates run. The second would answer what mathematical carrier is compared. The historical question arose in the gap between them.

Historical boundary

Nothing in the 2004 course is claimed here to imply geometric transport, vacuum descent, or a non-Fock representation. QFT II supplied the first physical machinery and the durable question (2).

2 The course for mathematicians: the question widens

2.1 The book that made the machinery strange again

Remembered. Continued engagement with *Quantum Fields and Strings: A Course for Mathematicians* pushed the question further. Its broad mathematical environment placed quantum field theory beside geometry, topology, representation theory, moduli, and categorical organization [3]. The effect was not to replace the physical RG learned in QFT II. It was to make that machinery strange again in a productive way.

If the effective description changes coherently with scale, perhaps the trajectory of couplings is only the coordinate shadow of a larger passage. One can then ask, without yet knowing how to answer,

$$\boxed{\text{Does renormalization transport an algebraic or geometric carrier?}} \tag{3}$$

The word *carrier* is modern shorthand. It means only the object whose data, symmetries, and physically meaningful distinctions are being compared. The historical question was less precise: something appeared to survive coarse-graining, and the running couplings did not seem to exhaust what that something was.

Retrospective recollection (2026)

That series was magical. QFT II had given me renormalization as physical machinery; *Quantum Fields and Strings* kept pushing the question farther. It made it natural to ask whether the machinery was the shadow of a geometric or algebraic passage I did not yet know how to name.

The volumes themselves do not propose the later construction. Their role in this history is formative: they held field theory, geometry, representation theory, topology, and categorical language close enough together that a question about "what flows" no longer had to be asked only in the coordinates of coupling space. The two-volume work records the 1996–97 Institute for Advanced Study program and was prepared specifically for a mathematical audience [3]. That is the documented setting; its effect on the author belongs to recollection.

2.2 Continued digestion rather than immediate theory

No geometric renormalization theory followed immediately. The question remained in the background, returning whenever a different formalism appeared to preserve more structure than its coordinates displayed. This long delay matters. The December 2025 ansatz was not the endpoint of a continuous twenty-year calculation. It was the point at which several intermittently active questions finally became coupled.

The intervening years were therefore not empty, but neither should they be made artificially continuous. Ideas were encountered, put down, recovered, and sometimes reinterpreted under a new pressure. The honest historical unit is not a twenty-year derivation. It is a twenty-year recurrence of one question in several mathematical dialects.

3 A vacuum without a familiar home

3.1 The non-Fock suspicion, 2016 onward

Remembered. From approximately 2016 onward, the author increasingly suspected that the Yang–Mills vacuum could not arrange itself globally inside an ordinary Fock space. The suspicion did not arise from a completed no-go theorem. It arose from the repeated mismatch between a single global particle presentation and the sector, locality, infrared, and scattering structures that seemed to matter.

The provisional word was *nonseparable*. It was a placeholder, not a diagnosis. It named the feeling that the carrier was too large, too sector dependent, or too poorly globalized for the naive Hilbert-space picture. In retrospect the word mixed several distinct possibilities:

- inequivalent representations of the observable algebra;
- infrared sectors without ordinary sharp-particle states;
- local normality without one preferred global presentation;
- and failure of local data to assemble by strict equality.

None of these possibilities was yet selected.

Retrospective recollection (2026)

I knew—or strongly suspected—that the vacuum could not simply arrange itself in the Fock space I had been taught to expect. “Nonseparable” was a placeholder. I knew the situation was odder than that, perhaps infraparticle- like in some respects, but I did not yet have the carrier.

The distinction matters. “The vacuum is nonseparable,” “the theory has inequivalent sectors,” “ordinary Fock asymptotics fail,” and “local representations do not come from one preferred global realization” are not synonyms. The early word compressed them because the correct type of failure was not yet visible. This account therefore preserves the word as evidence of a problem while refusing to promote it into a theorem.

The minimum historical claim

By 2016 the operative suspicion was only that the expected global Fock carrier was inadequate. The notes did not yet determine whether the obstruction was infrared, representation-theoretic, local-to-global, categorical, or some combination of these.

3.2 The infraparticle analogy

The closest available comparison was the infraparticle problem: a charged state may fail to define a sharp isolated mass shell because it is inseparable from its infrared field, and ordinary Fock asymptotics cease to be adequate [2]. The analogy did not assert that pure Yang–Mills vacuum structure was an infraparticle sector. It supplied a warning:

$$\boxed{\text{failure of a Fock description need not mean failure of physical structure.}} \tag{4}$$

The structure may live in representations, sectors, asymptotic relations, or local algebras rather than in a single global particle basis.

3.3 The 2016 local W^* foothold

Recorded/remembered; original source not recovered. A 2016 office diagram placed a local W^*-algebra between scattering geometry and a non-Fock vacuum. The surviving conceptual chain was approximately

$$\begin{aligned} \text{amplitude geometry} &\longrightarrow \text{surviving symmetry/Yangian data} \\ &\longrightarrow W^* \text{ (local)} \longrightarrow \text{vacuum (non-Fock).} \end{aligned} \tag{5}$$

This was a foothold, not a construction. No local net, vacuum module, comparison law, or spectral theorem had been supplied. Its importance is negative and directional: the vacuum question was already being redirected away from a globally chosen Fock basis and toward local operator-algebraic realizations.

The most important question mark in the diagram sat between the symmetry data and the local algebra. A scattering construction can exhibit remarkable geometry and symmetry while still failing to produce the local observable carrier of a quantum field theory. The office diagram did not bridge that gap. It preserved the location at which a bridge would eventually be needed.

Retrospective recollection (2026)

The local W^* algebra was a foothold. I could see a possible place for the vacuum to live without pretending that the scattering surface itself was the vacuum. But I had no descent law, no completion, and no reason yet that the arrow should exist.

Do not import the mature answer

The self-dual Hilbert W^*-module vacuum, admissible fibration, vacuum gerbe, sector trichotomy, and thick-to-thin passage belong to later work. They are not presented as structures known in 2016.

Audit 2: amplitude symmetry is not yet a Haag–Kastler net

Even a precisely defined scattering geometry and an exact symmetry action do not automatically produce a net

$$U \longmapsto \mathcal{A}(U)$$

with isotony, locality, covariance, and an admissible state. To earn the arrow from amplitude data to a local W^*-realization one would need, at minimum, a construction of the regional algebras, their inclusions, and a proof that the proposed surviving datum acts compatibly on them. The 2016 diagram located the desired target; it did not provide this reconstruction.

