Before the Mountain Again

A Glowing Screen, the Far Side of the Universe,

and the Geometry of What Survives

Why would anyone sane think scattering amplitudes
have anything to do with the structure of the vacuum?

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Research posture and claim boundary

This note combines historical reconstruction, elementary results proved here, established amplitudes mathematics, conditional recognition theorems, and an open source-side program. The amplituhedron is treated as an exact finite scattering geometry, not as the vacuum. The proposed operator-algebraic source and the bridge from that source to the positive geometry remain explicitly conditional.

Abstract

This note records an old amplitudes question and a modern attempt to formulate it without retrospective overclaiming. The historical route ran from Poincaré representation-theoretic localization through gluon amplitudes, spinor-helicity, BCFW recursion, twistors, Grassmannians, Yangian structure, and a conjectural W^*-type completion. The completion was never trusted as fundamental: the supersymmetric origin was too special, the global structure felt too stiff, and the map back from on-shell geometry to the vacuum remained dotted. The modern proposal is weaker and more local. The amplituhedron may be an exact geometry of the finite distinctions that survive a scattering interface, while the source realization remains richer, correspondence-valued, refinement-dependent, and potentially higher-coherent. We separate historical observations, elementary theorems proved here, established external mathematics, conditional bridge theorems, and open source-side conjectures. The central slogan is: *the shape is the geometry of what survives.*

The five evidentiary labels of the series

Recorded means supported by a surviving source. Remembered marks autobiographical chronology or motivation. Reconstructed marks mathematics rederived from stated assumptions. Modern reading identifies a later interpretation not attributed backward. Open marks a ridge that the present evidence does not cross. These historical labels are independent of the technical theorem-status words *established*, *proved here*, *conditional*, and *conjectural*. The first vocabulary says what kind of warrant supports a passage; the second says how much mathematics has actually been earned.

Reconstruction notice

The lost notebooks are not treated as evidence. The historical route is kept smaller than the surviving record; the modern interface and W^*-source language is not projected backward into 2012–2015. Every passage from finite scattering geometry to a source realization must declare what is preserved, what is forgotten, and what remains to be proved.

Series preamble: the trail and the view

A View from the Mountain returns to an earlier line of inquiry without pretending that the path was straighter than it was. The point is not to make the past look inevitable. It is to recover the trail: the first question, the wrong turn, the calculation that survived, the intuition that was too large for the available mathematics, and the exact place where the evidence ended.

The view from later work is wider. It can disclose relationships that were not visible from the valley. But a wider view does not license us to move an old trail marker or to report a modern theorem as an earlier discovery. Memory, surviving record, present reconstruction, and modern interpretation therefore remain visibly separated throughout the series.

There is also a methodological reason for preserving the questions. Insight is not an optional ornament attached after a proof. Intuition may be odd, imperfect, and wrong for years; it is not certification. Yet it is often the instrument that notices that two calculations smell alike, that a completion is too rigid, or that a beautiful answer has hidden the question that produced it. New tools can accelerate reconstruction, test lemmas, and expose missing arrows. They do not relieve the author of deciding what is physically meant, what is actually proved, or which distant outline is worth walking toward.

This fourth note therefore preserves both the theorem spine and the wonder that kept the question alive. The mountain metaphor is not decoration. It is the discipline of telling the reader where the path was, where the present view stands, and which ridges remain beyond reach.

Contents

1 Why preserve the archaeology?

Remembered / reconstructed

The temptation, looking backward, is to compress the history. That would be a mistake. There were correct intuitions, wrong completions, mathematically useful failures, and questions that outlived the first answers proposed for them. The point of this note is not to manufacture inevitability after the fact. It is to preserve the actual path.

The strongest retrospective claim is not

"I had the answer in 2014."

It is

$$\boxed{\text{the questions were already pointing at the missing interface.}}$$

The vocabulary was incomplete. The categorical architecture was absent. But the repeated discomforts—too rigid, too global, too symmetric, too many equivalent finite descriptions, no trustworthy inverse—were real mathematical symptoms.

1.1 The older route

The sequence was approximately

$$\boxed{\begin{array}{c}\text{Poincaré localization} \to \text{worked gluon amplitudes} \to \text{spinor-helicity} \to \text{BCFW} \\ \to \text{twistors} \to \text{Grassmannian} \to \text{Yangian} \dashrightarrow W^*\text{-type realization}\end{array}}$$

The final arrow was dotted. It meant suspected relation, unknown mechanism. It was not a theorem, not a completed equivalence, and not a claim that the special supersymmetric scattering sector was globally fundamental.

The representation-theoretic localization route had proved too stiff for the scattering question being asked. BCFW was the real philosophical disturbance: why should a one-complex-parameter continuation of two on-shell legs expose enough factorization data to reconstruct a complete tree amplitude? The Grassmannian sharpened the question by organizing many finite reconstruction histories of one scattering object. The operator-algebraic completion appeared because state, completion, and composition structure seemed to demand something deeper. But the $\mathcal{N} = 4$ origin and the broader warning supplied by Coleman–Mandula made naive globalization difficult to trust.

Mountain statement

The right questions matter more than the first answers, because the right question survives contact with a better mathematics.

2 The modern proposal

Modern reading

The modern proposal is not that the amplituhedron is the vacuum. It is more precise:

$$\boxed{\begin{array}{c}\text{the amplituhedron may be the exact positive geometry induced by}\\ \text{the finite distinctions that survive the scattering interface}\end{array}}$$

A local vacuum realization may be substantially richer. It may be operator-algebraic, carried by correspondences, dependent on refinement, and higher-coherent. The scattering interface may retain only cyclic provenance, integrable on-shell passage directions, positive orientation data, factorization incidence, and the positive external configuration Z.

The resulting geometry can therefore be exact without being ontologically complete.

2.1 Three levels of admissibility

The first modern correction is that three notions must be separated:

$$\boxed{\begin{array}{c}\text{linear admissibility} \neq \text{finite integrability}\\ \neq \text{global reconstruction completeness}\end{array}}$$

We denote the corresponding continuation classes by

$$\mathfrak{J}_{\mathrm{tot}} \subseteq \mathfrak{J}_{\mathrm{int}} \subsetneq \mathfrak{J}_{\mathrm{lin}}.$$

A tangent direction may satisfy every linearized physical constraint and still fail to integrate to a finite passage. An exact passage may expose finite factorization poles and still leave a boundary contribution at infinity. BCFW becomes special only at the final level.

3 Two channels are enough to ask the first real question

Reconstructed

If the modern interpretation is correct, the amplituhedron should leave a detectable fingerprint before one ever reaches a Grassmannian. The smallest place to look is the exact continuation geometry of two massless channels.

Let $p_a, p_b \in \mathbb{C}^{3,1}$ be generic non-collinear null momenta. Consider an infinitesimal momentum-conserving deformation

$$\delta p_a = q, \qquad \delta p_b = -q.$$

Linearized mass-shell preservation gives

$$p_a \cdot q = 0, \qquad p_b \cdot q = 0.$$

For generic independent p_a, p_b, this is a complex two-dimensional plane.

Proposition 3.1 (Linearized two-leg deformation dimension)**.** *For generic non-collinear complex null momenta p_a, p_b,*

$$T_{ab}^{\mathrm{lin}} = \{q \in \mathbb{C}^4 : p_a \cdot q = 0,\ p_b \cdot q = 0\}$$

has complex dimension two.

This breaks the naive rank-one conjecture. BCFW is not one-dimensional because the linearized physical tangent space is one-dimensional.

