05
SERIES NO. 5

From the Amplituhedron to a W*-Payload

The 2014–2015 Line, Reconstructed First;

the Modern Operator-Algebraic Program, Separated and Sharpened

View from the Mountain notes from a whiteboard

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Research posture and claim boundary

Planar $\mathcal{N} = 4$ super Yang–Mills theory is used here as a controlled exact witness, not as a globally fundamental ontology. The proposed W*-foothold is a represented closure of a specified bounded realization, not a declaration that the abstract super Yangian is a von Neumann algebra. The conjecture begins after that foothold, with localization, states, sectors, and removal of the special assumptions.

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Abstract

This note first reconstructs the narrower 2014–2015 line of investigation whose original notebooks and whiteboard calculations did not survive the author's move from Chillicothe to Columbus. The historical problem was not to derive full QCD, a global scattering theory, or a Haag–Kastler net from the amplituhedron. It was to ask whether a planar, color-ordered, stripped scattering kernel still carried evidence of the positive-energy Poincaré representation data from which a local quantum theory would have to be built. The note therefore reconstructs the missing comparison in detail: a globally realized scattering theory has a physical Hilbert space, asymptotic wave operators, Poincaré intertwining, operator unitarity, completeness, and causal separation, whereas the planar positive geometry retains only a rigid color-ordered scattering interface. The experimental regularity of that interface makes it physically meaningful; its projection and stripping make it partial. The reconstructed chain runs from planar positive geometry, through stripped scattering kernels and Yangian-organized intertwiners, to surviving Poincaré representation data and a *possible* local W*-realization. This was an evidentiary stop line, not a theorem of reconstruction. Only after that historical layer is fixed does the note state the modern analytic program: a specified $*$-compatible represented sector, controlled unbounded generators, a bounded functional-calculus realization, and bicommutant closure. The later language of presheaves, descent, cohomological gluing, and type III factors is explicitly excluded from the claims attributed to 2014–2015. A modern coda then distinguishes the amplituhedron preimage fiber from the admissible projection Grassmannian of GAT I. It identifies the fiber as the carrier of a sectorial selection, not of a von Neumann type, and records the type III_1 branch as a conditional downstream identification.

Central correction

The super Yangian is not declared to be a von Neumann algebra. The object called a W*-payload is the represented bicommutant closure of a specified bounded realization of a specified sector. "Section" is used only when an actual splitting map has been constructed.

Evidence labels used throughout

Established means supported by published mathematics within its stated hypotheses. **Recorded** means supported by a surviving dated source. **Remembered** marks historical recollection rather than mathematical evidence. **Reconstructed** means rederived here from declared assumptions. **Modern reading** marks later language not attributed to 2014–2015. **Open** identifies a bridge or interpretation not yet earned.

Contents

1 Reconstruction notice

1.1 Lost calculations and surviving lessons

The missing paper trail does not license invention, and missing paper does not erase the question. The notebooks, whiteboard sequences, and calculations underlying this investigation were lost in the move from Chillicothe to Columbus. Their loss removes the documentary order in which the argument was first assembled and prevents the present reconstruction from establishing an earlier date. No claim of priority is made on that basis.

The purpose is narrower than recovery by transcription and stronger than reminiscence. A statement that can be rederived now is presented as a present derivation. A remembered step that cannot be rederived is labeled historical memory or omitted. The central lesson retained from the original work is:

Lesson preserved

Representation before closure; closure before interpretation. A successful interface may expose genuine structure without exhausting the source from which it was cut.

1.2 Controlled witness, not global ontology

Planar $\mathcal{N} = 4$ super Yang–Mills theory is exceptionally rigid. Its positive Grassmannian organization, dual superconformal symmetry, and Yangian structure make it a useful controlled witness. This note does not infer that the planar theory, exact supersymmetry, or the super Yangian is globally fundamental. It asks instead whether a carefully typed fragment survives a change of mathematical category and, later, a removal of those assumptions.

1.3 Six evidentiary labels

Label	Meaning
Established	A published result used within its hypotheses.
Recorded	A claim supported by a surviving dated source or artifact.
Remembered	A historical recollection used for chronology or motivation, not as independent mathematical evidence.
Reconstructed	A present derivation from declared assumptions, not a recovery of missing wording.

Label	Meaning
Modern reading	A later interpretation that is not attributed to the 2014–2015 investigation.
Open	A conjecture, bridge claim, or physical identification requiring new mathematics or input.

2 The 2014–2015 line: reconstruction under a strict standard

2.1 Why the problem was hard

The difficulty did not lie in one missing calculation. It lay in a refusal. A beautiful on-shell object was not going to be allowed to answer questions it had not been built to answer. A formula exhibiting poles, residues, and an enormous symmetry algebra could still fail to exhibit a Hilbert space, a positive-energy representation, causal localization, or a unitary scattering operator. For a reader formed by Wigner's particle classification, Haag's local quantum physics, and Schroer's modular-localization program, those were not optional decorations [1, 2, 3, 4].

This was therefore a translation problem between mathematical cultures. The amplitudes literature compressed exactly the structures that the operator-algebraic literature ordinarily asks one to expose. In retrospect, the fact that the problem consumed weekends and afternoons is unsurprising. The admissible mathematics had been given a very narrow door.

The mathematics allowed through the door

The working standard required, at minimum: a declared positive-energy Poincaré representation; a Hilbert-space interpretation of unitarity; causal or modular localization rather than locality by nomenclature; explicit operator domains when generators were unbounded; and a representation before any operator-algebraic closure. Elegance of the scattering formula could not substitute for any of these obligations.

The severity of that standard is part of the history. It explains both why the original conclusion was modest and why reaching it was difficult.

2.2 John's calibration: intuition, evidence, and available time

When the author began explaining the proposed connection to John Klauder, the idea was still being developed at the edge of a demanding applied research program.

Klauder's response is remembered not as an objection to the reality of the connection, but as an exact calibration of the distance between the intuition and the mathematics then on the page.

> *"I do not know, Scott. Maybe. But I cannot see it. You have on-shell scattering shapes, supersymmetry, and many different things happening at once. There is a great deal to sort through. Just because I cannot see it does not mean it is not real. Keep at it; do not become discouraged. But I do not know how you are going to bridge these gaps without devoting more time than you have. Your intuition is in another universe from what you have shown."*
>
> Reconstructed from memory: the substance and cadence are preserved, but this is not a verbatim transcript.

The response neither granted the bridge nor dismissed the possibility. It separated three objects that the investigation itself still needed to separate. In the shortest form,

$$\textit{intuition} \neq \textit{displayed mathematics} \neq \textit{proof achievable in the available time.}$$

The visible ingredients were already substantial: on-shell scattering shapes, superconformal and Yangian organization, and the intuition of a local operator-algebraic carrier. The missing passages were larger still: Hilbert-space realization, positivity and domain control, localization, coherent gluing, and the represented closure. Klauder saw the joints before there was enough language to expose them.