Audit 3: a local W^*-algebra is not yet a vacuum

A von Neumann algebra can serve as a local observable carrier, but the word *vacuum* additionally asks for state, representation, covariance, spectral, and compatibility data. Depending on formulation, one would need a positive normal state or weight, a cyclic representation or module-level replacement, and a rule relating the local realizations. The early W^* foothold was therefore a candidate home, not a constructed resident.

4 The Green Field

4.1 The dream, approximately fall 2024

Remembered. The exact date remains open, provisionally fall 2024. The dream was simple in its scenery and difficult in its atmosphere. There was a green field. A fog moved with the dreamer, not merely around him. It behaved almost mathematically: movement disclosed forms. Beyond the field the old mountain was visible and recognizable. The dream carried a sense not of arrival but of permission—an invitation to return.

> There was a green field and a moving fog. As I moved, forms became visible. The old mountain was there. I recognized it. The feeling was not that I had reached the mountain. It was that I was being permitted to return.

On waking, the author wrote one word followed by a question mark:

$$\boxed{\text{Presheaf?}} \tag{6}$$

He did not know why.

Retrospective recollection (2026)

"Presheaf?" That note came after the dream, which is very odd. I had no idea what it might become. It was not an answer. It was scarcely even a question I could yet formulate.

4.2 The evidentiary boundary

The dream is not mathematical evidence. It does not prove that presheaves describe vacuum data, that a later construction was anticipated, or that a physical conclusion is true. It is evidence only for the biography of the question. The chronological oddity is that the word entered the record before the conscious developments that later made it mathematically relevant.

If the original note survives and can be dated, the note itself will be labeled Recorded; until then, its content and placement remain Remembered. The distinction is deliberately strict. Personal meaning and mathematical warrant are allowed to coexist without being identified.

The discipline here is not meant to drain the event of meaning. It is meant to preserve two different truths at once. The dream mattered to the author's decision to return to the problem; it does not carry the burden of proving any mathematical statement. The note matters historically because it fixes a word in discovery order; the later presheaf construction stands or falls on its own definitions and proofs.

4.3 A word without a role

A presheaf assigns local data contravariantly to regions and supplies restriction maps. That definition is not attributed to the dream. It is included only to mark the later mathematical role of the word:

$$U \longmapsto \mathcal{F}(U), \qquad V \subseteq U \Longrightarrow \rho_{UV} : \mathcal{F}(U) \to \mathcal{F}(V). \tag{7}$$

In fall 2024 there was no declared $\mathcal{F}$, no region category, no restriction law, and no descent theorem. There was a word and an unanswered question.

5 Re-outlining Yang–Mills in 2025

5.1 An inventory rather than a proof

Remembered; documentary audit pending. In 2025 the author began re-outlining what he knew, what he suspected, and what he had merely been carrying as an image of the Yang–Mills problem. The distinction among those levels was not always clean in the notes, but it can be reconstructed without pretending that every suspicion was already a claim.

Status	Historical content	Missing structure
Technically grounded	QFT, AQFT, renormalization, scattering, operator algebras, and known gauge-theoretic inputs	One construction relating the inputs
Suspected	A non-Fock vacuum carrier and geometric data surviving scattering and scale change	A declared object and comparison law
Unknown	What flowed, where it lived, and how local realizations assembled	Carrier, gluing, refinement, completion, and spectral realization

The inventory did not yet say that admissibility and refinement were distinct operations. It did not contain a sector trichotomy or a coefficient-band associator. Its value was preparatory: it placed the unresolved questions on one table where they could begin to interact.

The working questions in the 2025 re-outline

The re-outline can be reconstructed around five questions:

1. What is known from ordinary renormalization and local quantum field theory?
2. What survives from a scattering description after the amplitude itself has been evaluated?
3. Where can a vacuum live if one preferred global Fock realization is not available?
4. What is actually changed, and what is preserved, under coarse-graining?
5. If the physically meaningful data are local, what licenses their assembly into a global statement?

These questions are a present reconstruction of the outline's pressure points, not a verbatim transcription of one page.

Retrospective recollection (2026)

In 2025 I re-outlined what I knew—or thought I knew—about the problem. The important part was admitting the difference between what I could calculate, what I strongly suspected, and what I possessed only as a picture.

5.2 The return was not full-time

The work occurred alongside a demanding professional and family life. This historical fact matters because the chronology contains long periods of continued digestion rather than uninterrupted manuscript production. The return to the mountain was real, but it occurred in available hours, often after the day's primary obligations had ended. The eventual speed of December should not be mistaken for the absence of the preceding years.

6 The sock descends

6.1 May–June 2025: why it was called the sock

Remembered; contemporary corroborating note or diagram not recovered. The name *sock* began literally. During May–June 2025, while the author was studying at his computer, he was asked to find Sawyer's red baseball sock. He became frustrated, put the computer down hard, and walked through the house. After sitting down abruptly, the color red permeated his field of view. When ordinary sight returned, he perceived a shadow-like tube with strings moving back and forth from ceiling to floor. He paused and checked his physical condition before returning to the mathematics.

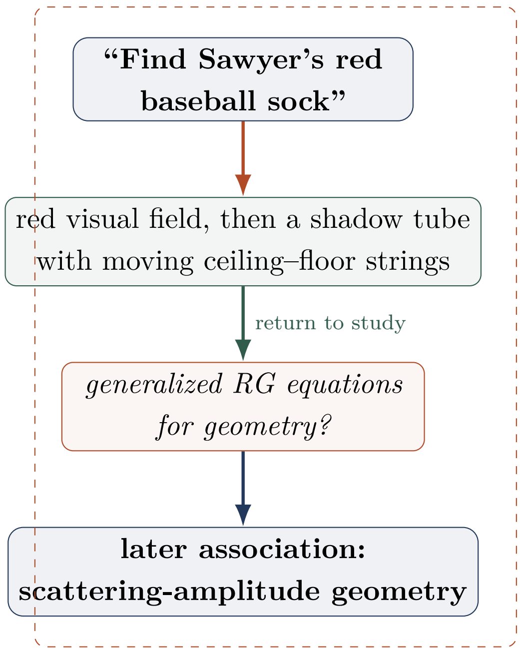
Retrospective recollection (2026)

"Scott, find Sawyer's red baseball sock." I became angry, slammed my computer down, and stomped through the house. I sat down abruptly and red permeated my field of view. When my sight reappeared, I saw a shadow tube with strings moving back and forth from ceiling to floor. I checked to make sure I was not having a stroke. When I returned to studying, it dawned on me: I was looking for a generalization of the renormalization-group equations for geometry.