Now impose exact affine continuation:

$$p_a(z) = p_a + zq, \qquad p_b(z) = p_b - zq.$$

Exact null preservation for all z requires

$$q^2 = 0, \qquad p_a \cdot q = 0, \qquad p_b \cdot q = 0.$$

Write

$$p_a = \lambda_a \tilde{\lambda}_a, \qquad p_b = \lambda_b \tilde{\lambda}_b.$$

Then the nonzero solutions split into two crossed spinor lines.

Theorem 3.2 (Exact two-line continuation decomposition)**.** *For generic non-collinear complex null momenta p_a, p_b, the nontrivial affine momentum-conserving deformations preserving both mass shells for all z form precisely*

$$\mathbb{C}(\lambda_a \tilde{\lambda}_b) \cup \mathbb{C}(\lambda_b \tilde{\lambda}_a).$$

Proof. Write $q = \lambda_q \tilde{\lambda}_q$. The orthogonality constraints factor as

$$\langle aq\rangle[qa] = 0, \qquad \langle bq\rangle[qb] = 0.$$

For generic a, b, the nonzero compatible solutions are

$$\lambda_q \parallel \lambda_a, \quad \tilde{\lambda}_q \parallel \tilde{\lambda}_b,$$

or

$$\lambda_q \parallel \lambda_b, \quad \tilde{\lambda}_q \parallel \tilde{\lambda}_a.$$

□

Thus the correct geometry is

$$\boxed{\mathbb{C}^2 \supset \{q : q^2 = 0\} = L_+ \cup L_-.}$$

One-dimensionality is branchwise finite integrability, not tangent-space dimensionality.

Remark 3.3 (The useful failure)**.** The first guess was that the admissible tangent space itself should already be a line. It was not. The calculation opened a plane. Then one additional exact condition, $q^2 = 0$, drew two lines through it. There is something almost childlike in the economy of this: a small algebraic demand does not merely reduce the possibilities; it reveals two paths that were invisible in the linear view. The failed conjecture was useful because it forced the question to become more precise. BCFW is not the only direction one may begin to move. It is one of the two directions along which the motion can remain exactly on shell.

4 A source-side toy model: the split nilpotent cone

Let $\mathcal{E}$ be a self-dual Hilbert W^*-module over a von Neumann algebra M, with two selected orthogonal corners $p_a\mathcal{E}$ and $p_b\mathcal{E}$. Take opposite off-diagonal directions

$$N_+ \in p_b \operatorname{End}_M(\mathcal{E}) p_a, \qquad N_- \in p_a \operatorname{End}_M(\mathcal{E}) p_b.$$

Orthogonality gives

$$N_+^2 = N_-^2 = 0.$$

For

$$N(x, y) = xN_+ + yN_-,$$

one has

$$N(x, y)^2 = xy\Big(N_+N_- + N_-N_+\Big).$$

Proposition 4.1 (Bidirectional nilpotent cone)**.** *If*

$$S := N_+N_- + N_-N_+ \neq 0,$$

then

$$N(x,y)^2 = 0 \iff xy = 0.$$

Hence the square-zero locus is the union of the two complex lines $\mathbb{C}N_+ \cup \mathbb{C}N_-$*.*

This is an exact algebraic model of the BCFW branch geometry. It is not yet a physical derivation. The physical burden is to identify the actual source constraint and the finite evaluator that map this split cone to the BCFW null cone.

Remark 4.2 (What the toy remembers)**.** The toy remembers the feature we asked it to remember: two individually harmless passages whose mixed use feeds back into the diagonal corners. It forgets almost everything else—localization, spectra, analytic domains, states, and the physical meaning of the corners. That selective forgetting is precisely why it is useful and precisely why it cannot be promoted into a source theorem. It is a lantern held over one patch of ground, not a map of the mountain.

4.1 Mixed feedback

Define the mixed feedback operator

$$\mathfrak{F}_{ab} := N_+N_- + N_-N_+.$$

The pure triangular directions are individually integrable. Their mixture produces diagonal feedback. This suggests that the source second-order finite-integrability obstruction may be the scalar shadow of $\mathfrak{F}_{ab}$.

Let ε_σ be a sectorial scalar evaluator on the diagonal feedback sector. Set

$$Q_{\mathrm{src}}(N) = \varepsilon_\sigma(N^2).$$

Then

$$Q_{\mathrm{src}}(xN_+ + yN_-) = xy\,\varepsilon_\sigma(\mathfrak{F}_{ab}).$$

At the kinematic level, if

$$q_+ = \lambda_a\tilde{\lambda}_b, \qquad q_- = \lambda_b\tilde{\lambda}_a,$$

then

$$(xq_+ + yq_-)^2 = 2xy\,q_+\cdot q_-.$$

Both are split quadratic forms.

Theorem 4.3 (Quadratic intertwining on the selected two-plane)**.** *Suppose a linear finite interface map satisfies*

$$\Phi_\sigma(N_+) = c_+ q_+, \qquad \Phi_\sigma(N_-) = c_- q_-,$$

with nonzero constants, and suppose

$$\varepsilon_\sigma(\mathfrak{F}_{ab}) \neq 0.$$

Then there is a unique nonzero scalar c_σ such that

$$Q_{\mathrm{src}}(N) = c_\sigma \, Q_{\mathrm{kin}}(\Phi_\sigma N)$$

for every N in the selected two-plane. In particular, the source and kinematic null cones coincide projectively.

The important consequence is negative as much as positive: one does not need to derive the entire Lorentz metric from the source in order to recognize the BCFW branch cone. The two null lines plus one nonzero mixed scalar suffice.

5 Future-stable visibility

An arbitrary finite projection is not a legitimate physical quotient. Data invisible now may be rotated back into visible coordinates by later admissible continuations. The correct kernel must be stable under the whole continuation class.

Definition 5.1 (Future-stable scattering-null space)**.** Let Λ_σ be a finite evaluator and $\mathfrak{J}_{\mathrm{adm}}$ a composition-closed admissible continuation semigroup containing the identity. Define

$$K_\sigma^\infty := \bigcap_{J \in \mathfrak{J}_{\mathrm{adm}}} \ker(\Lambda_\sigma \circ J).$$

Theorem 5.2 (Future-stable kernel invariance)**.** *For every $N \in \mathfrak{J}_{\mathrm{adm}}$,*

$$N K_\sigma^\infty \subseteq K_\sigma^\infty.$$

Proof. If $x \in K_\sigma^\infty$, then for every admissible J,

$$\Lambda_\sigma(JNx) = 0$$

because $JN \in \mathfrak{J}_{\mathrm{adm}}$. Hence $Nx \in K_\sigma^\infty$. □

Thus the primitive finite interface object is

$$\mathcal{V}_\sigma^{\mathrm{phys}} = \mathcal{E}/K_\sigma^\infty,$$

not the image of one arbitrary finite projection.

Remark 5.3 (Visibility has a future tense)**.** It is not enough for a distinction to be invisible at the present stage. If an admissible continuation can turn it back toward the detector, it was never physically null. The intersection defining K_σ^∞ gives the word *survival* its needed rigor: what disappears is what no permitted future experiment in the continuation calculus can make visible again.

6 Chiral branch recognition

For each massless external channel i, the scattering interface sees one-dimensional chiral lines S_i^L and S_i^R, with

$$p_i = \lambda_i \otimes \tilde{\lambda}_i.$$

The two crossed passage tensors are

$$q_+ = \lambda_a \otimes \tilde{\lambda}_b, \qquad q_- = \lambda_b \otimes \tilde{\lambda}_a.$$

Each is rank one and therefore null; each is orthogonal to both endpoint momenta. Their endpoint little-group weights are opposite. If the corresponding finite weight spaces are one-dimensional, branch identification is unique up to scale.