The work was slow for a practical reason. It was pursued only on the side and only during hours in which the author was not already exhausted. By then the author had produced roughly sixty journal-quality nuclear-technology articles that had undergone peer review within the applied program, although they had not entered ordinary journal publication. The paradox was that this substantial applied authorship could still feel like intellectual stagnation: the foundational question advanced only in rested weekends and afternoons, while most technical energy was committed elsewhere.

In retrospect, that applied work was not irrelevant to the result. It imposed an evidentiary discipline: an observed spectral or scattering shape could be trusted only after its carrier, interface, admissibility conditions, and limits of inference had been declared. That discipline is precisely what prevented the historical argument from calling the abstract super Yangian a W*-algebra merely because an operator-algebraic silhouette had become visible. The intuition had outrun the construction; the later transport and gluing program began bringing the construction into the same universe.

2.3 The same words carried different burdens

The obstruction can be seen before any calculation. Several familiar words had sharply different evidentiary force on the two sides of the investigation.

Term	Planar on-shell usage	Haag–Wigner burden
Locality	Singularities and factorization compatible with local interactions.	Spacelike commutation for algebras assigned to spacetime regions.
Unitarity	Factorization, cuts, and correct residues of amplitudes or integrands.	A unitary operator on a physical Hilbert space, with asymptotic interpretation and completeness questions stated.
Representation	Differential or oscillator action on functions, forms, or distributions.	A positive-energy unitary representation on a Hilbert space, with domains and adjoints controlled.
Particle	An on-shell leg carrying momentum, helicity, and supermultiplet labels.	A Wigner irreducible representation, or a controlled infraparticle/generalized replacement.
Global object	A cyclically ordered planar amplitude or integrand.	A covariant net or global scattering operator assembled from localized physics.

Thus *locality and unitarity emerge* could be accepted as a statement about analytic structure without being accepted as a construction of local quantum physics. The two claims are not rivals. They simply live in different categories.

2.4 What the shape of a realized scattering theory would have been

The word *shape* was not being used visually. It named the architecture that a globally realized scattering theory is expected to possess before its matrix elements are stripped into calculational kernels.

For a constrained theory, Klauder's lesson enters first: the kinematic carrier is not yet the physical Hilbert space. One must earn a physical subspace by a controlled enforcement of the constraints, schematically

$$\mathcal{H}_{\mathrm{kin}} \xrightarrow{E_{\mathrm{phys}}} \mathcal{H}_{\mathrm{phys}}, \qquad E_{\mathrm{phys}}^2 = E_{\mathrm{phys}} = E_{\mathrm{phys}}^*, \tag{1}$$

where the exact regularized projector depends on the constraint problem [6]. The physical scattering spaces must then be constructed, not presumed. In the standard asymptotic picture one seeks wave operators

$$\Omega_\pm : \mathcal{H}_{\mathrm{as}} \longrightarrow \mathcal{H}_{\mathrm{phys}}, \qquad S := \Omega_+^* \Omega_- : \mathcal{H}_{\mathrm{as}} \longrightarrow \mathcal{H}_{\mathrm{as}}. \tag{2}$$

When the wave operators are isometric and have the appropriate common scattering range, S is unitary on the asymptotic scattering space. Asymptotic completeness is the stronger statement that the relevant physical sector is exhausted by that range; it is not contained in the notation.

The Poincaré representation is part of this architecture. If $U_{\mathrm{as}}(g)$ and $U_{\mathrm{phys}}(g)$ are the asymptotic and physical positive-energy representations, covariance requires

$$U_{\mathrm{phys}}(g)\Omega_\pm = \Omega_\pm U_{\mathrm{as}}(g), \qquad [S, U_{\mathrm{as}}(g)] = 0. \tag{3}$$

Thus the particle labels are not annotations on an amplitude. They arise from the irreducible content of the asymptotic representation and are transported through the scattering map.

Writing $S = I + iT$, operator unitarity has the form

$$-i(T - T^*) = T^*T. \tag{4}$$

After insertion of a complete family of physical intermediate states, (4) yields the optical-theorem pattern

$$2\ \mathrm{Im}\langle i|T|i\rangle = \sum_f \left|\langle f|T|i\rangle\right|^2, \tag{5}$$

with the sum understood to include the appropriate integrals and conservation distributions. Experiment meets the theory only after flux, phase space, preparation, and detection are restored:

$$\mathrm{d}\sigma_{i\to f} = \frac{1}{\text{flux}}\left|\langle f|T|i\rangle\right|^2 \mathrm{d}\Phi_f. \tag{6}$$

A realized scattering theory also has a causal large-distance shape. Widely separated experiments should decouple in the relevant limit, schematically

$$S_{\alpha\sqcup\beta} \longrightarrow S_\alpha \otimes S_\beta \qquad \text{as the two scattering arrangements separate.} \tag{7}$$

The exact formulation belongs to cluster decomposition, macrocausality, or an underlying local net. Pole factorization resembles one boundary consequence of this structure; it is not by itself the entire structure.

Equations (1)–(7) describe the silhouette the planar positive geometry did not display: a physical projection, an asymptotic Hilbert space, positive-energy Poincaré covariance, a unitary operator identity, completeness of intermediate states, and causal separation.

2.5 Four voices, one methodological demand

The historical reasoning combined four sensibilities that are easy to flatten when the result is stated only as a chain.

Voice	The demand it placed on the problem
Schroer	Causal localization and Hilbert-space positivity constrain what may count as a quantum field theory; an S-matrix is not an arbitrary table of invariant numbers [5].
Klauder	Constraints must be enforced by a controlled physicalization map; kinematic formulas cannot silently be read as physical states or operators [6].
Wheeler	A recorded answer is produced through a physically posed question. The interface participates in the form of the answer, without making that answer unreal [7].
Feynman	Physics retains an experimental veto: whatever interpretive picture is adopted must return the probabilities and cross sections that experiments can test [8].

Feynman—Wheeler's *other son*, in the author's affectionate shorthand[1]—kept the argument from becoming purely ontological. If an observed scattering regularity persists, it is telling us something real. Wheeler prevents *real* from being confused with *view from nowhere*. Klauder asks which projection earned the physical carrier. Schroer asks whether the result can coexist with positivity and causal localization.

Reconciliation principle

An observation is neither a transparent photograph of the whole theory nor a mere artifact of representation. It is a real constraint produced through a specified physical interface. A satisfactory theory must explain both why the observed shape is stable and why that shape is only partial.