The domestic trigger, the perceptual image, and the mathematical interpretation must be kept in that order. The tube did not arrive already labeled as a scattering-amplitude construction. The amplitude-surface interpretation was the mathematical association made as the author resumed work.

Epistemic boundary of the sock episode

The visual episode belongs to the history of discovery, not to the warrant of the mathematics. It supplied an image and prompted a question. Definitions, equations, composition laws, and proofs had to be constructed independently.



On returning to the mathematical problem, the image suggested that the amplitude surface might not merely encode a number. It might carry a shape, incidence structure, symmetry, measure, or other geometric datum whose transformation under scale could be modeled. The question was immediate:

$$\boxed{\text{What would generalized renormalization-group equations for geometry be?}} \tag{8}$$

6.2 A passage without a category of passage

The sock did not specify its source and target. It did not say whether the flowing object was a variety, a measure, an algebra, a state, a representation, or a system of local sections. It supplied no composition law and no proof that information surviving on the amplitude surface survived the descent.

That incompleteness is the historical point. The picture contained a direction and a payload but no category of transport:

$$\text{surface datum} \dashrightarrow \text{local/vacuum datum.} \tag{9}$$

The dashed arrow is not a theorem. It is the question that the later theory would repeatedly try to type.

The picture also changed the direction of inquiry. Instead of asking only how an amplitude was computed, it asked what structure remained available after the amplitude had done its ordinary job. The surviving object might have been a symmetry representation, an incidence relation, a positivity domain, an operator-algebraic shadow, or nothing sufficiently invariant to transport. The sock was useful precisely because it kept the payload blank.

6.3 The shape is the geometry of what survives

Modern reading. The later maxim "the shape is the geometry of what survives" captures what the sock was trying to see, but the maxim should not be attributed verbatim to May 2025 without a dated source. The early picture asked whether some geometric payload survived the passage from scattering to local realization. It did not yet identify the survivor.

7 What could be flowing?

The word *renormalization* was now carrying several candidate objects. They should not be collapsed.

Picture	Candidate flowing datum	Unresolved question
Conventional RG	Couplings and effective actions	Is the parameter flow the whole physical passage?
Measure-theoretic	Measures and measure classes	Which distinctions survive change of measure or coarse-graining?
Operator-algebraic	Algebras, states, and representations	Is there one global carrier or only compatible local realizations?
Geometric	Shapes, incidence data, and symmetry payloads	What part of the geometry is invariant under descent?
Local-to-global	Sections and their comparison data	What gluing law, if any, is physically licensed?

This table is a modern reconstruction of the alternatives, not a transcription of one 2025 page. Its purpose is to show why a single phrase such as “geometric RG” could not yet be a definition. The candidate payloads had different mathematical types and required different comparison maps.

Lesson 7.1 (The carrier question precedes the flow law). One cannot define a geometric renormalization passage merely by drawing an arrow between scales. One must first declare what occupies each endpoint, which distinctions are physical, and what comparison is permitted. This lesson is modern; the sock is the historical picture that exposed the need for it.

Technical interlude: the five unearned arrows

Modern audit. By late summer 2025 the discovery pictures contained five arrows that looked plausible but had not been mathematically earned. The point of naming them is not to criticize the pictures. It is to show exactly what work those pictures were asking the later theory to perform.

Historical arrow	What the picture supplied	What remained unearned
Running parameters $\rightsquigarrow$ transported carrier	Scale dependence and effective comparison	Endpoint objects, comparison maps, composition, and invariants
Scattering geometry $\rightsquigarrow$ local W^* data	A symmetry and geometric payload that appeared to survive the amplitude	Reconstruction of regional algebras, locality, isotony, and compatible action

Historical arrow	What the picture supplied	What remained unearned
Local W^* data $\rightsquigarrow$ vacuum carrier	A plausible operator-algebraic home outside one naive Fock picture	State or module, covariance, spectral condition, and local compatibility
Local assignment $\rightsquigarrow$ glued global object	The word "presheaf" and a local-to-global direction	Covering structure, overlap comparisons, cocycle/coherence law, and effective descent
Gluing data $\rightsquigarrow$ RG-persistent obstruction	The expectation that something global survived coarse-graining	Equivariance, stability estimate, invariant target, and proof of persistence

Why the arrows cannot be composed informally

The codomain of one historical arrow was not yet the domain of the next. A point in coupling space is not an algebra. An amplitude surface is not a local net. A von Neumann algebra is not a vacuum state. A presheaf is not a glued object. A cocycle-shaped datum is not automatically stable under renormalization. Thus the attractive composite

$$\mathrm{RG} \dashrightarrow \text{geometry} \dashrightarrow W^*_{\mathrm{loc}} \dashrightarrow \mathbf{Vac} \dashrightarrow \text{persistent obstruction} \tag{10}$$

was a research program written as an arrow, not a single deduction.

The anti-smuggling rule for this history

Whenever two early pictures share a noun—*flow*, *geometry*, *vacuum*, *local*, or *obstruction*—the shared word does not license composition. Composition becomes available only after source, target, comparison law, and physical interpretation have been declared at compatible types.

A minimum typing ledger

The following ledger gives the least information that would have been needed to turn each picture into a mathematical claim. It is deliberately agnostic about the mature solution.

Word in the notes	Minimum declaration	Minimum witness
Flow	A family of endpoint objects and directed comparison maps	Identity and composition law, exact or controlled
Carrier	The algebraic/geometric object bearing the relevant distinctions	A criterion identifying which distinctions are physical
Local	A region category and restriction assignment	Isotony/restriction and compatibility on overlaps
Vacuum	A state, representation, module, or explicitly declared replacement	Positivity, covariance, and spectral/vacuum selection data
Gluing	Descent data relative to a declared cover or topology	Cocycle or higher coherence plus an effectivity statement
Persistence	An invariant or quantitative target compared across scales	Equivariance, stability, or monotonicity estimate

This ledger is the main mathematical correction. It preserves the audacity of the early pictures while preventing later success from laundering their missing types.

8 Measure mashing returns

8.1 August 2025: a measure is geometric data

Remembered. In August 2025 the carrier question turned back toward Klauder's measure-mashing program. The observation was elementary and productive:

$$\boxed{\text{A measure is itself geometric data.}} \tag{11}$$

Retrospective recollection (2026)

A measure is like geometric datum. What about Klauder's measure-mashing program? I had already wanted to understand renormalization in terms of what actually flowed; here was a concrete carrier whose geometry genuinely changed.