Proposition 6.1 (Crossed bispinor passage)**.** *The tensors q_+ and q_- are null and satisfy*

$$p_a \cdot q_\pm = p_b \cdot q_\pm = 0.$$

Hence they generate exact affine two-line continuations.

The source need not be fundamentally spinorial. It need only produce the correct two weighted null passage lines under the already four-dimensional scattering interface.

7 From an integrable branch to a complete transversal

A BCFW branch is complex. The positive bridge coordinate that will appear much later is real and oriented. Between those two statements lies the part of the story that the first compact freeze moved through too quickly.

The philosophical disturbance was never merely that BCFW computes amplitudes. It was that one complex parameter appears to know where a much larger factorization geometry is located. The parameter does not fill that geometry. It crosses it. The correct object is therefore a marked transversal.

Fix a physical channel I. Along the chosen two-leg deformation write

$$\widehat{p}_a(z) = p_a + zq, \qquad \widehat{p}_b(z) = p_b - zq,$$

and define

$$\epsilon_I = \begin{cases} +1, & a \in I,\ b \notin I, \\ -1, & b \in I,\ a \notin I, \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\widehat{P}_I(z) = P_I + \epsilon_I z q$$

and, since $q^2 = 0$,

$$\widehat{P}_I(z)^2 = P_I^2 + 2\epsilon_I z P_I \cdot q.$$

Only channels with $\epsilon_I \neq 0$ are directly exposed. A finite transverse mark also requires $P_I \cdot q \neq 0$.

7.1 The source channel-defect line

Let L_χ^{src} be an integrable source branch with affine coordinate z, and let $\mathscr{D}_I^{\mathrm{src}}$ be a channel-defect carrier. Its physically meaningful version is not the raw carrier but the future-stable quotient

$$\overline{\mathscr{D}}_I^{\mathrm{src}} := \mathscr{D}_I^{\mathrm{src}} / \mathscr{K}_{I,\infty}^{\mathrm{sc}}.$$

We seek a section

$$\Delta_I^{\mathrm{src}} : L_\chi^{\mathrm{src}} \longrightarrow \overline{\mathscr{D}}_I^{\mathrm{src}}$$

whose effective scattering shadow is

$$\boxed{\mathrm{Ev}_\sigma[\Delta_I^{\mathrm{src}}(z)] = \widehat{P}_I(z)^2.}$$

The equation $\Delta_I^{\mathrm{src}}(z_I) = 0$ must initially be read in this quotient. Exact representative-level vanishing is stronger and is not assumed.

Definition 7.1 (Source factorization mark)**.** A point $z_I \in L_\chi^{\mathrm{src}}$ is a source factorization mark for I if the effective channel defect vanishes there, the zero is transverse, the source datum admits an admissible separating degeneration, and scattering evaluation maps that degeneration to the physical boundary $\widehat{P}_I(z_I)^2 = 0$.

The first subtlety is easy to miss. Vanishing after evaluation does not imply vanishing in the effective source carrier unless evaluation is faithful on the line actually swept out by the defect. A scalar pole may be visible while a hidden transverse source defect remains.

Theorem 7.2 (Marked degeneration and channel-defect recognition)**.** *Assume that along* L_χ^{src}*:*

(i) the effective defect varies affinely,

$$[\Delta_I^{\mathrm{src}}(z)] = A_I + zB_I;$$

(ii) its image lies in a one-dimensional channel line $\ell_I \subset \overline{\mathscr{D}}_I^{\mathrm{src}}$*;*

(iii) $\mathrm{Ev}_\sigma |_{\ell_I}$ *is injective;*

(iv)

$$\mathrm{Ev}_\sigma(A_I) = P_I^2, \qquad \mathrm{Ev}_\sigma(B_I) = 2\epsilon_I P_I \cdot q;$$

(v) $P_I \cdot q \neq 0$.

Then the effective defect has a unique simple zero at

$$\boxed{z_I = -\frac{P_I^2}{2\epsilon_I P_I \cdot q}.}$$

If the zero also admits an evaluation-compatible separating degeneration and a simple-pole normal form, its residue is the correct lower-point factorization residue.

Proof. The finite shadow is the affine function

$$P_I^2 + 2\epsilon_I z P_I \cdot q,$$

which has the unique simple zero displayed above. Faithfulness on ℓ_I lifts scalar vanishing to effective source vanishing. For the residue, use

$$\widehat{P}_I(z)^2 = 2\epsilon_I P_I \cdot q\,(z - z_I)$$

and the assumed separating normal form. The result is

$$\operatorname*{Res}_{z=z_I} \mathcal{A}_n(z) = \frac{1}{2\epsilon_I P_I \cdot q} \sum_\alpha \mathcal{A}_L^{I,\alpha}(z_I)\mathcal{A}_R^{I,\alpha}(z_I),$$

up to fixed propagator and orientation conventions. □

This theorem is proved as an abstract recognition statement. Construction of the physical defect line, its faithful evaluator, and its separating W^*-source degeneration remains open.

Remark 7.3 (Why one dimension can be enough)**.** A branch is not complete because it fills factorization geometry. It is complete in the way a path across a hillside can be complete: it meets the ridges that matter, and each crossing tells us how the terrain continues on either side. The marks are therefore more important than the size of the branch. Completeness belongs to the recursively unfolding pattern of crossings, not to the ambient volume swept out by the parameter.

7.2 A quadratic sufficient model

Suppose the source germ carries channel and branch-direction data $\mathcal{P}_I^{\mathrm{src}}$, $\mathcal{Q}_\chi^{\mathrm{src}}$, and a symmetric source pairing B_I^{src}. Set

$$\widehat{\mathcal{P}}_I^{\mathrm{src}}(z) = \mathcal{P}_I^{\mathrm{src}} + \epsilon_I z \mathcal{Q}_\chi^{\mathrm{src}}$$

and

$$\Delta_I^{\mathrm{src}}(z) = B_I^{\mathrm{src}}(\widehat{\mathcal{P}}_I^{\mathrm{src}}(z), \widehat{\mathcal{P}}_I^{\mathrm{src}}(z)).$$

Affine variation in the effective quotient follows if

$$\left[B_I^{\mathrm{src}}(\mathcal{Q}_\chi^{\mathrm{src}}, \mathcal{Q}_\chi^{\mathrm{src}})\right] = 0.$$

This is the source analogue of $q^2 = 0$. It is a finite-integrability condition, not a consequence of linear admissibility.

8 The finite marked divisor and recursive closure

Let $\mathfrak{C}_n^{\mathrm{phys}}$ be the finite physical channel set and define

$$\mathfrak{C}_\chi^{\mathrm{dir}} = \{I \in \mathfrak{C}_n^{\mathrm{phys}} : \epsilon_I \neq 0,\ P_I \cdot q \neq 0\}.$$

For each recognized channel let z_I be its mark.

Definition 8.1 (Marked source factorization divisor)**.** The marked divisor of the branch is

$$\mathfrak{D}_\chi^{\mathrm{src}} = \sum_{I \in \mathfrak{C}_\chi^{\mathrm{dir}}} m_I [z_I; I],$$

where the channel label is retained and m_I is the effective vanishing multiplicity.

The label is essential. At exceptional kinematics two different channels can meet the one-dimensional branch at the same value of z. Equality of mark position does not imply equality of degeneration mechanism. The unlabelled divisor forgets precisely the provenance the interface calculation still needs.