2.6 The planar kernel had a different silhouette

A color-dressed loop amplitude has a schematic decomposition

$$\mathcal{M}_n^{(L)} = \sum_\rho C_\rho^{(L)} A_\rho^{(L)}, \qquad \mathcal{M}_n^{(L)} \longmapsto A_{n,\mathrm{planar}}^{(L)}(1,\ldots,n) \tag{8}$$

when the leading-color planar, cyclically ordered component is selected. Similarly, a planar superamplitude is commonly separated into universal conservation data and a reduced object,

$$\mathcal{A}_{n,k}^{\mathrm{planar}} = \delta^{(4)}(P)\delta^{(8)}(Q)\,\mathcal{R}_{n,k}, \qquad \mathcal{R}_{n,k} \rightsquigarrow \Omega(\mathcal{A}_{n,k,4}), \tag{9}$$

where the final arrow is schematic: it denotes the appropriate residue, pushforward, or canonical-form extraction, not an equality without conventions.

This is an extraordinarily informative compression. It retains cyclic order, little-group weight, supermultiplet data, singularity loci, factorization channels, and Yangian constraints. It does not display the wave operators in (2), the operator identity (4), or the causal assignment of algebras to spacetime regions.

Signature	Globally realized scattering	Planar positive-geometric object
Physical carrier	$\mathcal{H}_{\mathrm{phys}}$, asymptotic space, physical projection, and domains.	External on-shell representation data are present; the global carrier is not constructed.
Poincaré structure	Positive-energy unitary representation and intertwining wave operators.	Level-zero Ward identities and little-group weights survive, often nonmanifestly.

[1]An intellectual-lineage joke, not a biographical claim.

Signature	Globally realized scattering	Planar positive-geometric object
Unitarity	$S^*S = SS^* = I$ with a complete physical intermediate-state resolution.	Cuts, residues, and factorization test sectorial consequences of unitarity.
Causal separation	Cluster decomposition or derivation from a local covariant net.	Pole and boundary factorization are visible; spacelike commutation is not supplied.
Color and topology	All required color structures, planar and nonplanar sectors, and channel completion.	A cyclically ordered leading-color planar sector is selected.
Experimental output	Prepared states, flux, phase space, detector classes, and probabilities.	A kernel or form that must still be paired with those structures.

The mismatch is precise. The amplituhedron has the shape of a highly rigid scattering *interface*, not the shape of a globally realized scattering operator or local quantum field theory.

2.7 Why the mismatch was evidence rather than failure

The experimental lesson forbids dismissing the planar kernel. Its factorization, covariance, and singularity structure are not decorative; they control quantities that enter real scattering predictions. The operator-algebraic lesson forbids overreading it. A partial amplitude cannot inherit a global Hilbert space, complete unitarity sum, or causal net merely because it exhibits their projected shadows.

The 2014–2015 question was therefore not which picture should defeat the others. It was how all of them could be true at once:

$$\boxed{\begin{aligned} \text{experimental regularity} &\Longrightarrow \text{the planar shape is physically meaningful,}\\ \text{projection and selection} &\Longrightarrow \text{the planar shape is necessarily partial,}\\ \text{causal local quantum physics} &\Longrightarrow \text{the missing global structure still matters.} \end{aligned}} \tag{10}$$

The observation was telling us something real about reality. The burden was to identify exactly what had survived the question asked of it.

2.8 Step one: identify what the planar object was not

The amplituhedron organizes a planar, color-ordered sector of $\mathcal{N} = 4$ super Yang–Mills scattering. The corresponding working object is stripped: overall conservation distributions and some kinematic redundancy are solved, factored, or geometrically encoded. In momentum-twistor variables, for example, momentum conservation is built into the presentation and ordinary Poincaré covariance is not the manifest organizing principle [9].

This does *not* mean that the Poincaré group has literally been quotiented as a group. It means that the displayed finite geometry is a compressed interface. By construction it does not supply the nonplanar color completion, a global physical Hilbert space, a vacuum, a net of local observable algebras, or a fully realized scattering operator. The historical search never expected full QCD to be hiding in this planar data, and it did not require a global C^*-algebraic setting to count as successful.

This negative identification was the first positive result: the shape mismatch was real. A planar kernel should not be judged as though it were already the global S-matrix whose causal and representation-theoretic structure Haag and Schroer demand.

2.9 Step two: find the Poincaré representations in the rubble

Let the ith external leg carry a massless positive-energy Wigner representation $\mathcal{H}_i = \mathcal{H}_{0,h_i}$, with helicity h_i (and the corresponding supermultiplet data). A stripped n-point object is not thereby a bounded operator on $\bigotimes_i \mathcal{H}_i$. More cautiously, it may be treated schematically as a distributional multilinear kernel

$$\mathcal{K}_n : \mathcal{D}_1 \widehat{\otimes} \cdots \widehat{\otimes} \mathcal{D}_n \longrightarrow \mathbb{C}, \tag{11}$$

on suitable test domains. Level-zero covariance then has the intertwining form

$$\mathcal{K}_n\left(\sum_{i=1}^{n} J_{i,a}\psi\right) = 0, \qquad \psi \in \mathcal{D}_1 \widehat{\otimes} \cdots \widehat{\otimes} \mathcal{D}_n. \tag{12}$$

Little-group homogeneity records the helicity labels even after the overall conservation distribution has been stripped. On a physical factorization boundary, the residue has the schematic form

$$\operatorname*{Res}_{P_I^2=0} \mathcal{K}_n \sim \sum_{\lambda} \mathcal{K}_L(\ldots, P_I^{\lambda}) \mathcal{K}_R(-P_I^{-\lambda}, \ldots), \tag{13}$$

where the sum or supersum ranges over intermediate on-shell states.

Equations (12) and (13) were the relevant evidence. The external and factorization data still knew how to carry and compose particle representations. Poincaré representation data were therefore not simply *in the rubble* of the stripped planar presentation. This conclusion is weaker than construction of a global unitary S-operator, but it is stronger than the observation that the final formula is Lorentz invariant.

2.10 Step three: let the Yangian organize, but not replace, the intertwiners

The super Yangian is too large for the historical question to have been a search through all of its abstract subalgebras. What was visible was its action in the scattering representation. The Grassmannian residues and on-shell functions obey, schematically,

$$J_a^{(0)}\mathcal{I} = 0, \qquad J_a^{(1)}\mathcal{I} = 0, \tag{14}$$

with the generators acting through a chosen tensor-product or evaluation representation [13, 14]. At level zero, this includes the superconformal organization containing ordinary Poincaré covariance; at level one, the nonlocal coproduct further constrains how the sitewise data combine.

The defensible historical inference was therefore organizational: Yangian-invariant scattering objects behave as highly constrained intertwiners among external representation data. The Yangian did not replace the Poincaré representations, and algebraic Yangian invariance did not produce a unitary Hilbert-space representation of the abstract Yangian. It supplied a map through the debris.

2.11 Step four: ask for a local W* foothold of the right size

The endpoint sought in 2014–2015 was deliberately partial. If a physically meaningful represented fragment could be isolated on $\mathcal{D} \subset \mathcal{H}$, then a controlled bounded family extracted from it could, in principle, generate

$$\mathcal{M}_{\pi,S} = \mathcal{B}_{\pi,S}'' \subset B(\mathcal{H}). \tag{15}$$

This is the size of claim compatible with local quantum physics: a representation-dependent von Neumann-algebraic realization, not a declaration that the abstract Yangian is a W*-algebra and not a claim that one local algebra already determines the theory.