It determines null sets, typical configurations, integrability, concentration, and which limiting operations remain meaningful. Changing a measure is not merely changing coefficients in a completed calculation. It can change the effective geometry of the configuration space.

The working objective was therefore sharper than "make the measure flow." The author was trying to identify which information-geometric or measure-geometric datum was being transformed: the measure class, its radial and angular concentration, its distinguished variation directions, its moments and correlations, or some stable combination of these. Measure mashing was the first setting in which that carrier question could be posed by equations rather than by the sock image alone.

Klauder's program had long emphasized that nonperturbative quantization may require changing the measure-theoretic architecture rather than attempting to repair perturbation theory term by term [5, 7]. The author returned to that literature, reacquainted himself with the established results, and provisionally trusted technical work that Klauder himself had trusted. This was a working decision, not a claim that measure mashing already solved the Yang–Mills problem.

One concrete reason the program was suggestive is that, in infinite-dimensional field problems, parameter changes may otherwise produce mutually singular measures. Klauder's later scalar-field formulation described a modified ground-state measure intended to replace that singular behavior by equivalence and thereby reorganize the divergence problem [6]. No part of that scalar-field result is transferred here to Yang–Mills. The historical lesson is narrower: a change in the carrier measure can be structural rather than cosmetic.

Retrospective recollection (2026)

I had gone farther than using measure mashing as a picture. I began with John's definitions and worked through the calculation. I constructed renormalization-group equations, spent several weeks modeling their behavior, and varied the mashing terms to see how the geometric data transformed under flow. The compositions could be read from those calculations. It was already a calculation program, not merely an analogy for one.

8.2 The radial mashing mechanism

Reconstructed mathematical skeleton; original notation not recovered. The simplest finite-dimensional model begins with N field coordinates and their polar decomposition

$$\phi_k = \kappa\eta_k, \qquad \kappa \geq 0, \qquad \sum_{k=1}^{N} \eta_k^2 = 1.$$

The flat product measure contains the high-dimensional radial factor

$$\prod_{k=1}^{N} d\phi_k = \kappa^{N-1}\, d\kappa\, d\Omega_{N-1}(\eta). \tag{12}$$

As N grows, this power is one source of concentration and of singular behavior between parameter-dependent measures. A minimal mashing density has the form

$$m_{N,R}(\kappa,\eta) = c_{N,R}(\eta)\,\kappa^{R-N}, \tag{13}$$

so that

$$m_{N,R}\prod_{k=1}^{N} d\phi_k = c_{N,R}(\eta)\,\kappa^{R-1}\,d\kappa\,d\Omega_{N-1}(\eta). \tag{14}$$

The enormous exponent $N-1$ has been replaced by the controlled exponent $R-1$. Klauder's actual lattice constructions encode the mashing through a local ground-state factor rather than by treating (13) as a complete field theory. The displayed formula isolates the radial mechanism that made the RG calculation geometrically suggestive.

Proposition 8.1 (Elementary radial mashing identity)**.** *Suppose a measure on $\mathbb{R}^N$ has radial-angular density $f(\kappa,\eta)\kappa^{N-1}$. Multiplication by a positive weight $c(\eta)\kappa^{R-N}$, followed by normalization when finite, produces a measure with radial power κ^{R-1}.*

Proof. The claim follows by adding the radial exponents: $(N-1)+(R-N) = R-1$. Positivity is inherited from the weight and the original density; division by the finite total mass gives normalization. □

This elementary identity is not the whole measure-mashing program. It is the finite-volume algebraic seed explaining why a change of density can alter the effective geometry before any continuum limit is taken.

8.3 The RG equation in a mashing frame

Let $t = \log(\mu/\mu_0)$ denote scale time, let $\mathscr{X}_t$ be a regulated configuration space, and let $\mathbb{P}_t$ be its effective probability measure. A coarse-graining map $B_{s\leftarrow t} : \mathscr{X}_t \to \mathscr{X}_s$, for $s \geq t$, induces the measure-level renormalization map

$$\mathcal{R}_{s\leftarrow t}(\mathbb{P}_t) := (B_{s\leftarrow t})_{\#}\mathbb{P}_t. \tag{15}$$

For a positive mashing density m_t with finite nonzero normalizer, define

$$\mathcal{M}_t(\mathbb{P})(d\phi) := \frac{m_t(\phi)\,\mathbb{P}(d\phi)}{\int_{\mathscr{X}_t} m_t\,d\mathbb{P}}. \tag{16}$$

Proposition 8.2 (Probability and equivalence under a finite mash)**.** *Let $\mathbb{P}$ be a probability measure, let m be measurable, and assume $0 < m < \infty$ $\mathbb{P}$-almost everywhere with $0 < \int m\,d\mathbb{P} < \infty$. Then $\mathcal{M}(\mathbb{P})$ is a probability measure equivalent to $\mathbb{P}$, with*

$$\frac{d\mathcal{M}(\mathbb{P})}{d\mathbb{P}} = \frac{m}{\int m\,d\mathbb{P}}.$$

Proof. Normalization follows by integration. The Radon–Nikodym derivative is strictly positive and finite almost everywhere, so the two measures have the same null sets. □

On a measure class where $\mathcal{M}_t$ is invertible, the RG evolution seen in mashing coordinates is the conjugated passage

$$\widetilde{\mathcal{R}}_{s\leftarrow t} := \mathcal{M}_s \circ \mathcal{R}_{s\leftarrow t} \circ \mathcal{M}_t^{-1}. \tag{17}$$

Equation (17) is present notation for the minimal structure of the remembered calculation. It is not asserted to reproduce the notation or every term of the unrecovered 2025 manuscript. Its importance is structural: the mashing map changes the frame in which the ordinary measure-level RG is represented.

Proposition 8.3 (Composition inherited in the mashing frame)**.** *Assume $\mathcal{R}_{u\leftarrow s} \circ \mathcal{R}_{s\leftarrow t} = \mathcal{R}_{u\leftarrow t}$ and assume the mashing maps are invertible on the declared measure classes. Then*

$$\widetilde{\mathcal{R}}_{u\leftarrow s} \circ \widetilde{\mathcal{R}}_{s\leftarrow t} = \widetilde{\mathcal{R}}_{u\leftarrow t}.$$

Proof. Insert (17). The middle factors $\mathcal{M}_s^{-1}\mathcal{M}_s$ cancel, and the two ordinary RG maps compose:

$$\mathcal{M}_u \mathcal{R}_{u\leftarrow s} \mathcal{M}_s^{-1} \mathcal{M}_s \mathcal{R}_{s\leftarrow t} \mathcal{M}_t^{-1} = \mathcal{M}_u \mathcal{R}_{u\leftarrow t} \mathcal{M}_t^{-1}.$$

□

The proposition explains why composition was readable directly from the calculation: once the ordinary scale passages and the mashing frame at each endpoint were written explicitly, the intermediate frame cancelled. The conditional wording belongs only to the present reconstruction's hypotheses; the historical recollection is that the compositions themselves were worked out in the measure-mashing calculation.