At a marked point, the source datum separates:

$$\mathfrak{s}_n(z_I) \rightsquigarrow \mathfrak{s}_L^I \underset{I}{\boxtimes} \mathfrak{s}_R^I.$$

Each non-elementary factor inherits a new continuation problem. Iteration produces a rooted factorization tree.

Definition 8.2 (Recursive factorization closure)**.** The recursive closure

$$\mathrm{Cl}_{\mathrm{fac}}(\mathfrak{C}_\chi^{\mathrm{dir}})$$

is the smallest collection containing every directly marked channel and every admissible lower-point factorization of every marked residue, iterated until elementary seeds are reached.

Direct exposure and completeness are therefore different notions. A branch need not expose every physical channel as a first-generation pole. It must expose enough poles that their recursively generated closure contains the full physical factorization architecture.

Theorem 8.3 (BCFW branch as a minimal-dimensional complete transversal)**.** *Let $\mathcal{A}_n(z)$ be meromorphic along a selected integrable branch. Assume finite transverse marking, physical exhaustion of finite poles, residue compatibility, recursive factorization closure, termination on elementary seeds, and*

$$\operatorname*{Res}_{z=\infty} \frac{\mathcal{A}_n(z)}{z} = 0.$$

Then

$$\boxed{\mathcal{A}_n(0) = \sum_{I \in \mathfrak{C}_\chi^{\mathrm{dir}}} \sum_{\alpha} \mathcal{A}_L^{I,\alpha}(z_I) \frac{1}{P_I^2} \mathcal{A}_R^{I,\alpha}(z_I).}$$

The recursively expanded right-hand side contains the complete physical factorization data even when some channels are not directly marked on the original branch.

Proof. Apply the residue theorem to $\mathcal{A}_n(z)\,dz/z$. The residue at infinity vanishes. At a finite mark,

$$-\frac{1}{z_I}\frac{1}{2\epsilon_I P_I \cdot q} = \frac{1}{P_I^2},$$

and residue compatibility supplies the lower-point product. Recursive closure and termination finish the reconstruction. □

The geometry behind the formula is now visible:

$$\boxed{\begin{array}{c}\text{minimal dimension} + \text{transverse marking} + \text{recursive closure}\\ +\text{no leakage} = \text{complete transversal}\end{array}}$$

9 Admissible refinement and the strictification of paths

Finite marking gives the poles. It does not yet make the boundary calculus intrinsic. The same lower-dimensional physical stratum can be reached through different ordered degenerations.

For two compatible channels consider

$$\mathfrak{s} \xrightarrow{D_I} \mathfrak{s}_I \xrightarrow{D_{J|I}} \mathfrak{s}_{IJ}^{(I)}$$

and

$$\mathfrak{s} \xrightarrow{D_J} \mathfrak{s}_J \xrightarrow{D_{I|J}} \mathfrak{s}_{IJ}^{(J)}.$$

The terminal source data need not be literally equal. Choose an admissible common refinement $\widetilde{\mathfrak{s}}_{IJ}$ and comparison maps r_I, r_J. Define

$$L_{I,J} = r_I D_{J|I} D_I, \qquad R_{I,J} = r_J D_{I|J} D_J.$$

Definition 9.1 (Direct/refined comparison defect)**.** On an effective comparison corner where the inverse exists, set

$$\Theta_{I,J}^{\mathrm{dir/ref}} = R_{I,J}^{-1} L_{I,J}.$$

It is scattering-invisible when

$$\Theta_{I,J}^{\mathrm{dir/ref}} \equiv 1 \pmod{\mathscr{K}_\infty^{\mathrm{sc}}}.$$

Equality under one evaluation is not enough. A later admissible continuation must not recover the route provenance.

Definition 9.2 (Admissible source adjacency)**.** Two channel marks satisfy $I \sim_{\mathrm{adm}} J$ when both ordered degenerations exist, their endpoints admit a continuation-complete common refinement, and their comparison defect is future-stably scattering-invisible.

Planar compatibility is then a finite shadow rather than the primitive definition. Under evaluation compatibility,

$$I \sim_{\mathrm{adm}} J \Longrightarrow I \sim_{\mathrm{planar}} J.$$

The converse is an open source-completeness statement.

Pairwise squares are not enough. For three compatible degenerations the two bracketings may differ by an associator

$$\Omega^{\mathrm{src}}_{I,J,K} = c^{-1}_{I(JK)} c_{(IJ)K}.$$

Pairwise invisibility of every $\Theta_{I,J}$ does not imply invisibility of $\Omega_{I,J,K}$. Four-fold comparison then requires the pentagon relation. One finite pentagon is a witness, not a global H^3-class.

Let $\mathcal{P}^{\mathrm{src}}_{\mathrm{fac}}$ be the finite path category generated by marked degenerations and admissible refinements, and let $\mathcal{N}^{\mathrm{sc}}_{\infty}$ be the future-stable null congruence.

Theorem 9.3 (Finite factorization-path strictification)**.** *Assume finite termination, local common refinement, scattering-null pairwise comparison defects, scattering-null associators, pentagon coherence modulo the null congruence, and stability of nullity under composition, refinement, and recursive descent. Then any two finite admissible factorization paths with the same initial datum and the same labelled physical boundary target agree in*

$$\overline{\mathcal{P}}^{\mathrm{src}}_{\mathrm{fac}} := \mathcal{P}^{\mathrm{src}}_{\mathrm{fac}} / \mathcal{N}^{\mathrm{sc}}_{\infty}.$$

Proof. Any two finite paths with the same boundary data differ by finitely many replacements of common refinements, adjacent compatible interchanges, reassociations, and refinement identities. Pairwise nullity identifies the elementary interchanges; associator nullity identifies the bracketings; higher coherence makes the replacement independent of the order in which it is performed. Null stability allows every local identification to be composed with the rest of the path. □

The source may remain twisted. Only its finite scattering shadow must strictify:

$$\boxed{\text{twisted source descent} \longrightarrow \text{strict scattering incidence.}}$$

Remark 9.4 (No scar downstairs)**.** Two routes may genuinely disagree in the source. They may pass through different refinements, carry different correspondence data, and be related only by a nontrivial associator. Strictification does not erase that history from the source. It says something narrower and more physical: after every admissible future continuation, the disagreement leaves no scar in scattering. The clean boundary poset is the geometry of that scarless remainder.

10 The effective incidence poset

After strictification, a boundary stratum is the effective endpoint of a compatible defect family $\mathcal{I} = \{I_1, \ldots, I_r\}$. Write it as $\mathfrak{B}^{\mathrm{src}}_{\mathcal{I}}$. Its effective codimension is the rank of the differential of the joint defect map, not merely the number of refinement steps.

Define

$$\mathfrak{B}^{\mathrm{src}}_{\mathcal{I}} \preceq \mathfrak{B}^{\mathrm{src}}_{\mathcal{J}}$$

when the latter is obtained from the former by introducing additional effective vanishing defects.

Theorem 10.1 (Finite effective incidence poset)**.** *Assume finite path strictification, strict increase of effective codimension under every nontrivial degeneration, endpoint separation by effective defect data, and persistence of already-vanishing defects. Then the scattering-effective boundary strata form a finite graded poset.*

This poset is not yet a positroid poset. Ordinary factorization incidence remembers compatible propagator boundaries. Positroid stratification also remembers rank conditions, Pluecker support, cell dimensions, positive orientations, and network equivalence moves. A positive-network lift remains necessary.