At the historical stage, equation (15) was a possible foothold, not a completed construction. Its value was evidentiary. It offered a mathematically recognizable place where the surviving representation data might live after the planar compression, section by section, without demanding that the finite scattering geometry already possess the shape of a global theory.

2.12 The 2015 stop line

The reconstructed line is therefore:

Historical chain

$$\begin{aligned} \text{planar positive geometry} &\longrightarrow \text{stripped scattering kernel} \\ &\longrightarrow \text{Yangian-organized intertwiners} \\ &\longrightarrow \text{surviving Poincaré representation data} \\ &\longrightarrow \text{a possible local W}^*\text{-realization.} \end{aligned}$$

Its arrows do not all carry the same force.

Passage	Status	Content
Positive geometry → stripped kernel	established interface	Canonical forms, residues, or pushforwards encode planar scattering data.
Kernel → Yangian-organized intertwiners	established in the scattering representation	Level-zero and level-one invariance constrain the distributional object.
Intertwiners → surviving Poincaré data	historical inference, reconstructed here	External-leg covariance, little-group weight, and factorization expose representation content.
Poincaré data → possible local W* realization	conditional research foothold	A declared represented sector and controlled bounded realization would be required.

The line supplied one more piece of evidence, not the full proof: the planar quotient or compression had not annihilated every trace of the positive-energy particle representations needed on the other side.

2.13 What was not claimed

The original burden did not include any of the following:

- recovery of full QCD or the nonplanar theory;
- construction of a global quasi-local C^*-algebra or a complete Haag–Kastler net;
- construction of a single globally realized Poincaré-covariant scattering operator;
- proof that a Yangian subalgebra has a canonical W*-completion;
- a presheaf, Čech cocycle, descent theorem, or cohomological gluing mechanism;
- classification of any resulting local algebra as type III.

Historical/modern firewall

The author did not formulate the 2012–2015 investigation in presheaf, descent, or cohomological language. Those structures became visible only through substantially later work. The original investigation reached the more limited conclusion that the planar positive-geometric scattering kernel retained Yangian-organized intertwiners and, through them, evidence of surviving Poincaré representation data compatible with a possible local von Neumann-algebraic realization. The present presheaf and type III formulations are modern sharpenings of that earlier insight, not claims about the terminology or results possessed at the time.

3 Modern coda: the fiber did not carry the factor

The 2014–2015 reconstruction ends at a possible local W*-realization. The present Geometry of Admissible Transport construction (GAT I) [17] does not retroactively prove that historical possibility. It supplies a later, more precise mechanism by which a selected Grassmannian locus can support a sectorial operator-algebraic carrier. The decisive correction is categorical:

The fiber did not carry the factor

The admissible Grassmannian fiber carries the *selection* through which a sectorial local von Neumann algebra may become realizable. Type III is a property of that downstream algebra in a specified representation; it is not a property of a Grassmannian point or projection by itself.

3.1 Two Grassmannian fibers, not one

The amplituhedron side already has a genuine fiber. With Φ_Z as in (28), it is

$$F_{Y,Z} = \{[C] \in \mathrm{Gr}_{\geq 0}(k,n) : [CZ] = Y\} = \Phi_Z^{-1}(Y). \tag{16}$$

GAT I uses a different object. If H_α is a self-dual Hilbert W*-module and $P_{\alpha,s}$ is an admissible projection selecting a sector s, then

$$P_{\alpha,s} \in \mathrm{Gr}_{\mathrm{adm}}(H_\alpha), \qquad H_{\alpha,s} := P_{\alpha,s} H_\alpha. \tag{17}$$

Here $\mathrm{Gr}_{\mathrm{adm}}(H_\alpha)$ denotes the admissible projection Grassmannian in the reconstruction/refinement category C_{adm}.

Question	Amplituhedron preimage fiber	GAT admissible projection fiber
Ambient object	$\mathrm{Gr}_{\geq 0}(k, n)$ with fixed external data Z.	A self-dual Hilbert W*-module H_α with admissible reconstruction and refinement data.
Point	A positive k-plane $[C]$ satisfying $[CZ] = Y$.	An admissible projection $P_{\alpha,s}$ selecting the carrier $P_{\alpha,s}H_\alpha$.
Role	Records the nonuniqueness of positive-Grassmannian presentations of the same amplituhedron point.	Selects a sector in which transport, comparison, and local algebra formation may be tested.
Controls	Positive geometry, cells, canonical-form presentations, and boundary data.	Correspondence transport, destination comparison, linking-algebra descent, and sectorial localization.
Does not imply	A canonical Hilbert-space representation or local observable algebra.	A type III factor without the downstream transport, localization, and modular hypotheses.

Warning 3.1 (No bridge by homonymy). Equations (16) and (17) define different fibers in different categories. The common word *Grassmannian* supplies no canonical map between them. Any amplituhedron-to-GAT comparison must construct and type that map or correspondence explicitly.

3.2 The selected locus: "not on all, but on a subset"

The relevant modern claim is sectorial. One does not attach a local W*-algebra to every point of an unrestricted Grassmannian. One first selects an admissible projection $P_{\alpha,s}$ and then tests the selected module or correspondence carrier for coherent transport. In particular, a selected correspondence becomes an onto spatial algebra transport only when the local graph reductions descend coherently through the linking algebra. The phrase *on a Grassmannian fiber* therefore means: on the admissible projection locus satisfying these transport and coherence conditions.

This is the mathematical content of the remembered qualification that the connection was visible on a subset, not on all of the Grassmannian. The subset is doing selection work. It is not already an algebra and does not acquire a Murray–von Neumann type merely by being selected.

3.3 The GAT I core: destination-valued spectral trichotomy

Let D_{tot} denote the total spatial–refinement differential and $\Delta_{s\to s'}$ the destination-sector comparison operator. Under the closedness, homogeneity, completeness, compactness-modulo-transport, and finite-window stability hypotheses of GAT I, the source-adapted schematic decomposition is

$$\ker(D_{\mathrm{tot}}|_s) = K_s^{\mathrm{tr}} \oplus K_s^{\mathrm{mig}}, \qquad K_s^{\mathrm{esc}} \text{ lies in residual spectrum separated from zero.} \tag{18}$$

Here K_s^{tr} is same-sector transport, K_s^{mig} is inequivalent-sector migration, and K_s^{esc} is inadmissible escape. Equivalently, the escape branch obeys a positive spectral-gap condition of the form

$$\mathrm{dist}(0, \sigma_{\mathrm{esc},s}) \geq \gamma_s > 0. \tag{19}$$

The complete atomic destination family and norm-stable destination-window Riesz projections prevent a selected zero mode from leaking unnoticed into the escape branch. This destination-valued spectral trichotomy is the core result. The Grassmannian realization is formal only after the transport and coherence data are in place.