Group language versus irreversible coarse-graining

The historical notes used "renormalization group." Equation (17) may be an invertible change of frame on a selected measure class, but a physical coarse-graining map need not be invertible. The safest mathematical statement is therefore a semigroup or directed composition law unless an actual inverse has been constructed.

8.4 Infinitesimal form

On a fixed regulated configuration space, choose a common reference measure λ and write the unmashed density as $p_t = d\mathbb{P}_t/d\lambda$. Suppose its infinitesimal evolution is

$$\partial_t p_t = \mathcal{L}_t^* p_t. \tag{18}$$

Let M_t denote multiplication by the positive density m_t, and first ignore normalization by setting $q_t = M_t p_t$. Then

$$\partial_t q_t = \left(\dot{M}_t M_t^{-1} + M_t \mathcal{L}_t^* M_t^{-1}\right) q_t =: \mathcal{A}_t q_t. \tag{19}$$

The first term is the scale variation of the mashing frame; the second is the ordinary RG generator expressed in that frame. If $\nu_t = q_t / \int q_t \, d\lambda$, normalization adds the scalar correction

$$\partial_t \nu_t = \mathcal{A}_t \nu_t - \nu_t \int \mathcal{A}_t \nu_t \, d\lambda. \tag{20}$$

Here and below, ν_t denotes both the normalized density relative to λ and, when it appears under an integral such as $d\nu_t$, the corresponding probability measure.

Consequently every test observable F in the dual domain satisfies

$$\frac{d}{dt}\langle F \rangle_{\nu_t} = \int F \, \mathcal{A}_t \nu_t \, d\lambda - \langle F \rangle_{\nu_t} \int \mathcal{A}_t \nu_t \, d\lambda. \tag{21}$$

Thus normalization subtracts precisely the component parallel to the constant observable.

For the radial model $m_t(\kappa) = \kappa^{\alpha(t)}$, the contribution from the changing mashing exponent alone is

$$\partial_t \log \frac{d\nu_t}{d\mathbb{P}_t} = \dot{\alpha}(t) \left(\log \kappa - \langle \log \kappa \rangle_{\nu_t}\right), \tag{22}$$

where the centered term enforces normalization. The full RG equation also contains the transformed generator in (19).

8.5 Variations of the mashing term

The remembered program did not use a single frozen mash. To represent the variation calculation, let $\theta = (\theta^1, \ldots, \theta^d)$ parameterize a positive family $m_{t,\theta}$ and set

$$d\nu_{t,\theta} := \frac{m_{t,\theta} \, d\mathbb{P}_t}{Z_t(\theta)}, \qquad Z_t(\theta) := \int m_{t,\theta} \, d\mathbb{P}_t. \tag{23}$$

The centered logarithmic variation

$$S_a(t,\theta) := \partial_{\theta^a} \log m_{t,\theta} - \langle \partial_{\theta^a} \log m_{t,\theta} \rangle_{\nu_{t,\theta}} \tag{24}$$

is the infinitesimal geometric response of the normalized carrier to a change in the mashing term.

Proposition 8.4 (Observable response to a mashing variation)**.** *Assume differentiation may be passed under the integral in* (23)*. Then every integrable test observable F satisfies*

$$\partial_{\theta^a} \langle F \rangle_{\nu_{t,\theta}} = \langle F S_a(t,\theta) \rangle_{\nu_{t,\theta}} = \mathrm{Cov}_{\nu_{t,\theta}}(F, \partial_{\theta^a} \log m_{t,\theta})\,. \tag{25}$$

Proof. Differentiate the numerator and denominator of $\langle F \rangle_{\nu_{t,\theta}}$. The derivative of $\log Z_t(\theta)$ is the $\nu_{t,\theta}$-expectation of $\partial_{\theta^a} \log m_{t,\theta}$; subtracting it gives the centered score in (24). □

For a radial-angular family

$$m_{t,\theta}(\kappa,\eta) = c_{t,\theta}(\eta)\, \kappa^{\alpha(t,\theta)}, \tag{26}$$

the uncentered response splits as

$$\partial_{\theta^a} \log m_{t,\theta} = \partial_{\theta^a} \log c_{t,\theta}(\eta) + \partial_{\theta^a} \alpha(t,\theta) \log \kappa. \tag{27}$$

Thus variations of the angular factor probe redistribution along the sphere, while variations of the exponent probe radial concentration. Coupled with the scale equation (20), these identities make precise the program's governing question: not merely how parameters run, but how the measure's geometric data respond while the carrier is transported through scale.

8.6 Which information geometry is transformed?

Remembered objective; present mathematical organization. The variation scores in (24) determine the positive semidefinite information tensor

$$g_{ab}(t,\theta) := \langle S_a(t,\theta) S_b(t,\theta) \rangle_{\nu_{t,\theta}} = \mathrm{Cov}_{\nu_{t,\theta}}(\partial_{\theta^a} \log m_{t,\theta}, \partial_{\theta^b} \log m_{t,\theta})\,. \tag{28}$$

This is not asserted to be the notation used in the original calculation. It gives a precise present representative of the question the author remembers asking: which distinguishable directions in the family of mashed measures are preserved, contracted, or amplified by RG flow?

If S_aS_b lies in the time-dependent observable domain, then (21) gives

$$\partial_t g_{ab} = \langle \partial_t(S_aS_b)\rangle_{\nu_{t,\theta}} + \int S_aS_b\, \mathcal{A}_t\nu_{t,\theta}\, d\lambda \\ - g_{ab}\int \mathcal{A}_t\nu_{t,\theta}\, d\lambda. \tag{29}$$

The three terms separate change of the variation directions themselves, transport by the mashed RG generator, and the normalization correction. In the radial-angular family (26), this evolution tracks whether scale change alters principally the radial concentration, the angular distribution, or their coupling.