11 Positive networks and projection-surjective atlases

Suppose the strictified passage system admits a planar network realization with positive coordinates w_e. Boundary measurement gives

$$C_{\mathcal{N}}(w) \in Gr_{\geq 0}(k, n).$$

Definition 11.1 (Admissible positive-network atlas)**.** An admissible positive-network atlas $\mathfrak{U}^+_\chi$ assigns network charts to all recursively required effective strata, sends admissible source degenerations to positroid boundary degenerations, sends source comparison equivalences to boundary-measurement-preserving network moves, and contains every cell required by the chosen BCFW reconstruction.

The source does not need to realize every point of the positive Grassmannian. Let $\mathcal{U}^+_\chi$ be the union of represented cells and let

$$\pi_Z(C) = CZ.$$

Definition 11.2 (Projection-surjective atlas)**.** The atlas is projection-surjective for a target geometry $\mathcal{Y}_Z$ when

$$\pi_Z(\mathcal{U}_\chi^+) = \mathcal{Y}_Z.$$

It is BCFW-complete when it supports every cell and recursive boundary required by the BCFW reconstruction.

Proposition 11.3 (Positive-atlas sufficiency)**.** *If the source-effective realization is surjective onto $\mathcal{U}_\chi^+$ and $\pi_Z(\mathcal{U}_\chi^+) = \mathcal{Y}_Z$, then the composite source-to-target map is surjective onto $\mathcal{Y}_Z$, regardless of whether the source is surjective onto all of $Gr_{\geq 0}(k,n)$.*

This is the non-surjectivity firewall:

$$\boxed{\text{BCFW-complete} + \text{positive} + \text{projection-surjective} \quad \text{is sufficient.}}$$

12 Positivity as sign-coherent survival

Positivity is not yet a property of the full source. The source may carry phases, operator-valued passages, and nontrivial higher coherence. Positivity can arise only after the passage system has reduced to finite effective coefficient lines.

Remark 12.1 (Positivity arrives late)**.** This order matters. We did not begin by declaring the hidden source positive and then read the amplituhedron out of it. We first removed distinctions that scattering can never recover, strictified the surviving paths, and oriented their effective coefficient lines. Only then did nonnegative coordinates appear. Positivity is not being installed as a metaphysics. It is recognized as the sign-coherent form taken by what remains.

For an elementary passage e, suppose

$$[T_e]_{\mathrm{sc}} = w_e \ell_e^+$$

on an oriented one-dimensional effective line. Call the passage interface-positive when $w_e \geq 0$. A passage system is sign-coherent when composition multiplies these coefficients, compatible alternatives combine subtraction-freely, refinements act by positive rational transformations, and internal rescalings are positive gauge changes.

For an oriented network flow $\mathcal{F}$, let

$$w(\mathcal{F}) = \prod_{e \in \mathcal{F}} w_e.$$

Under the standard planar boundary-measurement convention,

$$\Delta_J(C_{\mathcal{N}}) = \sum_{\mathcal{F}\in\mathrm{Flow}(J)} w(\mathcal{F}) \geq 0.$$

Theorem 12.2 (Positive-carrier recognition)**.** *If a finite strictified source passage calculus admits a planar network realization with oriented scalar-line passages, nonnegative effective coefficients, subtraction-free composition, and boundary-measurement-preserving refinement moves, then its boundary-measurement matrix lies in $Gr_{\geq 0}(k,n)$. The positive interior lies in a positroid cell, and admissible zero-weight limits move to positroid boundaries.*

This does not say that the amplitude is a nonnegative number, nor that quantum interference is absent. Positivity belongs to the finite carrier coordinates after the relevant phases and provenance have been organized or forgotten.

12.1 Three losses of provenance

The complete finite passage contains at least three distinct identifications:

$$\mathfrak{S}^{\mathrm{src}} \longrightarrow \overline{\mathfrak{S}}^{\mathrm{sc}} \longrightarrow \mathcal{U}^+_\chi \xrightarrow{\pi_Z} \mathcal{Y}_Z.$$

The first removes future-stable scattering-null source distinctions. The second removes network presentation and internal gauge provenance. The third removes distinctions in C that the fixed external data Z cannot resolve.

Define $s \sim_Z s'$ when $\Psi_\chi(s)Z = \Psi_\chi(s')Z$. Then

$$\overline{\mathfrak{S}}^{\mathrm{sc}}/\sim_Z \cong \mathrm{Im}(\pi_Z \circ \Psi_\chi).$$

If the atlas is projection-surjective onto the amplituhedron, this quotient is the amplituhedron image. The statement is elementary. Its interpretation is not: the final shape is exactly the quotient by every distinction the chosen scattering interface cannot preserve.

13 Canonical forms from logarithmic defects

The region determines where the finite data live. It does not yet determine the differential form carried by that region.

At a simple mark,

$$[\Delta_I^{\mathrm{src}}(z)] = B_I(z - z_I), \qquad B_I \neq 0,$$

and hence

$$d\log[\Delta_I^{\mathrm{src}}] = \frac{dz}{z - z_I}.$$

The logarithmic pole is the differential shadow of transverse marked degeneration.

For a positive chart with coordinates $w_1, \dots, w_d > 0$, set

$$\omega_\alpha = \bigwedge_{j=1}^{d} d\log w_j.$$

Its boundary residues are the lower-dimensional logarithmic forms. However, chart changes also carry an orientation line. Let $\varepsilon_{\alpha\beta} \in \{\pm 1\}$ be the orientation comparison on overlaps. A nontrivial surviving orientation cocycle would produce a twisted top form rather than an ordinary global canonical form.

Definition 13.1 (Scattering-effective orientability)**.** The atlas is scattering-effectively orientable when its orientation cocycle becomes trivial in the future-stable scattering quotient, so compatible chart orientations may be chosen globally.

Internal facets must then occur with opposite induced signs. If $F_{\alpha\beta}$ is a shared nonphysical facet,

$$\operatorname*{Res}_{F_{\alpha\beta}} \omega_\alpha = -\operatorname*{Res}_{F_{\alpha\beta}} \omega_\beta.$$

Thus path strictification supplies equality of the underlying boundary data, while orientation supplies cancellation of the spurious residue.

Theorem 13.2 (Interface canonical-form recognition)**.** *Assume a finite positive atlas with dimension-matched logarithmic cell forms, effective orientability, pairing of all internal facets with opposite induced orientation, physical boundary completeness, recursively factorizing physical residues, no additional singularities, and no residue at infinity. Then*

$$\Omega_{\mathcal{Y}_Z} = \sum_\alpha (f_\alpha)_* \omega_\alpha$$

is the canonical form of the projected geometry, normalized by the elementary seed forms.

No leakage at infinity now has a geometric reading. A nonzero residue at $z = \infty$ is an unaccounted boundary of the chosen projective transversal. The equality

$$\text{no pole at infinity} = \text{projective completeness of the marked branch}$$

is therefore a recognition principle. Its physical source proof remains open.

14 Branch independence and common interface geometry

Different BCFW branches expose different finite divisors and different cell decompositions. Branch independence cannot mean equality of directly exposed poles.

Call two branches factorization-equivalent when their recursive closures contain the same labelled physical strata, their boundary residues agree, they terminate on the same normalized seeds, and neither leaks at infinity.

Let X_Z be the ambient projected space and $\mathcal{D}_{\mathrm{phys}}$ the physical divisor. Define the residue kernel

$$\mathscr{K}_{\mathrm{res}}(X_Z, \mathcal{D}_{\mathrm{phys}}) = \{\eta : \mathrm{Res}_{D_I} \eta = 0 \ \forall I,\ \mathrm{Res}_{\infty} \eta = 0\}$$

inside the prescribed class of rational top forms.