3.4 From the selected carrier to a thin local algebra

When the selected correspondence transports coherently through the linking algebra, local graph reductions can descend to an onto spatial algebra transport. Only then does one obtain the thin local von Neumann algebra

$$\mathcal{O} \longmapsto M_s(\mathcal{O}) \subset B(\mathcal{H}_s) \tag{20}$$

in the selected sector. Admissible self-replication of the glued carrier can then support proper infiniteness. This is the point at which a factor-type question becomes well typed: the subject of the question is $M_s(\mathcal{O})$, not $P_{\alpha,s}$.

The logical order is therefore

$$\boxed{\begin{aligned} P_{\alpha,s} \in \mathrm{Gr}_{\mathrm{adm}}(H_\alpha) &\longrightarrow \text{selected module/correspondence carrier} \\ &\longrightarrow \text{coherent linking-algebra descent} \\ &\longrightarrow M_s(\mathcal{O}). \end{aligned}} \tag{21}$$

3.5 The conditional type III_1 branch

Proper infiniteness alone does not prove type III_1. Nor does a type III label by itself identify the hyperfinite factor. The GAT I application branch requires the additional semidiscrete expectation-tower and modular information. Its defensible form is the conditional schema

$$\left.\begin{array}{l}\text{coherent selected local algebra with proper infiniteness,}\\ \text{semidiscrete control and exhaustion of the finite-window}\\ \qquad\text{expectation tower,}\\ \text{cornerwise modular saturation with type III}_1\text{ spectrum}\end{array}\right\} \Longrightarrow \quad M_s(\mathcal{O}) \cong R_\infty, \tag{22}$$

where R_∞ is the hyperfinite type III_1 factor. Equation (22) is recorded as a downstream conditional identification, not as a universal theorem about every GAT sector and not as a statement about the amplituhedron fiber.

3.6 The bridge still owed

The modern mechanism does not erase the central comparison problem. To connect the finite positive geometry to the admissible projection locus, one needs at least a typed construction of the form

$$U_{Y,Z} \subset F_{Y,Z}, \qquad \Xi_{Y,Z} : U_{Y,Z} \longrightarrow \mathrm{Gr}_{\mathrm{adm}}(H_\alpha), \tag{23}$$

or a correspondence of equivalent force. Such a bridge must preserve enough of the external representation data, boundary factorization, destination transport, and recovery maps to justify the passage. Shared Grassmannian language is not a substitute for these checks.

The carrier mechanism has therefore been identified more sharply: a selected admissible Grassmannian locus can make a sectorial local algebra visible. The specific amplituhedron-to-GAT comparison map in (23) remains a theorem obligation unless and until it is constructed.

3.7 Claim-status ledger

Claim	Status	Exact burden
2014–2015 scattering line	reconstructed historical inference	The planar kernel retains Yangian-organized evidence of surviving Poincaré representation data and suggests a possible local W* foothold.
Sectorial Mosco–Riesz no-leakage	recovered in the GAT sectorial setting	Destination-window projections are norm-stable and escape remains uniformly separated from zero.

Claim	Status	Exact burden
Admissible Grassmannian realization	formal after hypotheses	It follows after transport and coherence have been established; the Grassmannian does not create those hypotheses.
Destination-valued spectral trichotomy	GAT I core theorem	Zero modes split into transport and migration, while escape occupies the separated residual branch.
Type III_1 identification	conditional downstream application	Requires the modular-saturation and expectation-tower conditions, together with the stated semidiscrete control.
Amplituhedron-to-GAT map	open bridge in this note	Construct $\Xi_{Y,Z}$ or an equivalent correspondence and prove compatibility with representation and boundary data.

3.8 Historical-to-modern line

The result is a two-level chain whose dating and logical force remain visible:

$$\boxed{\begin{gathered}\text{planar positive geometry} \longrightarrow \text{stripped scattering kernel}\\ \longrightarrow \text{surviving Poincaré data} \longrightarrow \text{possible local W}^*\text{-realization} \quad (2014\text{–}2015),\\ \text{selected admissible Grassmannian fiber} \longrightarrow \text{coherent sectorial transport}\\ \longrightarrow \text{local von Neumann algebra} \longrightarrow \text{conditional type III}_1 \text{ identification} \quad (\text{GAT I}).\end{gathered}} \tag{24}$$

4 Phase I: the right-arrow audit

4.1 The displayed chain

The motivating display is

$$\begin{aligned}\boxed{\text{amplituhedron}} &\longrightarrow \boxed{\text{positive Grassmannian presentations}}\\ &\longrightarrow \boxed{\text{super-Yangian organization}} \longrightarrow \boxed{\text{representable sector}}\\ &\longrightarrow \boxed{\text{bounded realization}} \longrightarrow \boxed{\text{W}^*\text{-payload}}.\end{aligned} \tag{25}$$

The arrows in (25) are not all functions. Confusing image maps, coordinate presentations, residue constructions, symmetry actions, representations, and closures would make the chain rhetorically smooth but mathematically false.

4.2 Arrow types

Type	Notation	Required data
Map	$X \xrightarrow{f} Y$	Domain, codomain, rule, and equivalence relations.
Presentation	$X \rightsquigarrow P$	Coordinates or cells and their redundancy.
Extraction	$P \rightsquigarrow I$	Contour, residue, pushforward, or evaluation prescription.
Action	$A \curvearrowright V$	Algebra, representation space, and operator domain.
Representation	$A \xrightarrow{\pi} \mathrm{End}(\mathcal{D})$	Real form, involution, Hilbert space, and common invariant core.
Closure	$S \mapsto S''$	A bounded represented set $S \subset B(\mathcal{H})$ and topology.

4.3 Typed ledger

ID	Passage	Type	Obligation
R_0	positive matrix C to $[C] \in \mathrm{Gr}_{\geq 0}(k,n)$	quotient	Check invariance under $GL(k)$ and boundary conventions.
R_1	$[C] \mapsto [CZ]$	map	Fix Z, positivity, ranks, and the target Grassmannian.
R_2	$\mathrm{Gr}_{\geq 0}(k,n) \to \mathcal{A}_{n,k,m}(Z)$	surjection	This is the defining image map; it is not generally injective.
R_3	positroid cell to coordinates	presentation	State charts, orientations, and boundary identifications.
R_4	coordinates to canonical form	differential form	Establish logarithmic singularities and normalization.
R_5	form to scattering object	extraction	Specify residue or pushforward, contour, and external data.
R_6	scattering object to Yangian invariant	property	Specify the representation in which the generators act.
R_7	abstract Yangian to candidate sector	inclusion, quotient, or image	Do not say “section” without a splitting.
R_8	sector to operators on $\mathcal{D} \subset \mathcal{H}$	representation	Give the $*$-relation, grading, domains, and closability.
R_9	unbounded charges to bounded family	functional calculus	Use unitary groups, resolvents, or bounded transforms.