The broader geometric ledger remains deliberately plural:

$$\mathfrak{G}(\nu_{t,\theta}) = \Big([\nu_{t,\theta}],\ g_{ab}(t,\theta),\ \{\langle F\rangle_{\nu_{t,\theta}}\}_{F\in\mathcal{O}},\ \text{concentration data}\Big). \tag{30}$$

The historical calculation need not have combined these entries into one object. The ledger records the question that drove the modeling: what geometric information of the measure is actually carried through the flow?

8.7 Properties placed under study

Remembered at the program level; exact lemmas and numerical examples await the original notes. Positivity, normalization, finite-measure equivalence, and the composition law are consequences of the reconstructed system under the displayed hypotheses. The historical testimony is stronger than a list of questions: the author studied the properties of the RG system, its compositions, and families of mashing terms for several weeks. The surviving reconstruction places the following items in that calculation ledger:

(i) **positivity and normalization:** when does (16) remain a probability measure?;

(ii) **equivalence versus singularity:** when does $m_t > 0$ preserve a measure class, and when can a limiting mash change it?;

(iii) **composition:** how the mashing factors and ordinary scale maps combine, with the intermediate frame cancellation displayed above;

(iv) **fixed measures:** can a mashed measure satisfy $\widetilde{\mathcal{R}}_{s\leftarrow t}(\nu_t) = \nu_s$, or become stationary after rescaling?;

(v) **moment and correlation flow:** how do $\int F\, d\nu_t$ and selected correlations evolve under the dual generator?;

(vi) **mashing-term response:** how do radial exponents, angular weights, moments, and concentration profiles respond to variations of $m_{t,\theta}$?;

(vii) **information geometry:** which variation directions and information-metric components are preserved, contracted, or amplified by scale transport?; and

(viii) **stability:** which perturbations of the density or mashing exponent grow, decay, or remain neutral under iteration?

Source boundary for the reconstructed equations

The existence of an RG-equation calculation and a study of its properties is now recorded as first-person testimony. It is also recorded that the starting definitions were Klauder's, that composition was obtained from the calculations, and that variations of the mashing terms were explicitly modeled. Equations (15)–(30) give a mathematically consistent reconstruction of that program's minimal structure. They must be compared against the original manuscript before any symbol, generator, fixed point, or further proved property is attributed verbatim to the 2025 work.

8.8 Why the program was the right toy

Measure mashing supplied a concrete response to the carrier question. One could ask what changed under coarse-graining while keeping track of the measure class and of the distinctions it supported. The program offered:

- a nonperturbative object that genuinely changed;
- a way to think about equivalence and singularity of descriptions;
- a model for retaining structure across changes of scale;
- and a reminder that the underlying carrier is part of the physics.

The role was methodological. Following Klauder's training, once a feature became visible, the author sought the smallest model in which it could be isolated before restoring the full physical pressure:

$$\boxed{\text{isolate the feature} \longrightarrow \text{build a toy} \longrightarrow \text{calculate} \longrightarrow \text{restore the omitted pressures.}} \tag{31}$$

Retrospective recollection (2026)

This was my training from John: once you have a feature you are trying to understand, go immediately to a toy. Remove the full physicality long enough to see the feature itself; then bring the other pressures back.

This habit explains why the December work would begin with Ising rather than with a direct assault on four-dimensional Yang–Mills. The toy was not an attempt to pretend that the theories were the same. It was a controlled way to ask whether a local-to-global vacuum obstruction could survive a scale passage at all.

8.9 A scaffold that would disappear

The specific measure-mashing map did not survive the later move to the first W^*-vacuum toy. There was no evident operator-algebraic analogue of the map in the form first imagined. The disappearance should not be described as failure. A scaffold may be essential to discovering a structure without belonging to the completed structure.

What survived was more abstract:

$$\text{scale-dependent passage} + \text{a carrier whose distinctions must be tracked.} \tag{32}$$

The concrete measure map vanished. The question of transport remained.

Its disappearance is historically informative. The first operator-algebraic toy did not present an evident analogue of the measure-mashing map, so the map could not be retained merely because it had been useful one month earlier. What transferred was the investigative discipline and the intuition that the carrier participates in the physics. What did not transfer was allowed to fall away.

Open bibliographic item

The exact chain of measure-mashing papers revisited in August 2025, including the identity and role of Klauder's Italian colleague in the author's recollection, must be fixed from the surviving reading notes or bibliography. No name is assigned here by inference.

9 The presheaf question returns

9.1 One global object or many local realizations?

The sock supplied a descending payload. Measure mashing supplied an example in which the carrier mattered. The non-Fock suspicion supplied a warning against assuming one preferred global representation. Together they made a new question possible:

$$\boxed{\text{What if the flowing datum is represented only locally?}} \tag{33}$$

This was the point at which the unexplained word from the Green Field could begin to resonate. A presheaf could, in principle, assign local data to regions and remember restriction. But restriction is not gluing, and gluing is not physical realization. The word still did not answer:

- what the local objects were;
- whether their overlap comparisons were unique;

- whether the comparisons composed strictly;
- how scale entered the base;
- or which global object, if any, they determined.

Retrospective recollection (2026)

Presheaf? I still did not know what the object was. But now the word was standing beside three pressures at once: the vacuum might be only locally represented, the scattering surface might carry a descending datum, and the renormalization group required comparison across scales.

9.2 Two directions were present before their compatibility

Modern reading. Once local assignment and scale change are placed in the same picture, two distinct arrows appear:

$$\begin{aligned} U \subseteq V: &\quad \mathcal{F}(V,\mu) \longrightarrow \mathcal{F}(U,\mu), &&\text{regional restriction,} \\ \mu_1 \succcurlyeq \mu_2: &\quad \mathcal{F}(U,\mu_1) \dashrightarrow \mathcal{F}(U,\mu_2), &&\text{scale passage.} \end{aligned} \tag{34}$$

The notation was not present in the early notes. It is used here to expose a problem that was already latent: even if both arrows exist, it does not follow that restricting and coarse-graining commute, agree up to a controlled comparison, or preserve the same physical distinctions. The early coherence guess reconstructed in No. 8 will begin exactly where this incompatibility becomes impossible to ignore.