Theorem 14.1 (Canonical-form branch independence)**.** *If two factorization-equivalent branches produce forms in the same rational class, with the same allowed pole divisor and vanishing residue at infinity, and if $\mathscr{K}_{\mathrm{res}} = 0$, then the two forms are equal.*

Equality of forms does not automatically imply equality of arbitrary semialgebraic regions. Geometry independence follows from the stronger existence of a scattering-effective common positive refinement covering both projected images with equal multiplicity and coherent orientation.

No direct map between the two source branches is required. They may be faithful local descriptions that forget different source information while still having the same interface image:

$$\begin{tikzcd} L^{\mathrm{src}}_{\chi} \arrow[d, "F_{\chi}"'] & L^{\mathrm{src}}_{\chi'} \arrow[d, "F_{\chi'}"] \\ \mathcal{Y}_Z \arrow[r, equal] & \mathcal{Y}_Z. \end{tikzcd}$$

This is a concrete branch-level witness of the representational-boundary principle. Failure of a canonical cross-branch map is not failure of the underlying physical passage.

15 The conditional recognition theorem

The architecture is now complete enough to state the global implication. It is important to name it correctly. This is not yet a physical derivation of the amplituhedron from a W^*-source. It is a recognition theorem: if the source supplies a precise finite survivor package, then the downstream geometry is forced.

Definition 15.1 (Finite survivor package)**.** For fixed positive external data Z and scattering type (n, k, m), a finite survivor package consists of:

(i) an analytic source continuation germ whose finite shadow is a chiral BCFW branch;
(ii) faithful affine source channel-defect lines with simple physical marks;
(iii) evaluation-compatible lower-point factorization and recursive termination;
(iv) no boundary contribution at infinity;
(v) admissible common refinements with scattering-null comparison and associator defects;
(vi) a sign-coherent positive-network atlas for the strictified finite passage calculus;
(vii) BCFW completeness and Z-projection-surjectivity;
(viii) oriented logarithmic cell forms with cancellation of internal facets and correct physical residues;
(ix) branch independence by common refinement or residue determinacy.

Theorem 15.2 (Interface-induced amplituhedron recognition)**.** *Assume that a physical source admits a finite survivor package. Then:*

(1) its future-stable scattering quotient carries a finite graded boundary-incidence poset;
(2) that poset admits a positive-network realization inside $Gr_{\geq 0}(k, n)$;
(3) the quotient by Z-indistinguishability is canonically identified with the amplituhedron image,

$$\overline{\mathfrak{S}}^{\mathrm{sc}}_{n,k}/\sim_Z \cong \mathcal{A}_{n,k,m}(Z);$$

(4) the strictified source residue calculus descends to the canonical form of that image;
(5) the final geometry and form are independent of the complete continuation branch used;
(6) source surjectivity onto all of $Gr_{\geq 0}(k, n)$ is unnecessary.

Proof. Channel-defect recognition gives a finite labelled divisor. Recursive completeness and the vanishing residue at infinity turn the selected branch into a complete transversal. Refinement and higher-defect invisibility strictify its finite path calculus and produce the effective incidence poset. Scalar-line orientation and nonnegative subtraction-free passage weights produce a positive network atlas. Projection-surjectivity identifies the Z-fiber quotient with the amplituhedron image. Logarithmic cell forms glue because internal facets cancel and physical residues factorize. Finally, common refinement or vanishing of the residue kernel gives branch independence. □

The theorem is proved as a conditional implication. It does not prove that the hypotheses occur in the physical source.

What the recognition theorem does not say

It does not identify the amplituhedron with the vacuum. It does not identify the source with the positive Grassmannian. It does not require the source to be globally strict, globally positive, or totally reconstructible from scattering. It allows nontrivial Z-fibers and nontrivial source associators. It concludes only

$$\boxed{\text{amplituhedron} = \text{complete projected geometry of surviving scattering data.}}$$

This is the exact strength of the thesis. The geometry can be complete at the interface while remaining incomplete as a description of the source ontology.

16 The physical source frontier

The conditional theorem relocates the mystery. The amplituhedron side is no longer the unknown part of the bridge. The open problem is the construction of the finite survivor package from a physical source.

The first target is deliberately smaller than a Grassmannian. It is one channel defect.

16.1 An analytic firewall

A generic BCFW shift uses complex z. It is not physical real-momentum evolution and should not automatically be represented as a one-parameter group of normal $*$-automorphisms. Requiring

$$\zeta(z) = \mathrm{Ad}(e^{izH})(\zeta_0)$$

for a self-adjoint physical generator would impose unitary dynamics on an analytic continuation device.

The appropriate object is an analytic germ over a physical W^*-source.

Definition 16.1 (Analytic W^*-source germ)**.** Let $\mathfrak{M}^{\mathrm{src}}$ be a von Neumann algebra or the endomorphism algebra of a self-dual Hilbert W^*-module $\mathscr{E}^{\mathrm{src}}$. An analytic source germ consists of a dense common analytic core $\mathscr{E}_{\mathrm{an}}$, source passages $T(z)$ defined on that core, holomorphic coefficient functions

$$z \longmapsto \langle \xi, T(z)\eta \rangle, \qquad \xi, \eta \in \mathscr{E}_{\mathrm{an}},$$

compatibility with the physical datum at $z = 0$, and a scattering evaluation on the germ or its effective coefficient quotient.

The physical object remains a W^*-object. The complex continuation lives in its analytic enlargement.

16.2 Three strengths of realization

The strongest target would be an intrinsic affiliated source operator $\boldsymbol{\Delta}_I^{\mathrm{src}}(z)$. A weaker target is a closable source quadratic form on a common analytic core. The minimal target needed here is only a natural section of a scattering-effective defect line.

This hierarchy prevents an unnecessary demand. Channel momentum may be intrinsically asymptotic or interface-defined. A source-side defect can be natural along a scattering passage without being a globally intrinsic observable of the undifferentiated source algebra.

16.3 The momentum-form route

Suppose the source germ carries forms $\mathcal{P}_I^{\mu,\mathrm{src}}$ and $\mathcal{Q}_\chi^{\mu,\mathrm{src}}$ on a common core. Set

$$\widehat{\mathcal{P}}_I^{\mu,\mathrm{src}}(z) = \mathcal{P}_I^{\mu,\mathrm{src}} + \epsilon_I z \mathcal{Q}_\chi^{\mu,\mathrm{src}}$$

and form

$$\mathfrak{q}_I^{\mathrm{src}}(z) = g_{\mu\nu} \widehat{\mathcal{P}}_I^{\mu,\mathrm{src}}(z) \widehat{\mathcal{P}}_I^{\nu,\mathrm{src}}(z).$$

To obtain affine variation after future-stable reduction one needs

$$\boxed{[g_{\mu\nu} \mathcal{Q}_\chi^{\mu,\mathrm{src}} \mathcal{Q}_\chi^{\nu,\mathrm{src}}]_{\mathrm{sc}} = 0.}$$

One also needs evaluation to send the constant and mixed terms to P_I^2 and $P_I \cdot q$. This locates the correct zero. It does not yet turn that zero into a source factorization.

16.4 The degeneration-operator route

Introduce an analytic family of source passage operators $\mathfrak{D}_I(z)$. The desired mechanism is

$$\text{defect zero} \Longrightarrow \text{loss of invertibility} \Longrightarrow \text{factorization correspondence.}$$

At an isolated simple noninvertibility point one seeks

$$\mathfrak{D}_I(z)^{-1} = \frac{\mathfrak{R}_I}{z - z_I} + \mathfrak{H}_I(z),$$

with $\mathfrak{H}_I$ regular and

$$\operatorname{Ran}\mathfrak{R}_I \cong \mathscr{E}_L^I \overline{\otimes}_{\mathfrak{A}_I} \mathscr{E}_R^I.$$

Scattering evaluation should carry $\mathfrak{R}_I$ to the correct internal-state factorization sum.