ID	Passage	Type	Obligation
R_{10}	bounded family to $\mathcal{M} = S''$	closure	This is automatic once $S \subset B(\mathcal{H})$ is fixed.
R_{11}	$\mathcal{M}$ to local/vacuum physics	interpretation	Requires a net, covariance, state, spectrum, and locality; closure alone is insufficient.

4.4 Composition hazards

Warning 4.1. The reverse of the image arrow is not a canonical inverse. If $\dim \mathrm{Gr}_{\geq 0}(k, n) > \dim \mathcal{A}_{n,k,m}(Z)$, regular fibers have positive dimension.

Warning 4.2. Yangian invariance is a statement about a vector, function, distribution, or form in a representation. It is not membership in the abstract Yangian.

Warning 4.3. Unbounded charges do not belong to $B(\mathcal{H})$. Bicommutant notation applies only after a bounded realization has been chosen.

Phase gate

Phase I is passed only when every arrow has a declared mathematical type and the data needed for its composition are explicit. The chain is a research architecture, not a single functor already constructed.

5 Phase II: finite geometry and symmetry data

5.1 Positive data and the forward map

Fix integers $n \geq k + m$ and a full-rank external-data matrix $Z \in \mathbb{R}^{n\times(k+m)}$ satisfying the positivity hypotheses appropriate to the bosonized tree amplituhedron. For $I \subset \{1, \ldots, n\}$ with $|I| = k$, write $\Delta_I(C)$ for the corresponding Pluecker coordinate.

Definition 5.1 (Positive and nonnegative Grassmannians)**.**

$$\mathrm{Gr}_{>0}(k, n) = \{[C] \in \mathrm{Gr}(k, n) : \Delta_I(C) > 0 \textit{ for every } I\}, \tag{26}$$

$$\mathrm{Gr}_{\geq 0}(k, n) = \{[C] \in \mathrm{Gr}(k, n) : \Delta_I(C) \geq 0 \textit{ for every } I\}. \tag{27}$$

Definition 5.2 (Bosonized amplituhedron)**.** *The map determined by Z is*

$$\Phi_Z : \mathrm{Gr}_{\geq 0}(k, n) \longrightarrow \mathrm{Gr}(k, k + m), \qquad [C] \longmapsto [CZ]. \tag{28}$$

Its image is

$$\mathcal{A}_{n,k,m}(Z) := \Phi_Z(\mathrm{Gr}_{\geq 0}(k, n)). \tag{29}$$

Equations (28)–(29) are the direct geometric arrow. The notation suppresses technical choices on boundary strata where rank must be checked. The physical tree-level specialization uses $m = 4$; the finite geometric statements are clearer with m retained.

Established background

The amplituhedron was introduced as a positive-geometric reformulation of planar scattering data, with $Y = CZ$ as its defining map [9]. Positive Grassmannian cells organize on-shell functions and residues [10], while canonical forms provide the natural differential-form language [11].

5.2 Dimension and fibers

The ambient dimensions are

$$\dim \mathrm{Gr}(k, n) = k(n - k), \qquad \dim \mathrm{Gr}(k, k + m) = km. \tag{30}$$

Proposition 5.3 (Regular-fiber dimension count)**.** *Let $[C]$ be a point at which Φ_Z has rank km, and let $Y = \Phi_Z([C])$. Then the local fiber*

$$F_{Y,Z} := \{[C'] \in \mathrm{Gr}_{\geq 0}(k, n) : [C'Z] = Y\} \tag{31}$$

has dimension

$$\dim F_{Y,Z} = k(n - k - m). \tag{32}$$

Proof. At a regular point, the constant-rank theorem gives $\dim F_{Y,Z} = \dim \mathrm{Gr}(k, n) - \mathrm{rank}(\,\mathrm{d}\Phi_Z)$. Substituting (30) and $\mathrm{rank}(\,\mathrm{d}\Phi_Z) = km$ yields $k(n - k) - km = k(n - k - m)$. □

Thus Φ_Z is generically many-to-one whenever $n > k + m$. A triangulation, positive parametrization, or fiber description can supply a presentation of a point or form on $\mathcal{A}_{n,k,m}(Z)$; it does not convert Φ_Z into a global bijection. Fiber-based approaches make this distinction explicit [12].

5.3 Three noninterchangeable presentation layers

Layer	Object	What must not be inferred
Point layer	$Y = [CZ] \in \mathcal{A}_{n,k,m}(Z)$	A point does not determine a unique $[C]$.
Cell layer	positroid stratum and positive coordinates	A chart is not the geometry itself; overlaps and boundaries matter.

Layer	Object	What must not be inferred
Form layer	canonical form or residue representative	Equivalent forms may differ by pushforward, contour, or presentation data.

For a positive chart with coordinates $\alpha_1, \ldots, \alpha_d$, an on-shell form often takes the logarithmic shape

$$\omega = \bigwedge_{a=1}^{d} \frac{\mathrm{d}\alpha_a}{\alpha_a}. \tag{33}$$

The passage from (33) to the canonical form on the image is a pushforward or residue statement. It requires orientation and multiplicity data and is not merely the substitution $Y = CZ$.

5.4 Grassmannian residues

A standard schematic Grassmannian integral for planar $\mathcal{N} = 4$ data is

$$\mathcal{I}_{n,k}(\mathcal{Z}) = \int_{\Gamma} \frac{\mathrm{d}^{k\times n}C}{\mathrm{vol}\, GL(k)} \frac{\delta^{k\times(4|4)}(C\cdot\mathcal{Z})}{(1\cdots k)(2\cdots k{+}1)\cdots(n\cdots k{-}1)}. \tag{34}$$

Here $(i \cdots i+k-1)$ denotes a cyclic $k \times k$ minor and Γ is part of the definition, not decorative notation. Different contours select different residues or combinations. Reality conditions and distributional interpretation must be supplied before (34) is treated as an analytic integral.

5.5 Yangian action is representation-relative

Let $J_{i,a}$ denote a representation of the level-zero Lie-superalgebra generators on site i. In a standard evaluation-type realization, the first two levels have the schematic form

$$J_a^{(0)} = \sum_{i=1}^{n} J_{i,a}, \tag{35}$$

$$J_a^{(1)} = f_a{}^{bc} \sum_{1\le i<j\le n} J_{i,b}J_{j,c} + \sum_{i=1}^{n} u_i J_{i,a}, \tag{36}$$

with graded signs understood and evaluation parameters u_i. The statement that an object $\mathcal{I}$ is Yangian invariant means, in the chosen representation and distributional domain,

$$J_a^{(0)}\mathcal{I} = 0, \qquad J_a^{(1)}\mathcal{I} = 0 \quad \text{for all } a. \tag{37}$$

The Yangian origin of the Grassmannian integral and the Yangian symmetry of planar amplitudes are established in the relevant scattering representation [13, 14]. Neither result supplies a canonical Hilbert-space $*$-representation of the entire abstract super Yangian.