Audit 4: the first square was not known to commute

For $U \subseteq V$ and $\mu_1 \succcurlyeq \mu_2$, the first compatibility test would have been the square

$$\begin{tikzcd} \mathcal{F}(V,\mu_1) \arrow[r, "\rho^{\mu_1}_{VU}"] \arrow[d, "\Phi^V_{12}"'] & \mathcal{F}(U,\mu_1) \arrow[d, "\Phi^U_{12}"] \\ \mathcal{F}(V,\mu_2) \arrow[r, "\rho^{\mu_2}_{VU}"'] & \mathcal{F}(U,\mu_2). \end{tikzcd}$$

Strict commutation would require

$$\rho^{\mu_2}_{VU} \circ \Phi^V_{12} = \Phi^U_{12} \circ \rho^{\mu_1}_{VU}.$$

No such equality had been established. Nor had a specified comparison 2-cell, defect estimate, or obstruction class been constructed. The square is a modern display of the missing coherence, not an early theorem.

9.3 Before cohomology

The historical order must stop short of the later calculation. At this stage there was no declared vacuum torsor in $\check{H}^1$, no central lifting class, no C3 coherence system, no associator in H^3, and no Clearance estimate. There was only a local-to-global possibility:

$$\{\text{local datum on } U_i\} \quad + \quad \{\text{comparison on } U_i \cap U_j\} \quad \dashrightarrow \quad ? \tag{35}$$

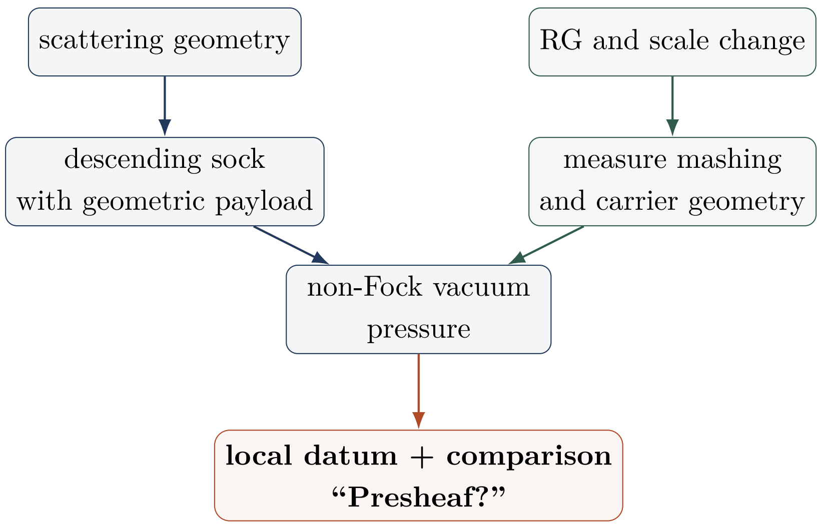
The final question mark is historically accurate. It is not a missing symbol to be repaired by the mature theory.

Audit 5: presheaf data are not yet effective descent

A presheaf supplies restriction. Even compatible-looking overlap data need not determine a global object unless a topology, descent condition, and effectivity statement have been provided. If overlap comparisons hold only up to isomorphism, one must additionally identify the relevant groupoid or higher coherence. None of this follows from writing "Presheaf?" The word identified the direction of the problem; it did not solve the gluing problem.

10 Two long threads meet

By late 2025 the prehistory can be represented by two lines that had developed largely independently, joined by a third pressure from the vacuum:



The diagram is a Reconstructed map of questions, not a claim that the presheaf word was deduced from either line. Chronologically, the word had already been written. Mathematically, the later developments supplied the first reasons it might matter.

10.1 The non-Fock pressure joins the intersection

The vacuum suspicion altered the local-to-global question. If one global Fock carrier was unavailable, then local data could not be treated merely as restrictions of an object already known to exist. The existence of the global object became part of the problem:

$$\boxed{\text{local realizations} \not\Rightarrow \text{one globally represented vacuum carrier.}} \tag{36}$$

This statement is written in modern notation, but the pressure behind it was present before the formal distinction was earned.

10.2 What a first toy would have to earn

The audit makes clear why a toy model was necessary. Before any Yang–Mills claim could be attempted, one needed a setting in which the following pieces could at least be written without ambiguity:

(a) a declared base of regions or patches;
(b) explicit local vacuum data;
(c) overlap comparisons and a gluing obstruction;
(d) a coarse-graining operation acting on those data;
(e) an observable or sector distinction detecting the global class; and
(f) a boundary separating what was exact in the toy from what remained conjectural in gauge theory.

The toy-model admission test

The toy would be informative only if it closed at its own level. It could not borrow the desired Yang–Mills conclusion as an assumption, and exact solvability could not be mistaken for universality. Its legitimate role was to show that the proposed obstruction-persistence mechanism was mathematically coherent in at least one controlled model.

11 At the threshold of December

By the threshold of the December work, the following ingredients were active or available:

(1) RG as physical scale machinery;
(2) the mathematical question of a transported carrier;
(3) a long-standing non-Fock vacuum suspicion;
(4) a local W^*-algebraic foothold;

(5) scattering geometry and the descending sock;
(6) measure mashing as a temporary geometric-RG model;
(7) and the unexplained word "Presheaf?"

They did not yet form a theorem. They did not even form one well-typed definition. What had formed was enough internal pressure to demand a toy. The threshold question was something like

What if the gluonic datum lives on local sections whose vacuum data must be glued?

Returning to a toy model was therefore neither a retreat nor an accidental simplification. It was the practiced response to an over-pressured question. The full Yang–Mills theory simultaneously carried locality, gauge symmetry, infrared structure, operator-algebraic representation, scale change, and spectral demands. A toy could temporarily remove most of that physicality and ask one sharply limited question: can local vacuum inequivalence be represented as gluing data that remains meaningful under coarse-graining?

Retrospective recollection (2026)

There were already things I had failed to explore before the second toy. That is what the speed can hide. Many ideas were present at once, but presence is not understanding. The first night supplied an onset, not the completed construction.

The wording is reconstructed. The next note will preserve the staircase moment, the Ising model, the immediate W^*-vacuum leap, the Wilson-loop clustering estimates, and the first conjecture called Clearance. This note ends one step earlier.

The prehistory did not supply the first calculation. It supplied the reasons that, when the calculation appeared, so many old questions entered it at once.

12 What survived, what vanished, what changed type

This section is explicitly a Modern reading. Survival does not mean identity, and resemblance does not establish descent from one historical object to another.