The momentum form answers where the denominator vanishes. The degeneration operator must answer why the residue is a lower-point product. Those questions should not be compressed into one analogy.

Conjecture 16.2 (One-channel W^*-defect realization target)**.** *There exists an analytic W^*-source germ, a future-stable faithful channel-defect line, a source-null branch direction, and an analytic degeneration family such that the unique simple defect zero occurs at the BCFW value and the meromorphic source residue factors through the correct lower-point correspondence.*

This is the first major physical theorem target. It is open.

16.5 The simple crossing problem

Let

$$K_I = \ker \mathfrak{D}_I(z_I), \qquad K_I^\vee = \ker \mathfrak{D}_I(z_I)^*.$$

The derivative $\dot{\mathfrak{D}}_I(z_I)$ induces a crossing map from K_I to the dual cokernel carrier. In ordinary analytic Fredholm theory, nondegeneracy yields a simple pole of the inverse. In the intended self-dual Hilbert-module setting one must also establish complementability of the kernel, closedness of the range, regularity or Fredholmness, module compatibility, and normality of the residue correspondence. These are the analytic conditions separating a formal zero from a closed physical degeneration.

17 Topology in the forgotten fibers

Open

The recognition theorem identifies the amplituhedron with a quotient of surviving scattering data. It does not identify the topology of the quotient with the topology of the carrier above it. This leaves open a sharper possibility:

the visible positive image may be contractible,
while the scattering carrier above it has nontrivial cycles.

This distinction is necessary. Important tree amplituhedra are known, or expected, to be homeomorphic to closed balls. That ball-like behavior does not force the fibers of the source-to-interface projection to be contractible, nor does it trivialize the compactified degeneration loci that appear after loop variables, cuts, and integration contours are admitted.

There are external clues, but not yet a bridge theorem. Oriented homology has been used to organize Yangian invariants in the amplituhedron; elliptic leading singularities require genuine homological and cohomological contour data; and families of higher-loop traintrack integrals have been related to elliptically fibered varieties, including a K3 surface and conjectural higher Calabi–Yau geometries.[1] These results do not prove that the amplituhedron itself has hidden handles. They do show that nontrivial cycles and higher algebraic geometry are native to the surrounding amplitude problem and can reappear precisely where finite rational charts cease to be the whole story.

Conjecture 17.1 (Topology in the forgotten fibers)**.** *For suitable scattering type and loop depth, there exists a stratified or derived scattering carrier*

$$\widetilde{\mathfrak{S}}^{\mathrm{sc}}_{n,k,L}$$

and a projection

$$\rho_Z : \widetilde{\mathfrak{S}}^{\mathrm{sc}}_{n,k,L} \longrightarrow \mathcal{A}^{(L)}_{n,k,m}(Z)$$

such that the positive interface image may be ball-like while the fibers, singular fibers, or compactified degeneration loci of ρ_Z carry nontrivial homology or monodromy. The canonical scattering form is obtained from an oriented pushforward or fiberwise residue calculus, and elliptic or higher Calabi–Yau structures arise as periods of the cycles retained by the lifted carrier but forgotten by the finite positive image.

The conjecture is intentionally broader than the statement that a particular displayed amplituhedron is a torus. A picture on a screen is already a projection of a projection. Apparent folds may be coordinate overlap, self-intersection of the display map, boundary identification, or evidence of a genuine cycle upstairs. The first task is to distinguish those possibilities by computing fibers and their incidence, not by interpreting the rendering literally.

The conjecture suggests a refinement of the central slogan:

$$\boxed{\text{The shape is what survives. The topology records how it survived.}}$$

[1] See J. L. Bourjaily, *Computational Tools for Trees in Gauge Theory and Gravity*, arXiv:2312.17745; J. L. Bourjaily, N. Kalyanapuram, C. Langer, and K. Patatoukos, *Prescriptive Unitarity with Elliptic Leading Singularities*, arXiv:2102.02210; and J. L. Bourjaily et al., *Traintracks Through Calabi–Yaus: Amplitudes Beyond Elliptic Polylogarithms*, arXiv:1805.09326.

17.1 The cloud line: supersymmetric sectors that do not globalize

There is a more speculative possibility. The supersymmetric sectors used to expose the finite geometry may be locally exact without assembling into a single global physical sector of the source vacuum.

Let $\{U_i\}$ be an admissible cover of the relevant continuation or kinematic carrier, and let $\mathscr{S}_i$ denote a supersymmetric realization over U_i. Pairwise equivalences on overlaps need not satisfy strict higher coherence on triple and quadruple overlaps. Their residual may define a globalization obstruction

$$[\omega_{\mathrm{SUSY}}]$$

in an appropriate nonabelian, categorical, or higher-cohomological coefficient system.

Conjecture 17.2 (Local supersymmetry, obstructed globalization)**.** *The supersymmetric sectors relevant to amplituhedron reconstruction form a coherent local atlas or higher descent object, but need not globalize as one physical source sector. Their globalization obstruction can remain nontrivial in the lifted scattering carrier while becoming invisible after passage to the finite positive interface quotient. Consequently, the amplituhedron may inherit exact supersymmetric and Yangian organization without requiring a globally supersymmetric vacuum ontology.*

This is not an assertion that standard planar $\mathcal{N} = 4$ super Yang–Mills is mathematically inconsistent, nor is it a derivation of supersymmetry breaking. It is a conjecture about the relation between local reconstruction sectors and a more general physical source. The relevant coefficient object, descent site, and obstruction class have not yet been constructed.

This lies beyond the present riverbend. The terrain is not mapped, and the claim should remain written in pencil—on dust of clouds unsettled. Still, the possibility deserves to be recorded because it would explain why supersymmetric finite shadows can be extraordinarily rigid while every attempted globalization into the physical source feels too strong.

18 The theorem-status boundary

The expanded note contains four different kinds of mathematics. They must remain visibly distinct.

Proved here or elementary

- The generic linearized two-leg deformation space is complex two-dimensional.
- Exact affine null preservation splits it into the two chiral BCFW lines.
- Opposite two-corner square-zero directions give the same split quadratic cone.
- Split quadratic forms with matched null lines agree projectively.
- Future-stable kernels are invariant under composition-closed admissible continuation.
- The marked-degeneration, path-strictification, incidence-poset, projection-quotient, canonical-form, and branch-independence implications hold under their explicit hypotheses.
- The global interface-induced amplituhedron theorem is proved as a conditional recognition theorem.

Established external mathematics

- Spinor-helicity and standard BCFW meromorphic factorization.
- Positive network boundary measurement and positroid stratification.
- On-shell/plabic graph and BCFW bridge technology.
- The amplituhedron map $C \mapsto CZ$, canonical forms, and intrinsic sign-flip/winding descriptions in their standard domains.
- BCFW triangulation results for standard $m = 4$ amplituhedra.

Open physical source theorems

- Construct the analytic physical source germ and faithful one-channel defect line.
- Derive source nullity and the physical mixed-feedback obstruction.
- Convert defect vanishing into a closed correspondence-valued degeneration with the correct residue.
- Prove recursive inheritance, source control of infinity, and branch completeness.
- Prove direct/refined and associator invisibility on finite refinement nerves.
- Derive coherent orientation and nonnegative scalar-line passage weights.
- Construct a BCFW-complete, Z-projection-surjective positive atlas from the source.
- Prove source-level chiral branch independence.