Phase gate

Phase II establishes a controlled finite witness: positive Grassmannian data, its image geometry, residue presentations, and representation-relative Yangian invariance. The next phase must specify the algebraic sector and the analytic representation. No W*-claim has yet been made.

6 Phase III: the representable-sector problem

6.1 Why "section" needs a map

Let Y denote the abstract super Yangian under consideration. Three distinct constructions are possible:

(a) a subalgebra $S \hookrightarrow Y$;
(b) a quotient $Y \twoheadrightarrow Q$;
(c) a represented image $\pi(Y) \subset \mathrm{End}(\mathcal{D})$.

If $q : Y \twoheadrightarrow Q$ is a quotient, a *section* is a map $s : Q \to Y$ with $q \circ s = \mathrm{id}_Q$. Without such a pair (q, s), the defensible terms are subalgebra, quotient, evaluation sector, or represented image.

Definition 6.1 (Analytically admissible represented sector). *An analytically admissible represented sector consists of*

$$\mathfrak{S} = (S, \star, \mathcal{H}, \mathcal{D}, \pi, \mathcal{G}), \tag{38}$$

where:

(i) S is an explicitly identified algebraic subobject, quotient, or evaluation sector of Y;
(ii) $\star$ is an involution compatible with a declared real form;
(iii) $\mathcal{H}$ is a Hilbert space and $\mathcal{D} \subset \mathcal{H}$ is a common dense invariant core;
(iv) $\pi : S \to \mathrm{End}(\mathcal{D})$ respects products, grading, and $\star$ on $\mathcal{D}$;
(v) $\mathcal{G}$ is a generating family whose represented elements are closable, with stated self-adjointness or skew-adjointness control.

This definition is a proof obligation, not a declaration that the desired sector has already been found. In particular, algebraic evaluation representations familiar in the scattering literature must not be silently promoted to positive-energy unitary Hilbert-space representations.

6.2 Candidate sector tests

Any proposed sector should be tested against the following ledger.

Test	Question	Failure mode
S_1	Algebraic identity	The "sector" is only a collection of formulas, not a closed algebraic object.
S_2	Real form and $\star$	The adjoint operation is incompatible with the chosen physical real form.
S_3	Positivity	The inner product is indefinite or the representation is not unitarizable.
S_4	Common core	Products and commutators have no shared invariant domain.
S_5	Closure	Generators are not closable or lack controlled self-adjoint extensions.
S_6	Yangian survival	The bounded closure forgets coproduct, grading, or level structure.
S_7	Scattering recovery	The original invariants cannot be recovered as matrix coefficients, distributions, or boundary data.
S_8	Nontriviality	The construction yields only scalars or all of $B(\mathcal{H})$ without informative intermediate structure.

Present reconstruction

The historically remembered foothold is reconstructed as a search for an admissible tuple (38). The existence of familiar oscillator and evaluation realizations motivates the search, but does not discharge tests S_1–S_8.

7 Phase IV: from unbounded charges to a W^*

7.1 Three bounded realizations

Let A be a self-adjoint operator on $\mathcal{H}$. Standard bounded data associated with A include

$$U_A(t) = e^{itA}, \qquad t \in \mathbb{R}, \tag{39}$$
$$R_A(z) = (A - zI)^{-1}, \qquad z \in \mathbb{C} \setminus \mathbb{R}, \tag{40}$$
$$b(A) = A(I + A^2)^{-1/2}. \tag{41}$$

These choices encode the same spectral information when used with the full functional calculus. They need not produce identical generating sets if only selected parameters are retained.

Definition 7.1 (W*-payload)**.** *For an admissible represented sector (38), suppose each $g \in \mathcal{G}$ is represented by an essentially self-adjoint operator $A_g = \overline{\pi(g)}$ after multiplication by i where appropriate. Define*

$$\mathcal{M}_{\mathfrak{G}} := \left\{ e^{itA_g} : g \in \mathcal{G},\ t \in \mathbb{R} \right\}'' \subset B(\mathcal{H}). \tag{42}$$

The represented von Neumann algebra $\mathcal{M}_{\mathfrak{G}}$ is called the W**-payload of the sector and generating family.*

Proposition 7.2 (Existence of the represented closure)**.** *Under the hypotheses of the definition, $\mathcal{M}_{\mathfrak{G}}$ is a von Neumann algebra. Each A_g is affiliated with $\mathcal{M}_{\mathfrak{G}}$.*

Proof. Stone's theorem makes each e^{itA_g} unitary and hence bounded. The bicommutant theorem implies that the bicommutant in (42) is a unital weak-operator closed $*$-subalgebra of $B(\mathcal{H})$. Because the complete unitary group of A_g lies in $\mathcal{M}_{\mathfrak{G}}$, its spectral projections lie in $\mathcal{M}_{\mathfrak{G}}$ by the spectral theorem. This is equivalent to affiliation of A_g with $\mathcal{M}_{\mathfrak{G}}$. □

Remark 7.3. This proposition is analytically standard. It proves that the payload exists *after* the representation and self-adjointness hypotheses are met. It does not prove that a physically relevant Yangian sector satisfies those hypotheses, nor that the closure retains the desired Hopf-algebraic data.

7.2 What closure does and does not preserve

The von Neumann algebra remembers the bounded operator relations visible in the chosen representation and topology. It does not automatically remember an abstract coproduct

$$\Delta : Y \longrightarrow Y \widehat{\otimes} Y, \tag{43}$$

an R-matrix, spectral parameters, or the distinction between level-zero and level-one generators. Preservation of such structure requires extra normal maps, coactions, tensor-product representations, or affiliated-operator data.

Proposition 7.4 (Representation dependence)**.** *If π_1 and π_2 are inequivalent admissible representations of the same algebraic sector, their payloads $\mathcal{M}_{\mathfrak{S}_1}$ and $\mathcal{M}_{\mathfrak{S}_2}$ need not be isomorphic. Therefore "the* W**-closure of the Yangian" is not representation-independent notation.*

Proof. The bicommutant depends on the concrete image in $B(\mathcal{H})$. Distinct representations can have distinct kernels, central decompositions, commutants, and von Neumann types. No abstract uniqueness follows from the algebraic source alone. □

Phase gate

Phase IV passes at the conditional level: an admissible represented sector has a well-defined W*-payload. The substantive open task is to exhibit a nontrivial sector satisfying the analytic tests and to show that its payload retains recoverable Yangian and scattering information.

8 Phase V: where the conjecture begins

8.1 Not yet a vacuum algebra

A single von Neumann algebra $\mathcal{M} \subset B(\mathcal{H})$ is not a Haag–Kastler net. A local theory would at minimum require an assignment

$$\mathcal{O} \longmapsto \mathcal{M}(\mathcal{O}) \tag{44}$$

to spacetime regions, together with isotony, locality, covariance, and an appropriate state or representation. A vacuum interpretation further demands a distinguished vector or normal state, an implementation of translations, and a spectrum condition. None follows from bicommutant closure.