Early element	Historical role	Later fate
QFT II RG machinery	Physical understanding of scale-dependent effective description	Remained the dynamical and analytic background
The carrier question	Asked what survived beneath running parameters	Became a transport and refinement question
“Nonseparable”	Placeholder for a vacuum that would not fit the expected global carrier	Discarded as too crude
Infraparticle comparison	Warning that non-Fock need not mean nonphysical	Survived as an analogy, not an identification
Local W^* foothold	Candidate home between scattering data and vacuum	Developed into a much more precise operator-algebraic carrier
Green Field “Presheaf?”	A word without an assigned object	Later became local-to-global vacuum language
The sock	A visual carrier descending from an amplitude surface	Developed toward tube and transport geometry
Measure mashing	Concrete model of a carrier changed under nonperturbative quantization	Vanished as a specific map; survived as method and scale intuition
“What flows?”	The unresolved question joining the threads	Became the question of what passage preserves which physical distinctions

13 Open historical questions and retained source obligations

This edition freezes the account at the strongest level supported by the available record. The following source obligations remain open. Their nonresolution limits the precision of the chronology and the language of individual claims; it does not license stronger claims, and it does not prevent publication of the bounded reconstruction.

Historical proof obligation 13.1 (Date the Green Field note)**.** Locate the original “Presheaf?” note, journal entry, photograph, or device timestamp. Until then, “fall 2024” remains approximate.

Historical proof obligation 13.2 (Recover the 2025 outlines)**.** Identify the outlines in which the known, suspected, and unknown parts of the Yang–Mills problem were reorganized. Record their dates without rewriting their terminology.

Historical proof obligation 13.3 (Recover the sock)**.** Locate the May–June 2025 drawing or earliest surviving description of the descending sock. The clean diagram in this note is a reconstruction and must not replace the original artifact.

Historical proof obligation 13.4 (Fix the measure-mashing bibliography)**.** Determine exactly which Klauder papers and related works were reread in August 2025, and identify any collaborator only from documentary evidence.

Historical proof obligation 13.5 (Separate the 2016 layers)**.** Date the office diagram and distinguish what was drawn in 2016 from what was later remembered or added to its interpretation.

Historical proof obligation 13.6 (Recover the 2004 course and reading sequence)**.** Locate the QFT II syllabus, notebooks, problem sets, or dated marginalia that can distinguish the renormalization material encountered in the course from the later effect of *Quantum Fields and Strings*. This edition records influence, not a verified week-by-week chronology.

Historical proof obligation 13.7 (Distinguish contemporary words from 2026 testimony)**.** Retain each boxed retrospective passage as present-day testimony unless the same wording is found in a dated source. In particular, “what flowed,” “magical,” and the description of the sock should not silently become 2004 or 2025 quotations.

These are not peripheral archival chores. They protect the distinction between the history that can be demonstrated, the history the author remembers, and the meaning later mathematics makes available.

Unrecovered artifact	What it could settle	What it cannot settle alone
QFT II notes/syllabus	Date and technical scope of the first RG exposure	The full later meaning of that exposure
2016 office diagram	Original order and labels of the amplitude–W^*–vacuum chain	Whether any displayed arrow was mathematically justified
Green Field note	Date and exact wording of "Presheaf?"	Any theorem about presheaves or vacua
Sock sketch	Earliest geometry and terminology of the descending tube	A category or invariant for the passage
August reading notes	Exact measure-mashing sources and named contributors	An identification with the later transport construction

14 Conclusion: first glimpses

The December ansatz was not waiting in completed form inside the 2004 course, the mathematical literature, the 2016 diagram, the Green Field dream, the sock, or measure mashing. Each contributed a different pressure.

QFT II supplied renormalization as physical machinery. The course for mathematicians widened it into a question about structure. The non-Fock suspicion removed confidence in one ordinary global carrier. Scattering geometry supplied a payload and a direction of descent. Measure mashing supplied a concrete example in which the carrier mattered. The word "presheaf" appeared before any of these pieces had been assembled into a mathematical use.

By late 2025 the questions had converged but the construction had not. There was no declared local vacuum object, no earned gluing law, no refinement system, no continuum passage, and no spectral theorem. There was only the sense that the old formulation had hidden the object one needed to follow.

The genealogy is now sharper. Renormalization supplied passage without a declared traveler. The mathematical course supplied possible languages for a carrier. The non-Fock suspicion made the global carrier doubtful. Scattering geometry supplied a picture of surviving payload. The sock supplied a direction. Measure mashing supplied a concrete example in which changing the carrier changed the theory's effective geometry. "Presheaf?" supplied a local-to-global word before its role was known. None was the answer. Together they made the old question impossible to leave in its earlier form.

Retrospective recollection (2026)

Looking back, the naive part was not that I saw something. Something was there. The naive part was believing that one glimpse meant I was close to the summit. The view from the mountain now shows how many distinct problems were compressed inside the first excitement.

Then came the staircase.

The next note begins with a toy model.

Public release certificate

This Version 1.0 is the canonical public website edition of Research Note No. 7. It freezes the note's historical reconstruction and its claim boundaries while retaining every identified uncertainty about dates, sources, and unrecovered artifacts. Publication does not convert recollection into documentary evidence, reconstruction into recovered notation, or historical influence into a theorem or priority claim.

A Provisional chronology

Date or period	Evidence status	Event
2004	Remembered	First sustained RG exposure in QFT II
After 2004	Remembered / source open	*Quantum Fields and Strings* widens the RG question toward mathematical structure
2016 onward	Remembered, partly recorded	Non-Fock vacuum suspicion; "nonseparable" used as a placeholder; local W^* foothold
Fall 2024-ish	Remembered; note not recovered	Green Field dream; "Presheaf?" written on waking
Early 2025	Remembered; outlines not recovered	Re-outline of what was known, suspected, and unknown about the Yang–Mills problem
May–June 2025	Remembered; diagram not recovered	Descending sock carrying geometric datum from an amplitude surface; generalized geometric RG question
August 2025	Remembered; reading record not recovered	Return to Klauder's measure-mashing program and coarse-graining literature

Date or period	Evidence status	Event
Late 2025	Reconstructed	Presheaf, non-Fock vacuum, scattering geometry, and RG carrier questions converge at the threshold of the December toy

B Claim ledger

Statement	Status	Boundary
The first sustained RG exposure occurred in QFT II in 2004	Remembered	Course record may sharpen
The Green Field dream preceded the mathematical presheaf construction	Remembered	Original note/date not recovered
The sock appeared in May–June 2025	Remembered	Original drawing not recovered
Measure mashing served as a geometric-RG scaffold	Remembered / reconstructed	No identity with the mature transport theory is claimed
The December ansatz arose from converging older questions	Reconstructed	Causal interpretation, not theorem or priority claim
The later theory validates the dream	Not claimed	Mathematical warrant is independent of the dream

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