Programmatic topology conjectures

- Determine whether nontrivial homology and monodromy live in the fibers or degeneration compactifications above a ball-like positive image.

- Test whether elliptic, K3, and higher Calabi–Yau amplitude geometries can be obtained as periods or vanishing-cycle data of that lifted carrier.
- Construct the local supersymmetric descent object and determine whether its globalization obstruction is nonzero but scattering-invisible.

19 What would falsify the program?

The program fails if the relevant source quotient has the wrong dimension or branch geometry; if the mixed obstruction is future-stably invisible; if finite-shadow vanishing does not lift to an effective source defect; if the degeneration has the wrong residue module or multiplicity; if lower-point continuations do not inherit; if an irreducible visible nonadjacent passage survives; if comparison or associator defects remain scattering-visible; if no coherent positive orientation exists; if the source atlas misses a physical projected chamber or boundary; or if a residue remains at infinity.

A conjecture with clear ways to die is healthier than one protected by analogy. The purpose of the recognition architecture is not to make the central idea unfalsifiable. It is to say exactly where it can break.

20 Handoff to the next proof

The first compact freeze ended with a two-channel source-plane problem. The later analysis sharpens that target. The smallest physical object now required is

$$\boxed{\begin{array}{c}\text{one analytic source defect whose simple zero has shadow } \widehat{P}_I(z)^2 = 0 \\ \text{and whose residue is } L \boxtimes R\end{array}}$$

Only after this one-channel bridge is certified should one attempt the finite divisor, recursive inheritance, three-channel adjacency, four-channel coherence, and positive atlas.

The route around the mountain is therefore

$$\boxed{\begin{array}{rl}\text{1 channel}: & \text{marked degeneration,} \\ \text{2 directions}: & \text{chiral integrability,} \\ \text{3 channels}: & \text{refinement,} \\ \text{4 channels}: & \text{coherence,} \\ \text{finite closure}: & \text{networks and positive geometry.}\end{array}}$$

21 Open questions

The conditional recognition architecture is now complete enough to say exactly where the mathematics stops. The following questions are not rhetorical conclusions. They are the remaining research program.

Q1. What is the smallest analytic W^*-source germ that can carry a complex BCFW continuation without falsely identifying that continuation with physical unitary evolution?

Q2. Can one construct, for a single physical channel I, a one-dimensional future-stable defect line and a natural section $\Delta_I^{\mathrm{src}}(z)$ whose scattering shadow is

$$\widehat{P}_I(z)^2 = P_I^2 + 2\epsilon_I z P_I \cdot q?$$

Q3. Can the source-nullity relation

$$[\mathcal{Q}_\chi^2]_{\mathrm{sc}} = 0$$

be derived inside the source continuation calculus rather than imposed only after finite kinematic evaluation?

Q4. What operator, quadratic-form, correspondence, or analytic-Fredholm mechanism converts the simple zero of Δ_I^{src} into an actual separating degeneration?

Q5. Under what hypotheses does the residue of the degenerating source passage factor through the correct lower-point correspondence product?

Q6. Do lower-point factors inherit admissible continuation branches coherently enough to prove recursive factorization completeness and termination?

Q7. Can primitive planar adjacency be characterized intrinsically by admissible common refinement, and when does admissible adjacency exhaust ordinary planar compatibility?

Q8. Do the direct/refined comparison residuals and the factorization associators lie in the future-stable scattering-null congruence? Does the resulting coherence persist over all finite nerves and recursive descendants?

Q9. When does the strictified finite passage calculus admit coherent scalar-line orientations, nonnegative weights, and subtraction-free refinement maps?

Q10. Can those positive data be assembled into a BCFW-complete positive-network atlas whose Z-projection is surjective onto the amplituhedron, without requiring source surjectivity onto the full positive Grassmannian?

Q11. What source condition excludes a residue or boundary contribution at $z = \infty$? Is no leakage equivalent to projective completeness of the marked continuation branch in a physical source realization?

Q12. When do two complete chiral or shifted-leg branches admit a common scattering-effective refinement, and when are equal recursive residues sufficient to force the same projected geometry and canonical form?

Q13. Is the factorization associator the restriction or transgression of a more general vacuum higher-coherence class, or are these genuinely different obstruction systems that merely share a formal shape?

Q14. Is the positive amplituhedron a contractible visible shadow of a lifted scattering carrier with nontrivial homological fibers, monodromy, or vanishing cycles? Can the elliptic and Calabi–Yau structures appearing at loop level be derived from this topology rather than appended after integration?

Q15. Do the supersymmetric reconstruction sectors form only a local descent object? If so, what is the precise globalization obstruction, and why does the finite positive scattering quotient fail to see it?

These problems occur at different scales. Some are finite analytic or combinatorial projects. Some could form dissertation chapters. The full W^*-source realization is a program. They should not be blurred into one enormous conjecture simply because they belong to one line of thought.

Coda: before the mountain again

Remembered / modern reflection

When I first came to these questions, I remember looking at the screen as though it opened onto the other side of the universe. The formulas were finite. The structure behind them was not. Something was visible there, but only in outline—like seeing the far side of a mountain without yet knowing the path around it.

Years later, I find myself before the mountain again. I know more of the terrain now. I know which early paths ended, which intuitions were too global, which completions were too stiff, and which questions survived after their first answers failed. That is not the same as having reached the other side. But it is not nothing. A failed path, honestly mapped, becomes part of the geography.

Perhaps that is why this way of working has always felt familiar to me. Long before amplitudes, operator algebras, or positive geometry, I was a little boy following paths through the hills of Eastern Kentucky. I did not walk them because I already knew where they ended. I walked because the next bend might disclose another ridge, another hollow, another way through. The joy was never merely in arriving. It was in learning how the land fit together.

Research has not changed as much as our instruments have. Even in the age of large language models, the question remains more important than the answer. An LLM can accelerate a calculation, test a lemma, retrieve a forgotten theorem, organize a proof, or expose a missing arrow. It can help us travel faster. It cannot decide, on our behalf, which distant outline is worth walking toward. Nor can it replace the responsibility—or the delight—of understanding why a path works.

So the note should end with open questions. Not as an apology. Not as a failure of closure. This is how research is done. Every honest answer alters the landscape and leaves better questions at its boundary.

The screen is still glowing on this side of the mountain. The other side of the universe is still visible through it, though never all at once. There is more to learn, more room to grow, and always another path through the hills.

Mountain statement

The shape is the geometry of what survives.

And the question is what teaches us how to see the shape.

About *A View from the Mountain*

A View from the Mountain is a series of historical and mathematical research notes by J. Scott Little. Each note returns to an earlier line of inquiry, reconstructs what can still be certified, records what was learned from the failed or unfinished portions, and compares the original view with the wider geometry visible from later work.

The series uses five evidentiary labels throughout:

Recorded	supported by a surviving source;
Remembered	autobiographical chronology or motivation;
Reconstructed	mathematics rederived from stated assumptions;
Modern reading	a later interpretation not attributed backward;
Open	a ridge that the present evidence does not cross.

The purpose is not to make the past look inevitable. It is to preserve the progression of thought while allowing proof, source, and explicit uncertainty to determine what may be claimed.

Research Note No. 04

Before the Mountain Again

A glowing screen, the far side of the universe, and the geometry of what survives. The finite scattering recognition theorem is conditional; the physical source bridge and the topology in the forgotten fibers remain open. The shape is what survives. The topology may yet tell us how it survived.