Conjectural extension

There exists a nontrivial analytically admissible sector $\mathfrak{S}$ whose payload $\mathcal{M}_{\mathfrak{S}}$ admits additional localization and state data such that:

(i) a meaningful remnant of the Yangian organization survives as normal or affiliated structure;

(ii) the construction can be deformed away from exact planarity and $\mathcal{N} = 4$ supersymmetry without collapsing to triviality;

(iii) amplituhedron or positive-Grassmannian scattering data can be recovered as matrix coefficients, modular data, boundary values, or another explicitly defined interface shadow.

This conjecture deliberately leaves the recovery mechanism open. Choosing one prematurely would convert a research question into an unsupported assertion.

8.2 A falsifiable research program

P1. Classify candidate subalgebras, quotients, or evaluation sectors rather than searching the entire super Yangian at once.

P2. For each candidate, fix a real form, involution, Hilbert space, and common invariant core.

P3. Prove or disprove essential self-adjointness for a generating family.

P4. Compute the commutant, center, factoriality, and, where possible, the type of the generated payload.

P5. Test whether coproduct or Yangian-level information extends to normal maps or affiliated operators.

P6. Recover a known finite Yangian invariant from the represented data.

P7. Only after recovery, introduce localization and candidate normal states.

P8. Test stability under deformations that break planarity or supersymmetry.

Negative outcomes are informative. Failure at positivity isolates the real form; failure at domains isolates the analytic representation; failure at Yangian survival shows that bicommutant closure is too coarse; failure at recovery disconnects the payload from its finite geometric motivation.

9 Core theorem target

The long-term theorem should not assert merely that a bicommutant exists. A defensible target has the following form.

Target theorem schema

Construct an explicit analytically admissible sector $\mathfrak{S} = (S, \star, \mathcal{H}, \mathcal{D}, \pi, \mathcal{G})$ of a declared real form of the super Yangian. Prove that its payload $\mathcal{M}_{\mathfrak{S}}$ is nontrivial and characterize its commutant, center, and type. Construct additional normal or affiliated structure that retains a specified Yangian operation. Finally, recover a stated positive-Grassmannian or amplituhedron invariant from that operator-algebraic data by an explicit map.

Each clause is independently checkable. Removing the final recovery clause would leave a general operator-algebra construction with no demonstrated link to the amplituhedron. Removing the representation hypotheses would leave the W* language undefined.

10 Claim firewall

Permitted statement	Statement not yet permitted
$\mathcal{A}_{n,k,m}(Z)$ is an image of nonnegative Grassmannian data under $[C] \mapsto [CZ]$.	Every amplituhedron point has a canonical positive-Grassmannian preimage.
Planar scattering invariants carry Yangian symmetry in specified representations.	The abstract super Yangian is itself an algebra of bounded observables.
A specified bounded represented family generates a von Neumann algebra.	Bicommutant closure creates locality, a vacuum, or physical completeness.
Unbounded self-adjoint generators may be affiliated with the generated algebra.	The unbounded generators are elements of $B(\mathcal{H})$.
An explicit splitting may be called a section.	Any convenient subset or image is a section.
The planar theory is a controlled exact witness.	Planar $\mathcal{N} = 4$ SYM is assumed globally fundamental.

11 Conclusion

The reconstructed chain contains two secure kinds of structure and one open bridge. Positive Grassmannian data and the amplituhedron provide a finite geometric interface. Functional calculus and bicommutant closure provide a rigorous operator-algebraic endpoint once an admissible represented sector is given. The open bridge is the mathematically significant part: identify the sector, control its unbounded generators, prove that recognizably Yangian data survive the closure, and recover the finite scattering witness.

The remembered question was therefore correctly placed after the foothold: *where do we go from here?* The answer proposed by this note is not a premature vacuum ontology. It is a phased program with explicit failure modes, designed so that each successful step leaves a defensible mark even if the full conjecture fails.

References

[1] E. P. Wigner, *On Unitary Representations of the Inhomogeneous Lorentz Group*, *Annals of Mathematics* **40**, 149–204 (1939).

[2] R. Haag, *Local Quantum Physics: Fields, Particles, Algebras*, 2nd ed., Springer (1996).

[3] B. Schroer, *Wigner Representation Theory of the Poincaré Group, Localization, Statistics and the S-Matrix*, *Nuclear Physics B* **499**, 519–546 (1997), arXiv:hep-th/9608092.

[4] B. Schroer, *Modular Localization and the Holistic Structure of Causal Quantum Theory, a Historical Perspective*, arXiv:1107.1374.

[5] B. Schroer, *The Ongoing Impact of Modular Localization on Particle Theory*, *SIGMA* **10**, 085 (2014), arXiv:1407.2124.

[6] J. R. Klauder, *Path Integrals, and Classical and Quantum Constraints*, arXiv:quant-ph/0507222.

[7] J. A. Wheeler, *Information, Physics, Quantum: The Search for Links*, in W. H. Zurek (ed.), *Complexity, Entropy, and the Physics of Information*, Addison–Wesley (1990), archival text.

[8] R. P. Feynman, *The Character of Physical Law*, MIT Press (1965), Messenger Lectures archive.

[9] N. Arkani-Hamed and J. Trnka, "The Amplituhedron," *Journal of High Energy Physics* **2014**, 030 (2014), arXiv:1312.2007.

[10] N. Arkani-Hamed, J. L. Bourjaily, F. Cachazo, A. B. Goncharov, A. Postnikov, and J. Trnka, "Scattering Amplitudes and the Positive Grassmannian," arXiv:1212.5605.

[11] N. Arkani-Hamed, Y. Bai, and T. Lam, "Positive Geometries and Canonical Forms," *Journal of High Energy Physics* **2017**, 039 (2017), arXiv:1703.04541.

[12] F. Mohammadi, L. Monin, and M. Parisi, "Triangulations and Canonical Forms of Amplituhedra: a Fiber-Based Approach Beyond Polytopes," arXiv:2010.07254.

[13] J. M. Drummond and L. Ferro, "The Yangian Origin of the Grassmannian Integral," *Journal of High Energy Physics* **2010**, 010 (2010), arXiv:1002.4622.

[14] J. M. Drummond, J. M. Henn, and J. Plefka, "Yangian Symmetry of Scattering Amplitudes in $\mathcal{N} = 4$ Super Yang–Mills Theory," *Journal of High Energy Physics* **2009**, 046 (2009), arXiv:0902.2987.

[15] T. Bargheer, N. Beisert, and F. Loebbert, "Exact Superconformal and Yangian Symmetry of Scattering Amplitudes," arXiv:0905.3738.

[16] F. Loebbert, "Lectures on Yangian Symmetry," *Journal of Physics A* **49**, 323002 (2016), arXiv:1606.02947.

[17] J. Scott Little, *The Geometry of Admissible Transport—Grassmannian Gluing, Spectral Trichotomy, and Sectorial Type III Local Algebras*, companion manuscript (2026).